Resolving Hubble Tension with New Gravitational Scalar Tensor Theories

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[S. Banerjee, M. Petronikolou, E. N. Saridakis, arXiv:2209.02426]

Motivation

• ΛCDM model has been tested by numerous probes and has provided a remarkable explanation for cosmological observations such as CMB anisotropies and BAO.

• However, there has been a growing discrepancy between the measured values of H_0 as inferred from early Universe probes and late-Universe probes.

• SH0ES collaboration recently obtained $H_0 = 74.03 \pm 1.42$ km/s/Mpc. Other distance-ladder measurements lead to other values, most of them in rough agreement with SH0ES.

•The PLANCK collaboration, assuming ΛCDM model obtained $H_0 = 67.36 \pm 0.54$ km/s/Mpc→in 4.4σ tension with the value reported by SH0ES.

• Various theoretical solutions/models were hitherto suggested to solve the H_0 discrepancy.

Model Overview

- The action of the new gravitational scalar-tensor theories is a function of the metric and its derivatives only-

$$S = \int d^4\sqrt{-g}\, f\left(R, (\nabla R)^2, \Box R\right), \qquad (1)$$

with $(\nabla R)^2 = g^{\mu\nu}\nabla_\mu R \nabla_\nu R$ [A. Naruko1, D. Yoshida1 and S. Mukohyama, Class. Quantum Grav. **33** (2016)].

- The Lagrangian contains the Ricci scalar and its first and second derivatives, in a specific combination that makes them free of ghosts.
- In the Einstein frame they are proved to be a subclass of bi-scalar extensions of general relativity
- One can rewrite the action by converting the above Lagrangian, using double Lagrange multipliers, to actions of multi-scalar fields coupled minimally to gravity.

- Fixing the dependence of f on $\Box R = \beta$, in the present work, we consider theories with the following form of f

$$f(R,(\nabla R)^2,\Box R) = \mathscr{K}((R,(\nabla R)^2) + \mathscr{G}(R,(\nabla R)^2)\Box R. \quad (2)$$

- In this case, the action (1) transforms to

$$S = \int d^4x\sqrt{-\hat{g}}\Big[\frac{1}{2}\hat{R} - \frac{1}{2}\hat{g}^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi - \frac{1}{\sqrt{6}}e^{-\sqrt{\frac{2}{3}}\chi}\hat{g}^{\mu\nu}\mathscr{G}\nabla_\mu\chi\nabla_\nu\phi + \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\mathscr{K} + \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\mathscr{G}\hat{\Box}\phi - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\phi\Big] \quad (3)$$

where

$$\mathscr{K} = \mathscr{K}(\phi, B), \quad \mathscr{G} = \mathscr{G}(\phi, B), \quad B = 2e^{\sqrt{\frac{2}{3}}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \quad (4)$$

- The fields are introduced through the following conformal transformations $g_{\mu\nu} = \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\hat{g}_{\mu\nu}$, $\varphi \equiv f_\beta$.

Cosmological Behaviour

- For this we consider a flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \tag{5}$$

- Including the matter part, the metric field equations now become

$$\mathcal{E}_{\mu\nu} = \frac{1}{2} T_{\mu\nu}, \tag{6}$$

with $T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$ →perfect fluid.

- Friedmann equations can be written as

$$H^2 = \frac{1}{3}(\rho_{DE} + \rho_m) \tag{7}$$

$$2\dot{H} + 3H^2 = -(p_{DE} + p_m) \tag{8}$$

•With the effective dark energy and pressure defined as

$$\rho_{DE} \equiv \frac{1}{2}\dot{\chi}^2 - \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\mathcal{K} - \frac{2}{3}\dot{\phi}^2\left[\dot{\phi}\left(\sqrt{6}\dot{\chi} - 9H\right) - 3\ddot{\phi}\right]\mathcal{G}_B$$
$$+\frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\left[\dot{B}\dot{\phi}\mathcal{G}_B + \frac{\phi}{2} + \dot{\phi}^2\left(\mathcal{G}_\phi - 2\mathcal{K}_B\right)\right], \quad (9)$$

