Alleviation of H_0 tension in f(Q) gravity

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Contents

Why modified gravity?

The concordance model (ΛCDM) is well tested with a variety of probes and proved to be successful .
However:

- Value/physics of the cosmological constant
- Coincidence problem*
- Number of physical entities

Hubble constant and $\sigma 8$ problems

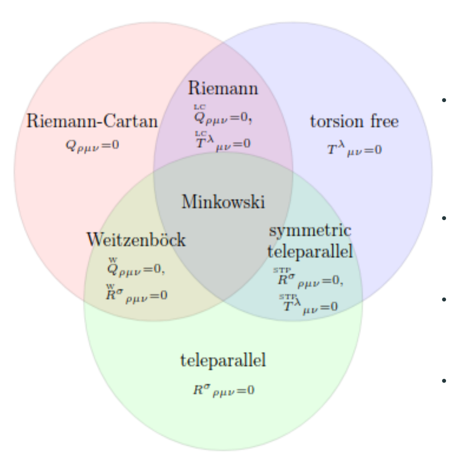
- "Direct" measurement †, $H_0 = 74.03 \pm 1.42\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$
- From PLANCK, $H_0 = 67.37 \pm 0.54\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$

*H. E. S. Velten, R. F. vom Marttens and W. Zimdahl, Eur. Phys. J. C **74**, no. 11, 3160 (2014)
†A. G. Riess et al, Astrophys. J. (2019) [arXiv:1903.07603 [astro-ph.CO]]

Possibilities

- Dark Energy as a geometrical effect
- Evasion of the initial singularity
- Joint description of Dark Energy and Dark Matter(?)
- A path towards renormalizable gravity[‡] (?)

‡K. S. Stelle, Phys. Rev. D **16** (1977)



from Järv et al, PRD, (2018), arXiv:1802.00492

- 3 geometrical entities: curvature, torsion, non-metricity
- General Theory of Relativity (GR): uses curvature
- Teleparallel equivalent of GR (TEGR): uses torsion
- What about non-metricity?

Friedman equations and over-density evolution

Similar with f(R) and f(T), one could construct the f(Q) gravity [§] From the field equations and in the case of FRWL metric, we obtain the Friedmann eqs:

$$
\begin{aligned}
6f_Q H^2 - \frac{1}{2}f &= 8\pi G(\rho_m + \rho_r), & (1)\\
\left(12H^2 f_{QQ} + f_Q\right)\dot{H} &= -4\pi G(\rho_m + p_m + \rho_r + p_r)\,, & (2)
\end{aligned}
$$

where $H \equiv \dot{a}/a$ the Hubble rate, $Q = 6H^2$ and $\rho_m,\ \rho_r, p_m, p_r$ are the energy densities and pressures for the matter and radiation fluid. For $f(Q) = Q - 2\Lambda$ we have ΛCDM

[§] J. Beltrán Jiménez, L. Heisenberg, T. S. Koivisto and S. Pekar, Phys. Rev. D (2020), arXiv:1906.10027
N. Frusciante, Phys. Rev. D (2021), arXiv:2101.09956

Perturbation analysis [¶] for a general f(Q) gives: $\left(\delta = \frac{\delta\rho}{\rho}\right)$

$$\delta''_{m} + \left(\frac{H'}{H} + \frac{3}{a}\right)\delta'_{m} = \frac{3\Omega_{m0}}{2H^2a^5}\frac{G_{eff}}{G}\delta_{m}, \tag{3}$$

The effective Newton constant is

$$G_{eff} = \frac{G}{f_Q(Q)} \tag{4}$$

[¶] J. Beltrán Jiménez, L. Heisenberg, T. S. Koivisto and S. Pekar, Phys. Rev. D (2020), arXiv:1906.10027

A new cosmological model

The exponential f(Q) model

We introduced

$$f(Q) = Qe^{\lambda \frac{Q_0}{Q}}, \tag{5}$$

where λ is a parameter, $Q_0 = 6H_0^2$ and H_0 the Hubble constant.

- $\lambda = 0 \rightarrow$ GR, without Λ
- For small Q_0/Q, there is correspondence with the model $f(Q) = Q^n$
- $z >> 1, \rightarrow$ GR, without Λ

Employing the Friedman equations for the particular model

$$\left(\frac{H^2}{H_0^2} - 2\lambda\right) e^{\lambda H_0^2/H^2} = \Omega_{m0} a^{-3} + \Omega_{r0} a^{-4}. \tag{6}$$

No extra free parameters

Considering $\lambda \neq 0$ and z = 0, $H = H_0$

$$\lambda = 0.5 + \mathcal{W}_0 \left(\frac{-\Omega_{m0} - \Omega_{r0}}{2e^{1/2}} \right) , \tag{7}$$

where $\mathcal{W}_0$ is the principal branch of the Lambert function.

