3-forms as a mean of resolving tensions:

let's be hopefull

Workshop on Tensions in Cosmology (Corfu)

Mariam Bouhmadi-López
IKERBASQUE & EHU (Spain)

September 11th, 2022

Table of contents

1. Introduction

2. Late-time acceleration of the Universe within GR: dark energy with a constant EoS

3. Late-time acceleration of the Universe within GR: A 3-form field

4. BHs and wormholes (Whs) supported by 3-forms

5. Conclusions

Introduction

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1. For example, this is the case for a wCDM model with w constant and $k = 0$.
- Such an acceleration could be due
  - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
  - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....

Late-time acceleration of the Universe within GR: dark energy with a constant EoS

Constant equation of state for DE: background-1-

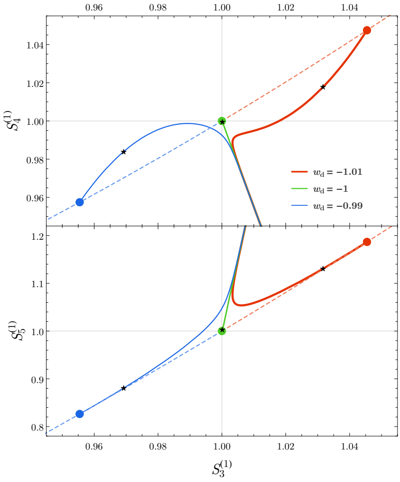
- Cosmic acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi \mathrm{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{de}} + 3p_{\mathrm{de}})$$

- Observation indicates that for $w_{\mathrm{de}} \sim -1$ where $w_{\mathrm{de}} = p_{\mathrm{de}}/\rho_{\mathrm{de}}$.
- Therefore, as soon as DE starts dominating the Universe starts accelerating, i.e. $\ddot{a} > 0$.
- Simplest cases ΛCDM or wCDM.

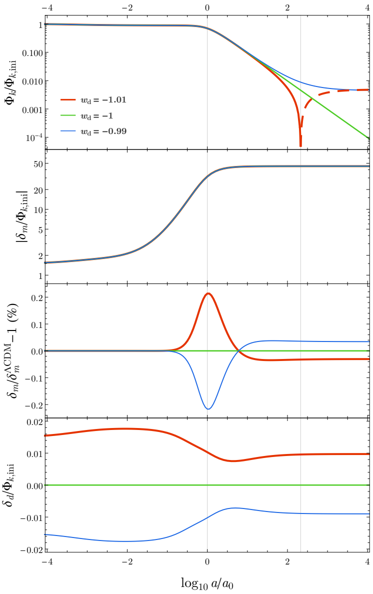
Constant equation of state for DE: background-2-

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
    - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0 \left(t - t_0\right)\right]^n$, where $A_n := a^{(n)}/(a\,H^n)$, $n \in \mathbb{N}$.
    - State finders parameters: $S_3^{(1)} = A_3$, $S_4^{(1)} = A_4 + 3\,(1 - A_2)$, $S_5^{(1)} = A_5 - 2\,(4 - 3A_2)\,(1 - A_2)$
    - $\Omega_\mathrm{m} = 0.309$, $\Omega_\mathrm{d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

- What about $f\sigma_8$ for the three different DE models?

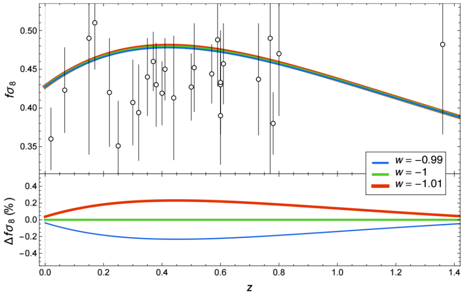


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\mathrm{m}}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\, k_{\sigma_8}\right) = \sigma_8\left(0,\, k_{\sigma_8}\right)\frac{\delta_{\mathrm{m}}\left(z,\, k_{\sigma_8}\right)}{\delta_{\mathrm{m}}\left(0,\, k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0, k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais [arXiv:1706.01484]

Late-time acceleration of the Universe within GR: A 3-form field

Can we have something more fundamental to describe DE?

