Alleviating H_0 Tension in Horndeski Gravity

(arxiv: 2110.01338, M. Petronikolou, S. Basilakos, E. Saridakis**)**

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Introduction

Motivation:

- The need to incorporate tensions such as the *H0* one.

Extensions/ modifications of the concordance cosmology

Alter the universe content and interactions while maintaining GR as the gravitational theory

Modify gravity, GR as a particular limit

Horndeski Gravity

- Horndeski Gravity is equivalent to Generalized Galileon theory

- The most general four-dimensional scalar-tensor theory

- With second-order field equations

- Free from Ostrogradski instabilities

Horndeski gravity

- The Lagrangian describing the theory is

$$L = \sum_{i=2}^{5} L_i \qquad (1)$$

- $L_2 = K(\varphi, X)$,

- $L_3 = -G_3(\varphi, X) \Box\varphi$,

- $L_4 = G_4(\varphi, X) R + G_{4,x} [(\Box\varphi)^2 - (\nabla_\mu\nabla_\nu\varphi) (\nabla^\mu\nabla^\nu\varphi)]$,

- $L_5 = G_5(\varphi, X) G_{\mu\nu} (\nabla^\mu\nabla^\nu\varphi) - \frac{1}{6} G_{5,X} [(\Box \varphi)^3 - 3(\Box \varphi) (\nabla_\mu\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi) + 2(\nabla^\mu\nabla_a \varphi) (\nabla^\alpha\nabla_\beta\varphi) (\nabla^\beta\nabla_\mu\varphi)]$.

Horndeski gravity

- The functions K and G_i (i = 3, 4, 5) depend on:
  - The scalar field φ
  - And its kinetic energy $X = -\partial^{\mu}\varphi\partial_{\mu}\varphi/2$

- The terms $G_{i,X}$ and $G_{i,\varphi}$ (i = 3, 4, 5) → $G_{i,X} \equiv \partial G_i/\partial X$ and $G_{i,\varphi} \equiv \partial G_i/\partial\varphi$

Horndeski gravity

- The total action is given by

$$S = \int d^4x \sqrt{-g}\,(L + L_m)\,, \qquad (2)$$

- L_m : describes the matter content of the universe, corresponding a perfect fluid
- g: the determinant of the metric $g_{\mu\nu}$

Imposing a flat Friedmann-Robertson-Walker (FRW) background metric:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \qquad (3)$$

where $a(t)$ → the scale factor

Horndeski gravity

- Varying the action (2) with respect to the metric, we get

$$2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m \quad (4)$$

$$K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4HG_{4,X} - 8\dot{H}XG_{4,X} - 8HX\dot{X}G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} + 2[2(\dot{H}X + H\dot{X}) + 3H^2X]G_{5,\phi} + 4HX\dot{\phi}G_{5,\phi\phi} = -p_m \quad (5)$$

The *Modified Friedmann equations*, with the Hubble parameter: $H \equiv \dot{a}/a$

Horndeski gravity

- Variation of (2) with respect to $\varphi(t)$ provides its equation of motion

$$\frac{1}{a^3}\frac{d}{dt}(a^3 J) = P_\phi \qquad (6)$$

- with

$$J \equiv \dot{\phi} K_{,X} + 6H X G_{3,X} - 2\dot{\phi} G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2X G_{4,XX}) \\ -12H X G_{4,\phi X} + 2H^3 X(3G_{5,X} + 2X G_{5,XX}) \\ +6H^2\dot{\phi}(G_{5,\phi} + X G_{5,\phi X}) \qquad (7)$$

$$P_\phi \equiv K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi} G_{3,\phi X}) + 6(2H^2 + \dot{H}) G_{4,\phi} \\ +6H(\dot{X} + 2HX) G_{4,\phi X} - 6H^2 X G_{5,\phi\phi} + 2H^3 X \dot{\phi} G_{5,\phi X} \qquad (8)$$

Horndeski gravity

We define:

- Pressure: $p_m = 0$, energy density: $\rho_m = \rho_{m0}/a^3$
- $K = -V_0\,\varphi + X$
- Cosmological redshift: $z = -1 + a_0/a$, $a_0 = 1$

We choose suitable sub-classes of the theory to obtain a cosmological behavior that is almost identical with that of ΛCDM at early times, but which at intermediate and late times deviates from it, so:

- $G_3 = 0$
- $G_4 = 1/(16\pi G)$
- We modify the parameter G_5, by varying the coefficients c, λ in the following cases: $G_5 = c\,X^2$, $G_5 = \lambda\,X^4$.
- (ΛCDM: arising from $G_4 = 1/(16\pi G)$, $K = -2\Lambda = \text{const}$, $G_3 = G_5 = 0$)

