# The $H_0$ tension alleviated through ultra-light primordial black holes:

# Information insight through gravitational waves

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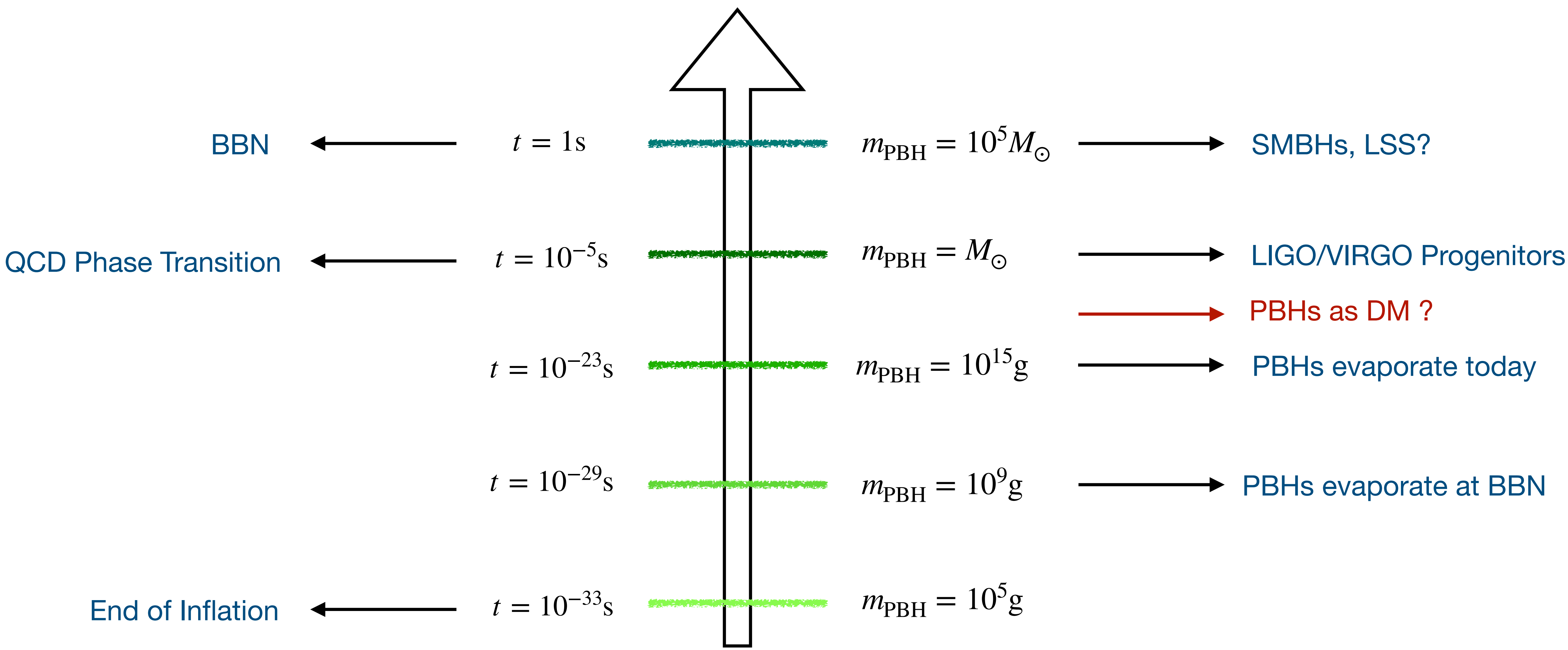
# Introduction

- Primordial Black Holes (PBHs) are formed out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_c(w \equiv p/\rho)$ [Carr - 1975].

$$m_{\rm PBH} = \gamma M_{\rm H} \propto H^{-1} \text{ where } \gamma \sim {\rm O}(1)$$




