BBN constraints in models that alleviate the H_0 tension

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Outline

Introduction

- Modified gravity is one of the two main ways that are being followed in order to explain the early and late accelerated phases of universe expansion with the other one being the introduction of inflaton or/and dark energy sectors
- Amongst the various classes of gravitational modifications that can fulfill the above cosmological motivation, theories that incorporate higher-order corrections to the Einstein-Hilbert Lagrangian have an additional motivation, namely the potential for improving the renormalizability of General Relativity.
- Such theories may naturally arise as (ghost-free) low-energy effective field-theory limits of String Theory and include Einstein gravity in the lowest-order in a derivative expansion.
- We can test such theories by using observational data

$f(G)$ Gravity

- Let us first consider quadratic terms in the Riemann tensor. The corresponding combination is the Gauss-Bonnet one, where

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{1}$$

- This term is topological in four dimensions and thus it cannot lead to any corrections in the field equations, so we use the extended action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + f\left(G\right)\right], \tag{2}$$

with $M_P \equiv 1/\sqrt{8\pi \mathrm{G_N}} = 2.4 \times 10^{18}$ GeV the (reduced) Planck mass, and $\mathrm{G_N}$ the gravitational constant, which corresponds to a new gravitational modification, namely $f(G)$ gravity.

$f(G)$ Gravity

- Applying it to a cosmological framework, namely to the FRW metric , and considering additionally the matter and radiation perfect fluids, we find the Friedmann equations

$$3M_P^2 H^2 = \rho_m + \rho_r + \rho_{DE} \tag{3}$$

$$-2M_P^2 \dot{H} = \rho_m + p_m + \rho_r + p_r + \rho_{DE} + p_{DE}\,, \tag{4}$$

with ρ_m and p_m respectively the energy density and pressure of the matter fluid, ρ_r and p_r the corresponding quantities for radiation sector, and where we have introduced the corresponding quantities of the effective dark energy sector as

$$\begin{aligned}\rho_{DE} \equiv \frac{1}{2}\Big[&-f\left(G\right) + 24H^2\left(H^2+\dot{H}\right)f'\left(G\right)\\ &-24^2 H^4\left(2\dot{H}^2 + H\ddot{H} + 4H^2\dot{H}\right)f''\left(G\right)\Big]\,,\end{aligned} \tag{5}$$

$$\begin{aligned}p_{DE} \equiv\ & f(G) - 24H^2\left(H^2+\dot{H}\right)f'\left(G\right)\\ &+8(24)^2\left(2\dot{H}^2 + H\ddot{H} + 4H^2\dot{H}\right)^2 f'''\left(G\right)\\ &+192H^2\Big(6\dot{H}^3 + 8H\dot{H}\ddot{H} + 24\dot{H}^2H^2\\ &\qquad +6H^3\ddot{H} + 8H^4\dot{H} + H^2\ddot{H}\Big)f''\left(G\right)\,,\end{aligned} \tag{6}$$

Gauss-Bonnet Dilaton Gravity

- A string inspired model is the modified gravity with quadratic curvature terms, which are coupled to dilatons Φ, with potential $V(\Phi)$. The effective action in this case reads

$$\int d^4x\sqrt{-g}\, M_P^2 \left[\frac{R}{2} - \frac{1}{4}\,\partial_\mu\Phi\,\partial^\mu\Phi + c_1 e^{\Phi}\, G - V(\Phi)\right], \tag{7}$$

  where c_1 a coefficient.
- Using a FRW metric we find

$$3H^2 = \frac{1}{4}\dot{\Phi}^2 + V(\Phi) - 24 c_1 e^{\Phi}\dot{\Phi}H^3 + M_P^{-2}(\rho_m+\rho_r), \tag{8}$$

$$2\dot{H} + 3H^2 = -\left[\frac{1}{4}\dot{\Phi}^2 - V(\Phi) + 16 c_1 e^{\Phi}\dot{\Phi}H(\dot{H}+H^2)\right.$$

$$\left. +8c_1 e^{\Phi}(\ddot{\Phi}+\dot{\Phi}^2)H^2 + M_P^{-2}(p_m+p_r)\right], \tag{9}$$

