STOCHASTIC COOLING OF ELECTRONS AND POSITRONS WITH EUV LIGHT

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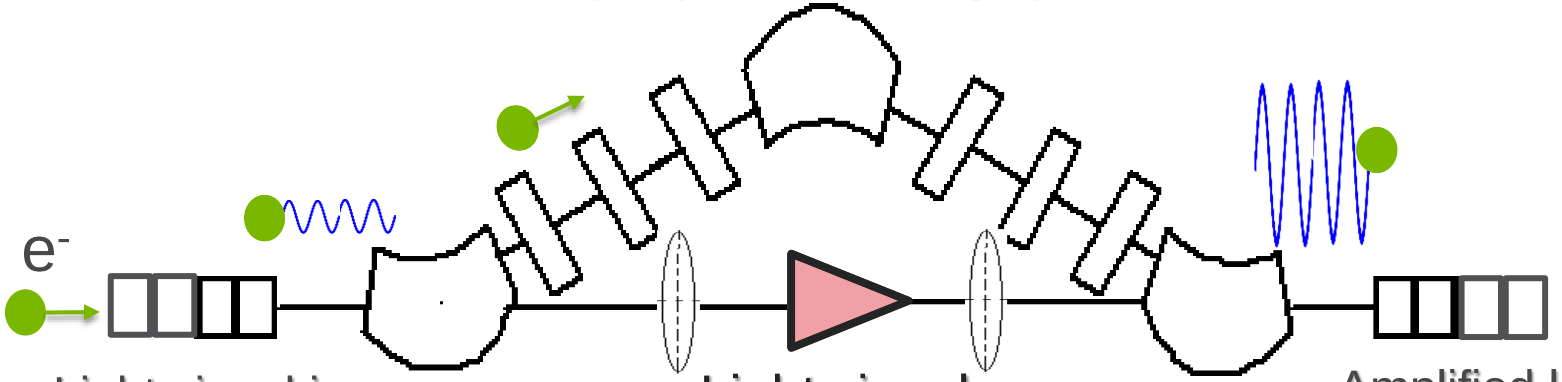
OSC in traditional approach



Light signal is produced in undulator

Light signal amplifier

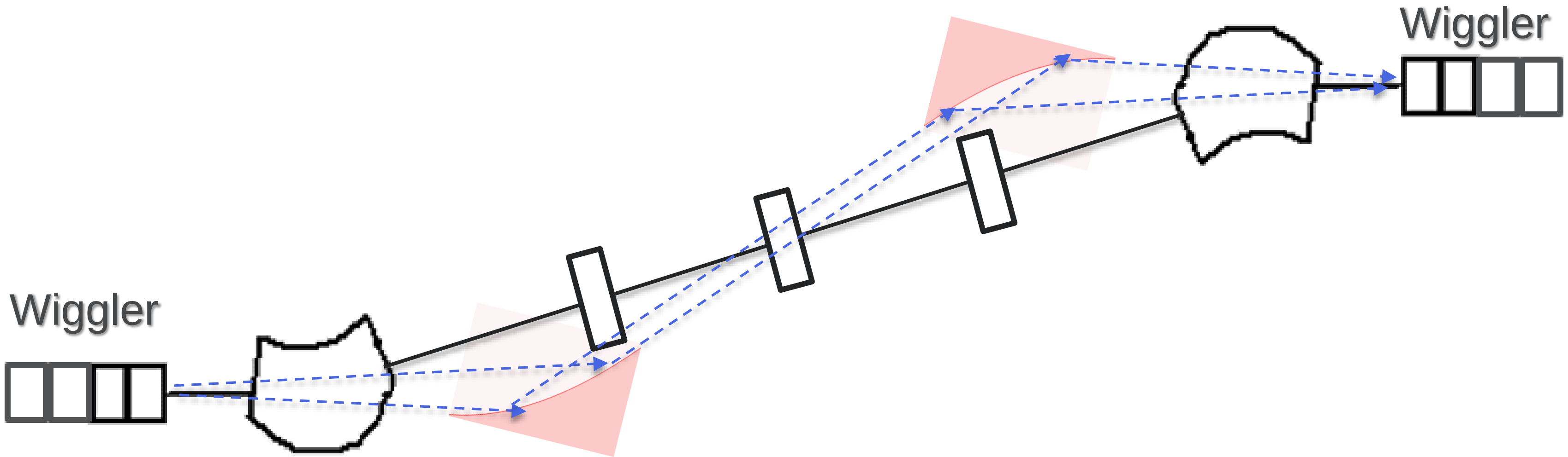
Amplified light signal is used in undulator to correct the offset

Limitations

- Amplifiers are available only for several IR wavelengths
- Amplifier and refractive lenses limit bandwidth of the system

1) A. Mikhailichenko, M. Zolotorev, “Optical Stochastic Cooling”, PRL, 71, 1993.
2) M. Zolotorev, A. Zholents, “Transient-time method of optical stochastic cooling”, PRE, V.50, 1994
3) J. Jarvis, V. Lebedev, et al., “Experimental demonstration of optical stochastic cooling”, Nature, 2022.