See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

# Hubble tension and ultra-light PBHs

- Black holes radiate energy by emitting particles through the process of Hawking evaporation (HE) [S. Hawking - 1974] with their mass loss rate being recast as

$$\frac{\mathrm{d}m_{\rm BH}}{\mathrm{d}t} \propto -\frac{g_{*,\rm H}(T_{\rm BH})}{m_{\rm BH}^2}, \text{ with } T_{\rm BH} \equiv \frac{M_{\rm Pl}^2}{8\pi m_{\rm BH}}.$$

- The factor $g_{*,\rm H}(T_{\rm PBH})$ counts *all the existing particle species* with $m < T_{\rm BH}$.

$$\text{For SM particles, } g_{*,\rm H}(T_{\rm BH}) \simeq \begin{cases} 108, & T_{\rm BH} \gg 100\mathrm{GeV}, m_{\rm BH} \ll 10^{11}\mathrm{g} \\ 7, & T_{\rm BH} \ll 1\mathrm{MeV}, m_{\rm BH} \gg 10^{16}\mathrm{g} \end{cases}$$

- HE is a **universal process** across all species. In particular, HE products with **feeble couplings to SM** if relatively **light**, they will contribute to **dark radiation** (DR) injecting energy to the primordial plasma, increasing $N_{\rm eff}$ [Hooper et al. - 2019, Nesseris et al. - 2019, Lunardini et al. - 2020] and subsequently the early value of $H$, hence alleviating the Hubble tension.

$$\Delta N_{\rm eff} \simeq 7.5 \times 10^{-2} \left(\frac{g_{\rm DR}}{4}\right)\left(\frac{\Omega_{\rm PBH,f}}{10^{-15}}\right)\left(\frac{m_{\rm PBH}}{10^9\mathrm{g}}\right)$$

# Phenomenology of ultra-light PBHs

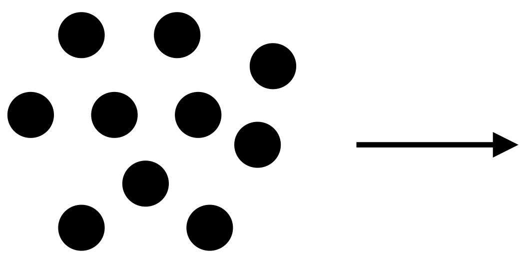
- Ultralight PBHs with $m_{\rm PBH} < 10^9 {\rm g}$ form in the early universe and evaporate before BBN.

- These ultralight PBHs can change drastically the standard cosmological scenario by giving rise to **an early matter dominated era (eMD)** and **driving the reheating process** through their evaporation [Zagorac et al. - 2019, Martin et al. - 2019, Inomata et al. - 2020].

- However, these very small PBHs **leave no direct observational imprint**, apart from possible Planckian relics, since they Hawking evaporate even before Big Bang Nucleosynthesis (BBN).

- A possible way to constrain them is to study the emitted secondary GWs induced from the gravitational potential of gas of PBHs which can propagate until today and leave an **indirect imprint** of the PBH past experience.

# Scalar Induced Gravitational Waves from PBH Poisson fluctuations



**Poisson Statistics** [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

**Same mass** [Dizgah, Franciolini & Riotto - 2019]

PBH dom: $\Omega_{\mathrm{PBH,f}} \geq 2\times 10^{-15}\left(\frac{10^9\mathrm{g}}{m_{\mathrm{PBH}}}\right) \Rightarrow \mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\mathrm{UV}}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_{\mathrm{d}}^2}\right)^{-2}$

[Papanikolaou et al. - 2020]

$$\Omega_{\mathrm{GW}}^{\mathrm{2nd\ order}}(\eta,k) \equiv \frac{1}{\rho_{\mathrm{tot}}}\frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} \propto \mathcal{P}_h(\eta,k) \propto \int \mathrm{d}k_1 \int \mathrm{d}k_2 \left(\int f(k_1,k_2,k\eta)\mathrm{d}(k\eta)\right)^2 \mathcal{P}_\Phi(k_1)\mathcal{P}_\Phi(k_2)\,.$$

[Baumann et al. - 2007, Ananda et al. - 2007]