G_{eff}

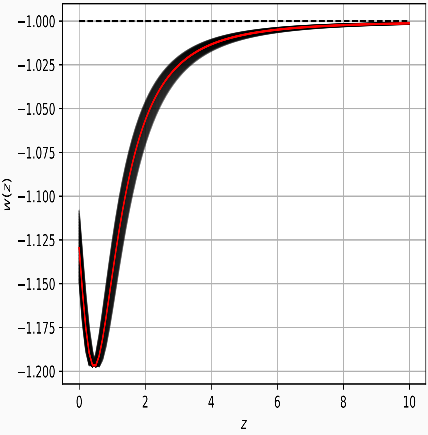
The effective Newton constant

$$G_{eff} = \frac{G}{e^{\lambda \frac{H_0}{H}} \left(1 - \lambda \frac{H_0}{H}\right)}. \tag{8}$$

- GR limit for $\lambda = 0$ and asymptotically for $H_0/H \to 0$
- For $\lambda < 0$, $G_{eff} > 0$ at all times
- For $H = H_0$, $G_{eff} < G$

Results

Observational results for exponential f(Q) model



F. K. Anagnostopoulos, M. Petronikolou, E. N. Saridakis, to appear

- **Overperforms** the concordance model

  F. K. Anagnostopoulos, S. Basilakos and E. N. Saridakis, PLB **822** (2021)

- $w(z) < -1$ recently, thus could solve the H_0 tension.

  L. Heisenberg, H. Villarrubia-Rojo and J. Zosso, PRD **106** (2022)

- Satisfies trivially the BBN constraints,

  F. K. Anagnostopoulos, V. Gakis, E. N. Saridakis and S. Basilakos, arXiv:2205.11445.

The exponential f(Q) model seems to be a very promising alternative to he concordance model.

- Is it actually better than the concordance one?
- Is able to solve both the tensions?

What Is To Be Done

- Exhaustive observational testing (CMB, weak lensing, full LSS spectrum, etc)
- Theoretical study of the f(Q) gravity
- N-body simulations

Thank you

The action is[?]

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}f(Q), \tag{9}$$

where the non-metricity scalar is

$$Q = -\frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} + \frac{1}{2}Q_{\alpha\beta\gamma}Q^{\gamma\beta\alpha} + \frac{1}{4}Q_\alpha Q^\alpha - \frac{1}{2}Q_\alpha \tilde{Q}^\alpha\,, \tag{10}$$

The geometrical quantities $Q_\alpha \equiv Q_{\alpha}{}^{\mu}{}_{\mu}$ and $\tilde{Q}^\alpha \equiv Q_{\mu}{}^{\mu\alpha}$, are contractions of the non-metricity tensor

$$Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu} \tag{11}$$

[?] J. Beltrán Jiménez et al, Phys. Rev. D **101** (2020) no.10, 103507 [arXiv:1906.10027 [gr-qc]].

After minimizing the action, the field equations are**,

$$\frac{2}{\sqrt{-g}}\nabla_\alpha\left\{\sqrt{-g}g_{\beta\nu}f_Q\left[-\frac{1}{2}L^{\alpha\mu\beta}+\frac{1}{4}g^{\mu\beta}\left(Q^\alpha-\tilde{Q}^\alpha\right)\right.\right.$$
$$\left.\left.-\frac{1}{8}\left(g^{\alpha\mu}Q^\beta+g^{\alpha\beta}Q^\mu\right)\right]\right\}$$
$$+f_Q\left[-\frac{1}{2}L^{\mu\alpha\beta}-\frac{1}{8}\left(g^{\mu\alpha}Q^\beta+g^{\mu\beta}Q^\alpha\right)\right.$$
$$\left.+\frac{1}{4}g^{\alpha\beta}\left(Q^\mu-\tilde{Q}^\mu\right)\right]Q_{\nu\alpha\beta}+\frac{1}{2}\delta^\mu_\nu f=T^\mu{}_\nu\,, \tag{12}$$

where $L^\alpha{}_{\mu\nu}=\frac{1}{2}Q^\alpha{}_{\mu\nu}-Q_{(\mu}{}^\alpha{}_{\nu)}$ is the disformation tensor, $T_{\mu\nu}$ is the (standard) energy momentum tensor, and $f_Q\equiv\partial f/\partial Q$.

J. Beltrán Jiménez et al, Phys. Rev. D **101 (2020) no.10, 103507 [arXiv:1906.10027 [gr-qc]].

EXTRAS - Find the most probable values of the free parameters

Given a set of observations, and supposing that the model at hand is true, what are the values of the free parameters?

Method: Likelihood analysis using Markov Chain Monte Carlo[††].

Data sets analyzed:

1. SNIa+CC data, with free parameters $\Omega_{m0}, h, \mathcal{M}$.
2. SNIa+CC+BAOs data, with free parameters $\Omega_{m0}, h, r_d, \mathcal{M}$.
3. SNIa+CC+RSD data, with free parameters $\Omega_{m0}, h, \sigma_8, \mathcal{M}$.

