Dynamical Vacuum Energy
and
Cosmological Tensions

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Effective Field Theory of Gravity and Dynamical Vacuum Energy

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Cosmological 'Constant' Problem

- Supernova Observations:
  Expansion of the Universe is accelerating
- ΛCDM Cosmology:

$$\Lambda_{SN} = \Omega_\Lambda \times 3 \left(\frac{H_0}{c}\right)^2 \simeq \left(\frac{\Omega_\Lambda}{0.70}\right)\left(\frac{H_0}{70\,\mathrm{km/sec/Mpc}}\right)^2 \left(\frac{3.1\times 10^{-122}}{L_{Pl}^2}\right)$$

- Order unity (0.7) in Hubble units but 10^{-122} in Planck units
- If Λ is due to (UV divergent) quantum zero-point energy,
  it 'should' naturally (?) be order unity in Planck units
- Most severe Fine-tuning 'Naturalness' problem in physics
- Cosmic 'Co-incidence' Problem: Why now?
- Or is the EFT of Gravity incomplete at low energies ?

Three Main Ingredients

1) Conformal Anomaly contributes to Macroscopic Gravity

- Additional scalar conformalon **φ** degree of freedom in EFT of Low Energy Gravity beyond GR

2) Λ as a 4-form gauge field ***F*** & integration constant

- $\Lambda_{\rm eff} = 0$ vanishes *identically* in flat space
- Solves Λ Naturalness Problem

3) Extension of fermion conformal anomaly to spacetimes with Torsion generates coupling of ***F*** to anomaly **φ**

$\Rightarrow$ Dynamical Dark Energy

Consequences for Cosmological Tensions & New Cosmological Models

What is the Effective Field Theory of Macroscopic Gravity ?

- **EFT** = Expansion of Effective Action in *Local* Invariants: Based on **Decoupling** of Short Distance from Long Distance Physics
- In modern terms, Einstein's **GR** is an **EFT** (2^{nd} order eqs.)
- **But Std. Model Stress Tensor is Quantum**

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \langle T_{\mu\nu} \rangle$$

- *Massless* Quantum Fields do <u>not</u> decouple
- Conformal Symmetry is *<u>Anomalous</u>:* Trace Anomaly in $\langle T^{\mu}{}_{\mu} \rangle$
- ***IR*** Sensitivity & **Macroscopic** Quantum Effects
- Effective Scalar Degree of Freedom in **Low Energy** Gravity

1) Conformal Anomaly & Induced Scalar

- Trace (Conformal) Anomaly

$$\langle T^{\mu}{}_{\mu} \rangle = \mathcal{A} = b\, C^2 + b' \left(E - \tfrac{2}{3}\Box R\right) + \sum_i \beta_i\, \mathcal{L}_i$$

- Local Covariant Effective Action in Terms of New Scalar Field

$$S_{\mathcal{A}}[g;\varphi] = \frac{b'}{2}\int d^4x\sqrt{-g}\left\{-(\Box\varphi)^2 + 2\,(\nabla_\mu\varphi)\left(R^{\mu\nu} - \tfrac{1}{3}Rg^{\mu\nu}\right)(\nabla_\nu\varphi)\right\} + \frac{1}{2}\int d^4x\sqrt{-g}\,\mathcal{A}\,\varphi$$

← Linear Coupling to Total Anomaly

- **Dynamical Scalar** in Conformal Sector: **'Conformalon'**

$$\Delta_4\varphi = \frac{1}{2b'}\,\mathcal{A}$$

$$b = \frac{\hbar}{120(4\pi)^2}\left(N_s + 6N_f + 12N_v\right)$$

$$b' = -\frac{\hbar}{360(4\pi)^2}\left(N_s + 11N_f + 62N_v\right)$$

$$\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \tfrac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu$$

IR Relevant Term in the Gravitational Action

The effective action for the conformal trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

GR Correction not given in powers of Local Curvature

$$S_{\rm eff}[g;\varphi] \supset S_{\rm EH}[g] + S_{\mathcal{A}}[g;\varphi]$$

*This is a non-trivial modification of classical General Relativity required by **first principle** quantum effects in the Std. Model*

A scalar-tensor effective theory but very different from e.g. Brans-Dicke, Hordenski,...