# GW Overproduction Constraints $\Rightarrow \Delta N_{\rm eff}$ constraints

$$\Omega_{\rm GW,tot}(\eta_{\rm evap}) \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 10^{-4}\left(\frac{10^9{\rm g}}{m_{\rm PBH}}\right)^{1/4}$$

[Papanikolaou et al. 2020]

$$\Delta N_{\rm eff} \simeq 7.5\times 10^{-2}\left(\frac{g_{\rm DR}}{4}\right)\left(\frac{\Omega_{\rm PBH,f}}{10^{-15}}\right)\left(\frac{m_{\rm PBH}}{10^9{\rm g}}\right)$$

**[Planck 2015]**

$$a)\ \Delta N_{\rm eff}^{\rm max} = 7.5\times 10^{9}\left(\frac{g_{\rm DR}}{4}\right)\left(\frac{m_{\rm PBH}}{10^9{\rm g}}\right)^{3/4} \leq 0.28 \Leftrightarrow m_{\rm PBH} < 0.2 M_{\rm Pl}\left(\frac{4}{g_{\rm DR}}\right)^{4/3} \leq M_{\rm Pl}$$

$$b)\ \Delta N_{\rm eff} \leq 0.28 \Rightarrow \Omega_{\rm PBH,f,max} = 3.5\times 10^{-15}\left(\frac{4}{g_{\rm DR}}\right)\left(\frac{10^9{\rm g}}{m_{\rm PBH}}\right) > \Omega_{\rm PBH,f,min} = 2\times 10^{-15}\left(\frac{10^9{\rm g}}{m_{\rm PBH}}\right) \Rightarrow g_{\rm DR} < 7$$

**Early PBH domination era ?**

*If not*

**To alleviate $H_0$ tension $\Rightarrow$High $g_{\rm DR}$**

# Conclusions

- We studied the scenario of alleviating the Hubble tension through the portal of ultralight PBHs due to DR production from PBH Hawking evaporation.

$$\Delta N_{\rm eff} \simeq 7.5 \times 10^{-2} \left(\frac{g_{\rm DR}}{4}\right)\left(\frac{\Omega_{\rm PBH,f}}{10^{-15}}\right)\left(\frac{m_{\rm PBH}}{10^{9}{\rm g}}\right)$$

- By avoiding overproduction of SIGWs associated to PBH Poisson fluctuations produced during an early PBH-dominated (PBHd) era, we find that $m_{\rm PBH} < M_{\rm Pl}$ which is rather questionable.

