The effect of peculiar velocities on the deceleration parameters

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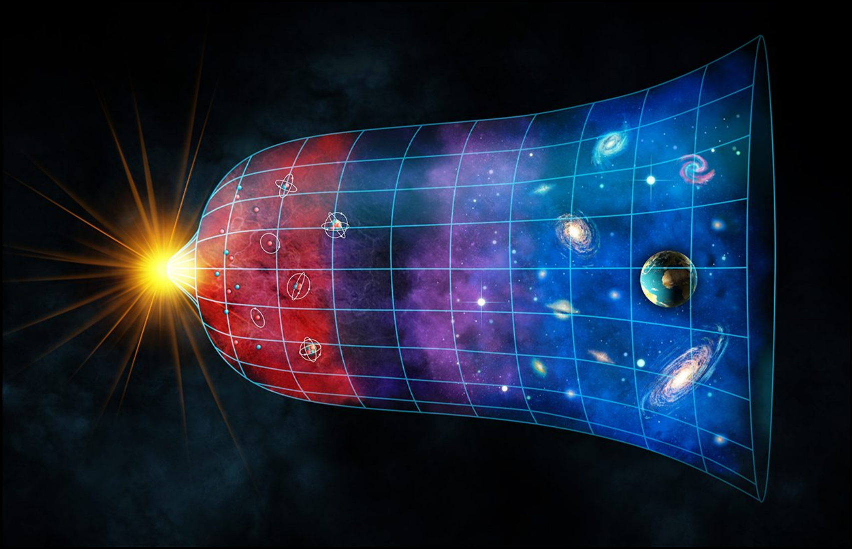
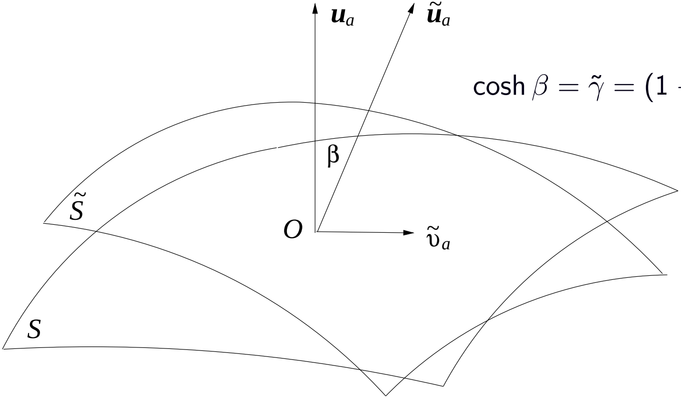


The tilted cosmological model



$\cosh\beta = \tilde{\gamma} = (1 - \tilde{v}^2)^{-1/2}$

Two families of observers: u^a represent observers following the Hubble flow and $\tilde{u}^a$ represent real observers.

The dynamics of a congruence of observers

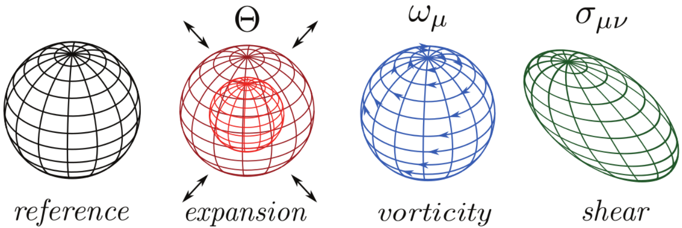
Given a congruence of observers with four-velocity u^a, we can define:

$$\nabla_b u_a = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab} - A_a u_b \, ,$$

where Θ, σ_{ab} and ω_{ab} are the *expansion*, *shear* and *vorticity*, respectively. A^a is the congruence's four-acceleration and

$$h_{ab} = g_{ab} + u_a u_b$$

is the projection operator.



Rezzolla & Zanotti (2013)

The dynamics of a congruence of observers

Clearly, a different congruence following the flow lines of $\tilde{u}^a$ will measure different parameters:

$$\nabla_b \tilde{u}_a = \frac{1}{3}\tilde{\Theta}\tilde{h}_{ab} + \tilde{\sigma}_{ab} + \tilde{\omega}_{ab} - \tilde{A}_a \tilde{u}_b \,. \tag{1}$$

Rewriting $\tilde{u}_a$ in terms of u_a:

$$\tilde{u}_a = \tilde{\gamma}(u_a + \tilde{v}_a) \approx u_a + \tilde{v}_a, \tag{2}$$

Non-relativistic ($\tilde{v}^2 \ll 1$) relative motion effects:

$$\tilde{\Theta} = \Theta + \tilde{\vartheta} \quad \longrightarrow \quad \tilde{\Theta}' = \dot{\Theta} + \tilde{\vartheta}', \quad (\tilde{\vartheta} = \tilde{\mathrm{D}}_a \tilde{v}^a)$$

$$\tilde{A}_a = A_a + v_a' + \frac{1}{3}\Theta v_a \quad \longrightarrow \quad \tilde{A}_a \neq 0 \ \text{ even if } \ A_a = 0$$