- Can we have something more fundamental to describe (phantom) DE models?
  - A possibility come in the form of 3-forms.
  - Inspired in string theory: Copeland, Lahiri,Wands (1995)
  - Massless 3-form as Cosmological Constant (solving CC problem): Turok, Hawking (1998)
  - Inflation or late time acceleration driven by self-interacting 3-forms: Koivisto, Nunes (2009) and (2010)
  - Non-Gaussianity: Kumar, Mulryne, Nunes, Marto, Moniz (2016)
  - Quantum cosmology with 3-forms: Bouhmadi-López, Brizuela, Garay (2018)
- The answer as we will see in a momment is yes: Phantom DE models (LSBR): Morais, Bouhmadi-López, Kumar, Marto, Tavakoli (2017) and Bouhmadi-López, Marto, Morais and Silva (2017)

Late-time acceleration of the Universe within GR: A 3-form field

Reviewing the 3-form field $A_{\mu\nu\rho}$

p-forms

A p-form is a totally anti-symmetric covariant tensor:

$$\omega_{\mu_1 \ldots \mu_p} = \omega_{[\mu_! \ldots \mu_p]} \,.$$

In D-dimensions, the number of degrees of freedom of a massive p-form is

$$\text{degrees of freedom} = \frac{(D-1)!}{(D-1-p)!p!} \,.$$

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
Section based on Morais, B.L, Kumar, Marto and Tavakoli, Phys. Dark Univ. 15, 7 (2017) [arXiv:1608.01679 [gr-qc]]

p-forms in Cosmology

In a 4-dimensional space-time:

- $p = 0$ (scalar field) $\Rightarrow$ 1 degree of freedom
- $p = 1$ (vector field) $\Rightarrow$ 3 degrees of freedom
- $p = 2 \Rightarrow$ 3 degrees of freedom
- $p = 3 \Rightarrow$ 1 degree of freedom

$\Rightarrow$ The scalar field and the 3-form are the only ones compatible with a homogeneous and isotropic universe (in an easy way).

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

The 3-form action

- We will consider the following action for a massive 3-form, $A_{\mu\nu\rho}$, minimally coupled to gravity

$$S^A = \int \mathrm{d}^4 x \sqrt{|\det g_{\mu\nu}|} \left[-\frac{1}{48} F^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} - V\left(A^{\mu\nu\rho} A_{\mu\nu\rho}\right) \right] .$$

- The strength tensor, a 4-form, is defined through the exterior derivative: $F_{\mu\nu\rho\sigma} \equiv 4\nabla_{[\mu} A_{\nu\rho\sigma]}$
- The equation of motion, obtained from variation of S^A, is

$$\nabla_\sigma F^\sigma{}_{\mu\nu\rho} - 12 \frac{\partial V}{\partial (A^2)} A_{\mu\nu\rho} = 0$$

- $\Rightarrow$ a massless 3-form is equivalent to a cosmological constanst

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
M. Duff and P. Van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)

3-form Cosmology

We consider a homogeneous and isotropic universe described by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \,.$$

t - cosmic time, $\dot{\{\}} = d\{\}/dt$
a - scale factor
x^i - comoving spatial coordinates (roman indices run from 1 to 3).

Only the purely spatial components of the 3-form are dynamical:

$$A_{0ij} = 0\,, \qquad A_{ijk} = a^3(t)\chi(t)\epsilon_{ijk}\,.$$

T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

3-form Cosmology: background equations

⇒ Friedmann Equation

$$3H^2 = \kappa^2 \rho_\chi = \kappa^2 \left[\frac{1}{2} \left(\dot{\chi} + 3H\chi \right)^2 + V(\chi^2) \right] .$$

⇒ Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_\chi + P_\chi \right) = -\frac{\kappa^2}{2} \chi \frac{\partial V}{\partial \chi} .$$

A 3-form can show phantom-like behavior if $\partial V / \partial \chi^2 < 0$.