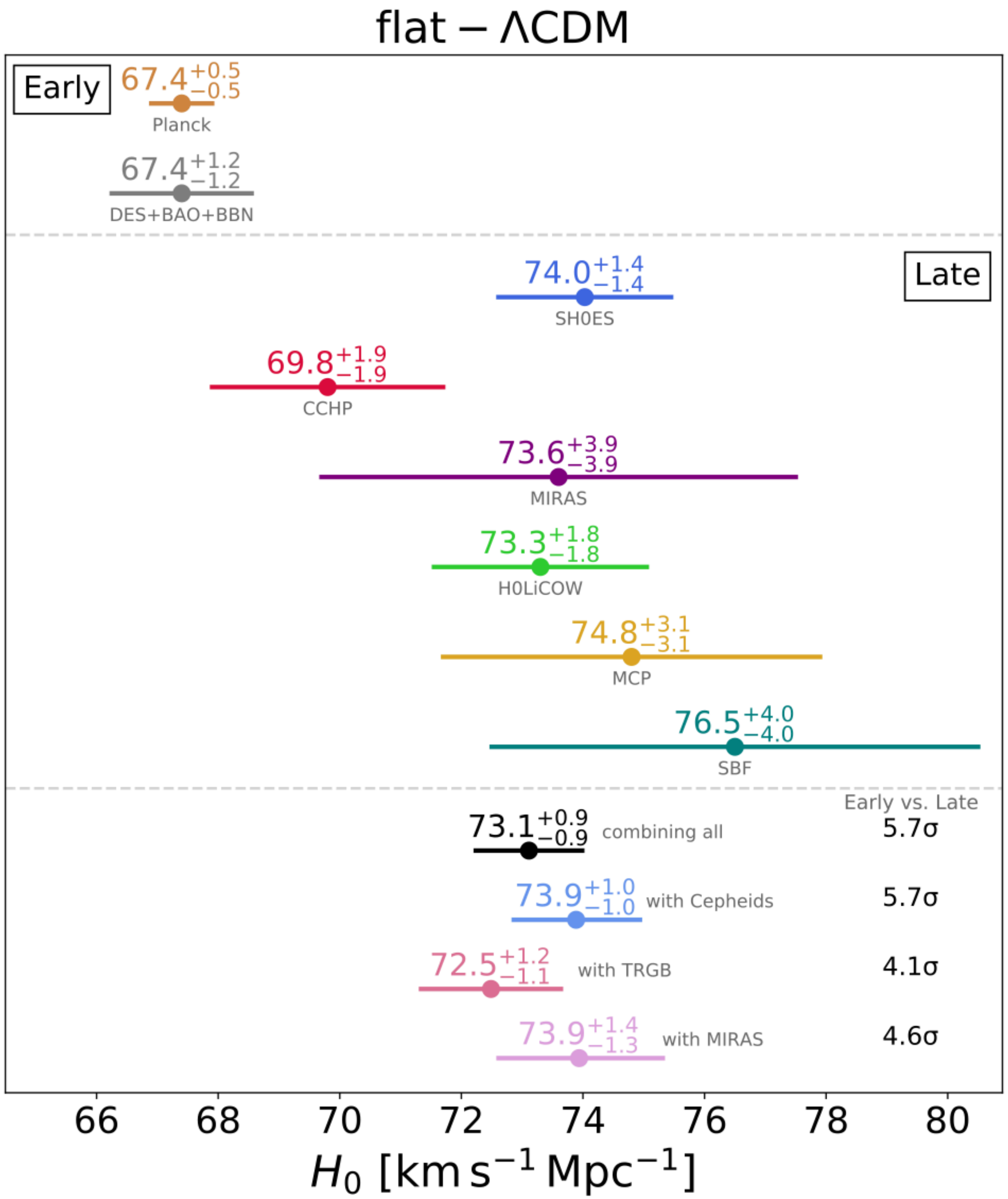
- By accounting as well for Planck upper bound constraints on $\Delta N_{\rm eff}$, we find an extremely small upper bound on $\Omega_{\rm PBH,f}$ very close to the lower bound so as to have early PBH domination.

- Thus, we conclude that the **early PBHd scenario is questioned**. Thus, in case of **no early PBHd era scenario** In order to have a sizeable $\Delta N_{\rm eff}$ and alleviate the Hubble tension, one should have a **high** $g_{\rm DR}$**.**

Thank you for your attention!

# Appendix

# The Hubble Tension




[Di Valentino et al. - 2020]

# GW signatures of PBHs

- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

  $h\Phi^2 \rightarrow$ GWs, PBHs

- 2) **Relic Hawking radiated gravitons** from PBH evaporation [Anantua et al. - 2008, Dong et al. - 2015].

- 3) **GWs** emitted **by PBH mergers** [Eroshenko - 2016, Raidal et al. - 2017].

- 4) **GWs induced** at second order **by PBHs themselves** [Papanikolaou et al. - 2020].

# The PBH Gravitational Potential Power Spectrum

- We assume that PBHs form in the radiation era, $\rho_{\rm PBH} \ll \rho_{\rm tot}$. Thus, PBH formation can be regarded as a transition of a fraction of radiation into dust matter.