New approach: cooling with EUV light

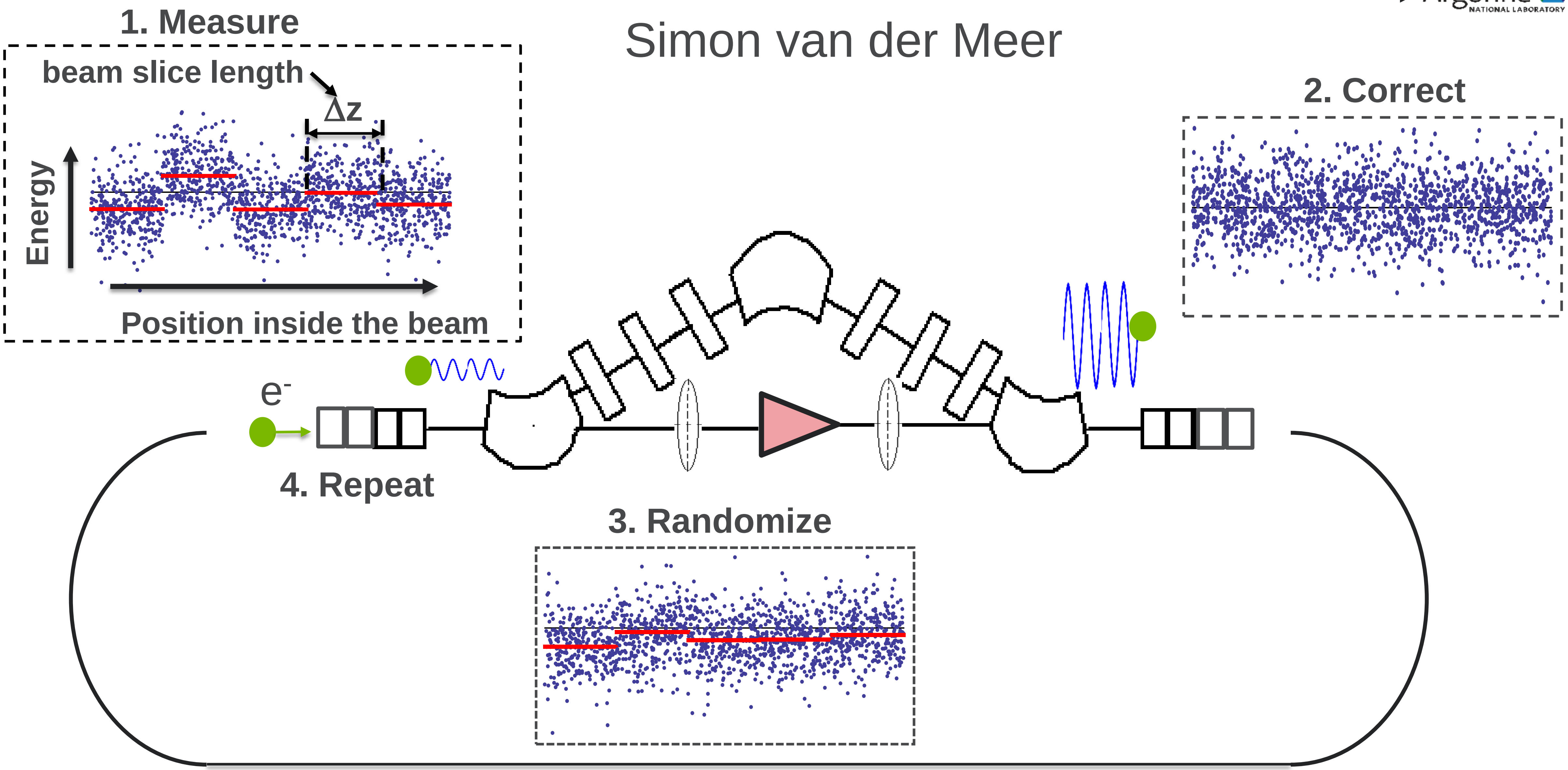


- No amplifier
- Reflective optics
- 100% relative bandwidth
- Use multiple light sources within one setup

Use light in extreme ultraviolet part of spectrum; bandwidth Δf = 7.5 PHz

Basics of stochastic cooling: measure, correct, randomize

Simon van der Meer



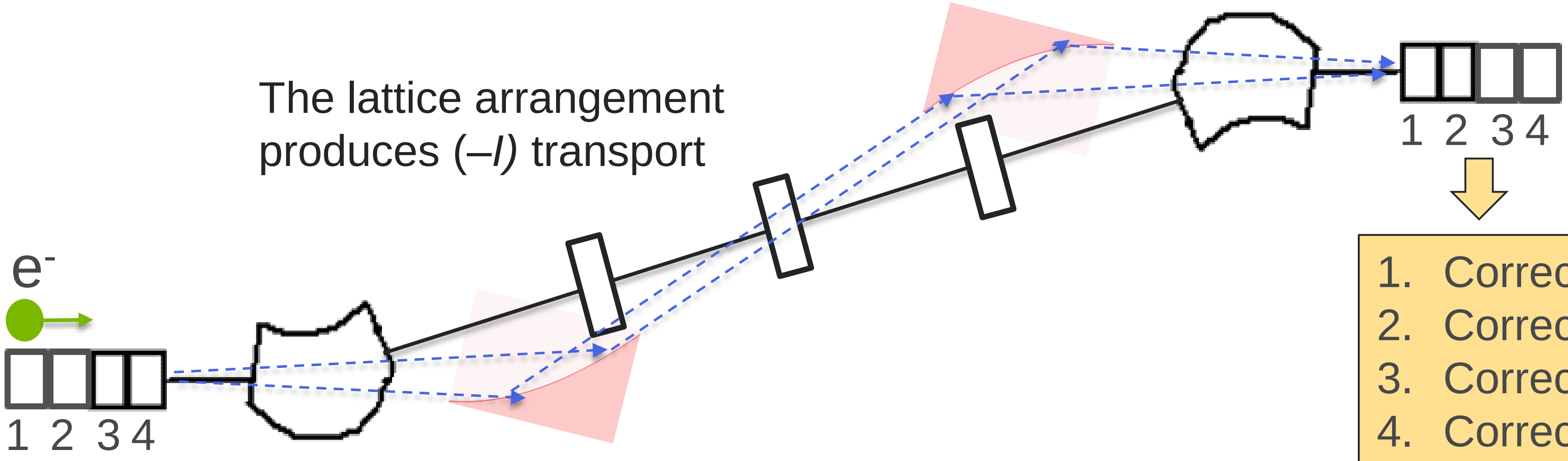
1. Measure

2. Correct

3. Randomize

4. Repeat

New approach: delayed correction



The lattice arrangement produces (–*I*) transport

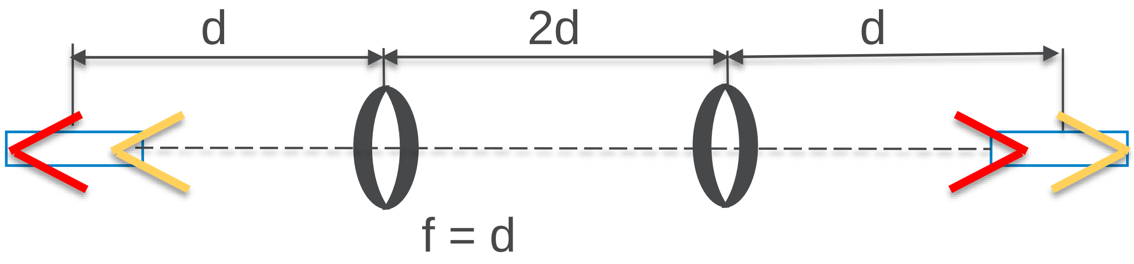
1. Measure, randomize
2. Measure, randomize
3. Measure, randomize
4. Measure, randomize

1. Correct
2. Correct
3. Correct
4. Correct

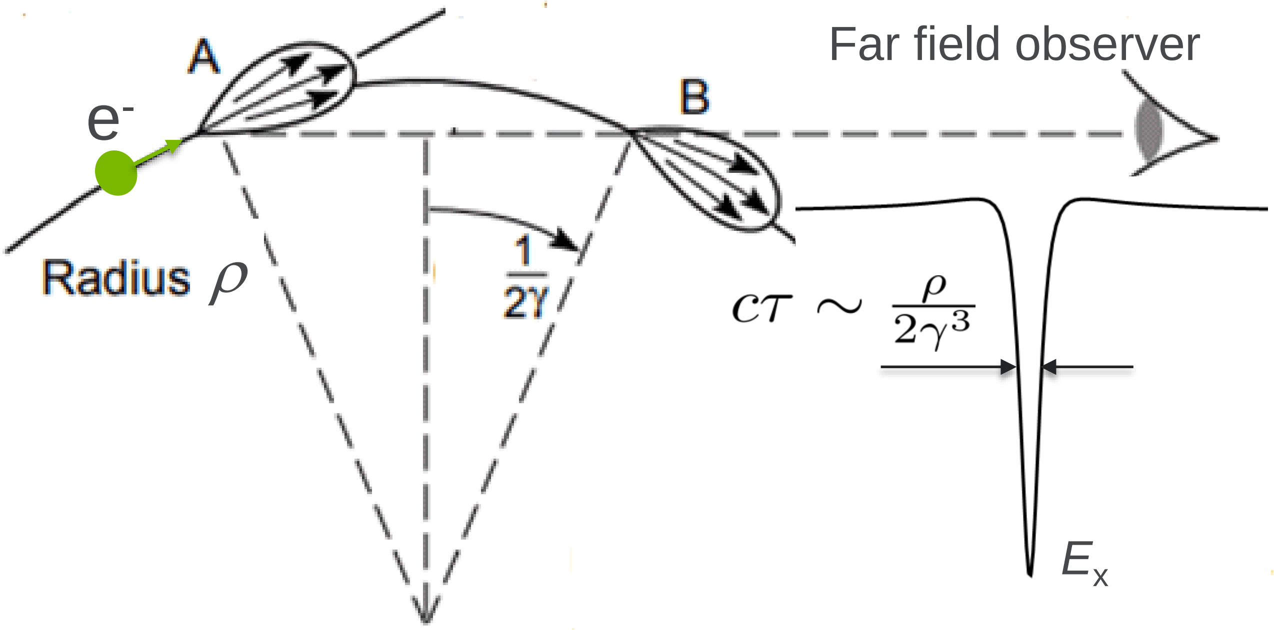
Illustration on the left shows use of four light sources within one setup