$$p_{DE} \equiv \frac{1}{2}\dot{\chi}^2 + \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\chi}\mathcal{K} + \frac{1}{2}e^{-\sqrt{\frac{2}{3}}\chi}\left(\dot{B}\dot{\phi}\mathcal{G}_B + \dot{\phi}^2\mathcal{G}_\phi - \frac{\phi}{2}\right). \quad (10)$$

→ The effective dark-energy equation of motion is given by

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0, \quad (11)$$

→with the dark-energy equation-of-state parameter given by:

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}}. \quad (12)$$

Model I

- We need to consider specific ansatzes for the functions $\mathcal{K}(\phi, B)$ and $\mathcal{G}(\phi, B) \to \mathcal{K}(\phi, B) = \frac{\phi}{2}$, $\mathcal{G}(\phi, B) = 0$ corresponds to the GR case.
- $$\mathcal{K}(\phi, B) = \frac{1}{2}\phi - \frac{\zeta}{2}B \quad \text{and} \quad \mathcal{G}(\phi, B) = 0. \tag{13}$$
  ζ is a coupling constant [E. N. Saridakis, M. Tsoukalas, Phys. Rev. D **93** (2016)].
- The corresponding Friedmann equations read as

$$3H^2 - \rho_m - \frac{1}{2}\dot{\chi}^2 + \frac{1}{8}e^{-2\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi + \zeta\dot{\phi}^2\right) = 0, \tag{14}$$

$$3H^2 + 2\dot{H} + p_m + \frac{1}{2}\dot{\chi}^2 + \frac{1}{8}e^{-2\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi - \zeta\dot{\phi}^2\right) = 0. \tag{15}$$

- The two scalar field equations become

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{2\sqrt{6}} e^{-2\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{2\sqrt{6}} e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi - \zeta\dot{\phi}^2\right) = 0, \quad (16)$$

$$\zeta\ddot{\phi} + \frac{1}{3}\zeta\dot{\phi}\left(9H - \sqrt{6}\dot{\chi}\right) - \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi} + \frac{1}{2} = 0. \quad (17)$$

- The effective dark-energy energy density and pressure become

$$\rho_{DE} = \frac{1}{2}\dot{\chi}^2 - \frac{1}{8} e^{-2\sqrt{\frac{2}{3}}\chi}\phi + \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi + \zeta\dot{\phi}^2\right), \quad (18)$$

$$p_{DE} = \frac{1}{2}\dot{\chi}^2 + \frac{1}{8} e^{-2\sqrt{\frac{2}{3}}\chi}\phi - \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi}\left(\phi - \zeta\dot{\phi}^2\right). \quad (19)$$

Evolution of Hubble Parameter

- We consider matter sector to be dust$\longrightarrow$ set $p_m \approx 0$.
- We set $z = -1 + a_0/a$ with the current value of scale factor being set $a_0 = 1$.
- The behaviour of the Hubble parameter in ΛCDM cosmology is given by

$$H_{\Lambda CDM}(z) = H_0\sqrt{\Omega_{m_0}(1+z)^3 + 1 - \Omega_{m_0}} \qquad (20)$$

We set $\Omega_{m_0} \approx 0.31$ and $H_0 \approx 67.3$km/s/Mpc.