- Given the random spatial distribution of PBHs, $\rho_{\rm PBH}$ **is inhomogeneous** while $\rho_{\rm tot}$ is homogeneous. Thus, the $\delta_{\rm PBH}$ can be viewed as **an isocurvature perturbation.**

- If $\Omega_{\rm PBH,f}$ is sufficiently large then PBHs will dominate the energy budget of the universe since $\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$ . Consequently, in the subsequent PBH domination era, **the isocurvature perturbation will convert to a curvature perturbation associated to a PBH gravitational potential $\Phi$.**

- By treating separately the sub and super-horizon scales, the PBH gravitational potential power spectrum reads as

$$\mathcal{P}_\Phi(k) = \frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{\mathcal{H}_{\rm d}^2}\right)^{-2} = \begin{cases} \propto k^3 \text{ for } k \ll \mathcal{H}_{\rm d} \\ \propto \frac{1}{k} \text{ for } k \gg \mathcal{H}_{\rm d} \end{cases}$$

# Basics of Scalar Induced Gravitational Waves

- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

- The equation of motion for the Fourier modes, $h_{\overrightarrow{k}}$, read as:

$$h^{s,''}_{\overrightarrow{k}} + 2\mathscr{H} h^{s,'}_{\overrightarrow{k}} + k^2 h^{s}_{\overrightarrow{k}} = 4S^{s}_{\overrightarrow{k}}$$

- The source term, $S_{\overrightarrow{k}}$ can be recast as:

$$S^s_{\overrightarrow{k}} = \int \frac{\mathrm{d}^3\overrightarrow{q}}{(2\pi)^{3/2}} e^s_{ij}(\overrightarrow{k}) q_i q_j \left[2\Phi_{\overrightarrow{q}}\Phi_{\overrightarrow{k}-\overrightarrow{q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\overrightarrow{q}} + \Phi_{\overrightarrow{q}})(\mathscr{H}^{-1}\Phi'_{\overrightarrow{k}-\overrightarrow{q}} + \Phi_{\overrightarrow{k}-\overrightarrow{q}})\right]$$

- At the end, the energy density of GWs can be recast as [M. Maggiore - 2000]:

$$\rho_{\mathrm{GW}}(\eta, \overrightarrow{x}) = \frac{M^2_{\mathrm{Pl}}}{8}\overline{\left(\partial_t h_{\alpha\beta}\partial_t h^{\alpha\beta} + \partial_i h_{\alpha\beta}\partial_i h^{\alpha\beta}\right)}.$$

