DYNAMIC APERTURE AND FREQUENCY ANALYSIS IN ELENA WITH ELECTRON COOLER MAGNETIC FIELD INCLUDED

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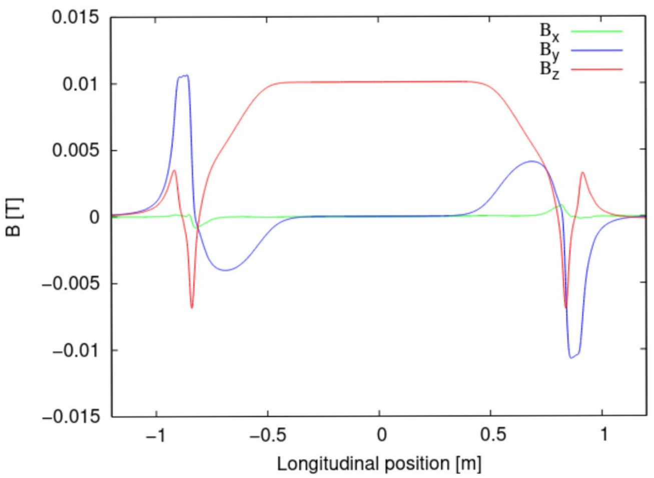
Motivation

- Long-time tracking of charged particles is an important topic for particle accelerators.
- Existing approaches often use simplifications, like kick codes.
- For arbitrary electromagnetic fields the problem was not fully solved. Ad hoc solutions and partially successful methods for short to medium term tracking exist.
- The effect of the magnetic system of the ELENA electron cooler on the beam dynamics was unknown.
- Previous attempts did not managed to perform long-time tracking in ELENA with the electron cooler magnets included.



Magnetic field of the ELENA electron cooler

Introduction

- Started to think about the problem in 2014.
- After many failed attempts with various approaches two requirements was clear:

  - Must have a physically valid representation of the fields obeying Maxwell's equations close to machine precision, otherwise there is a spurious energy drift during the tracking.

  - This implies a continuous description of the electromagnetic fields in the entire beam region without any cuts. Cutting the field even far from a magnet creates an nonphysical discontinuity.

- Existing approaches fail to satisfy one or both conditions above.
- For example : Field description on a 3D grid does not satisfy the Maxwell equation between the grid points. There is no interpolation method to do that, while being continuous between the grid cells.
- For static magnetic and electric fields following conditions must hold:

$\nabla \cdot \mathbf{B} = 0 \quad \nabla \cdot \mathbf{E} = 0$ ← Gauss's law, no sources in the beam region. Collective effects are ignored.

$\nabla \times \mathbf{B} = \mathbf{0} \quad \nabla \times \mathbf{E} = \mathbf{0}$ ← Ampere's and Faraday laws for the static case

$\nabla^2 \mathbf{B} = \mathbf{0} \quad \nabla^2 \mathbf{E} = \mathbf{0}$ ← Laplace's equation also holds. It implies the equations above.