⇒ Equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + \frac{\partial V}{\partial \chi} = 0 .$$

3-form Cosmology: evolution of χ-1-

Combining the Raychaudhuri equation and the equation of motion for χ:

$$\ddot{\chi} + 3H\dot{\chi} + \left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi} = 0\,.$$

The static solutions are:

- the critical points of the potential: $\dfrac{\partial V}{\partial \chi} = 0$,
- the limiting points: $\chi = \pm\chi_c$.

Once inside the interval $[-\chi_c, \chi_c]$, the field χ evolves towards a local minimum of V. However...

3-form Cosmology: evolution of χ-2-

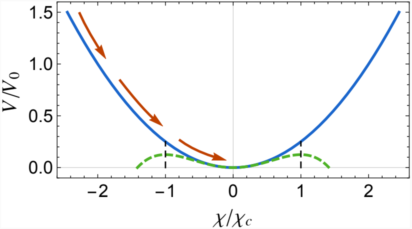
- Independently of the shape of a regular potential, in absence of DM interaction, the 3-form decays rapidly towards the interval $[-\chi_c, \chi_c]$ Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

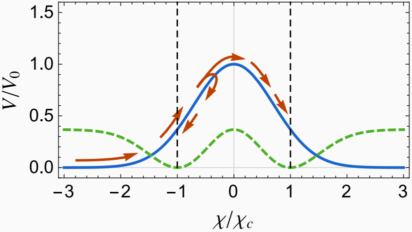
- In an expanding Universe, once inside the interval $[-\chi_c, \chi_c]$, the 3-form will end up in one of the minima of the potential (notice $V_{\rm eff} \neq V$).

- If the 3-form stops at the limits of this interval:

$$\chi = \pm\chi_c \quad \text{and} \quad \dot{\chi} = 0$$

- $\longrightarrow$ Universe heads towards a LSBR event $\left(\chi_c = \sqrt{2/3\kappa^2}\right)$





Late-time singularities and future abrupt events:

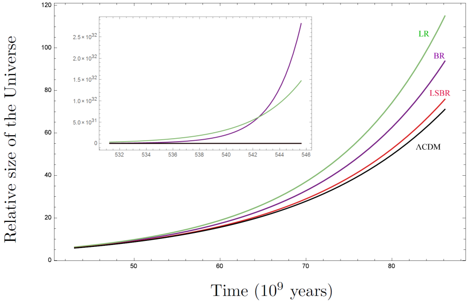
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H} \ldots$
Big Bang	0	0	∞	∞	∞
De Sitter (ΛCDM)	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

Asymptotic evolution of the scale factor



(1) A.A. Starobinsky. astro-ph 9912054; R.R. Cadwell astro-ph 9908168; Cadwell *et al.* astro-ph/0301273
(2) H. Štefančić. astro-ph 0411630; S. Nojiri, S. Odintsov and S. Tsujikawa. hep-th/0501025
(3) M. Bouhmadi-López , A. Errahmani, P. Martín-Moruno, T. Ouali and Y. Tavakoli. arXiv:1407.2446
(4) M. P. Dąbrowski, C. Kiefer and B. Sandhöfer. hep-th/0605229
(5) Bouhmadi-López, Kiefer, Martín-Moruno, arXiv:1904.01836 [gr-qc]
(6) Borislavov Vasilev, Bouhmadi-López, Martín-Moruno, arxiv: arXiv:2106.12050

The Little Sibling of the Big Rip

The Little Sibling of the Big Rip (LSBR) is a cosmological event that happens at infinite time and is characterised by

- $a\,(t \to \infty) \to +\infty$,
- $H\,(t \to \infty) \to \infty$,
- $\dot{H}\,(t \to \infty) \to \mathrm{constant}$.