This mirror arrangement produces (–*I*) transport in the geometrical optics, i.e.

$$M = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$



BENDING MAGNET



Pulse width ~ 6 nm

Condition of randomization

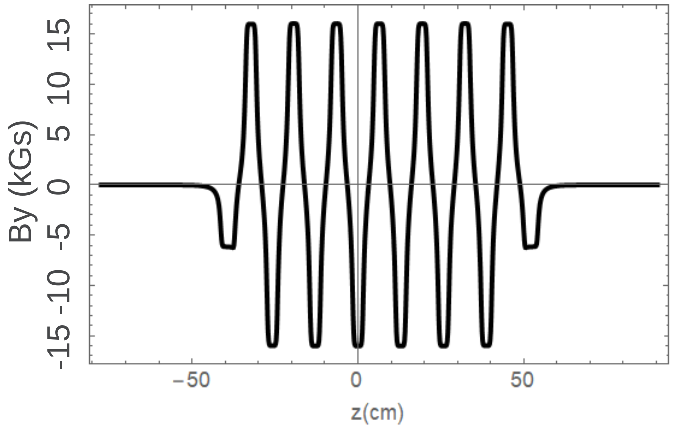
$$\frac{1}{\pi^2}\left(\gamma\phi\right)^3 \sigma_{\frac{\Delta E}{E}} \geq 1$$

Bending angle

Energy spread

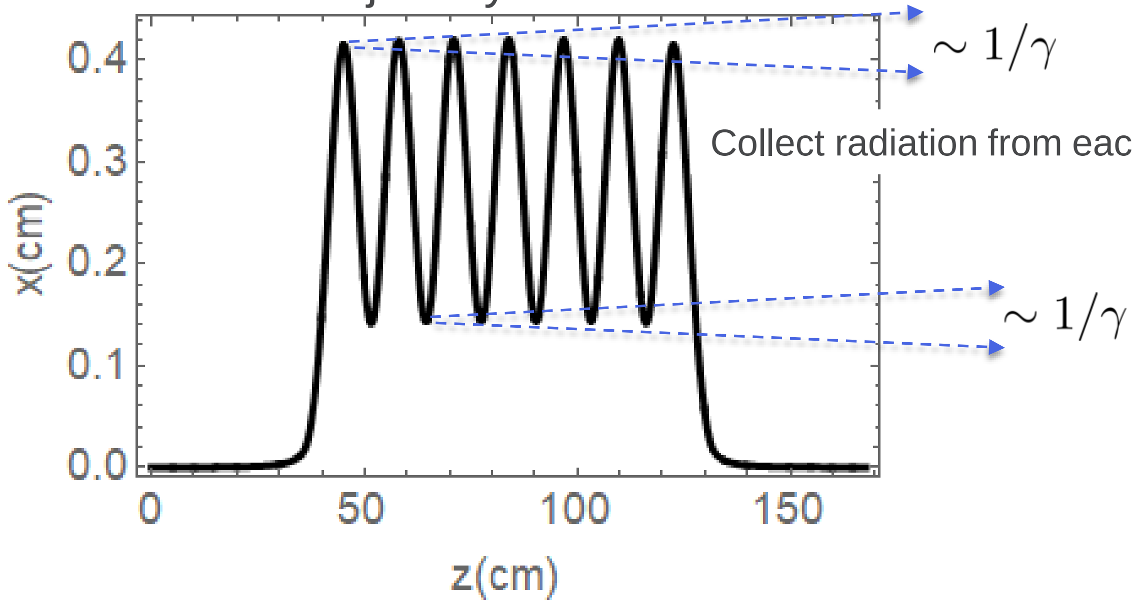
WIGGLER MAGNET

Field*) Period 12.9 cm -> 10.9 cm



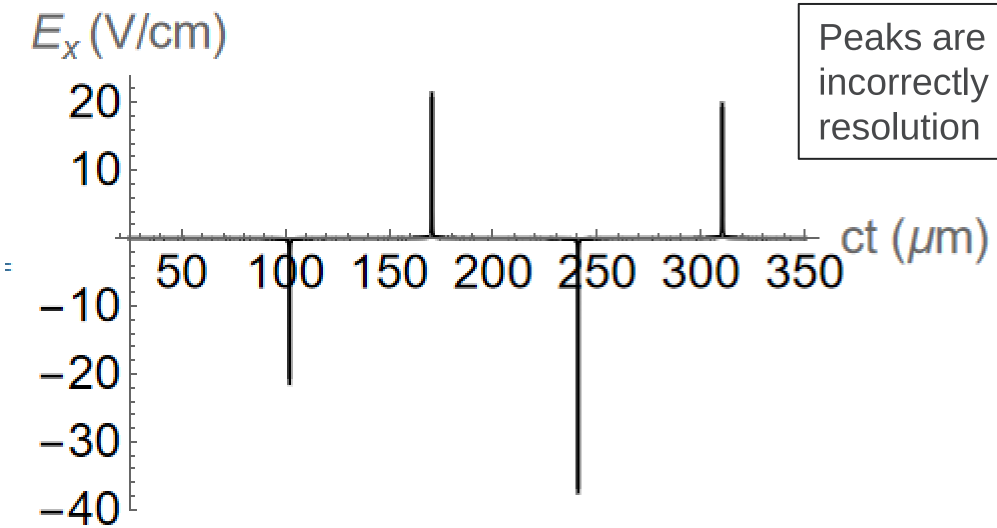
*) Designed by Maofei Qian (ANL)