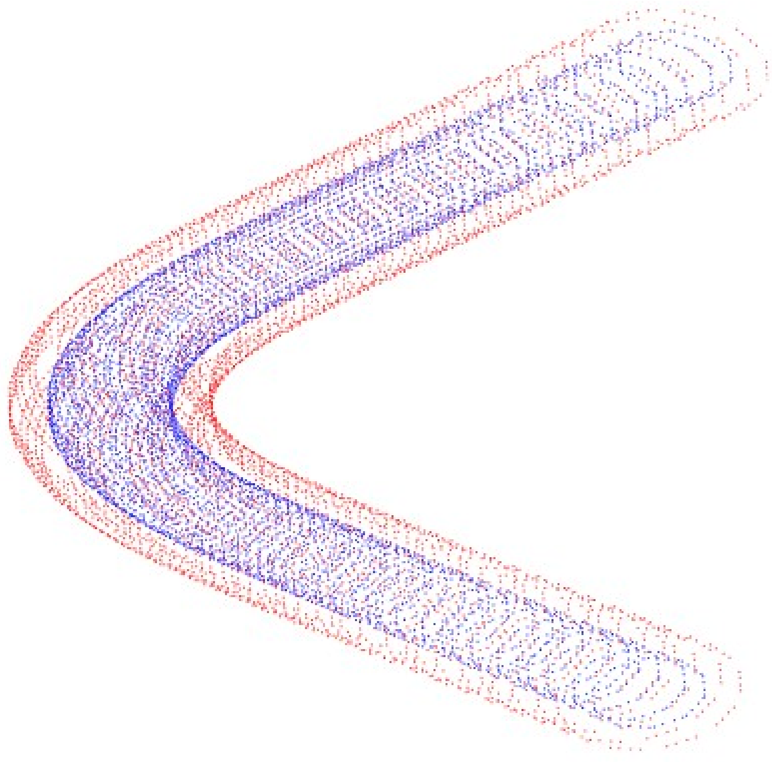
Harmonic potentials

- Potentials obeying Laplace's equation called harmonic potentials.
- Divergence Theorem: A gradient flow is volume-preserving if and only if the potential function is harmonic [1].
- In our algorithm called SIMPA, the static electromagnetic field is represented by harmonic electric and magnetic potentials.

$$\nabla^2 \mathbf{A} = 0 \qquad \nabla^2 \phi = 0$$

Surface method

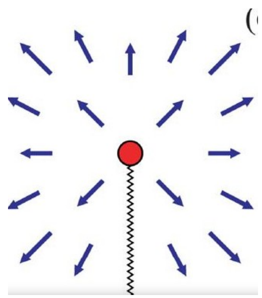
- Laplace’s equation is a linear PDE.
- It has a unique solution for a given boundary condition.
- Field values on the boundary determine the fields in a source-free region.
- Linearity implies superposition.
- The potentials are expressed analytically in terms of point sources.
- Sources are placed outside the volume at some distance above the boundary.
- Their strength is calculated by a system of linear equations.



Red points are point sources, blue dots are quadrature points on the boundary.

Point sources

- Electric fields sources are electrons.
- In early version of the algorithm published in [2,3] magnetic monopoles and current loops were used as sources for the magnetic field.
- Magnetic monopoles recently has been replaced by current point sources.
- These are two infinitesimally small wires perpendicular to each other and parallel to the boundary surface.
- Two tangential component are matched to the given magnetic field.
- No Dirac strings with the new current sources. This allows the use of the Fast Multipole Method [4] at the summation operation.



Dirac's monopole has singularity at the half line called Dirac string.

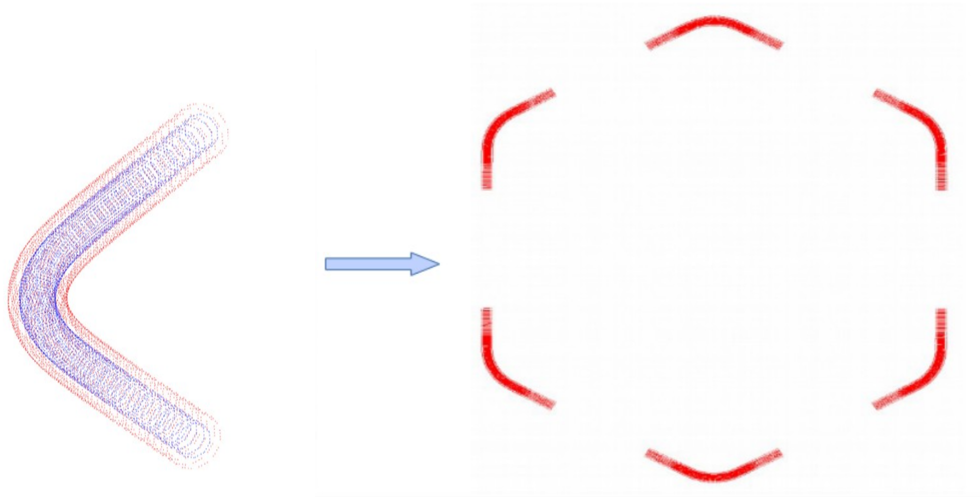
$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_\Omega \frac{\mathbf{J}(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'$$

$$\phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_\Omega \frac{\rho(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}^3\mathbf{r}'$$

$$\mathbf{B} = \nabla\times\mathbf{A}\,, \quad \mathbf{E} = -\nabla\phi$$

The surface method is too slow

- The beam line or ring can be assembled from the point sources.
- The relative precision of the reproduction of the magnetic field is typically about 1E-3.
- Can be made better by increasing the number of point sources.
- Regardless of the relative error respect to the input field, the potentials satisfy Laplace’s equation to machine precision and can be evaluated analytically anywhere inside the volume.