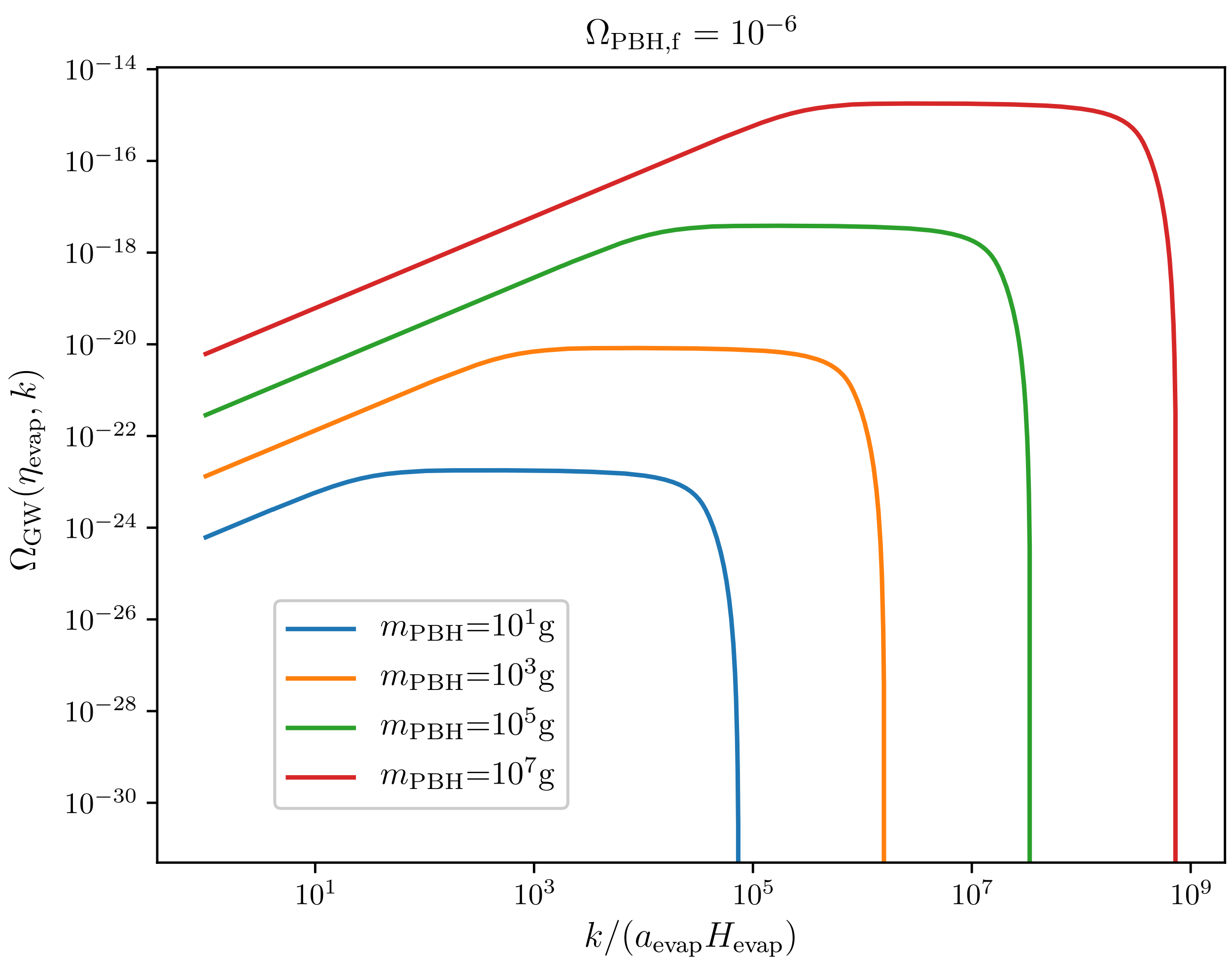
Kinetic Energy (KE) — Gradient Energy (GE)

# The Gravitational Wave Spectrum

- The spectral abundance, $\Omega_{\rm GW}(\eta, k)$ of GWs can be written as:

$$\Omega_{\rm GW}(\eta, k) \equiv \frac{1}{\rho_{\rm tot}} \frac{\mathrm{d}\rho_{\rm GW}}{\mathrm{d}\ln k} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \mathscr{P}_h(\eta, k)$$