- We can identify the effective dark energy density from (18) as

$$\rho_{DE} \equiv M_P^2\left[\frac{1}{4}\dot{\Phi}^2 + V(\Phi) - 24c_1 e^{\Phi}\dot{\Phi}H^3\right], \tag{10}$$

  and the dark energy pressure as

$$p_{DE} \equiv M_P^2\Big[\frac{1}{4}\dot{\Phi}^2 - V(\Phi) + 16c_1 e^{\Phi}\dot{\Phi}H(\dot{H}+H^2)$$

$$+8c_1 e^{\Phi}(\ddot{\Phi}+\dot{\Phi}^2)H^2\Big]. \tag{11}$$

$f(P)$ Gravity

- Now we proceed to cubic gravity. We write a general cubic combination invariant which is

$$\begin{aligned} P = {} & \beta_1 R_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma} R_{\rho}{}^{\gamma}{}_{\sigma}{}^{\delta} R_{\gamma}{}^{\mu}{}_{\delta}{}^{\nu} + \beta_2 R^{\rho\sigma}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma} R^{\mu\nu}_{\gamma\delta} \\ & + \beta_3 R^{\sigma\gamma} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho}{}_{\gamma} + \beta_4 R R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + \beta R_{\mu\nu\rho\sigma} R^{\mu\rho} R^{\nu\sigma} + \beta R^{\nu}_{\mu} R^{\rho}_{\nu} R^{\mu}_{\rho} \\ & + \beta R_{\mu\nu} R^{\mu\nu} R + \beta R^3. \end{aligned} \tag{12}$$

- We can construct a more general function of cubic P as in $f(G)$ gravity. The action is

$$\int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + f\left(P\right)\right]. \tag{13}$$

- In a FRW universe the dark energy density and pressure are

$$\rho_{DE} \equiv -f(P) - 18\tilde{\beta}H^4(H\partial_t - H^2 - \dot{H})f'(P), \tag{14}$$

$$\begin{aligned} p_{DE} \equiv {} & f(P) + 6\tilde{\beta}H^3\Big[H\partial_t^2 + 2(H^2 + 2\dot{H})\partial_t \\ & -3H^3 - 5H\dot{H}\Big] f'(P). \end{aligned} \tag{15}$$

Running Vacuum Models

- In running vacuum cosmology the vacuum energy density is expressed, as a result of general covariance, in terms of even integer powers of the Hubble parameter.
- The corresponding running vacuum model (RVM) energy density, which plays the role of the dark energy, reads as

$$\rho_{\rm RVM}^{\rm vac} = 3M_P^2\Big(c_0 + \nu H^2 + \frac{\alpha}{H_I^2}\, H^4 + \dots\Big), \tag{16}$$

- We can parametrize the last equation because it can be used in supergravity for renormizability purposes. So we have

$$\begin{aligned}\rho_{\rm RVM}^{\rm vac} \equiv \rho_{DE} \simeq 3M_P^2\Big\{ & c_0 + [\nu + d_1 \ln(M_P^{-2}H^2)]\, H^2 \\ & + \frac{\alpha}{H_I^2}[1 + d_2 \ln(M_P^{-2}H^2)]\, H^4 + \dots \Big\}\,,\end{aligned} \tag{17}$$

where $c_0 > 0, \nu > 0, \alpha > 0$ and $d_i, i = 1, 2$ are phenomenological parameters.