Now have a continuous analytic field for the machine and could track particles. Would be very slow, as the contribution of each source must be summed at each time step.

The role of the surface method is to provide a physically valid input for a faster method.
The input from CAD software does not satisfy Laplace’s equation and isn’t continuous !

Solid harmonics I.

- Orders of magnitude speed improvement can be achieved by using a local description of the potentials.
- The value of the potentials evaluated inside a sphere with radius R with the following expression.
- Information about the a potential is encoded in these pre-calculated coefficients.
- Regular solid harmonics are the canonical representation for harmonic functions inside a sphere.
- The potentials satisfy exactly the Laplace equation inside the spheres, regardless to the degree of the solid harmonics expansion.

$$f(r,\theta,\phi) = \frac{1}{R}\sum_{\ell=0}^{\ell_{max}}\sum_{m=-\ell}^{\ell} r^{\ell}\, c_{\ell m}\, \tilde{P}_{\ell}^{m}(\cos\theta)\,\Phi(\phi;m) \quad (1)$$

We denoted the ϕ dependent part as $\Phi(\phi;m)$ given by

$$\Phi(\phi;m) = \begin{cases} \sqrt{2}\sin(|m|\phi) \text{ if } m<0 \\ 1 \text{ if } m=0 \\ \sqrt{2}\cos(m\phi) \text{ if } m>0 \end{cases} . \quad (2)$$

In the expression above, $\tilde{P}_{\ell}^{m}(\cos\theta)$ is the orthonormalized associated Legendre polynomial, given as:

$$\tilde{P}_{\ell}^{m}(\cos\theta) = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-|m|)!}{(\ell+|m|)!}}\, P_{\ell}^{m}(\cos\theta). \quad (3)$$

$P_{\ell}^{m}(\cos\theta)$ is calculated without the Condon-Shortley phase.

In Eq.(1), $r = |\mathbf{r_e} - \mathbf{r_c}|/R$ is the scaled distance between the sphere center $\mathbf{r_c}$ and the evaluation point $\mathbf{r_e}$.

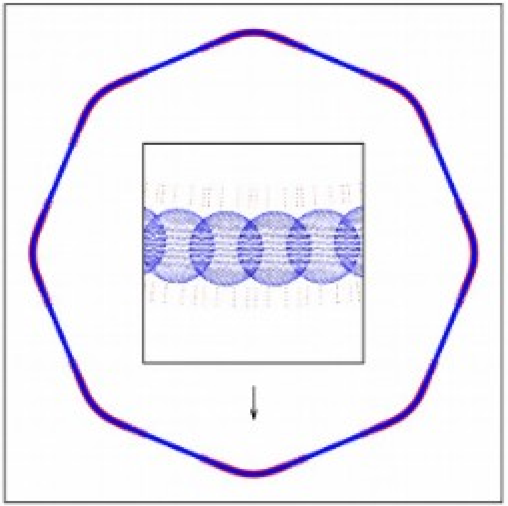
The real-valued coefficients $c_{\ell m}$ are pre-calculated by the following integral:

$$c_{\ell m} = \int_{\mathbf{S}^2} \varphi(\theta,\phi)\, \tilde{P}_{\ell}^{m}(\cos\theta)\,\Phi(\phi;m)\, d\Omega, \quad (4)$$

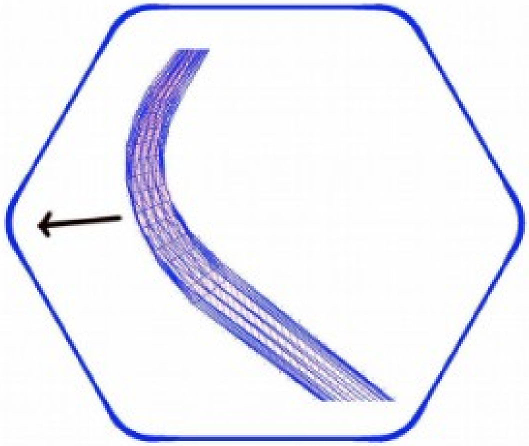
Solid harmonics II.

