The Massless Limit and Tension in Massive Gauge Theories

Anamaria Hell

Ludwig-Maximilians-Universität

Chair for Astroparticle Physics and Cosmology

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Based on:

[arXiv:2109.05030]

[arXiv:2111.00017]

The Puzzle of Massless Limit

- The Principle of Continuity
- Massive Yang-Mills Theory

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + m^2\,\mathrm{Tr}(A_\mu A^\mu)$$

$$\tilde{\Delta}^{ab}_{\mu\nu}(k) = \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}\right)\frac{i\delta^{ab}}{k^2 - m^2 + i\epsilon}$$

- Fierz-Pauli Massive Gravity
- vDVZ Discontinuity

- The Vainshtein Mechanism

$$r_V = \left(\frac{GM}{m_g^4}\right)^{1/5}$$

- The Vainshtein-Kriplovich conjecture

"...it appears highly probable that outside perturbation theory, a continuous zero-mass limit exists, and the theory is renormalizable."

- **The Goal**

Massive Yang-Mills Theory

- Quantum fluctuations

$$\delta\phi_L^2 = \langle 0|\,\phi(\vec{x},t)\,\phi(\vec{y},t)\,|0\rangle \sim \left.\frac{k^3}{\omega_k}\right|_{k\sim\frac{1}{L}}$$

$$\left.\delta\phi_L\right|_{k^2\gg m^2} \sim \frac{1}{L} \qquad\qquad L = |\vec{x}-\vec{y}|$$

- The decomposition

$$(A_0, A_i)$$

$$A_i = \zeta A_i^T \zeta^\dagger + \frac{i}{g}\zeta_{,i}\zeta^\dagger$$

$$A_{i,i}^T = 0 \qquad\qquad \zeta = e^{-ig\chi}$$

- The full perturbation theory

$$\mathcal{L}_0 = \mathrm{Tr}\left[-\chi\left(\partial^2+m^2\right)\frac{-\Delta m^2}{-\Delta+m^2}\chi - A_i^T\left(\partial^2+m^2\right)A_i^T\right]$$

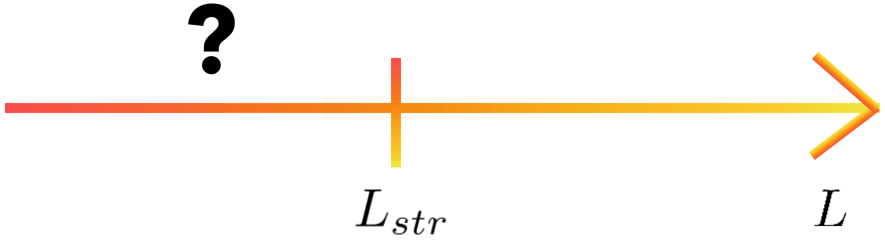
$$\mathcal{L}_{int} \sim \mathrm{Tr}\left\{-2igm^2 A_i^T\chi\chi_{,i} + \frac{m^2g^2}{6}\left(\chi_{,\mu}\chi\chi^{,\mu}\chi - \chi_{,\mu}\chi^{,\mu}\chi^2\right)\right\}$$

$$\sim \frac{g}{L^4} \qquad\qquad \sim \frac{g^2}{(mL)^2 L^4}$$

$$\delta A_L^T \sim \frac{1}{L} \qquad\qquad \delta\chi_L \sim \frac{1}{mL}$$

- The strong coupling scale $\quad L_{str} \sim \dfrac{g}{m}$

The Massless Limit and Duality



- Beyond the Strong Coupling Scale

$$\mathcal{L}_0 \sim \mathrm{Tr}\left[\frac{m^2}{g^2}\zeta^\dagger_{,\mu}\zeta^{,\mu} - A_i^T\left(\partial^2 + m^2\right)A_i^T\right]$$

$$\mathcal{L}_{int} \sim -\frac{2im^2}{g}\,\mathrm{Tr}\left(A_i^T\zeta^\dagger\zeta_{,i}\right)$$

- The leading corrections

$$A_i^{T(1)} \sim -i\frac{m^2}{g}\zeta^\dagger\zeta_{,i} \sim \frac{g}{L^3}\frac{L}{L_{str}}$$

- Proca theory

$$\mathcal{L}_P = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu \boxed{+\frac{g^2}{4}\left(A_\mu A^\mu\right)^2}$$

- Kalb-Ramond Theory

$$\mathcal{L}_{KB} = \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{m^2}{4}B_{\mu\nu}B^{\mu\nu} \boxed{+\frac{g^2}{16}\left(B_{\mu\nu}B^{\mu\nu}\right)^2}$$