Trajectory

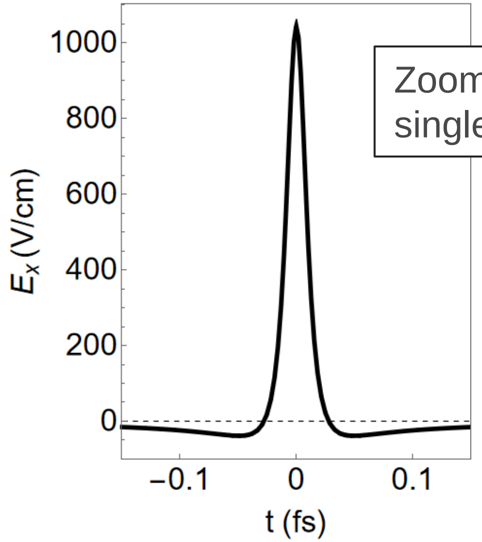


Collect radiation from each pole

Radiation field seen by an observer



Peaks are shown incorrectly due to plot resolution



Zoom on a single pulse

FIELD CALCULATION

$$E_{x_1}(t) = -\frac{e}{4\pi\epsilon_0 R_0}\frac{d^2\hat{x}}{c^2 dt^2}$$ *)

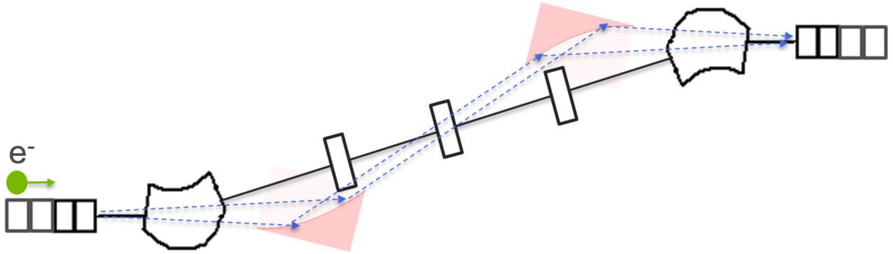
The field produced by one electron and seen by the observer at a distance R_0 >> wiggler length

$\hat{x}$ is the *apparent* trajectory of the electron

t is the time in the observer's frame

The transported field to the second wiggler

$$E_{x_2}(t) = -\frac{e}{4\pi\epsilon_0}\frac{1}{\gamma}\frac{1}{2.35\sigma_{dif}}\frac{d^2\hat{x}}{d(ct)^2}$$



$$\lambda_c = \frac{4\pi}{3}\frac{\rho}{\gamma^3}$$ critical radiation wavelength

$$\sigma_\theta = \frac{0.64}{\gamma}$$ radiation divergence at a critical radiation wavelength

$$\sigma_{dif} = \frac{\lambda_c}{4\pi\sigma_\theta}$$ rms size of focused light

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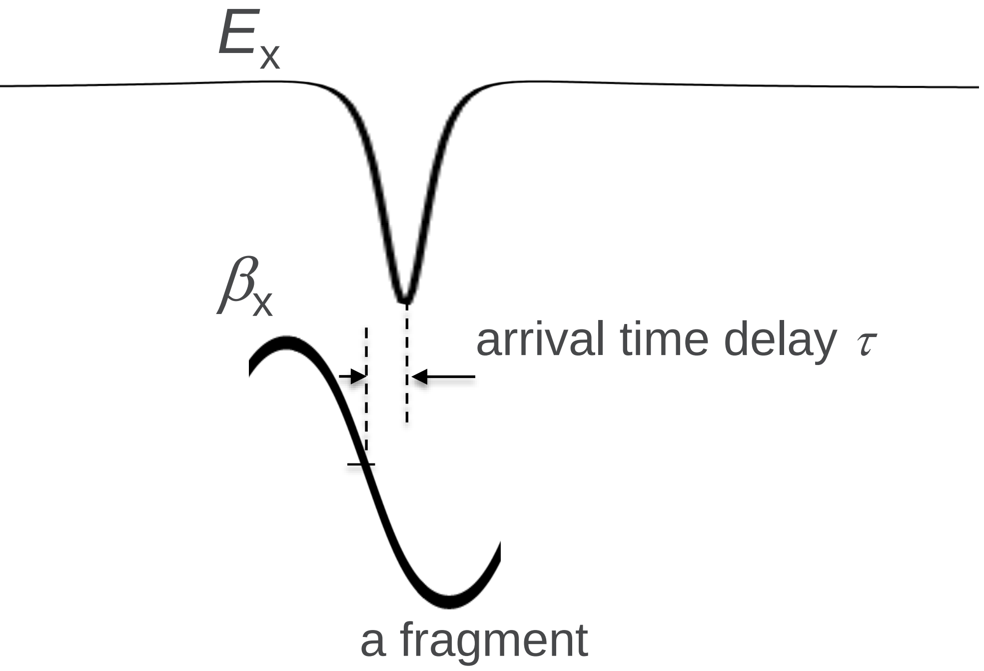
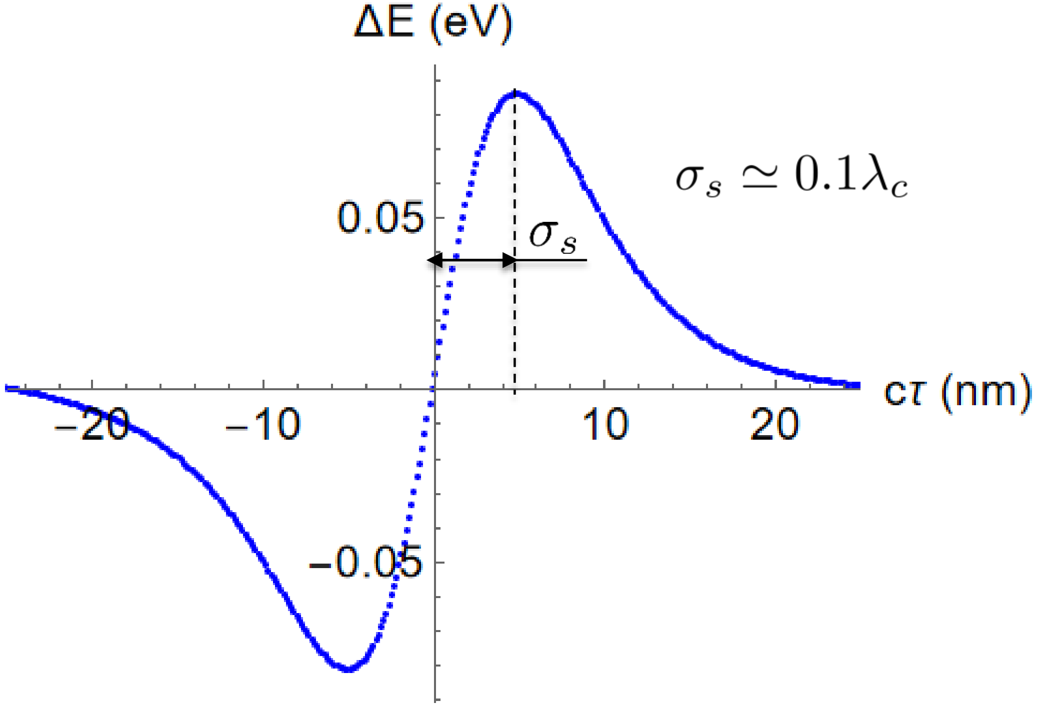
*) The Feynman Lectures on Physics (Addison-Wesley, New York, 1963), Vol. 1, Chapter 28