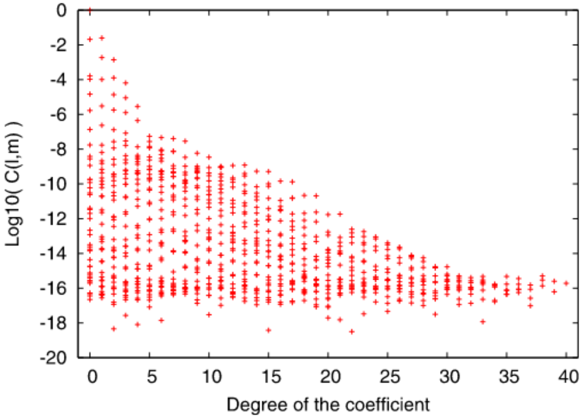
- A key characteristic of the algorithm is the description of vector and scalar potentials by solid harmonics inside a set of overlapping spheres covering the volume of interest.
- The covering can be a single row for an uniform aperture or HCP lattice of smaller spheres. The second gives faster but bigger field maps.
- HCP sphere packing is optimal.
- The discontinuity between the spheres decrease exponentially with the degree of solid harmonics expansion and can be easily kept close to machine precision.
- This gives a big advantage compared to 3D grid interpolation methods where the improvement of the discontinuity errors as the number of grid points is increased is slower than linear.

Single row cover



HCP lattice cover





Field map preparation workflow

- 1. Modeling individual magnets with CAD software or measurement data.
- 2. The strengths of the point sources are calculated for each type of magnet.
- 3. Assemble the sources according to the machine layout.
- 5. The beam region is filled with overlapping spheres, such that the entire beam region is covered without gaps using a single row or HCP lattice.
- 6. The point sources are organized into element groups. Usually, a group consists of magnets connected to the same power supply.
- 7. For each magnet group, a field map is produced by calculating the solid harmonics coefficients in each ball in the cover file.

$$c_{\ell m} = \int_{\mathbf{S}^2} \varphi(\theta, \phi)\, \tilde{P}_\ell^m(\cos\theta)\, \Phi(\phi; m)\, d\Omega.$$

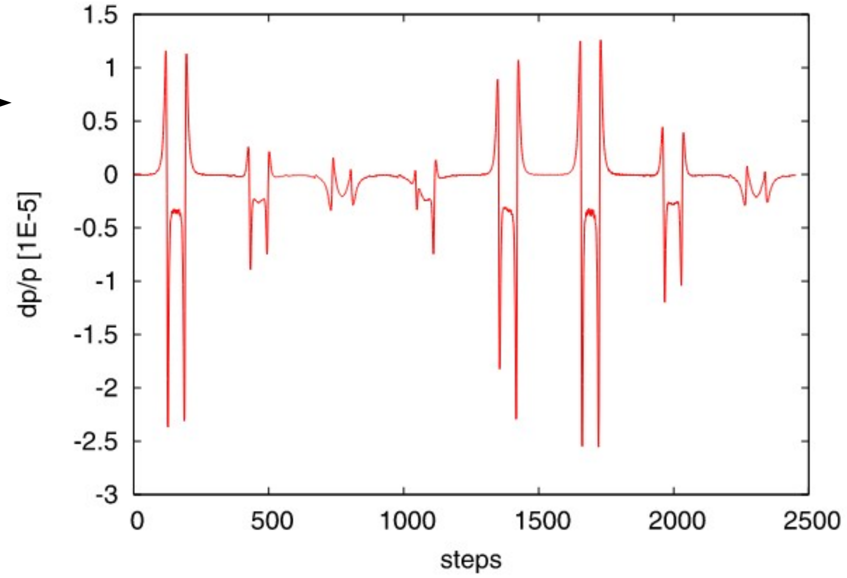
Symplectic tracking

- These field maps are scaled individually, then combined into a global field map.
- Once a global field map is prepared as described above, tracking can be performed. The motion of a charged particle calculated by the usual relativistic Hamiltonian.