BBN

- One of the constraints we have to test the modified theories is from radiation era.
- BBN is realized during the radiation epoch
- During that era the radiation dominates so the first friedmann equation can be written as

$$H^2 \approx \frac{M_P^{-2}}{3}\rho_r \equiv H_{GR}^2. \tag{18}$$

- The energy density of relativistic particles is

$$\rho_r = \frac{\pi^2}{30} g_* T^4, \tag{19}$$

where $g_* \sim 10$ the effective number of degrees of freedom and T the temperature.
- The last two give us

$$H(T) \approx \left(\frac{4\pi^3 g_*}{45}\right)^{1/2} \frac{T^2}{M_{Pl}}, \tag{20}$$

where $M_{Pl} = (8\pi)^{\frac{1}{2}} M_P = 1.22 \times 10^{19}$ GeV is the Planck mass.

BBN

- During the BBN, we have interactions between particles. For example we have interactions between neutrons, protons, electrons and netrinos.
- Now we assume that the various particles (neutrinos, electrons, photons) temperatures are the same, and low enough in order to use the Boltzmann distribution instead of the Fermi-Dirac one), and we neglect the electron mass compared to the electron and neutrino energies.
- The total conversion rate between protons and neutrons is

$$\lambda_{tot}(T) = 4A\, T^3(4!\, T^2 + 2 \times 3!\, QT + 2!\, Q^2)\,, \tag{21}$$

  where $Q = m_n - m_p = 1.29 \times 10^{-3}\text{GeV}$ is the mass difference between neutron and proton and $A = 1.02 \times 10^{-11}\ \text{GeV}^{-4}$.
- The corresponding freeze-out temperature will arise comparing the universe expansion rate $\frac{1}{H}$ with $\lambda_{tot}\,(T)$.
- In particular, if $\frac{1}{H} \ll \lambda_{tot}\,(T)$, namely if the expansion time is much smaller than the interaction time we can consider thermal equilibrium
- On the contrary, if $\frac{1}{H} \gg \lambda_{tot}\,(T)$ then particles do not have enough time to interact so they decouple.

BBN

- The freeze-out temperature T_f, in which the decoupling takes place corresponds to $H(T_f) = \lambda_{tot}(T_f) \simeq c_q\, T_f^5$, with $c_q \equiv 4A\,4! \simeq 9.8 \times 10^{-10}\,\mathrm{GeV}^{-4}$.
- If we use (20) and $H(T_f) = \lambda_{tot}(T_f) \simeq c_q\, T_f^5$, we find

$$T_f = \left(\frac{4\pi^3 g_*}{45 M_{Pl}^2 c_q^2}\right)^{1/6} \sim 0.0006\ \mathrm{GeV}. \tag{22}$$

- Using modified theories we obtain extra terms in energy density due to the modification of gravity. The form that the first friedmann equation appears is

$$3H^2 = M_P^{-2}\left(\rho_r + \rho_m + \rho_{DE}\right). \tag{23}$$

- In radiation era this becomes

$$3H^2 = M_P^{-2}\left(\rho_r + \rho_{DE}\right), \tag{24}$$

  where ρ_{DE} must be very small compared to ρ_r in order to be in accordance with the observation facts.
- So we can write (24) using (18) as

$$H = H_{GR}\sqrt{1 + \frac{\rho_{DE}}{\rho_r}} = H_{GR} + \delta H, \tag{25}$$

  where H_{GR} is the Hubble parameter of standard cosmology.

BBN

- Finally, since $H_{GR} = \lambda_{tot} \approx c_q\, T_f^5$ and $\sqrt{1+\frac{\rho_{DE}}{\rho_r}} \approx 1 + \frac{1}{2}\frac{\rho_{DE}}{\rho_r}$, we easily find

$$\frac{\delta T_f}{T_f} \simeq \frac{\rho_{DE}}{\rho_r}\frac{H_{GR}}{10 c_q\, T_f^5}. \tag{26}$$

- This theoretically calculated $\frac{\delta T_f}{T_f}$ should be compared with the observational bound

$$\left|\frac{\delta T_f}{T_f}\right| < 4.7 \times 10^{-4}\,, \tag{27}$$

  which is obtained from the observational estimations of the baryon mass fraction converted to ${}^4He$.
- In radiation era Hubble's parameter evolves as $H(t) = 1/(2t)$. So we can express the derivatives of the Hubble constant as powers of the Hubble parameter.
- In present we have

$$\Omega_{DE0} \equiv \rho_{DE0}/(3M_P^2 H_0^2) \approx 0.7, \tag{28}$$

  where this can be used to constraint one parameter of the modified theory
- Given a modified theory, putting the above constraint into the dark energy density equation, one can use (27) to constraint another parameter of the theory.