- We set the initial conditions such that $H(z \rightarrow z_{CMB}) \approx H_{\Lambda CDM}$ while $H(z \rightarrow 0) > H_{\Lambda CDM}(z \rightarrow 0)$. [S. Banerjee, M. Petronikolou, E. N. Saridakis, arXiv:2209.02426]

Evolution of w_{DE}

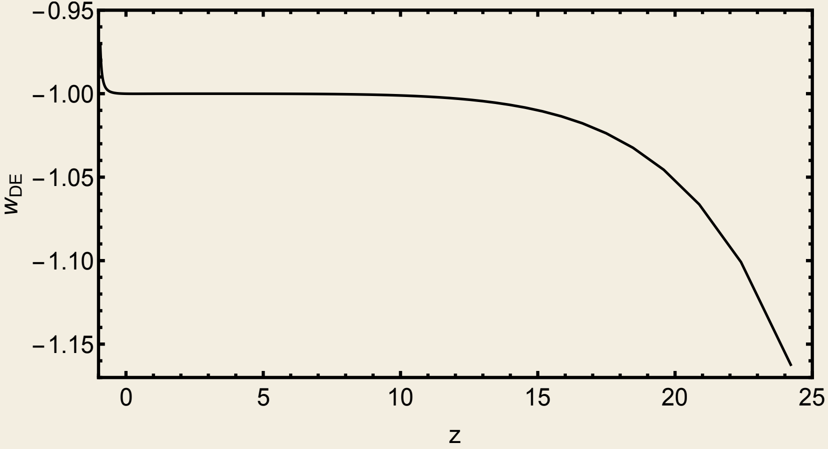

Figure: *The effective dark-energy equation-of-state parameter w_{DE} as a function of the redshift, for Model I for $\zeta = -10$ in H_0 units.*

Evolution of $H(z)$

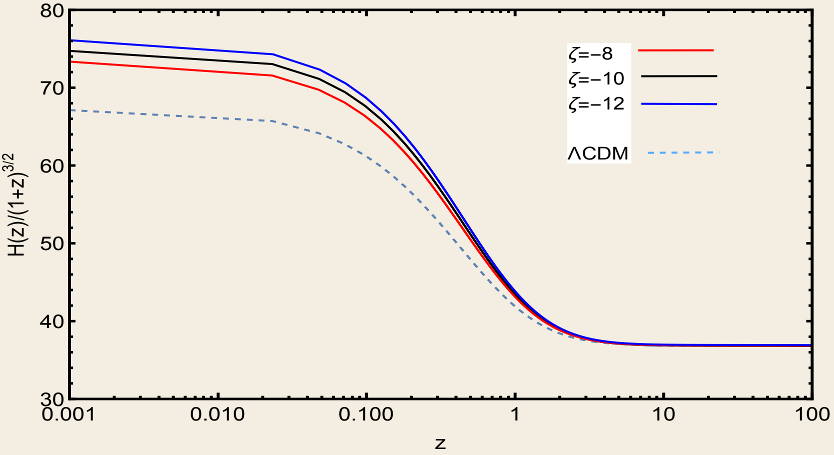

Figure: *The normalized* $H(z)/(1+z)^{3/2}$ *as a function of redshift.*

Results

- $w_{DE} < -1$ mostly, thereby depicting phantom evolution which implies faster expansion → one of the theoretical requirements that are capable of alleviating the H_0 tension [S. F. Yan, *et al.*, Phys. Rev. D **10**, 2020, L. Heisenberg, H. Villarrubia-Rojo and J. Zosso, arXiv:2201.11623].
- The present value of H_0 depends on the model parameter ζ.
- For value of ζ around -10 (in units of H_0), the present value of the Hubble parameter is around $H_0 \approx 74$km/s/Mpc, which is consistent with the direct measured value of the Hubble parameter.
- Values of ζ higher or lower than this give higher or lower values of H_0 respectively.