Alleviating the H_0 Tension in Horndeski framework

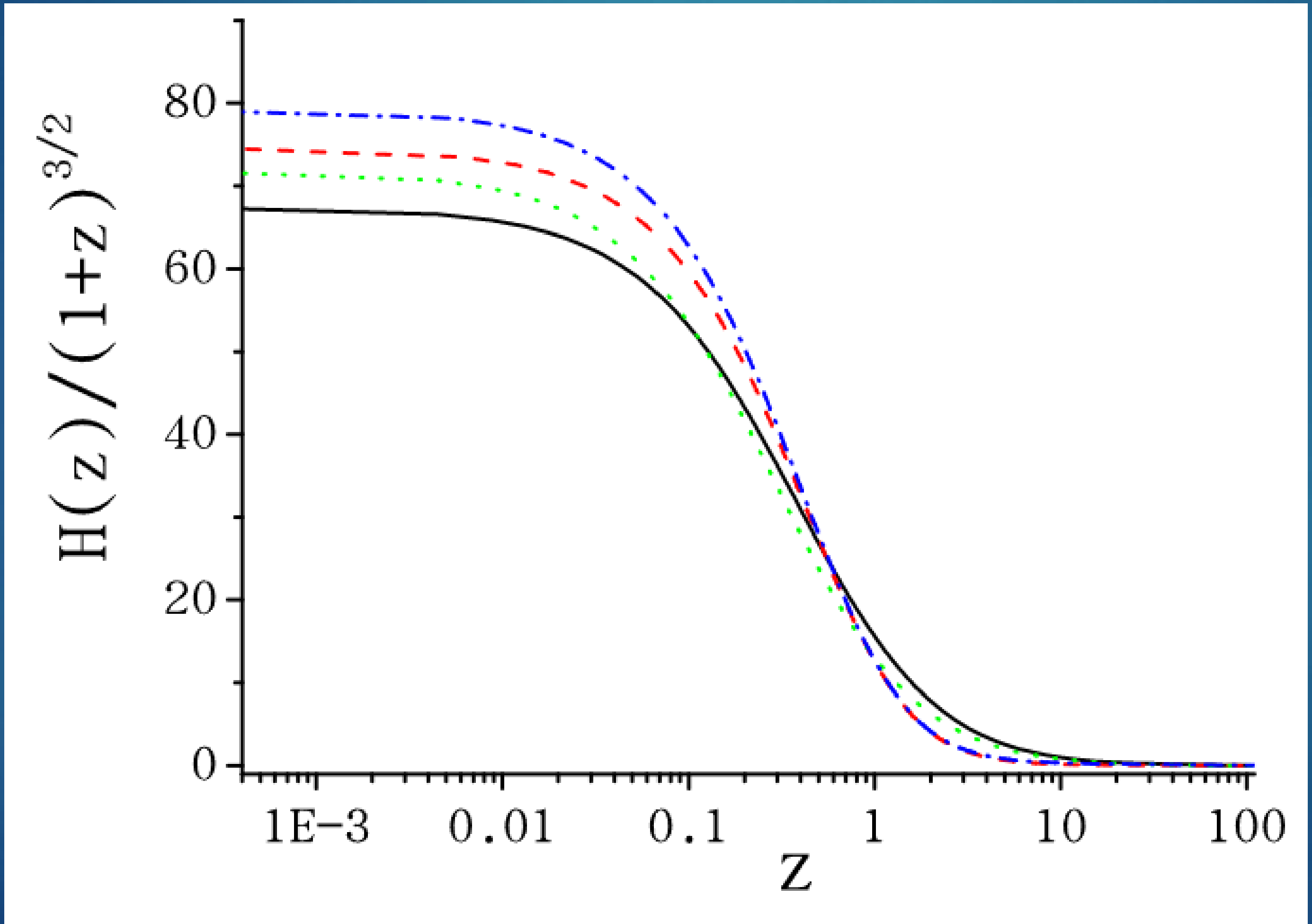
- We choose the initial conditions for the scalar field in order to obtain $H(z_{CMB}) = H_{\Lambda CDM}(z_{CMB})$ and $\Omega_{m0} = 0.31$.
- We solve numerically the equations (5), (6) with respect to φ and a, for each of the modifications on the G_5 and so we get the H parameter values.
- We compare the results of the H parameter obtained by the sub-classes of Horndeski theory with the corresponding one obtained by the ΛCDM model assuming a flat universe:

$$H(z) = H_0\sqrt{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}} \qquad (9)$$

with: $H_{0\Lambda CDM} = 67.27 \pm 0.6\ km/s/Mpc$

Results

- Model I: $G_5(X) = cX^2$



c=1 (Blue dashed dotted)

c=1.3 (red-dashed)