$$H = \sqrt{m^2c^4 + c^2(\mathbf{p} - q\mathbf{A})^2} + qV_E$$

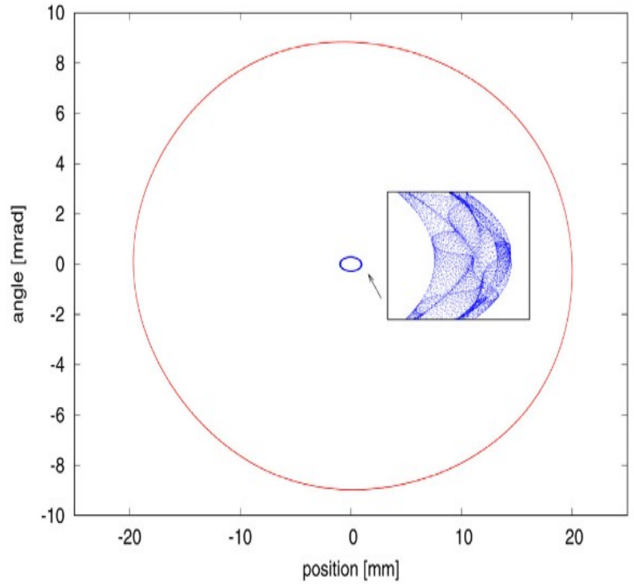
A symplectic integrator keep the errors bounded.

An explicit symplectic second-order method for general Hamiltonians, is described in [5] is implemented in SIMPA.



Long-time tracking

- No sign of any phase space volume change or energy drift in 1E+5 turns.
- Can track millions of turns.
- About 1 million field evaluations/ second/core, which is about 100 turns /second in ELENA (30 m circumference)



Frequency analysis in ELENA

- The drift in the tunes can serve as an early indicator of the long-term stability.
- A range of H and V tunes were scanned in 160 steps in both directions giving 25600 initial conditions.
- Each particle was tracked for 300 turns and the phase space variables were saved.
- The phase space data post-processed with the NAFF algorithm to get frequencies.
- Tune shifts were calculated as the frequency difference for two consecutive time intervals.
- The diffusion coefficient D was calculated:

$$D = log_{10}(\sqrt{\Delta Q_h^2 + \Delta Q_v^2})$$

Frequency analysis results

- The electron cooler introduced non-negligible magnetic perturbations, strengthening many resonance lines.

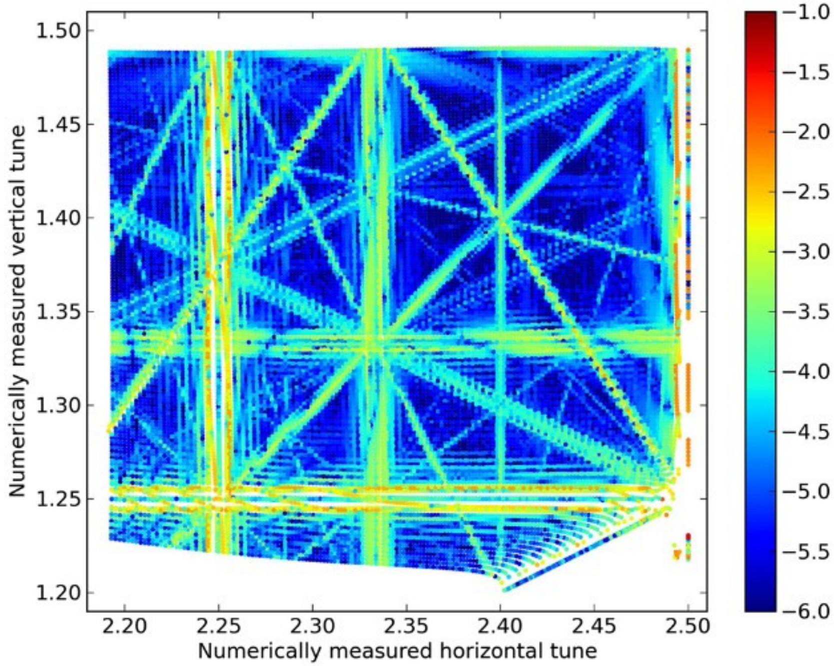


FIG. 5. *D* plotted as colors against the numerically measured tunes for the bare ELENA machine, consisting of only the main bendings and the three quadrupole families.