BBN constraints in $f(G)$ Gravity

- For example in $f(G) = \alpha G^n$ model we have two free parameters, α, n.
- We use the equation of the present dark energy density to constraint α and then we plug the equation into the radiation dark energy constraint. So we have[1]

$$\begin{aligned}\frac{\delta T_f}{T_f} = &-\Omega_{DE0}\,(\zeta)^{4n-1}\,(T_f)^{8n-7}\\ &\cdot\left[(-1)^n + n(-1)^{n-1} + 8n\,(n-1)\,(-1)^{n-2}\right]\\ &\cdot(H_0)^{2-2n}\left(H_0^2+\dot{H}_0\right)^{-n}\left[10c_q\,(n-1)\right]^{-1}\\ &\cdot\left[(1-2n)\left(\dot{H}_0^2+2\dot{H}_0H_0^2\right) - n\ddot{H}_0H_0 + H_0^4\right]^{-1}.\end{aligned} \tag{29}$$

- Constraints from BBN require $n < 0.45$.

[1] **P. Asimakis, S. Basilakos, N. E. Mavromatos, and E. N. Saridakis, Phys. Rev. D 105 (2022) no. 8, 084010, arXiv:2112.10863 [gr-qc].**

BBN constraints in $f(P)$ Gravity

- In the case of $f(P) = \alpha P^n$ model we have two free parameters, α, n.
- We use the equation of the present dark energy density to constraint α and then we find an expression for dark energy density in radiation era with the only free parameter n.
- We find[2]

$$\begin{aligned}\frac{\delta T_f}{T_f} &= 2.1\,(\zeta)^{6n-1}\,(T_f)^{12n-7} \\ &\cdot \left[(-24)^n - 216n\,(n-1)\,(-24)^{n-1} + 18n\,(-24)^{n-1}\right] \\ &\cdot (30c_q)^{-1}\,(6)^{1-n}\,(H_0)^{2-4n}\left(2H_0^2 + 3\dot{H}_0\right)^{2-n} \\ &\cdot \Big\{\left[216n\,(n-1) - 18\right]\dot{H}_0 H_0^2 \\ &\quad + 54n\,(n-1)\left(4\dot{H}_0^2 + \ddot{H}_0 H_0\right) - 12H_0^4\Big\}^{-1}. \end{aligned} \tag{30}$$

- Using the contraint from BBN we find $n < 0.31$.

[2] **P. Asimakis**, **S. Basilakos**, **N. E. Mavromatos**, **and E. N. Saridakis**, **Phys. Rev. D 105 (2022) no. 8**, **084010**, **arXiv:2112.10863 [gr-qc]**.

Conclusions

- The present analysis shows that models of higher-order modified gravity, apart from being closer to a renormalizable gravitational theory, they can be viable candidates of the description of Nature too, since they can quantitatively account for the dark energy sector and the late-time acceleration of the Universe, without altering the successes of the BBN epoch and the formation of light elements.
- In most of the cases the corresponding model parameters are constrained in narrow windows, which is expected since it is well known that BBN analysis imposes strong constraints on possible deviations from standard cosmology.
- The results of the present work reveal the capabilities of such constructions and offers a motivation for further investigation, at a more detailed level, of the evolution of cosmic perturbations and their role in the large-scale structure of the Universe.
- We hope to study such more general cases in future works!

Thank You!