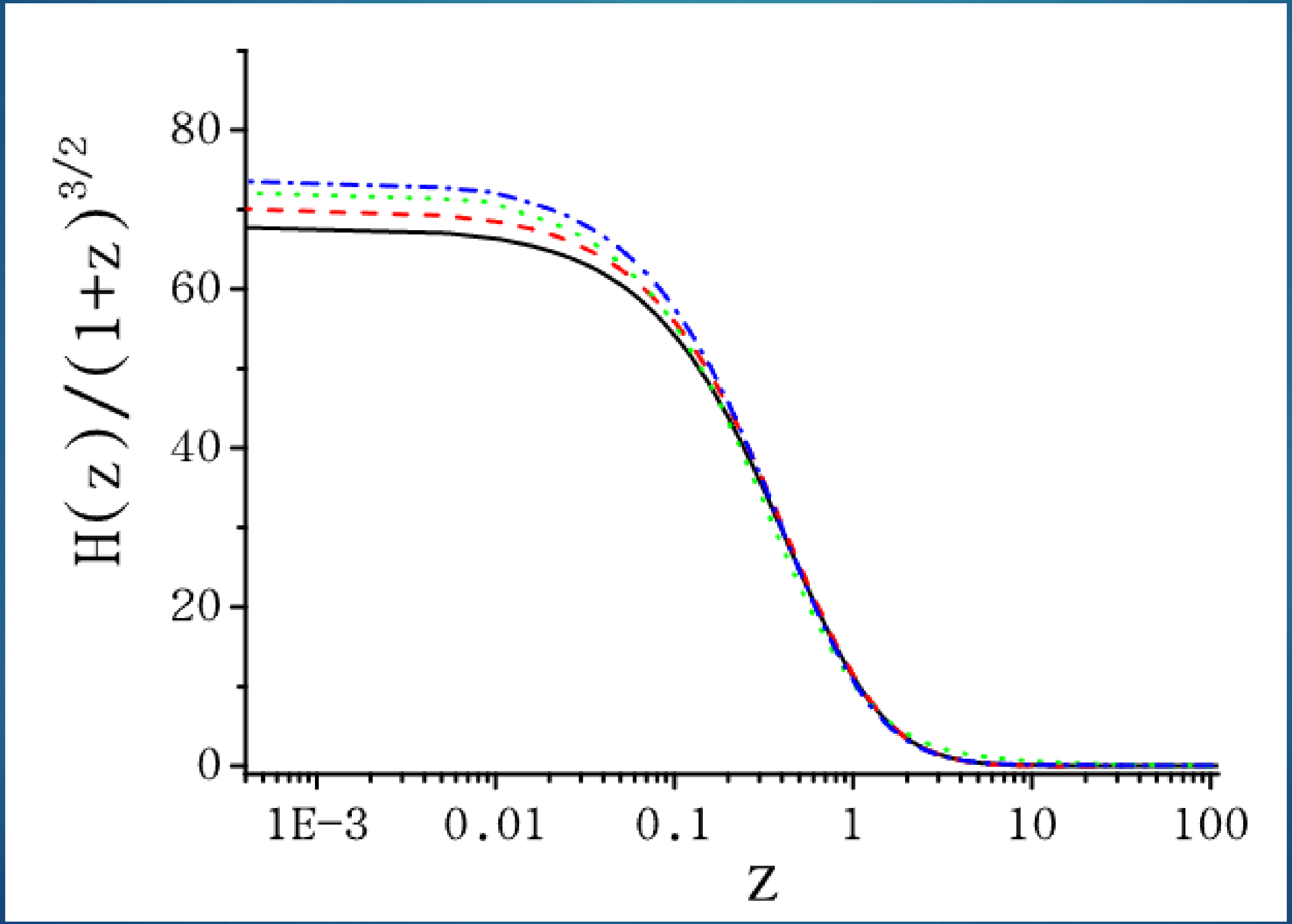
c=1.5 (green –dotted)

ΛCDM (black)

For c =1.3 → $H_0 \approx$ 74km/s/Mpc

Results

- Model II: $G_5(X) = \lambda X^4$



$\lambda = 1$ (blue-dashed-dotted)

$\lambda = 0.9$ (red-dashed)

$\lambda = 0.5$ (green-dotted)

ΛCDM (black)