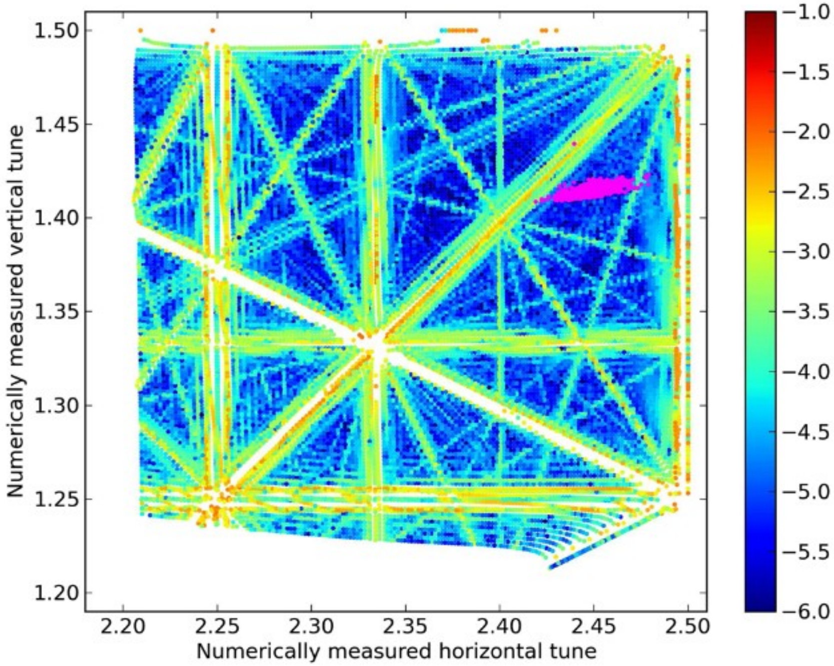
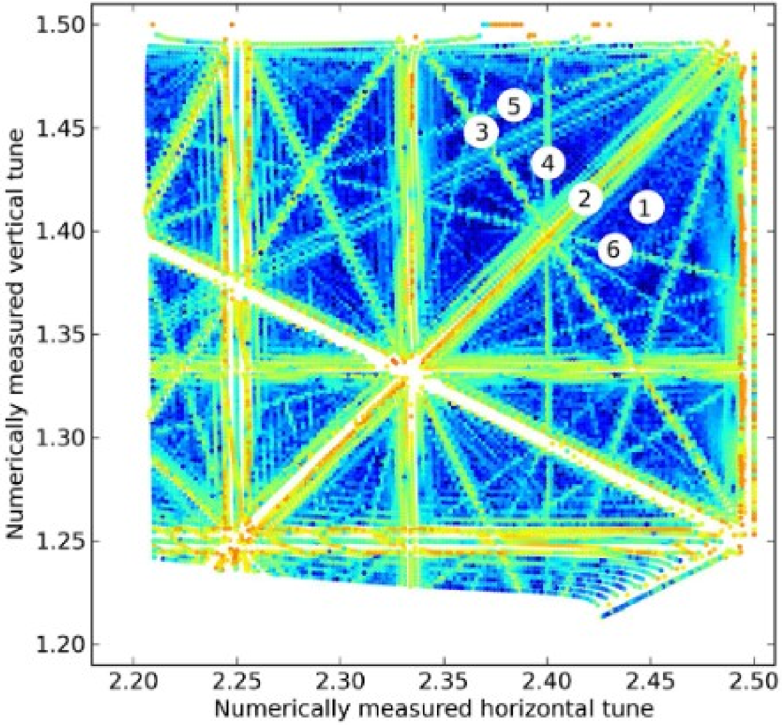


FIG. 7. *D* plotted as colors against the numerically measured tunes for the ELENA machine with electron cooler. The magenta dots in this plot correspond to the tunes of particles which survived 10^4 turns in a bunch filling the acceptance, without space charge included in the model.

Dynamic aperture results

- The dynamic aperture of six working points has been evaluated to compare resonance conditions with a non-resonant case. The selected set of tunes is indicated on the figure.
- All the results were calculated with 10000 turns, 20 polar steps, and 20 radial steps.