In general, this can be obtained with an equation of state:

$$p = -\rho - A \qquad (A > 0)\,.$$

Solving the conservation equation we find $H^2 \propto \log(a)$ and $\dot{H} = (\kappa^2/2)A$ (asymptotically).

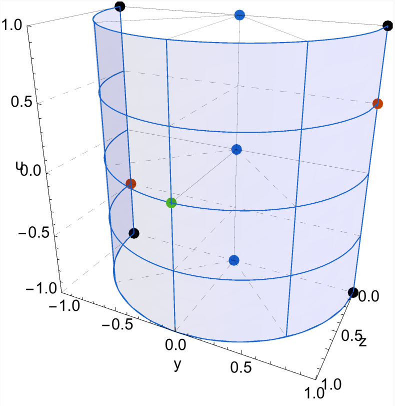
M.B.L, A. Errahmani, P. Martín-Moruno, T. Ouali, and Y. Tavakoli, Int. J. Mod. Phys. D 24, 1550078 (2015)

3-forms & with a Gaussian potential: a dynamical system approach

Using a Dynamical Systems representation Morais *et al* PofDU [arxiv:1608.01679]; BL *et al*, JCAP [arXiv:1611.03100]

$u = (\pi/2)\arctan(\chi/\chi_c) \qquad y = (\dot{\chi} + 3H\chi)/(3H\chi_c) \qquad z = \sqrt{\kappa^2 V/3H^2}$

- Three **matter era** points:
  two repulsive - $(\pm 1, 0, 0)$
  one saddle - $(0, 0, 0)$
- One potential dom. **de Sitter** point: saddle - $(0, 0, 1)$
- Two **LSBR event** points:
  attractor - $(\pm 1/2, \pm 1, 0)$
- Four **unphysical** points:
  saddles - $(\pm 1, \pm 1, 0)$



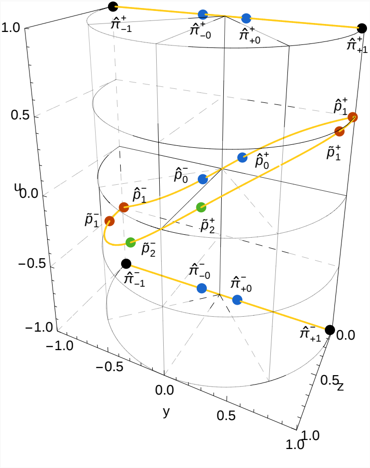
Fixed points in the non-interacting case

DM-DE interaction as a mean to remove a LSBR

- Turning on the interaction:

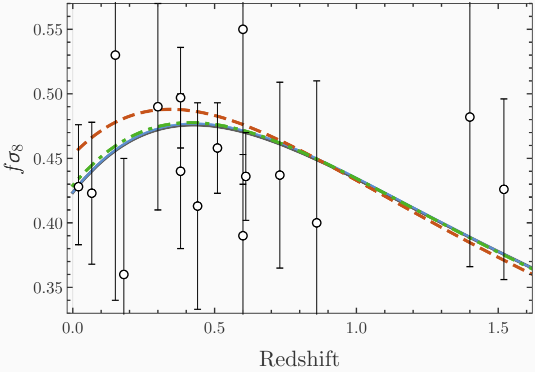
$$Q = 3H(\rho_m + \rho_\chi) \sum_{i=0}^{2} \alpha_i \left(\frac{\rho_\chi}{\rho_m + \rho_\chi} \right)^i$$

- The interaction decomposes each fixed point into pairs.
- Different $(\alpha_0, \alpha_1, \alpha_2)$ affect different fixed points.
- If $\alpha_0 + \alpha_1 + \alpha_2 \neq 0$ ($Q \not\propto \rho_m^i$), the LSBR event is removed.
- The end state is a de Sitter scaling solution with DE domination.