Anomaly Stress tensor

$$T_{\mathcal{A}}^{\mu\nu}[g;\varphi] \equiv \frac{2}{\sqrt{-g}}\,\frac{\delta}{\delta g_{\mu\nu}}\,S_{\mathcal{A}}[g;\varphi]$$

relevant on macroscopic scales of cosmology

2) Cosmological Term as a 4-Form Gauge Field

If a **Four-Form** Abelian Field Strength is <u>Exact</u>: ***F* = *d*A** (**like E&M**)

It can be Written in Terms of a Three-Form Gauge Potential **A** i.e.

$$F_{\alpha\beta\gamma\lambda} = 4\,\nabla_{[\alpha}A_{\beta\gamma\lambda]}$$

Its Dual is a Scalar—only one **'electric'** component

$$\tilde{F} \equiv \star F = \frac{1}{4!}\,\varepsilon_{\alpha\beta\gamma\lambda}F^{\alpha\beta\gamma\lambda} = F^{txyz}$$

'Maxwell' Action

$$S_F = -\frac{1}{2\varkappa^4}\int F\wedge\star F = -\frac{1}{48\varkappa^4}\int d^4x\sqrt{-g}\,F_{\alpha\beta\gamma\lambda}F^{\alpha\beta\gamma\lambda}$$

'Maxwell' Eq. (sourcefree 'Gauss' Law): the 1 electric component is **constant**

$$\frac{1}{\varkappa^4}\nabla_\lambda F^{\alpha\beta\gamma\lambda} = J^{\alpha\beta\gamma} = 0 \qquad \partial_\lambda\tilde{F} = 0 \quad \Rightarrow \quad F = const.$$

Its Stress Tensor is **Equivalent to a Non-Negative Cosmological Constant**

$$T_F^{\mu\nu} = -\frac{1}{2\varkappa^4}\,g^{\mu\nu}\,\tilde{F}^{\,2} \qquad\qquad \Lambda_{\rm eff} = \frac{4\pi G}{\varkappa^4}\,\tilde{F}^{\,2} \geq 0$$

Natural Solution of Naturalness Problem

Stress Tensor of F

$$T_F^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_F}{\delta g_{\mu\nu}} = -\frac{1}{2\varkappa^4}\, g^{\mu\nu}\, \tilde{F}^{\,2}$$

Equivalent Effective Cosmological Constant

$$\boxed{\Lambda_{\rm eff} = \frac{4\pi G_N}{\varkappa^4}\, \tilde{F}^{\,2} = const.}$$

Constant of Integration set by **Classical** Boundary Condition in Flat Space

—Absolute Minimum of Energy, Stable Ground State

$$T_F^{00}\Big|_{\rm flat} = \frac{1}{2\varkappa^4}\, \tilde{F}^{\,2} \quad \Rightarrow \quad \tilde{F} = 0$$

Required by Consistency with Einstein's eqs. in Flat Space limit

$$\left[R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}\right]_{\rm flat} = 0 = -\Lambda_{\rm eff}\Big|_{\rm flat}\, \eta_{\mu\nu}$$

No Infinite or **UV** Sensitive Zero Point Energy, No Fine Tuning

$\varkappa$ **Coupling constant still arbitrary**

The Current Source for *F*

When the conformal anomaly has a total derivative term

$$\mathcal{A} = k_f \nabla_\lambda V^\lambda + \dots$$

the linear term in the Anomaly Effective Action can be integrated by parts

$$\frac{k_f}{2}\int d^4x\sqrt{-g}\,(\nabla_\lambda V^\lambda)\,\varphi = -\frac{k_f}{2}\int d^4x\sqrt{-g}\,V^\lambda(\nabla_\lambda\varphi)$$

$$= \frac{1}{3!}\int d^4x\sqrt{-g}\,J^{\alpha\beta\gamma}A_{\alpha\beta\gamma} \equiv S_{\rm int}[\varphi; A]$$

3-Form Gauge Potential $\quad A_{\alpha\beta\gamma} = \varepsilon_{\alpha\beta\gamma\lambda}V^\lambda$

Conserved Current $\quad J^{\alpha\beta\gamma} = \dfrac{k_f}{2}\,\varepsilon^{\alpha\beta\gamma\lambda}\partial_\lambda\varphi$

Provides Source for 4-Form Field Strength in `Maxwell' eq.

$$\boxed{\nabla_\lambda F^{\alpha\beta\gamma\lambda} = \varkappa^4 J^{\alpha\beta\gamma}}$$

Immediate Solution $\quad \boxed{\tilde{F} = -\dfrac{\varkappa^4 k_f}{2}\varphi + const.}$

F now varies whenever φ does $\rightarrow$ ***Dynamical Dark Energy***

F is Related to Torsion

The 'Maxwell' eq. results iff A is an independent variable & can be varied separately from the metric $g_{\mu\nu}$

This means that the E or R total derivative terms in the conformal anomaly in usual curved space will not work.