CORRECTION “KICK” FOR ENERGY OFFSET

$$\frac{d\gamma(\tau)}{dt} = \frac{e}{m_e c^2} E_x \cdot \beta_x$$



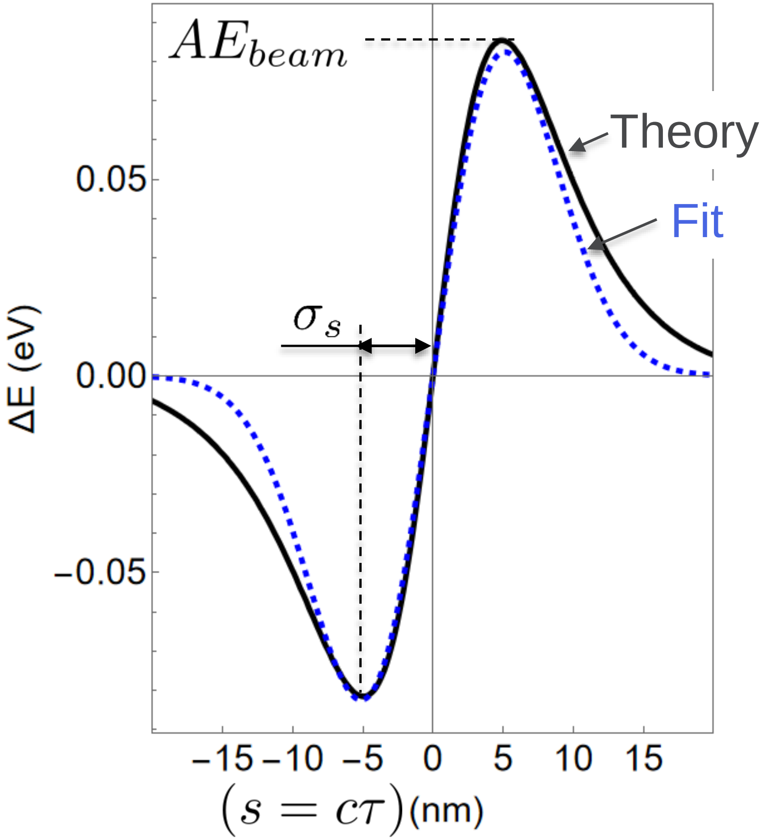


Energy gain/loss as a function of electron arrival time delay in interaction with a radiation pulse from a single pole

Bandwidth $\simeq \frac{c}{8\sigma_s}$ = 7.5 PHz

Warning!
The delay pathlength must be controlled with an unprecedented accuracy of a few nm

THEORY



Fit energy “kick” by the function

$$\frac{\delta E(s)}{E_{beam}} = A\left(e^{-\frac{(s-\sigma_s)^2}{2\sigma_s^2}} - e^{-\frac{(s+\sigma_s)^2}{2\sigma_s^2}}\right) = Af(s)$$

Consider delay that is only due to energy offset

$$s \rightarrow R_{56}\delta$$

Assume Gaussian energy distribution

$$\rho(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\frac{\delta^2}{2\sigma_\delta^2}}$$

$$\delta_{ic} = \delta_i - Af(R_{56}\delta_i)$$

$$y \rightarrow \frac{R_{56}\delta}{\sigma_s} \qquad \eta = \frac{R_{56}}{\sigma_s}$$

number of particles in the slice

$$\overline{\Delta\delta^2} = \overline{\delta_{ic}^2 - \delta_i^2} = -2A\underbrace{\frac{1}{\sqrt{2\pi}\sigma_y\eta}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma_y^2}} y f(y) dy}_{\text{cooling}} + \underbrace{A^2 N_s \overline{f(y)^2}}_{\text{heating}}$$

$$\boxed{\text{Cooling rate:} \qquad \alpha = \frac{\overline{\Delta\delta^2}}{\sigma_\delta^2}}$$

THEORY CONT'D

Find optimal A_{max} when cooling rate is at a maximum

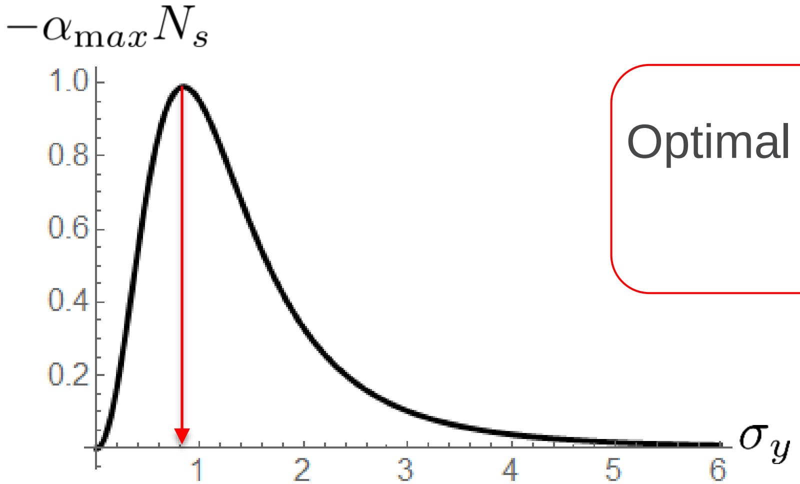
$$A_{max} = \frac{2\pi\sigma_y^2 e^{-\frac{1}{2+2\sigma_y^2}}}{N_s \eta (1+\sigma_y^2)^{3/2}}$$

$$\begin{cases} A < A_{max} & \text{insufficient cooling} \\ A > A_{max} & \text{heating dominates cooling} \end{cases}$$

Maximum decrement

$$\alpha_{max} = -\frac{4\pi\sigma_y^2 e^{-\frac{1}{1+\sigma_y^2}}}{N_s (1+\sigma_y^2)^3}$$

$$\boxed{\begin{aligned} A_{max} &= \frac{1.77\sigma_\delta}{N_s} \\ \alpha_{max} &= -\frac{1}{N_s} \end{aligned}}$$



$\sigma_y \simeq$ 0.84

Optimal R_{56} can be found from

$$\frac{R_{56}\sigma_\delta}{\sigma_s} \simeq 0.84$$

In Zolotorev, Zholents paper $\alpha_{max} = -\frac{0.18}{N_s}$

THEORY CONT’D

Cooling rate in case of “insufficient cooling” (when correction kicks are too weak)

$$\alpha = -\frac{A}{\sigma_\delta}\frac{4\sigma_y e^{-\frac{1}{2+2\sigma_y^2}}}{(1+\sigma_y^2)^{3/2}} \underset{\sigma_y = 0.84}{\simeq} -1.125\frac{A}{\sigma_\delta}$$

A straightforward interpretation.
Damping time is defined by a number of the kicks needed to zero the energy spread

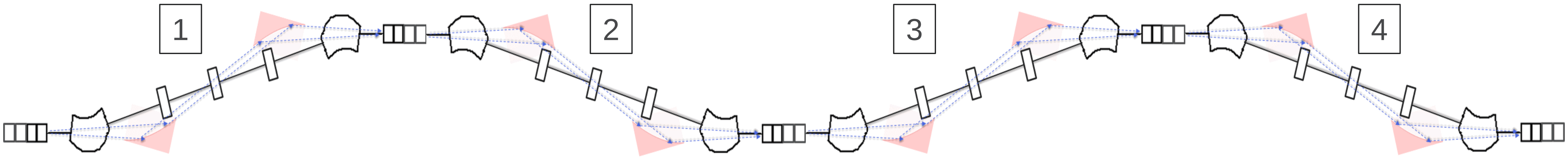
$$n_{kicks} = -\frac{\sigma_\delta}{1.125A}$$