Fixed points for a general quadratic interaction

Behaviour of fσ_8 and $3-$*forms*



Let me add that 3-forms can be quite interesting for further reasons as:

- They allow naturally for regular BHs (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC)
- They naturally support wormholes without changing the sign of the kinetic energy (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP

BHs and wormholes (Whs) supported by 3-forms

Why should we care about those strange objects: BHs and WHs?

- It is well known that GR predicts the existence of singularity
- Plenty of BHs have been detected so far there is even one at the center of our galaxy!!!
- About 70% of the current content of the universe is DE which is fully compatible with phantom DE, it even alliviate the H_0 tension. This matter is precisely what is needed close to the WHs mouths!!!
- Phantom or effective phantom matter implies repulsive forces:
  - (Regular) Black holes?
  - Wormholes?

BHs and wormholes (Whs) supported by 3-forms

Regular BHs supported by 3-forms

Our Goal within 3-forms

- Black holes (exterior) and naked singularity has been analysed in: Barros, Danila, Harko, Lobo (2020)
- We are rather interested in getting regular BHs; i.e. keeping the essence and avoiding the singularity.

BHs interior and KS space-time

- Inside the event horizon, the space-time of a static and spherically symmetric black hole can be described by the Kantowski-Sachs anisotropic metric
- The geometry is caracterised by 2 scale factors $a(t)$ and $b(t)$
- The matter sector will be described by the 3-form (incoded in χ) and its potential (incoded in $V(\chi)$)
- We assume that close to the event horizon we recover Schwarzschild solution.
- We assume a functional form for $b(t)$

Regular BHs supported by 3-forms: geometry and matter content

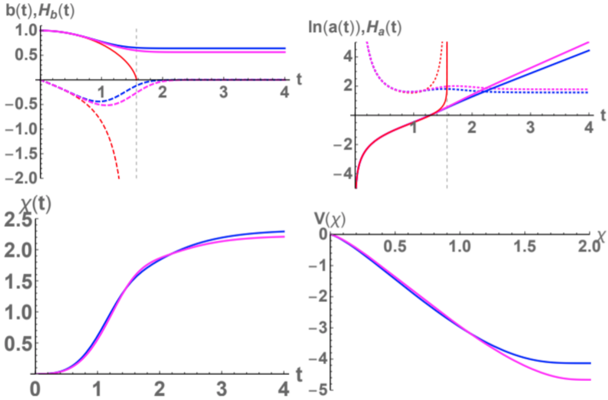
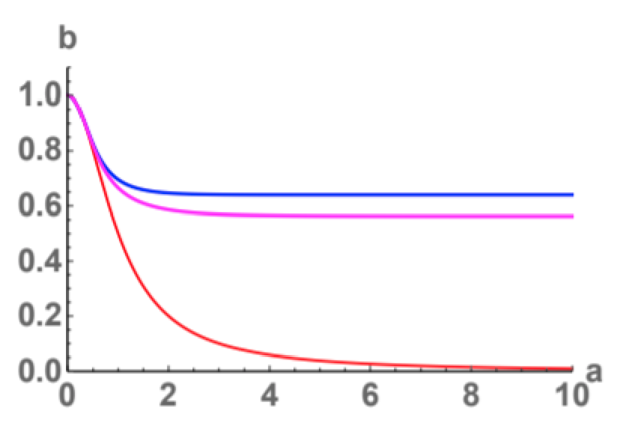


FIG. 1: The results of the regular black hole model given by Eq. (19) for $x = 4$ (blue) and $x = 3$ (magenta), respectively. The red curves are the results of the Schwarzschild spacetime. Top-left: $b(t)$ (solid) and $H_b(t)$ (dashed). Top-right: $\ln a(t)$ (solid) and $H_a(t)$ (dotted). The 3-form field $\chi(t)$ and the potential $V(\chi)$ are shown in the bottom-left and bottom-right panels, respectively. The vertical dashed line ($t = \pi/2$) stands for the singularity in the Schwarzschild black hole. Note that we have rescaled $t/r_s \to t$. In addition, the gravitational constant is set to $\kappa = 1$ in these plots. Likewise for the plots below.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: a closer look at the geometry



Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The curvature(s)

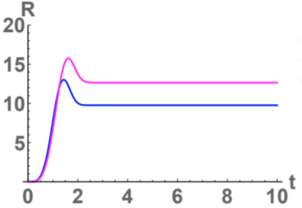


FIG. 3: The Ricci scalar R and the Kretschmann scalar K of the regular black hole model (the magenta and blue curves correspond to $x = 3$ and $x = 4$ respectively), compared to the Schwarzschild case in red. The Ricci scalar is of course identically zero in the Schwarzschild case, hence not shown in the plot.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The null energy condition

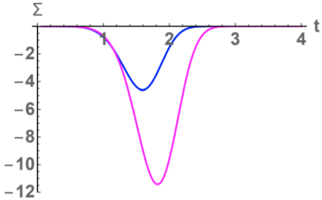


FIG. 5: The null energy condition is violated in the regular black hole model ($\Sigma := T_{\mu\nu}k^{\mu}k^{\nu} < 0$). The blue and the magenta curves correspond to $x = 4$ and $x = 3$, respectively.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC

Regular BHs supported by 3-forms: The Penrose diagram

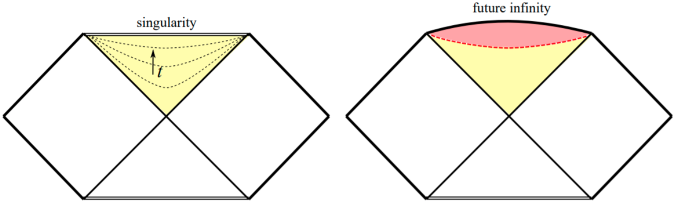


FIG. 4: Left: The causal structure of a maximally extended Schwarzschild black hole, where our coordinate covers inside the event horizon (yellow colored region), where the time coordinate varies from $t = 0$ (horizon) to $t = \pi/2$ (singularity). Right: The effects of the 3-form field is to modify the solution near the putative singularity. The areal radius approaches a constant and the singularity is replaced by the topology $dS_2 \times S^2$ (red colored region). Therefore, one can interpret that the internal structure will evolve to a spacelike future infinity rather than a spacelike singularity.

$$ds^2 = -dt^2 + \cosh^2\frac{t}{a}dy^2 + b^2 d\Omega_2^2,$$

- Instead of a singularity a Nariai space-time is reached asymptotically
- Geodesic completeness: One takes infinite proper time to reach the minimum value of $b(t)$; i.e. $b_m \neq 0$
- Notice that we are reaching an effective postive cosmological constant (de Sitter) for a negative potential (!!!!)

BHs and wormholes (Whs) supported by 3-forms

wormholes supported by 3-forms A

Model building

- We assume a quartic potential

$$V(\mathbf{A}^2) = \mu^2 \mathbf{A}^2 + \lambda \mathbf{A}^4$$

.

- The 3-form $\mathbf{A}$ gives rise to the d.o.f. χ
- Our geometry is described by a static, spherically symmetric metric:

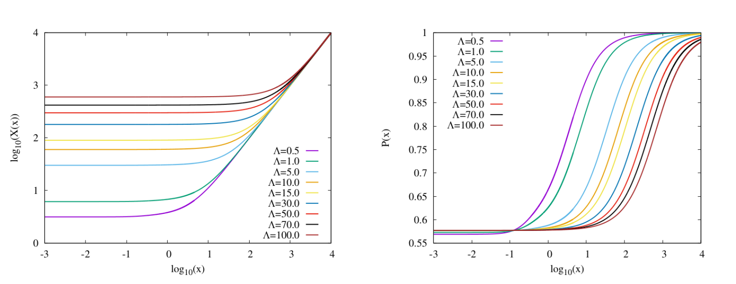
$$ds^2 = -P(r)^2 dt^2 + dr^2 + R(r)^2 d\Omega_2^2$$