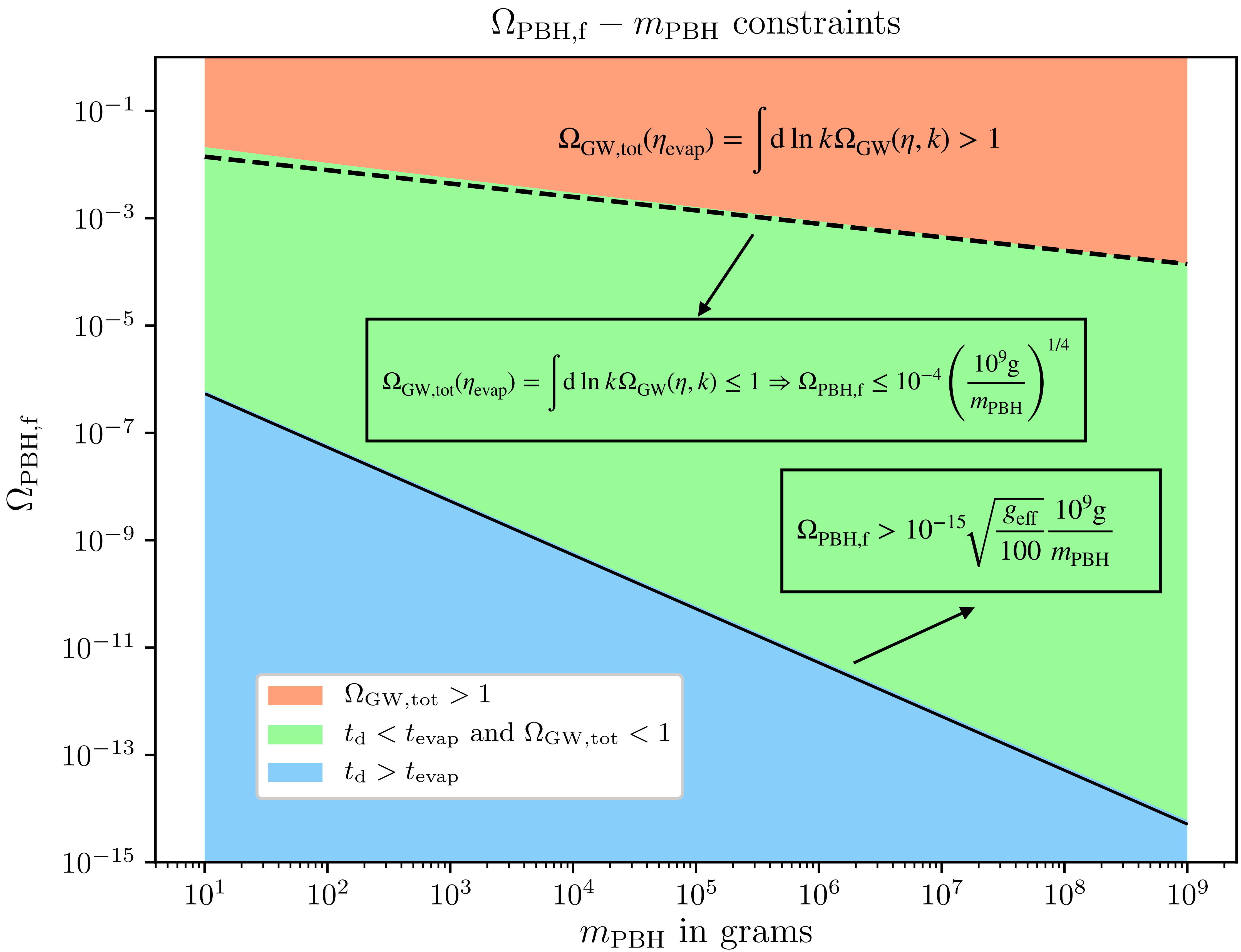
$$\text{with} \quad \mathscr{P}_h(\eta, k) \propto \int \mathrm{d}k_1 \int \mathrm{d}k_2 \left(\int f(k_1, k_2, \eta)\mathrm{d}\eta\right)^2 \mathscr{P}_\Phi(k_1)\mathscr{P}_\Phi(k_2)\,.$$




$$\Omega_{\rm GW}(\eta_{\rm evap}, k) \propto \left(\frac{m_{\rm PBH}}{10^9\mathrm{g}}\right)^{4/3} \Omega_{\rm PBH,f}^{16/3} \times \begin{cases} \frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d} \\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d} \end{cases}$$

$$\boxed{\Omega_{\rm GW,tot}(\eta_{\rm evap}) \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 10^{-4}\left(\frac{10^9\mathrm{g}}{m_{\rm PBH}}\right)^{1/4}}$$