For $\lambda = 1 \rightarrow H_0 \approx 74$ km/s/Mpc

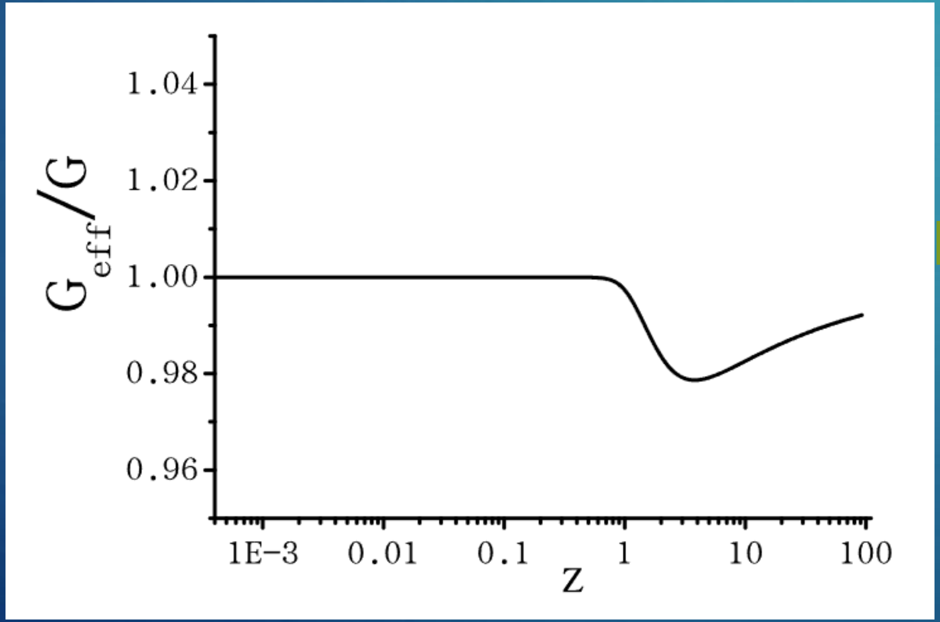
The mechanism behind H_0 alleviation

- In Horndeski theories we have

$$\frac{G_{eff}}{G} = \frac{1}{2}\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}HXG_{5,X}\right)^{-1} \quad (10)$$

(arxiv:1404.3713, arXiv:1712.00444)

And in our models this exhibits a decrease at intermediate redshifts



"weaker" gravity → "faster" expansion

(brought about in turn by the friction term)

Stability of the obtained solutions

- Condition for Horndeski/generalized Galileon theory in order to be free from Laplacian instabilities, associated with the scalar field propagation speed (c_s^2):

$$c_S^2 \equiv \frac{3(2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2 \dot{w}_2) - 6w_1^2[(1+w\ )\rho}{w_1(4w_1 w_3 + 9w_2^2)} \gtrsim 0 \qquad (11)$$

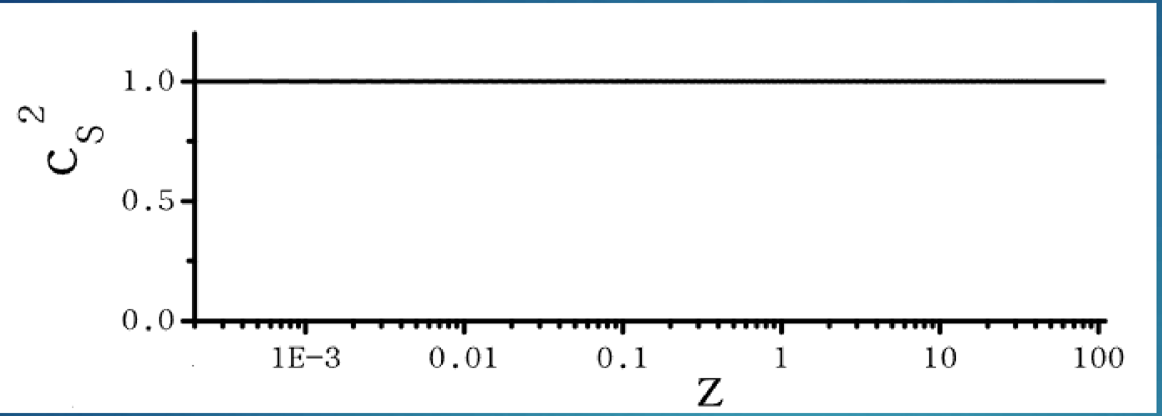
(De Felice,Tsujikawa , arXiv:1110.3878 [gr-qc])