But when generalized to Einstein-Cartan spaces with **Non-zero Torsion**, described by the anti-symmetric part

$$\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} \neq 0$$

of the Affine Connection

Spinor Fields, e.g. Dirac Fermions, are distinguished by coupling to Torsion $\tau_{\alpha\beta\gamma}$ even under minimal coupling to geometry, via the Spin Connection and al Vector coupling $W^{\lambda}\bar{\psi}\gamma_{\lambda}\gamma^{5}\psi$ where

$$W^{\lambda} = \frac{1}{8}\,\varepsilon^{\alpha\beta\gamma\lambda}\tau_{\alpha\beta\gamma}$$

3) Massless Fermion Anomaly with Torsion

Additional Contribution to Conformal Anomaly in Presence of Torsion

$$\langle \Delta \hat{T}^{\mu}{}_{\mu} \rangle \Big|_{\rm Dirac} = \frac{N_f}{12\pi^2}\, \nabla_\lambda V^\lambda - \frac{N_f}{24\pi^2}\, \left(\nabla_\mu W_\nu - \nabla_\nu W_\mu\right)^2$$

$$V^\mu = \nabla^\mu \big(W_\nu W^\nu\big) + W^\mu \big(\nabla_\nu W^\nu\big) - \big(W^\nu \nabla_\nu\big) W^\mu$$

The first total derivative term is exactly what is needed to define the 3-form gauge potential

$$A_{\alpha\beta\gamma} = \varepsilon_{\alpha\beta\gamma\lambda} V^\lambda \qquad\qquad k_f = \frac{N_f}{12\pi^2}$$

Result: ***F= dA*** is a 4-form gauge field that can be varied **independently** of the metric & 'Maxwell' Eq. now gives a consistent theory of finite **dynamical vacuum energy** coupled to conformalon **φ** (requiring massless fermions & torsion).

$$\Lambda_{\rm eff} = \frac{4\pi G}{\varkappa^4}\, \tilde{F}^{\,2} \geq 0 \qquad\qquad \tilde{F} = \star dA = \nabla_\lambda V^\lambda = -\frac{\varkappa^4 k_f}{2}\, \varphi$$

Dynamical Dark Energy & Hubble Tension

- Dynamical Dark Energy from Quantum Conformal Anomaly of 'Massless' Fermions in Spacetime with Torsion
- 'Massless' means lighter than all other energy scales
- Lightest Fermions are ν's with mass $m_\nu \leq 0.04$ eV corresponding to $T \leq 460\ {}^\circ K$ or $z \leq 170$
- Above this scale there is DDE (**Early DDE**)
- Below this lightest ν mass scale, **DE** 'freezes' out—on average.
- In homogeneous FLRW cosmology $z \leq 170$ is between the CMB last scattering $z=1090$ and non-linear structure formation $z\approx20$

$\Rightarrow$ Affects both the comparison between CMB & late time H measurements AND the growth of structure in between

- Since φ can be time and space dependent, there is no reason to restrict to FLRW models—Violation of Cosmological Principle
- Solving the Cosmic Coincidence problem $\Omega_\Lambda \approx 1$ requires spatial inhomgeneities on the Hubble scale at all times.