Therefore, cooling decrement in the ring with one cooling section employing the wiggler with N_w periods is

$$\alpha_1 = -2.25N_w\frac{A}{\sigma_\delta}f_0$$

f_0 is the revolution frequency

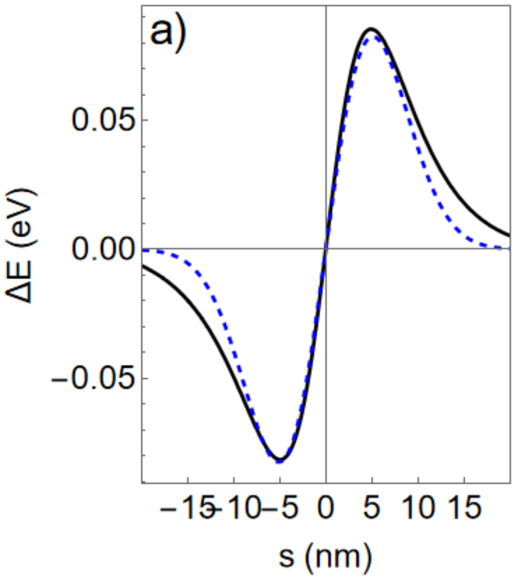
CASCADE-AMPLIFIED EUV STOCHASTIC COOLING



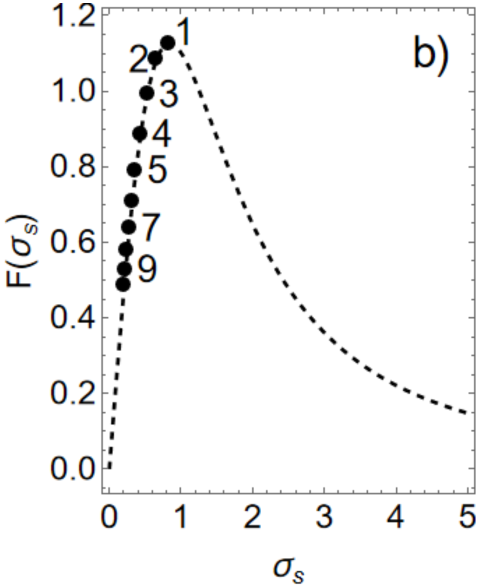
When N_c cooling sections is used back-to-back, the cooling rate is growing NOT as N_c, but as

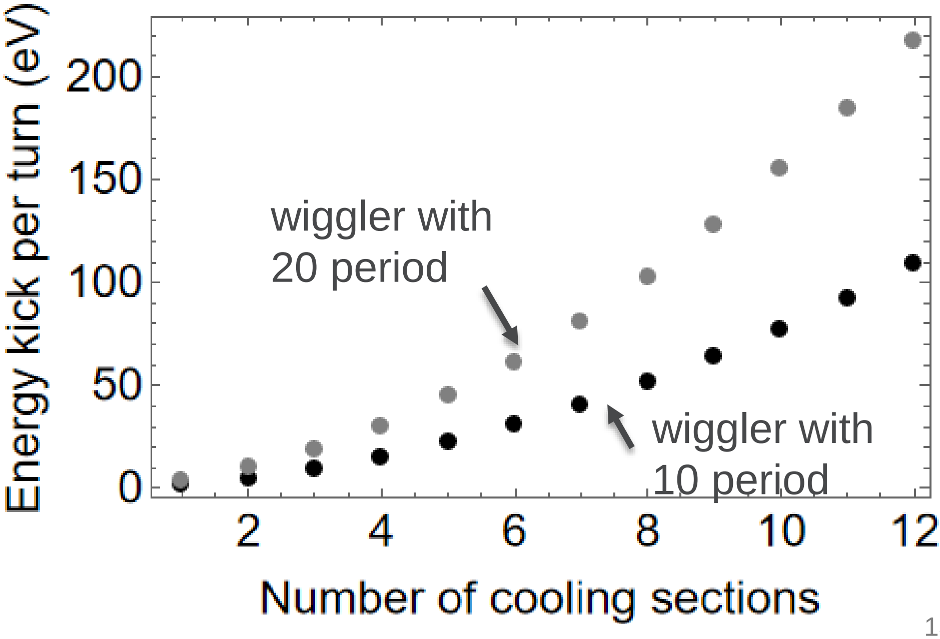
$$\alpha_{cascade} = -2N_w \frac{A}{\sigma_\delta}(0.75N_c + 0.41N_c^2)f_0$$

Energy kick from one magnet



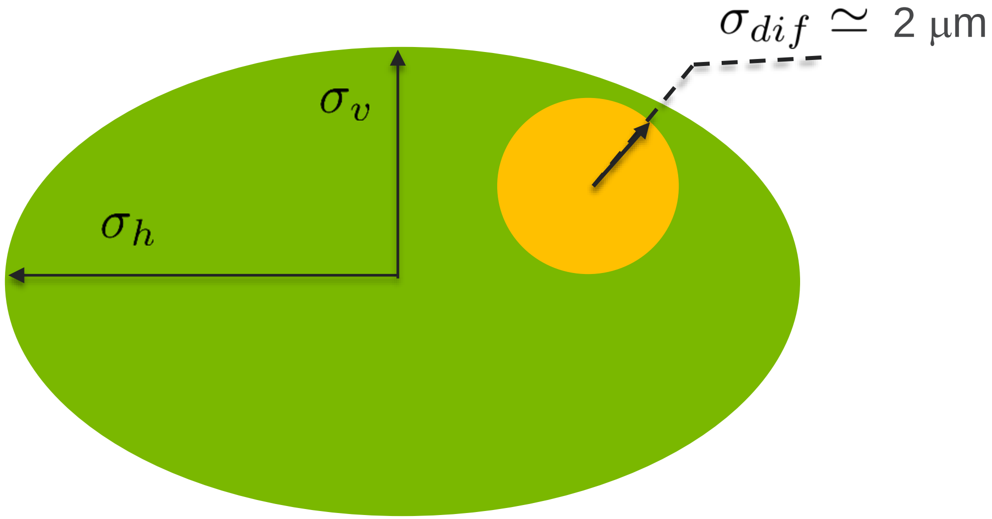
Optimal slippage





TRANSVERSE SLICING*)

Much simpler when operating at short wavelength



σ_v rms vertical beam size

σ_h rms horizontal beam size

Actual number of electrons in the slice

$$\hat{N}_s = N_s \frac{\sigma_{dif}^2}{\sigma_v \sigma_h}$$

N_s is the number of electrons in the longitudinal slice

It cannot help to reduce the damping time.
It only makes “insufficient cooling” even more “insufficient.