Model II

- $$\mathscr{K}(\phi, B) = \frac{1}{2}\phi \quad \text{and} \quad \mathscr{G}(\phi, B) = \xi B, \tag{21}$$

  with ξ the corresponding coupling constant.
- The corresponding Friedmann equations read as

$$3H^2 - \rho_m - \frac{1}{2}\dot{\chi}^2 + \frac{1}{8}e^{-2\sqrt{\frac{2}{3}}\chi}\left(1 - 2e^{\sqrt{\frac{2}{3}}\chi}\right)\phi$$
$$+\xi\dot{\phi}^3\left(\sqrt{6}\dot{\chi} - 6H\right) = 0, \tag{22}$$

$$3H^2 + 2\dot{H} + p_m + \frac{1}{2}\dot{\chi}^2 + \frac{1}{8}e^{-2\sqrt{\frac{2}{3}}\chi}\left(1 - 2e^{\sqrt{\frac{2}{3}}\chi}\right)\phi$$
$$-\frac{1}{3}\xi\dot{\phi}^2\left(\sqrt{6}\dot{\phi}\dot{\chi} + 6\ddot{\phi}\right) = 0. \tag{23}$$

- The two scalar field equations become

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{2\sqrt{6}} e^{-2\sqrt{\frac{2}{3}}\chi} \left(1 - e^{\sqrt{\frac{2}{3}}\chi}\right) \phi - \sqrt{6}\,\xi\dot{\phi}^2 \left(H\dot{\phi} + \ddot{\phi}\right) = 0, \quad (24)$$

$$\xi\dot{\phi} \left\{2\left(-6H + \sqrt{6}\dot{\chi}\right)\ddot{\phi} + \dot{\phi}\left[-6\dot{H} + 3H\left(-6H + \sqrt{6}\dot{\chi}\right) + \sqrt{6}\ddot{\chi}\right]\right\} + \frac{1}{8} e^{-2\sqrt{\frac{2}{3}}\chi} \left(1 - 2e^{\sqrt{\frac{2}{3}}\chi}\right) = 0. \quad (25)$$

- The effective dark-energy energy density and pressure become

$$\rho_{DE} = \frac{1}{2}\dot{\chi}^2 - \frac{1}{8} e^{-2\sqrt{\frac{2}{3}}\chi} \left(1 - 2e^{\sqrt{\frac{2}{3}}\chi}\right) \phi - \xi\dot{\phi}^3 \left(\sqrt{6}\dot{\chi} - 6H\right), \quad (26)$$

$$p_{DE} = \frac{1}{2}\dot{\chi}^2 + \frac{1}{8} e^{-2\sqrt{\frac{2}{3}}\chi} \left(1 - 2e^{\sqrt{\frac{2}{3}}\chi}\right) \phi - \frac{1}{3}\xi\dot{\phi}^2 \left(\sqrt{6}\dot{\phi}\dot{\chi} + 6\ddot{\phi}\right). \quad (27)$$

Evolution of w_{DE}

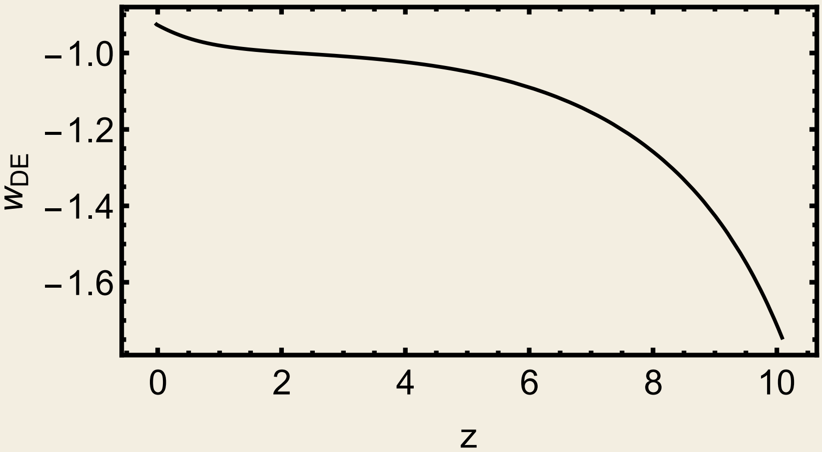

Figure: *The effective dark-energy equation-of-state parameter w_{DE} as a function of the redshift, for Model II for $\xi = -10$ in units of H_0.*