[††] D. Foreman-Mackey, et al Publ. Astron. Soc. Pac. **125** (2013), 306-312 doi:10.1086/670067 [arXiv:1202.3665 [astro-ph.IM]].

Suppose a set of models along with the corresponding results from the aforementioned analysis. Which model is preferred from the data?

There are many criteria, i.e. χ^2/dof. Akaike Information Criterion, Bayesian Evidence, Deviance Information criterion[‡‡],κλπ

The quantity $\Delta IC \equiv IC_i - IC_{min}$ is to be used to assign "degree of belief" to each model, using the following rule

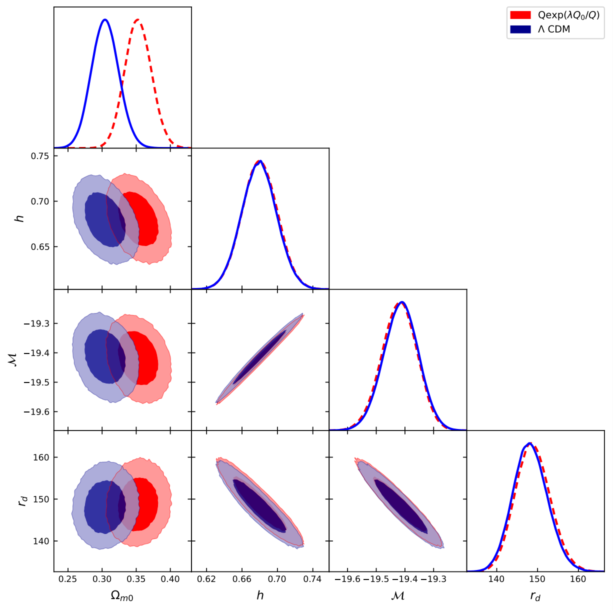
$$P_i = \frac{e^{-\Delta IC_i}}{\sum_{i=1}^{N} e^{-\Delta IC_i}} \tag{13}$$

[‡‡] F. K. Anagnostopoulos, S. Basilakos and E. N. Saridakis Phys. Rev. D (2019), arXiv:1907.07533

Extras - Possible explanations of Model Selection Results

- Error on the analysis (wrong data input etc)
- Numerical relics within the used datasets
- The results favouring ΛCDM could be caused by the circularity problem in the data

CC+SNIA+BAOS FOR ΛCDM+F(Q)



Model	AIC	ΔAIC	BIC	ΔBIC	DIC	ΔDIC
			SNIa/CC			
$Qe^{\lambda \frac{Q_0}{Q}}$	1039.390	0.1	1054.300	0.106	1039.320	0.115
ΛCDM	1039.290	0	1054.194	0.0	1039.205	0
			SNIa/CC/BAOs			
$Qe^{\lambda \frac{Q_0}{Q}}$	1043.650	0	1063.537	0	1043.542	0
ΛCDM	1043.994	0.344	1063.881	0.344	1043.888	0.346
			SNIa/CC/RSD			
$Qe^{\lambda \frac{Q_0}{Q}}$	1056.430	1.753	1076.3749	1.751	1056.3198	1.750
ΛCDM	1054.677	0	1074.624	0	1054.570	0

Values of the parameters

Model	Ω_{m0}	h	r_d	σ_8	$\mathcal{M}$	χ^2_{min}	χ_{min}/dof
			SNIa/CC				
$Qe^{\lambda\frac{Q_0}{Q}}$	0.349 ± 0.021	$0.6828^{+0.0203}_{-0.0201}$	–	–	-19.412 ± 0.062	1033.370	0.968
ΛCDM	0.299 ± 0.021	$0.6825^{+0.0203}_{-0.0201}$	–	–	$-19.406^{+0.061}_{-0.062}$	1033.267	0.968
			SNIa/CC/BAOs				
$Qe^{\lambda\frac{Q_0}{Q}}$	$0.353^{+0.020}_{-0.019}$	$0.6800^{+0.0201}_{-0.0199}$	$148.747^{+4.385}_{-4.144}$	–	$-19.419^{+0.066}_{-0.062}$	1035.613	0.966
ΛCDM	0.304 ± 0.020	0.6794 ± 0.0199	$148.141^{+4.350}_{-4.112}$	–	$-19.413^{+0.060}_{-0.062}$	1035.957	0.966
			SNIa/CC/RSD				
$Qe^{\lambda\frac{Q_0}{Q}}$	0.339 ± 0.020	$0.6864^{+0.0204}_{-0.0202}$	–	0.703 ± 0.0292	$-19.405^{+0.061}_{-0.062}$	1048.392	0.964
ΛCDM	0.292 ± 0.020	$0.6852^{+0.0202}_{-0.0201}$	–	$0.742^{+0.032}_{-0.031}$	$-19.400^{+0.060}_{-0.062}$	1046.640	0.963