- It is useful to introduce a dimensionless representation to numerically solve the equations from the throat outward:

$$r = \frac{x}{\mu}, \quad R = \frac{X}{\mu}, \quad \lambda = \kappa \mu^2 \Lambda, \quad \chi = \frac{\Psi}{\sqrt{\kappa}}.$$

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

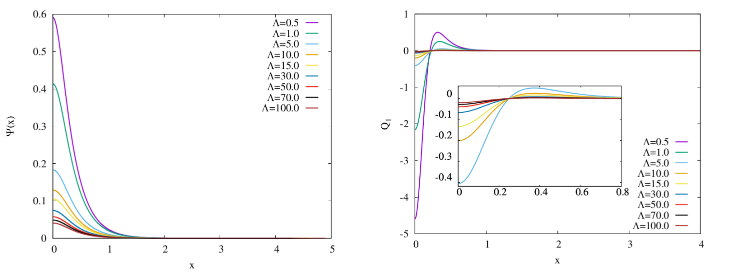
3-form wormholes: The geometry



At the throat: $(x = 0)$, the 2-sphere radius X is minimum and satisfies the flaring-out condition

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

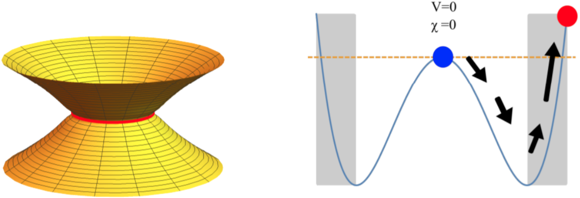
3-form wormholes: The matter content



3-form field evolution + the violation of NEC is concentrated at the throat.

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

3-form wormholes: A geometrical approach



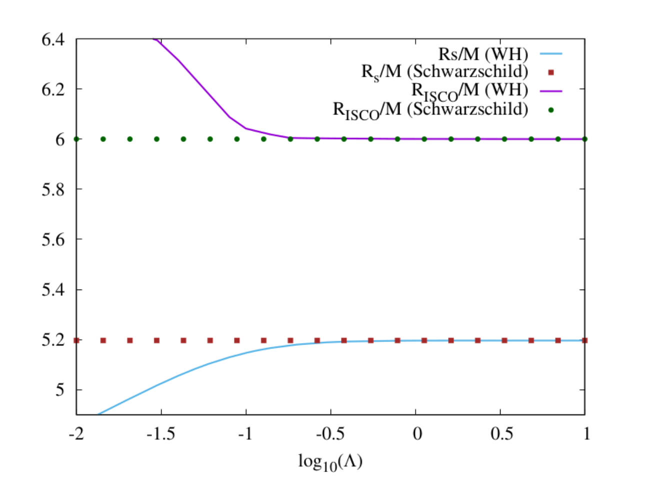
(a) (b)

Figure 2. (a) The embedding diagram of the wormhole spacetime supported by the 3-form field. The red curve indicates the wormhole throat. (b) This schematic plot shows the evolution of the 3-form field on the potential in the wormhole spacetime. The shaded regions represent the regions where the NEC is violated.

3-form field evolution

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

3-form wormhole as a Black hole mimicker



3-form wormhole as a Black hole mimicker

Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021

Conclusions

Conclusions

- We have described DE and in particular a phantom DE through a more fundamental field encoded in a 3-form.
- We have shown that 3-forms support regular black holes.
- We have shown that 3-forms support wormholes that can be excelent mimicker of BHs.

Thank you for your attention!!! Efcharisto!!!