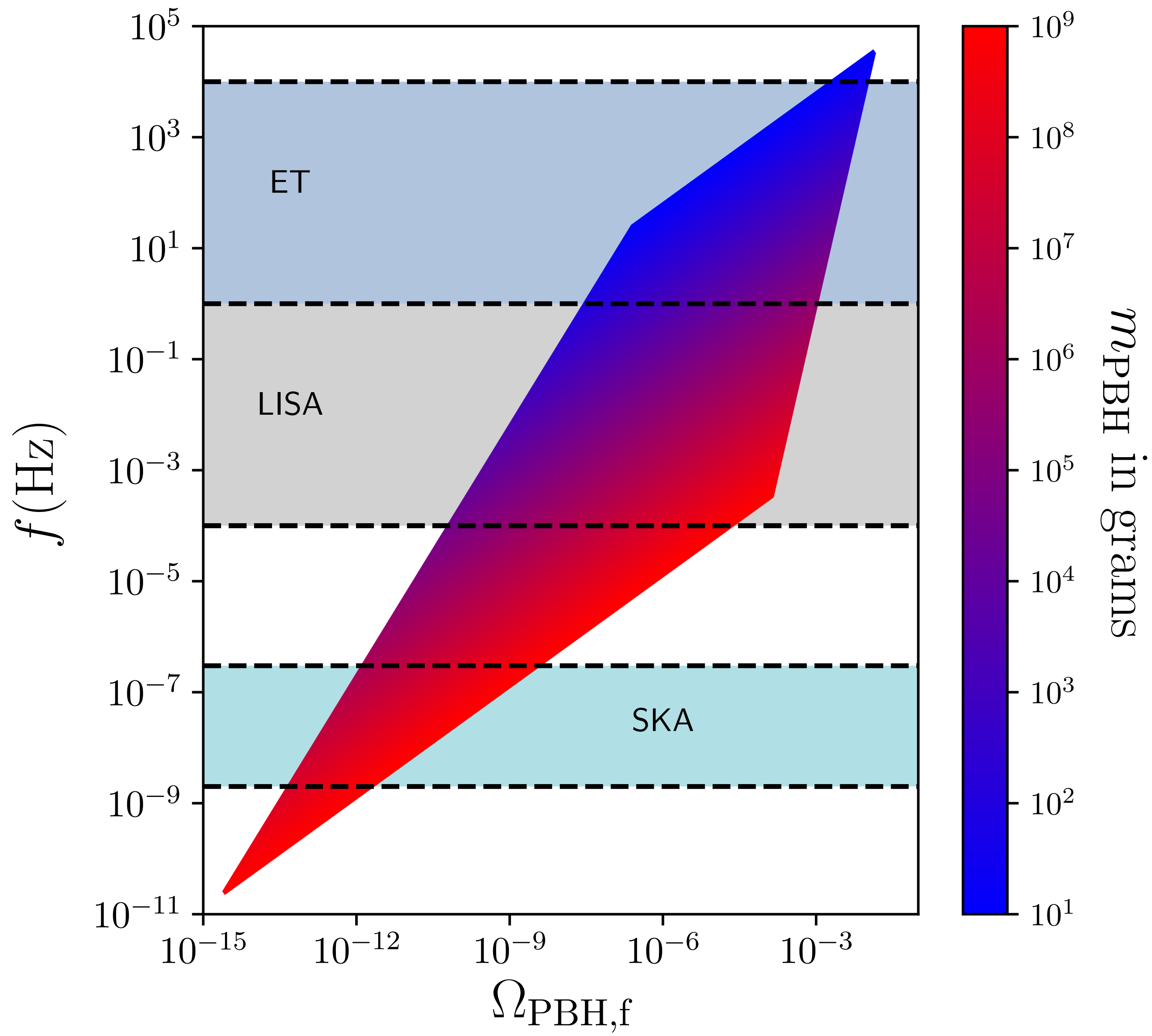
[Papanikolaou et al. - 2020]




$$\rho_{\rm f}^{1/4} < \rho_{\rm inf}^{1/4} < 10^{16}{\rm GeV} \Rightarrow m_{\rm PBH} > 10{\rm g}$$

$$\rho_{\rm evap}^{1/4} > \rho_{\rm BBN}^{1/4} \simeq 1{\rm MeV} \Rightarrow m_{\rm PBH} < 10^{9}{\rm g}$$

# GW Frequency




[Papanikolaou et al. - 2020]

- **GWs induced by a dominating gas of PBHs might still be detectable** in the future with gravitational-waves experiments.