# THE MASSLESS LIMIT AND TENSION

# IN MASSIVE GAUGE THEORIES

## ANAMARIA HELL

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Based on:

[arXiv:2109.05030]

[arXiv:2111.00017]

♦ THE PRINCIPLE OF CONTINUITY

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♦ MASSIVE YANG-MILLS THEORY

$$\mathcal{L} = -\frac{1}{2} \operatorname{Tr}(F_{\mu\nu}F^{\mu\nu}) + m^2 \operatorname{Tr}(A_{\mu}A^{\mu})$$

$$\tilde{\Delta}^{ab}_{\mu\nu}(k) = \left(-\eta_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{m^2}\right)\frac{i\delta^{ab}}{k^2 - m^2 + i\epsilon}$$

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$$r_V = \left(\frac{GM}{m_g^4}\right)^{1/5}$$

THE PRINCIPLE OF CONTINUITY

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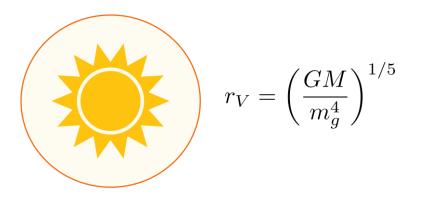
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"...it appears highly probable that outside perturbation theory, a continuous zero-mass limit exists, and the theory is renormalizable."

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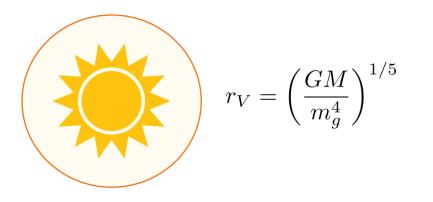
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#### THE GOAL

#### • QUANTUM FLUCTUATIONS

$$\delta \phi_L^2 = \langle 0 | \phi(\vec{x}, t) \phi(\vec{y}, t) | 0 \rangle \sim \left. \frac{k^3}{\omega_k} \right|_{k \sim \frac{1}{L}}$$
$$\delta \phi_L|_{k^2 \gg m^2} \sim \frac{1}{L} \qquad \qquad L = |\vec{x} - \vec{y}|$$

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#### THE DECOMPOSITION

$$(A_0, A_i)$$
$$A_i = \zeta A_i^T \zeta^{\dagger} + \frac{i}{g} \zeta_{,i} \zeta^{\dagger}$$
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$$\mathcal{L}_0 = \operatorname{Tr}\left[-\chi \left(\partial^2 + m^2\right) \frac{-\Delta m^2}{-\Delta + m^2} \chi - A_i^T \left(\partial^2 + m^2\right) A_i^T\right]$$

$$\mathcal{L}_{int} \sim \text{Tr}\left\{-2igm^2 A_i^T \chi \chi_{,i} + \frac{m^2 g^2}{6} \left(\chi_{,\mu} \chi \chi^{,\mu} \chi - \chi_{,\mu} \chi^{,\mu} \chi^2\right)\right\}$$

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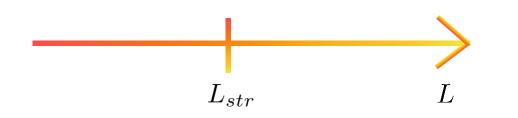
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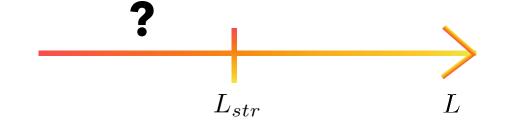
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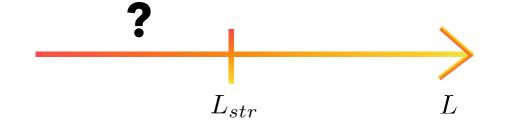
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The Strong Coupling Scale 
$$L_{str} \sim rac{g}{m}$$

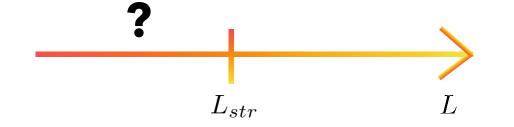






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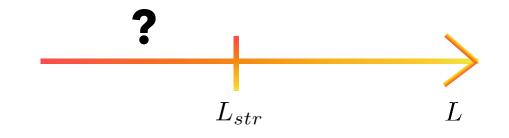


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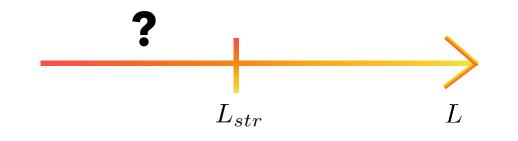
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## THE MASSLESS LIMIT AND DUALITY