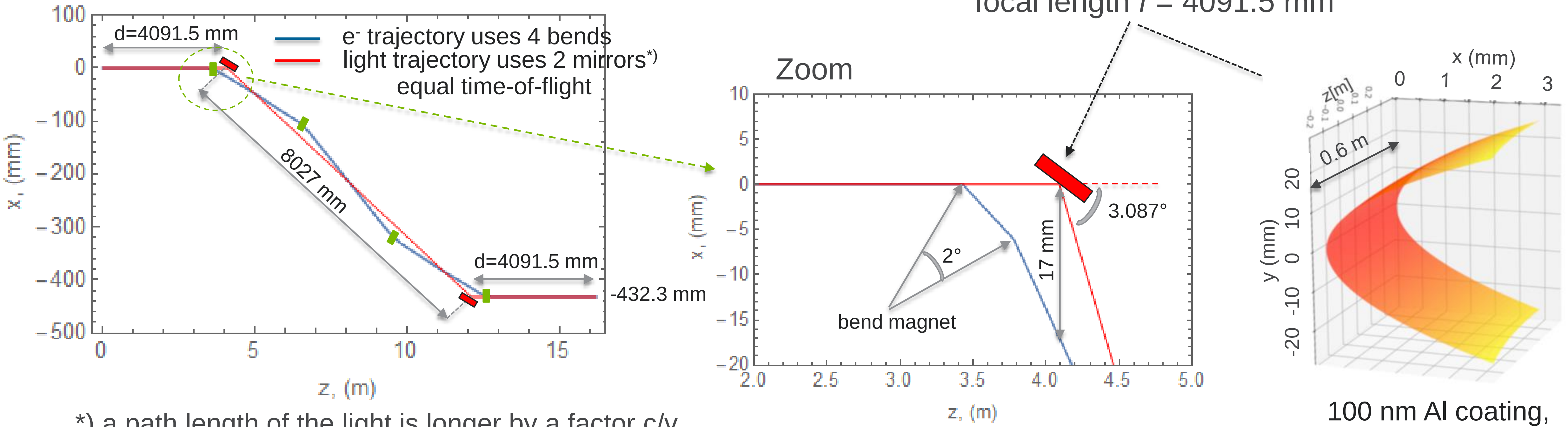
*) Zolotorev, Zholents, “Transient-time method of optical stochastic cooling”, PRE, V.50, N4, p.3087, 1994

LIGHT AND ELECTRON BEAM TRANSPORT

Image light source (and electron beam) from the upstream wiggler to the downstream wiggler

-*unit* transport matrix for e-beam (almost)

Parabolic mirror with the focal length *f* = 4091.5 mm



*) a path length of the light is longer by a factor c/v

100 nm Al coating,
92.5% reflectivity,
range 0.5-100 eV

Curtesy
L. Rebuffi and X. Shi

-*I* light ray's transport matrix (almost)

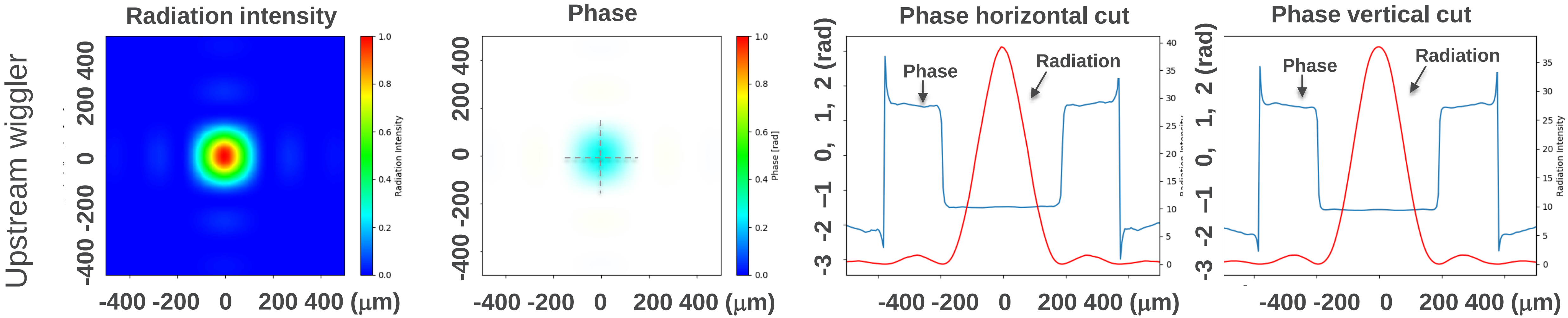
$$\begin{pmatrix} -1. & -3.55271\times 10^{-15} \\ -0.00931766 & -1. \end{pmatrix}$$

Beam energy = 147 MeV
Critical photon energy 23.7 eV
Wiggler period= 12.9 cm
Number of periods = 10

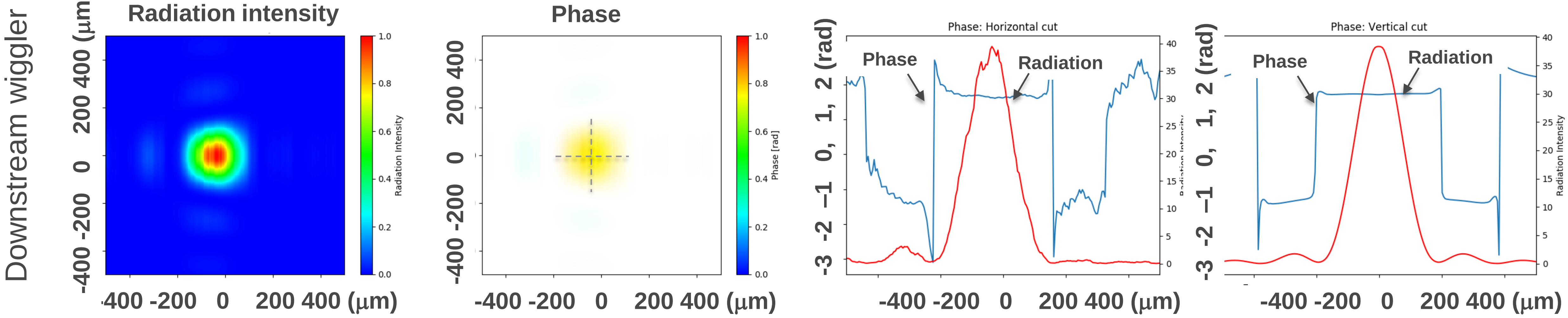
IMAGING OF A SINGLE ELECTRON RADIATION FROM THE CENTER OF THE UPSTREAM WIGGLER TO THE DOWNSTREAM WIGGLER

SRW calculations. Curtesy L. Rebuffi and X. Shi

At source (the center of the upstream wiggler)