where

$$\begin{aligned}
w_1 &\equiv 2\,(G_4 - 2\,X G_{4,X}) - 2X\,(G_{5,X}\dot{\phi}H - G_{5,\phi})\,,\\
w_2 &\equiv -2\,G_{3,X}X\dot{\phi} + 4\,G_4 H - 16\,X^2 G_{4,XX}H + 4(\dot{\phi}G_{4,\phi X} - 4H\,G_{4,X})X + 2\,G_{4,\phi}\dot{\phi}\\
&\quad + 8\,X^2 H G_{5,\phi X} + 2H\,X\,(6G_{5,\phi} - 5\,G_{5,X}\dot{\phi}H) - 4G_{5,XX}\dot{\phi}X^2 H^2\,,\\
w_3 &\equiv 3\,X(K_{,X} + 2\,X K_{,XX}) + 6X\,(3X\dot{\phi}H G_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\,H\dot{\phi}G_{3,X})\\
&\quad +18\,H(4\,H X^3 G_{4,XXX} - H G_4 - 5\,X\dot{\phi}G_{4,\phi X} - G_{4,\phi\phi} + 7\,H G_{4,X}X + 16\,H X^2 G_{4,XX} - 2\,X^2\dot{\phi}G_{4,\phi XX})\\
&\quad +6H^2 X\,(2\,H\dot{\phi}G_{5,XXX}X^2 - 6\,X^2 G_{5,\phi XX} + 13X\,H\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15\,H\dot{\phi}G_{5,X} - 18G_{5,\phi})\,,\\
w_4 &\equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}\,.
\end{aligned} \qquad (12)$$

Testing our models through the stability condition for each of the sub-classes, regarding c_s^2, we get:

- (E.g. Case c=1.3)



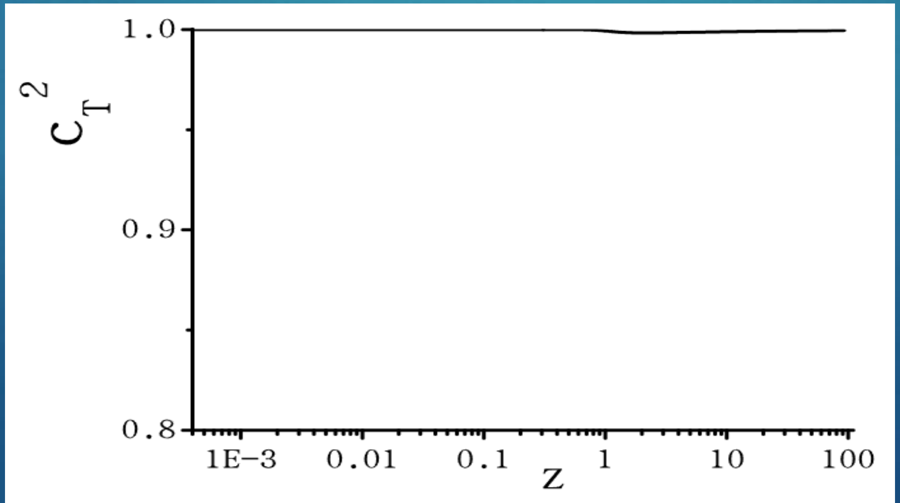
→ free from Laplacian instabilities

- Finally, in Horndeski theories the gravitational-wave speed square is

$$c_T^2 \equiv \frac{w_4}{w_1} \geq 0 \qquad (13)$$

(A. De Felice and S. Tsujikawa, JCAP 02, 007)

- For our models:



Conclusions-Prospects

- By choosing suitable sub-classes of the Horndeski theory we obtained a cosmological behavior that is almost identical with that of ΛCDM at early times, but which at intermediate and late times deviates from it due to the weakening of the gravitational interaction.

- There exist regions of the free parameters that are able to reproduce the observed Hubble function evolution and at late times potentially alleviate the H_0 tension, implying also the viability of the examined models.

- A detailed verification of viability for the proposed models and their results is necessary using observational data sets of SNIa, BAO, CMB (etc.) samples.

- Finally, if the tension isn't a result of unknown systematics, then one should indeed seek for alleviation in extensions of the standard lore of cosmology.

10 commandments for Hubble hunters

1. I am $H_0 \approx 74$ thy Goal
2. Thou shalt not fail to fit key data (BAO, SNeIa, polarization)...
3. ...or include a local H_0 prior in vain
4. Thou shalt not forget the true source of the tension (from the SH0ES side)
5. Honour H_0's central value, and keep an eye on your $\Delta\chi^2$/Bayesian evidence
6. Thou shalt not murder σ_8/S_8...
7. ...but aim to solve this and other tensions/puzzles at the same time
8. Thy solution shall come from a compelling particle/gravity model...
9. ...which makes verifiable predictions...
10. ...which later better be verified!

Credits: Gustave Doré

THANK YOU!