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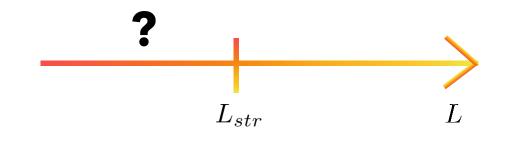
## THE MASSLESS LIMIT AND DUALITY

PROCA THEORY

$$\mathcal{L}_{P} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^{2}}{2}A_{\mu}A^{\mu}$$

Kalb-Ramond Theory

$$\mathcal{L}_{KB} = \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu}$$



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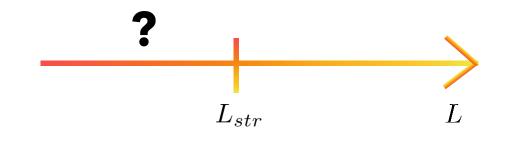
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DUAL THEORIES



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## **THE MASSLESS LIMIT AND DUALITY**

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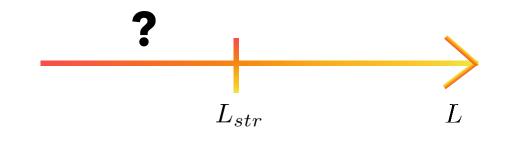
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DUAL THEORIES

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#### PROCA THEORY

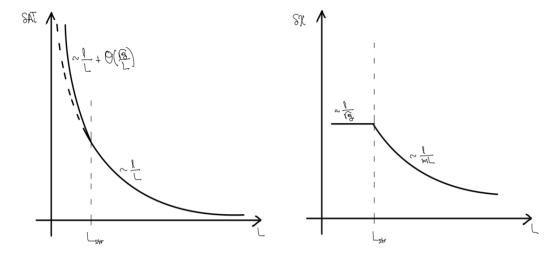
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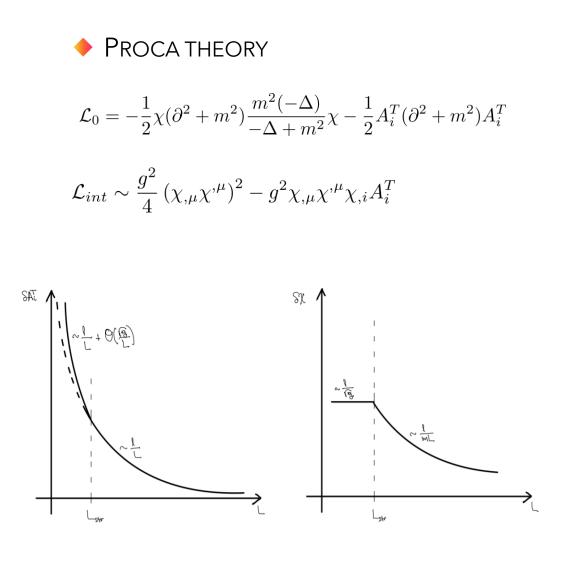
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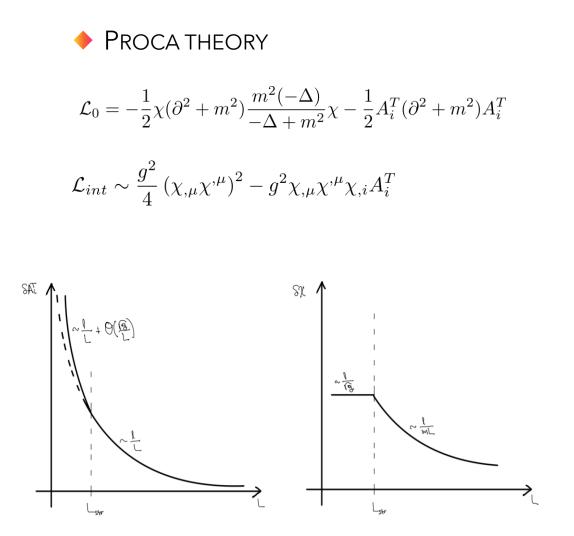




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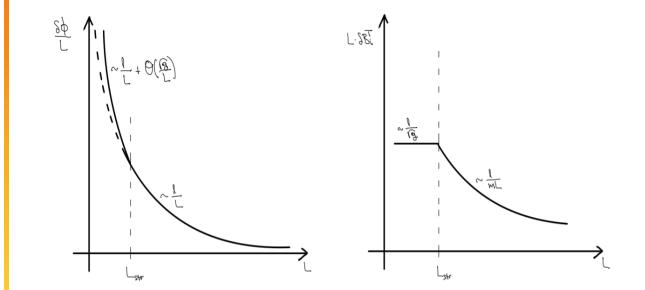
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**THANK YOU!** 

DUALITY WITHOUT SELF-INTERACTION