Possibility of Universe within a de Sitter-like Hubble Bubble (gravastar)

Summary

- Einstein's classical theory receives Quantum Conformal Anomaly relevant at macroscopic Distances
- First Principles modification of classical GR
- Scalar 'Conformalon' φ degree of freedom in EFT of Gravity
- Scalar-Tensor Theory (not inflaton or quintessence)
- Λ Term as 4-form classical gauge field strength
- $\Lambda_{eff} = 0$ fixed by condition of lowest energy in flat space
- Naturalness Problem Solved: No Fine Tuning, dependence on cutoffs or ultra high energy scales
- **Dynamical Dark Energy** by extending Conformal Anomaly of 'Massless' Fermions to Spacetime with Torsion
- Scale set by lightest ν mass intermediate between early time (CMB) and late time (SN) determinations of expansion rate, relevant for Hubble Tension
- Inhomogeneous Cosmology possible in which DDE is naturally tied to the Hubble scale solving Cosmic Coincidence Problem

Backup Slides

Effective Field Theory & Quantum Anomalies

- Expansion of Effective Action in *Local* Invariants assumes Decoupling of Short Distance from Long Distance Physics
- But *Massless* Modes *do not decouple*
- Axial & Conformal Symmetries are *Anomalous*
- ***IR*** (Light cone) Sensitivity to ***UV*** Physics
- Requires Special Addition (Wess-Zumino term) to EFT
- Collective Boson Deg. of Freedom in Low Energy EFT
- *Macroscopic* Effect of QFT in (Semi-)Classical Gravity
- Important on null horizons because of large blueshift↔ ↔redshift

Relevance of Torsion

Cartan's 2nd Eq. of Structure, defines the Torsion 2-form

$$\mathcal{T}^a \equiv \frac{1}{2}\,T^a{}_{bc}\,e^b \wedge e^c = de^a + \omega^a{}_b \wedge e^b$$

can also be solved for the affine connection $e^a = e^a{}_\mu\,dx^\mu$

$$\omega_{ab\,\mu} = v^\nu{}_a\,\eta_{bc}\left(\nabla_\mu e^c{}_\nu\right) - K_{abc}\,e^c{}_\mu$$

'Contortion' Tensor $K_{abc} = \frac{1}{2}\left(T_{abc} + T_{bca} - T_{cab}\right)$

Has 24 components independent of the metric, so 24 components of $\omega_{ab\,\mu}$ can now be varied separately from $g_{\mu\nu}$

This will allow *A* to be treated as an independent variable and its 4-form field strength *F=dA* to describe

Dynamical Dark Energy

Case I: EFT of Gravity in Absence of Massless Fermions/Torsion

$$S_{\rm eff}[g;\varphi;A] = S_{\rm EH} + S_{\mathcal{A}}[g;\varphi] + S_F[A;g]$$

This A uncoupled to φ or the anomaly & just equivalent to

$$\Lambda_{\rm eff} = \frac{4\pi G_N}{\varkappa^4}\,\tilde{F}^2 = const = 0$$

Boundary condition: Minimum energy in infinite flat space w/o fine tuning

Eq. for φ

$$\Delta_4\varphi = \nabla_\alpha\left(\nabla^\alpha\nabla^\beta + 2R^{\alpha\beta} - \tfrac{2}{3}Rg^{\alpha\beta}\right)\nabla_\beta\varphi = \frac{\mathcal{A}}{2b'}$$

$$= \tfrac{1}{2}\left(E - \tfrac{2}{3}\Box R\right) + \tfrac{1}{2b'}\left(bC^2 + \Sigma_i\beta_i\mathcal{L}_i\right)$$

Einstein Eq.

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = 8\pi G_N\Big\{T_{\mathcal{A}\,\mu\nu}[g;\varphi] + (T_{F\,\mu\nu} + T^{cl}_{\mu\nu})\Big\}$$

Stress Tensor of Anomaly

$$T_{\mathcal{A}}^{\mu\nu}[g;\varphi] \equiv \frac{2}{\sqrt{-g}}\,\frac{\delta}{\delta g_{\mu\nu}}\,S_{\mathcal{A}}[g;\varphi] =$$

$$b'E^{\mu\nu} + bC^{\mu\nu} + \Sigma_i\beta_i T^{(i)\,\mu\nu}$$

Case II: EFT of Gravity in Presence of Massless Fermions/Torsion

$$S_{\rm eff}[g;\varphi;A] = S_{\rm EH} + S'_{\mathcal{A}}[g;\varphi;W^{\perp}] + S_{\rm int}[\varphi;A] + S_F[A;g]$$