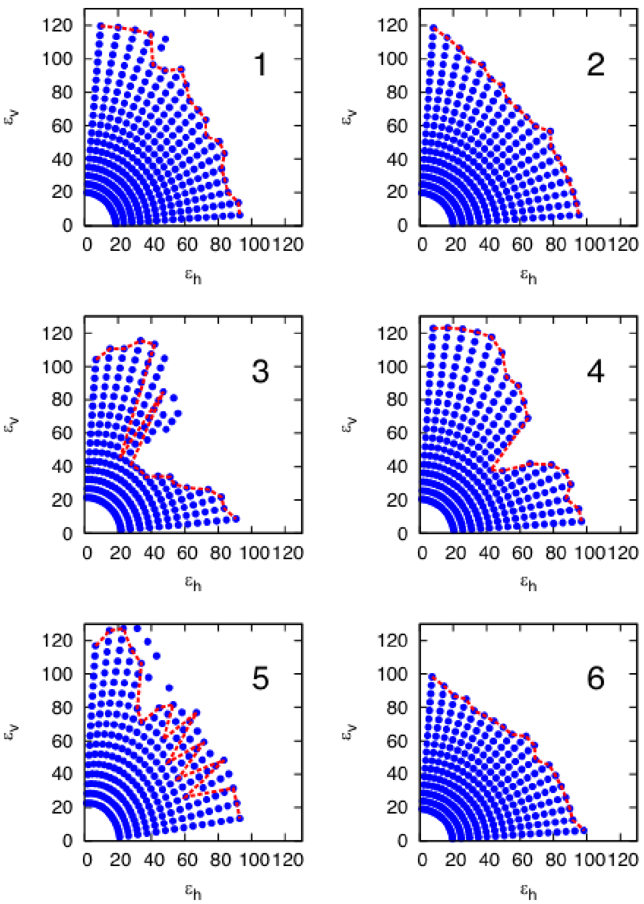


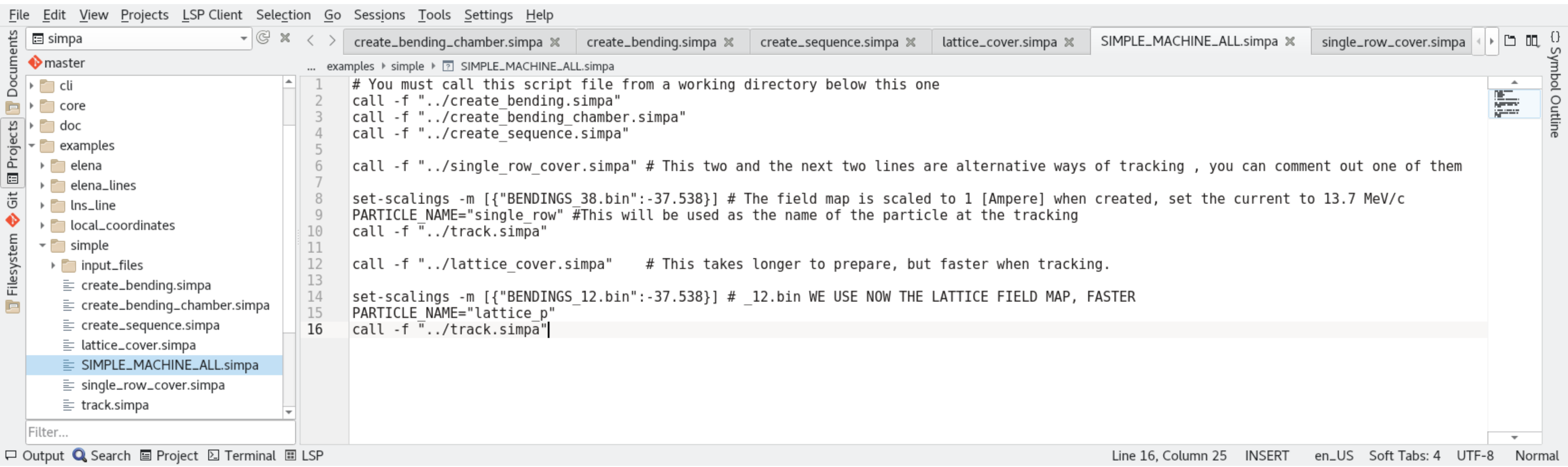
FIG. 9. Initial emittances plotted in units of π mm mrad for particle with $dp/p = 0$. The horizontal axis corresponds to the horizontal initial emittance and the vertical axis to the initial vertical emittance. Only those initial conditions are plotted which survived $N = 10^4$ turns. The numbers in the upper right corners correspond to the numbering in Fig. 8. The lines indicate the last connected initial emittances for each angle α_k.