Evolution of $H(z)$

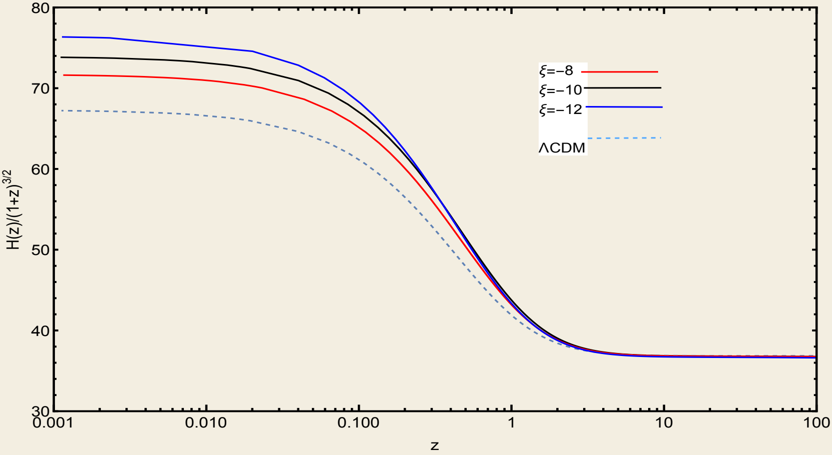

Figure: *The normalized* $H(z)/(1+z)^{3/2}$ *as a function of redshift.*

Results

- $w_{DE} < -1$ mostly, thereby depicting phantom evolution which implies faster expansion, thus serving as a mechanism for Hubble tension alleviation.
- For value of $\xi = -10$, the present value of the Hubble parameter is around $H_0 \approx 74$ km/s/Mpc, which is consistent with the direct measured value of the Hubble parameter.
- Values of ξ higher or lower than this give higher or lower values for H_0 respectively.

Cosmic Chromometer (CC) Data [R. Jimenez and A. Loeb, Astrophys. J. 573, 2002]

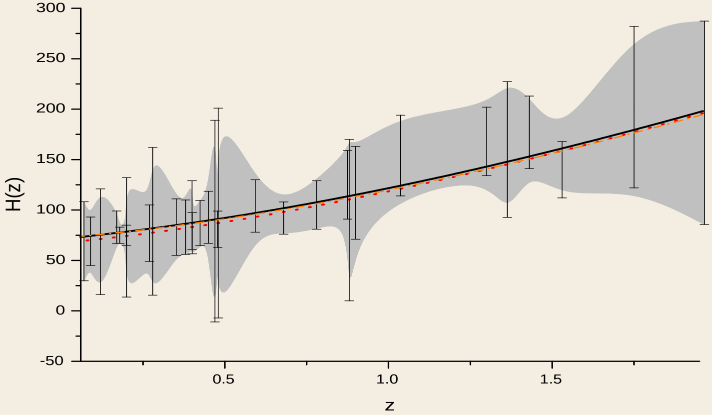

Figure: *Evolution of* $H(z)$ *as function of redshift. Red dotted line:* Λ*CDM model, orange dashed-dotted line: model I with* $\zeta = -10$ *and black line: Model II with* $\xi = -10$ *on top of the CC data points at 1σ confidence level.*

Summary

- We investigated the possibility of resolving the Hubble tension using the new gravitational scalar tensor theories.
- We studied the cosmological behaviour of two specific models, imposing as initial conditions at high redshifts the coincidence of the behaviour of the Hubble function with that predicted by ΛCDM cosmology.
- We showed that as time passes, the effect of bi-scalar modifications become important and thus at low redshifts the Hubble function acquires increased values in a controlled way resulting to $H_0 \approx 74 km/s/Mpc$ for particular parameter choices.
- We further confronted our models with CC data at 1σ level confidence and find they are viable and in agreement with the observed data.

Thank you for your attention!!!