The Hubble observers

We can define the Hubble and deceleration parameters as:

$$H(t) = \frac{\Theta}{3}, \quad \text{and} \quad q(t) = -\left(1 + \frac{3\dot{\Theta}}{\Theta^2}\right),$$

In this way, the timelike deceleration parameters measured by the real (bulk-flow) observers is:

$$\tilde{q} = -\left(1 + \frac{3\tilde{\Theta}'}{\tilde{\Theta}^2}\right),$$

Relative motion effects on q

The relation between q and $\tilde{q}$ can be shown to be:

$$\tilde{q} = q - \frac{\tilde{\vartheta}'}{3H^2}.$$

Where again $\tilde{\vartheta} = \tilde{\mathrm{D}}_a \tilde{v}^a$. Using linear perturbation theory (with $p = 0$) *,

$$\tilde{\vartheta}' + 2H\tilde{\vartheta} = \frac{1}{3H}\tilde{\mathrm{D}}^2\tilde{\vartheta} - \frac{1}{3a^2}\left(\frac{\tilde{\Delta}'}{H} + \frac{\tilde{\mathcal{Z}}}{H}\right).$$

*Ellis, et.al., Relativistic Cosmology

Applying standard scalar harmonic decomposition [1] and $\Omega \simeq 1$, we see that this difference is scale-dependent:

$$\tilde{q}^{\pm}_{(n)} = q_{(n)} \pm \frac{1}{9}\left(\frac{\lambda_H}{\lambda_n}\right)^2 \frac{|\tilde{\vartheta}_{(n)}|}{H}.$$

For $\lambda_n = a/n \to \lambda_H = 1/H$, we have $\tilde{q} \to q$, while well inside the Hubble horizon the perturbation term can play an important role.

[1] $\tilde{\vartheta} = \sum_n \tilde{\vartheta}_{(n)} Q^{(n)}$, with $\tilde{\mathrm{D}}_a \tilde{\vartheta}_{(n)} = 0$ and $Q^{(n)\,\prime} = 0$ while $\tilde{\mathrm{D}}^2 Q^{(n)} = -(n/a)^2 Q^{(n)}$.

Relative motion effects on q

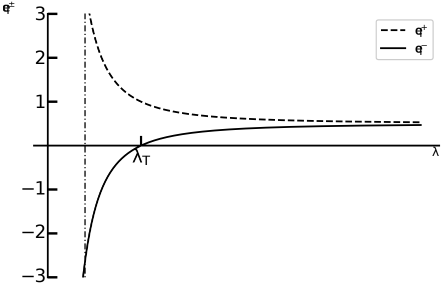
Transition length

$$\lambda_T = \sqrt{\frac{1}{9q}\,\frac{|\tilde{\vartheta}|}{H}}\,\lambda_H\,.$$

Using the above definition, we have

$$\tilde{q}^{\mp} = q\left[1 \mp \left(\frac{\lambda_T}{\lambda_n}\right)^2\right]\,,$$

where the negative/positive sign corresponds to contracting/expanding bulk flows respectively.



$$\tilde{q}^{\mp} = 0.5\left[1 \mp \left(\frac{\lambda_T}{\lambda_n}\right)^2\right]\,,$$

Note that there is always a lower threshold below which our linear analysis no longer holds. Typically, this nonlinear cutoff is set around the 100 Mpc mark.

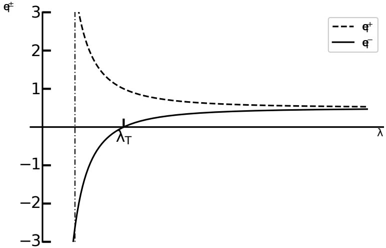
Relative motion effects on q

Unsing dimensional analysis:

$$\tilde{\vartheta} \simeq \frac{\sqrt{3}\langle\tilde{v}\rangle}{\lambda_n},$$

we have:

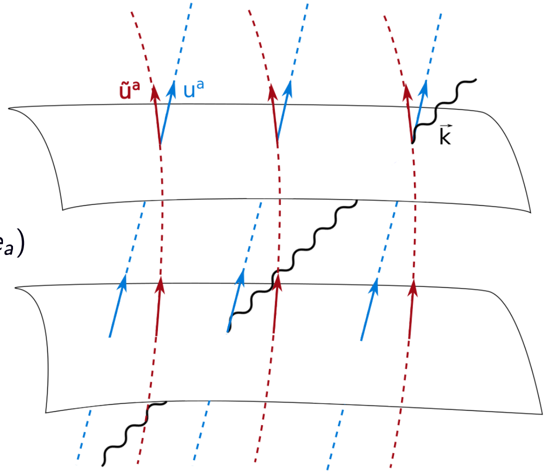
$$\tilde{q} = 0.5 + \frac{\sqrt{3}}{9}\left(\frac{\lambda_H}{\lambda_n}\right)^3 \langle\tilde{v}\rangle\,.$$