This A is coupled to φ & the anomaly by

$$S_{\rm int}[A;\varphi] = \frac{1}{3!}\int d^4x\sqrt{-g}\,J^{\alpha\beta\gamma}A_{\alpha\beta\gamma} = -\frac{b'}{12}\int d^4x\sqrt{-g}\,\varepsilon^{\alpha\beta\gamma\lambda}A_{\alpha\beta\gamma}\,\partial_\lambda\varphi$$

'Maxwell' Eq. $\nabla_\lambda F^{\alpha\beta\gamma\lambda} = \varkappa^4 J^{\alpha\beta\gamma} = -\frac{b'}{2}\varkappa^4\varepsilon^{\alpha\beta\gamma\lambda}\nabla_\lambda\varphi$

Immediately solved $\tilde{F} = \frac{b'}{2}\varkappa^4\,\varphi + const. \to \frac{b'}{2}\varkappa^4\,\varphi$

Eq. for φ $\Delta_4\varphi + \frac{b'}{4}\varkappa^4\varphi = \frac{1}{2b'}\left(bC^2 + \Sigma_i\beta_i\mathcal{L}_i\right)$

Einstein Eq. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_N\left\{-\frac{b'^2}{8}\varkappa^4\varphi^2 g_{\mu\nu} + b'E^{(2)}_{\mu\nu} + C_{\mu\nu} + T^{cl}_{\mu\nu}\right\}$

Dynamical Dark Energy ↑ through F and φ

Eq. for $W^{\perp}$ $\nabla_\mu\left(\nabla^\mu W^{\perp\nu}\varphi\right) = 0 \;\Rightarrow\; W^{\perp\nu} = 0$

Torsion no longer appears explicitly

Stress Tensor of the Anomaly

$$T_{\mu\nu}[\varphi] = b'\, E_{\mu\nu} + b\, C_{\mu\nu} + \sum_i \beta_i\, T^{(i)}_{\mu\nu}[\varphi]$$

Euler-Gauss-Bonnet—Quadratic & Linear in φ

$$\begin{aligned} E_{\mu\nu} \equiv & -2\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\,\Box\varphi) + 2\nabla^{\alpha}\big[(\nabla_\alpha\varphi)(\nabla_\mu\nabla_\nu\varphi)\big] - \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big] \\ & + \tfrac{2}{3}\,R_{\mu\nu}\,(\nabla_\alpha\varphi)(\nabla^\alpha\varphi) - 4\,R^{\alpha}_{\ (\mu}\,\big[(\nabla_{\nu)}\varphi)(\nabla_\alpha\varphi)\big] + \tfrac{2}{3}\,R\,(\nabla_{(\mu}\varphi)(\nabla_{\nu)}\varphi) \\ & + \tfrac{1}{6}\,g_{\mu\nu}\,\left\{-3\,(\Box\varphi)^2 + \Box\big[(\nabla_\alpha\varphi)(\nabla^\alpha\varphi)\big] + 2\left(3R^{\alpha\beta} - Rg^{\alpha\beta}\right)(\nabla_\alpha\varphi)(\nabla_\beta\varphi)\right\} \\ & - \tfrac{2}{3}\,\nabla_\mu\nabla_\nu\,\Box\varphi - 4\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R^{\alpha}_{(\mu}\nabla_{\nu)}\nabla_\alpha\varphi + \tfrac{8}{3}\,R_{\mu\nu}\,\Box\varphi + \tfrac{4}{3}\,R\,\nabla_\mu\nabla_\nu\varphi \\ & - \tfrac{2}{3}\,\left(\nabla_{(\mu}R\right)\nabla_{\nu)}\varphi + \tfrac{1}{3}\,g_{\mu\nu}\,\left[2\,\Box^2\varphi + 6\,R^{\alpha\beta}\,\nabla_\alpha\nabla_\beta\varphi - 4\,R\,\Box\varphi + (\nabla^\alpha R)\nabla_\alpha\varphi\right] \end{aligned}$$

Weyl—Purely Linear in φ

$$C_{\mu\nu} = -4\,\nabla_\alpha\nabla_\beta\left(C_{(\mu\ \ \nu)}^{\ \ \alpha\ \ \beta}\,\varphi\right) - 2\,C_{\mu\ \nu}^{\ \alpha\ \beta}\,R_{\alpha\beta}\,\varphi$$

Trace is $2\Delta_4\varphi$: Full Anomaly recovered using Eq. of Motion for φ