The SIMPA tracking tool

- At the time of writing papers [2,3] the code was rather experimental.
- Since then it has been turned into a tracking tool.
- Its mostly JAVA code with some external native library written in Fortran (FMM3D, LAPACK)
- Runs under LINUX.
- It is in the development phase, but its already usable.
- The first stable release “1.0” is expected around the end of 2023.
- It is an open source code with the rather permissive MIT license.
- There is a website for the project with the relevant links to the code, documentation (partial at the moment) and examples..
- https://simpa-project.web.cern.ch/

The SIMPA CLI interface

- SIMPA has a command line interface (CLI)
- Input and outputs are mostly text files. Easy to display, debug or process.
- It has also a Java API. No Python interface yet, but any external programs can be called from the CLI, including Python scripts.
- Most of the tasks can be done from the CLI with SIMPA commands. These are usually organized into text command files, similar to MAD-X.
- You can find example scripts in GITLAB in the simpa/examples directory.



References

- [1] John Lee, Introduction to Smooth Manifolds, Volume 218 of Graduate Texts in Mathematics, p. 424
- [2] L. Bojtár. Efficient evaluation of arbitrary static electromagnetic fields with applications for symplectic particle tracking. Nuclear Instruments and Methods A, 948:162841, 2019.
- [3] L. Bojtár. Frequency analysis and dynamic aperture studies in a low energy antiproton ring with realistic 3d magnetic fields. Phys. Rev. Accel. Beams, 23:104002, Oct 2020.
- [4] Rokhlin, Vladimir (1985). "Rapid Solution of Integral Equations of Classic Potential Theory." J. Computational Physics Vol. 60, pp. 187–207.
- [5] M. Tao, Explicit symplectic approximation of nonseparable hamiltonians, Phys. Rev. E 94 (2016) 043303.
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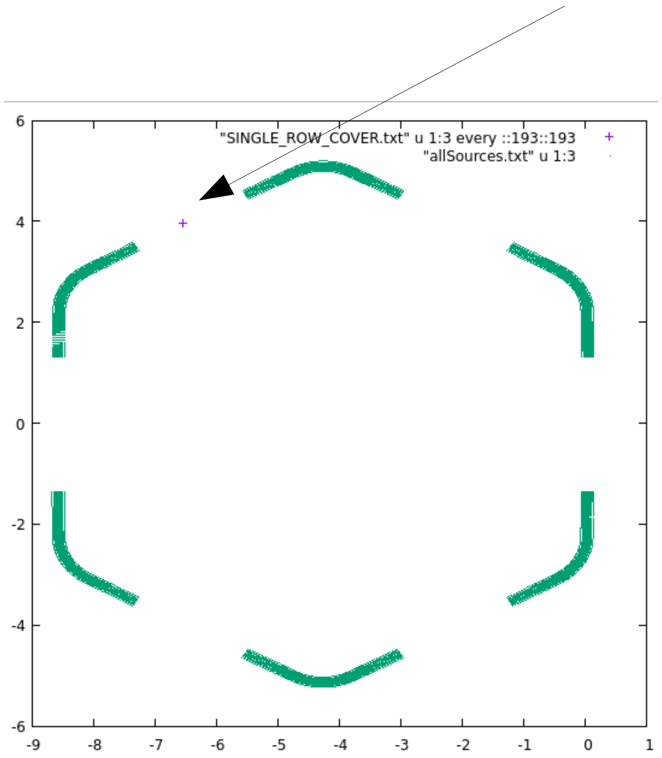
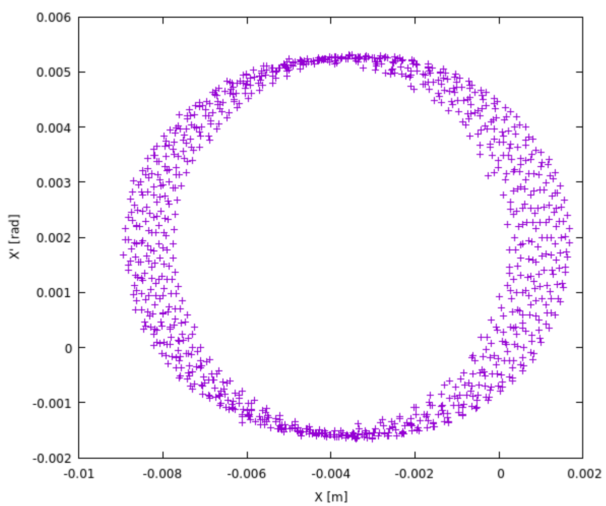
The importance of field continuity

- The quality of the field map is crucial. This includes the continuity of the field.
- The test below demonstrate the effect of discontinuity of the field map by setting the field of one ball out of 574 in a drift space to zero.



With one ball set to zero the energy drift is visible in 1000 turns



Original
1000 turns