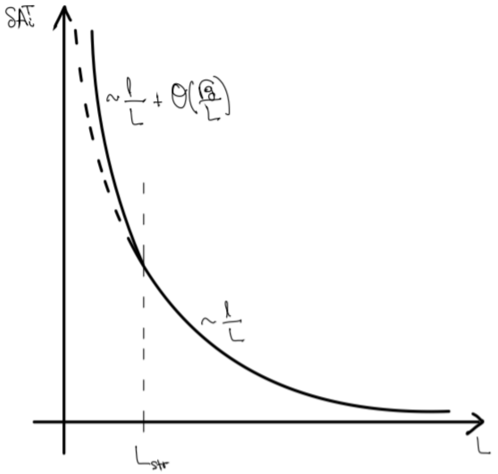
- Dual Theories
- The Modification
- $\boxed{L_{str} \sim \frac{\sqrt{g}}{m}}$

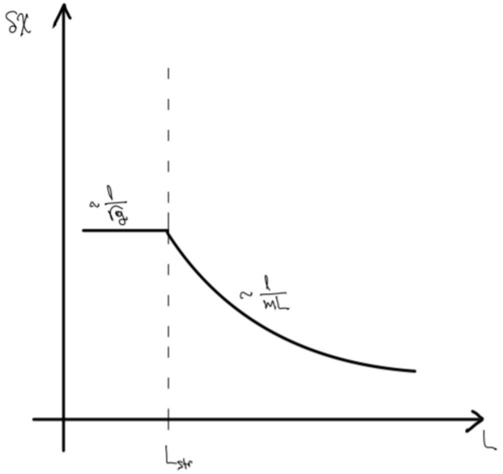
The Comparison

Proca theory

$$\mathcal{L}_0 = -\frac{1}{2}\chi(\partial^2 + m^2)\frac{m^2(-\Delta)}{-\Delta + m^2}\chi - \frac{1}{2}A_i^T(\partial^2 + m^2)A_i^T$$

$$\mathcal{L}_{int} \sim \frac{g^2}{4}\left(\chi_{,\mu}\chi^{,\mu}\right)^2 - g^2\chi_{,\mu}\chi^{,\mu}\chi_{,i}A_i^T$$

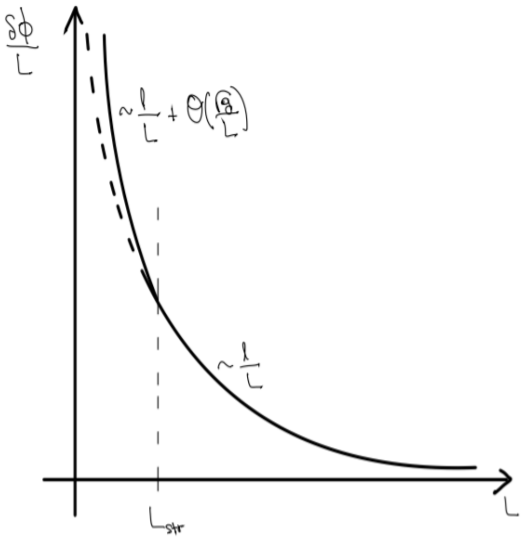


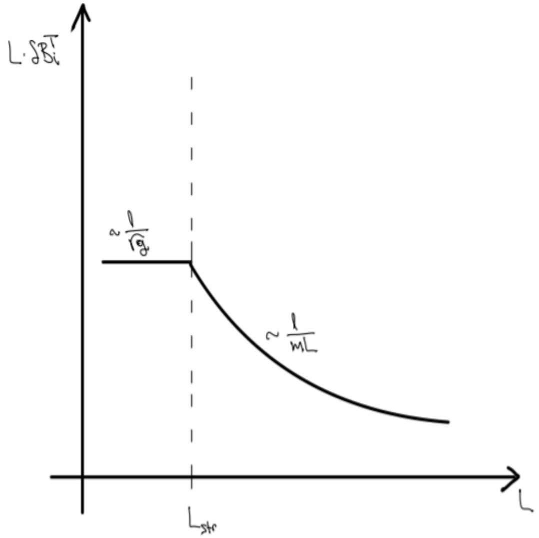


Kalb-Ramond Theory

$$\mathcal{L}_0 = -\frac{1}{2}B_i^T(\partial^2 + m^2)\frac{m^2}{-\Delta + m^2}B_i^T - \frac{1}{2}\phi_n(\partial^2 + m^2)\phi_n$$

$$\mathcal{L}_{int} \sim g^2\left(B^T\right)^4 + g^2\left(B^T\right)^3\phi_n$$





Conclusion

Duality?

- The analysis indicates that the theories are not dual.
- The origin lies in the minimal level of quantum fluctuations of the original fields.

The massless limit of MYM?

- The longitudinal modes enter a strong coupling regime at $L_{str} \sim \frac{g}{m}$
- Beyond it, the transverse modes are weakly coupled.
- The massless theory is restored up to small corrections that vanish in the massless limit.

What Next?

Extension to p-forms in D-dimensions

Hermitian Gravity

Duality without self-interaction

Thank You!