Tsagas C., Eur.Phys.J.C 81, 753 (2021)

Survey	λ	$\langle\tilde{v}\rangle$	$\tilde{q}^{(+)}$	$\tilde{q}^{(-)}$	λ_T
Nusser & Davis	280	260	+1.01	−0.01	282
Colin, et al	250	260	+1.24	−0.24	304
Scrimgeour, et al	200	240	+1.81	−0.81	323
Ma & Pan	170	290	+3.05	−2.05	384
Turnbull, et al	140	250	+4.58	−3.58	400
Feldman, et al	140	410	+7.08	−6.08	508

Taking the null point of view



$$k_a = E(u_a - e_a)$$

$$E = -k_a u^a, \quad e_a u^a = 0$$

Taking the null point of view

For a photon travelling in an FLRW spacetime:

$$\frac{dE}{d\lambda} = -E^2 H,$$

Similarly now we have:

$$\frac{\mathrm{d}E}{\mathrm{d}\lambda} = -E^2\mathfrak{H}\,, \qquad \mathfrak{H}(e) = \frac{1}{3}\Theta - A_a e^a + \sigma_{ab}e^a e^b\,.$$

which gives us some physical intuition for defining the *null expansion* and *null deceleration* parameters as:

$$\mathfrak{H} \equiv -\frac{1}{E^2}\frac{dE}{d\lambda} \qquad \text{and} \qquad \mathfrak{Q} \equiv -1 - \frac{1}{E\mathfrak{H}^2}\frac{d\mathfrak{H}}{d\lambda}\,,$$

Asta Heinesen JCAP05(2021)008

The null deceleration parameter

A multipole representation of $\mathfrak{Q}(e)$

$$\mathfrak{Q}(e) = -1 - \frac{1}{\mathfrak{H}^2(e)} \left(\overset{0}{\mathfrak{q}} + \overset{1}{\mathfrak{q}}_a e^a + \overset{2}{\mathfrak{q}}_{ab} e^a e^b + \overset{3}{\mathfrak{q}}_{abc} e^a e^b e^c + \overset{4}{\mathfrak{q}}_{abcd} e^a e^b e^c e^d \right)$$

In the following, we will be focusing on the multipole component only, therefore giving us:

$\mathfrak{Q}$ for tilted and Hubble observers

$$\tilde{\mathfrak{Q}} = \tilde{q} - \frac{1}{3H^2} \tilde{\mathrm{D}}^a \tilde{A}_a \qquad \text{and} \qquad \mathfrak{Q} = q - \frac{1}{3H^2} \mathrm{D}^a A_a$$

Relative motion effects on $\mathfrak{Q}$

Comparison between relative motion effects

$$\tilde{q} = q + \frac{1}{9}\left(\frac{\lambda_H}{\lambda_n}\right)^2 \frac{\tilde{\vartheta}}{H} \quad \text{vs.} \quad \tilde{\mathfrak{Q}} = \mathfrak{Q} + \frac{2}{9}\left(\frac{\lambda_H}{\lambda_n}\right)^2 \frac{\tilde{\vartheta}}{H}$$

Transition scales

$$\lambda_T = \sqrt{\frac{1}{9q}\frac{|\tilde{\vartheta}|}{H}}\,\lambda_H\,, \quad \text{vs.} \quad \lambda_{\mathcal{T}} = \sqrt{\frac{2}{9\mathfrak{Q}}\frac{|\tilde{\vartheta}|}{H}}\,\lambda_H\,,$$

Survey	λ	$\langle \tilde{v} \rangle$	$\tilde{q}^-$	λ_T	$\tilde{\mathfrak{Q}}^-$	$\lambda_{\mathcal{T}}$
Nusser & Davis	280	260	-0.01	282	-0.51	399
Colin, et al	250	260	-0.24	304	-0.97	429
Scrimgeour, et al	200	240	-0.81	323	-2.11	457
Ma & Pan	170	290	-2.05	384	-4.60	543

Conclusions

- Observers inside bulk flows can measure an apparent recent acceleration in the expansion rate only due to their peculiar motion;
- The *local* effect creates the *illusion* of a global acceleration.

For the null deceleration parameter:

- *Qualitatively* the effects due to peculiar velocities persist, *quantitatively* it is enhanced in the null case;
- The transition length is increased by a factor of $\sqrt{2}$.

Thank you for your attention

arXiv: 2203.01126