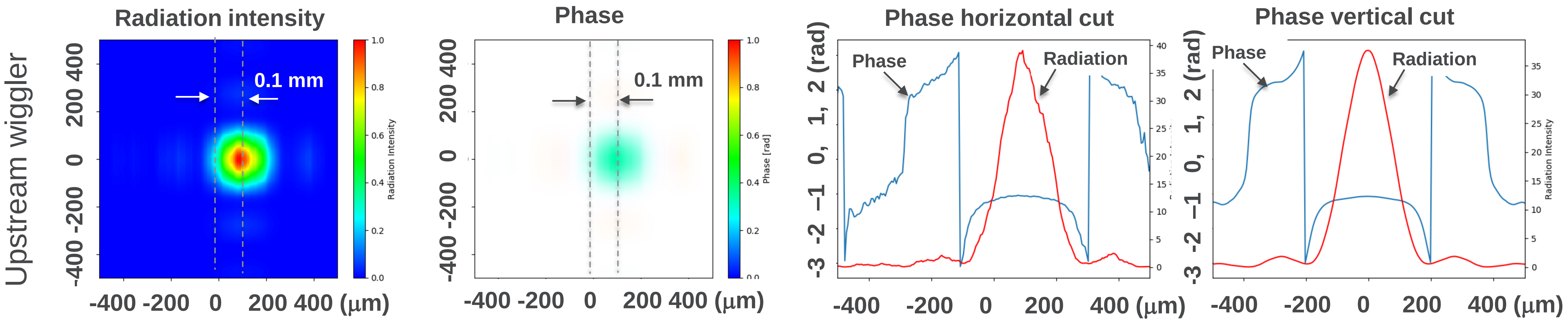
At focus (the center of the downstream wiggler)



LIGHT PROPAGATION STUDIES CONT'D

Intensity and phase at shifted longitudinally (-105 mm) and shifted vertically by 0.1 mm

At source in the upstream wiggler (- 105 mm shifted)



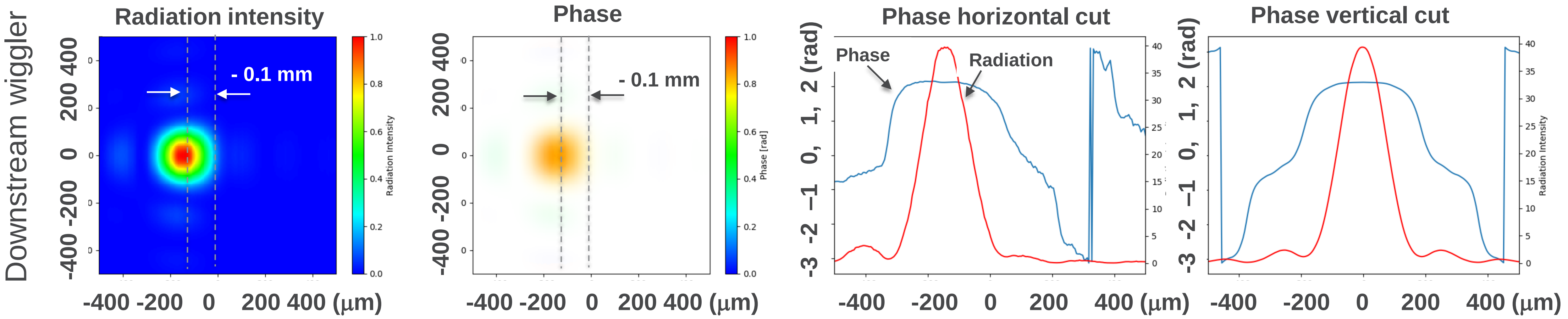
Radiation intensity

Phase

Phase horizontal cut

Phase vertical cut

At focus in the downstream wiggler (- 105 mm shifted)



Radiation intensity

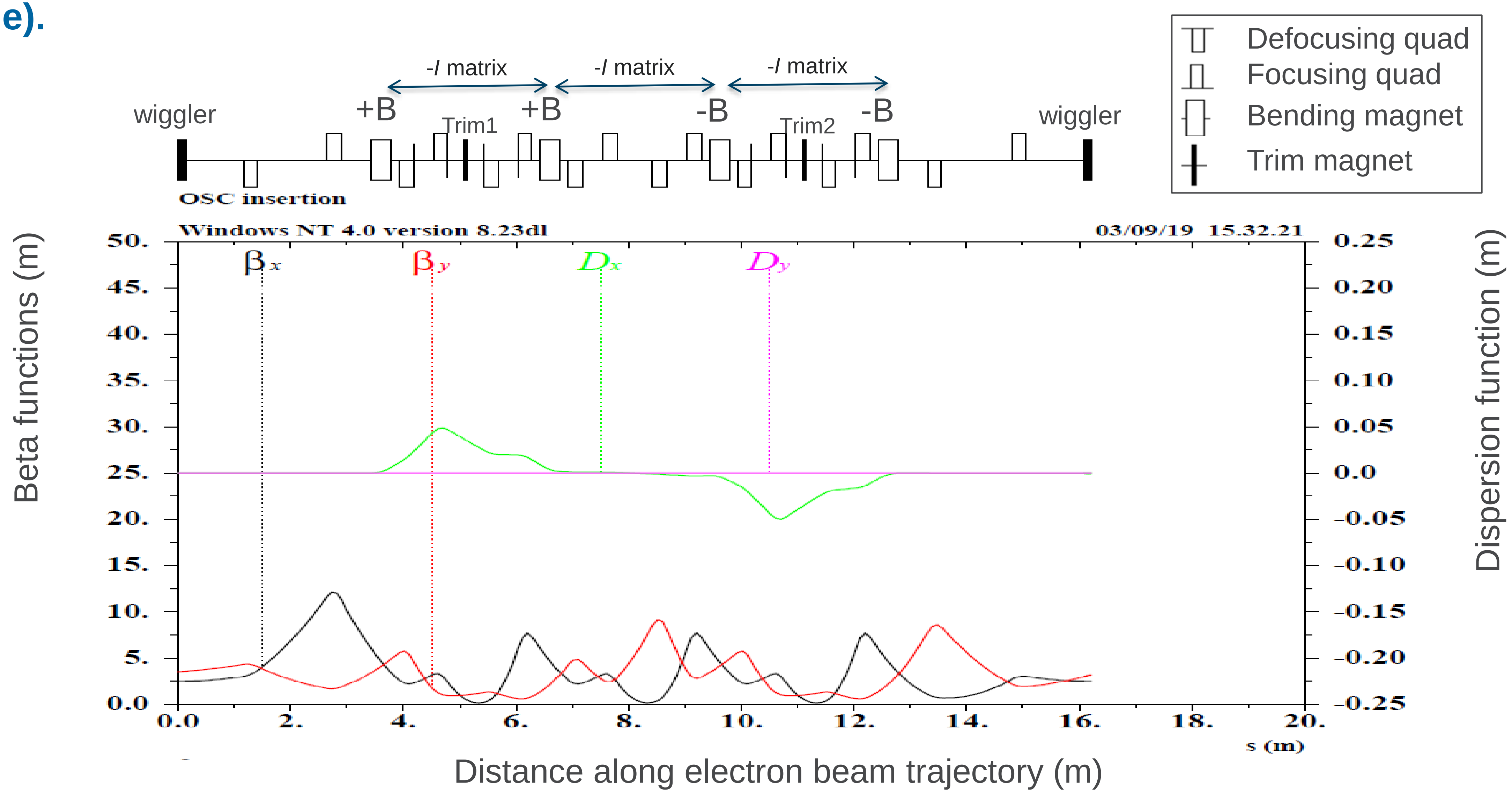
Phase

Phase horizontal cut

Phase vertical cut

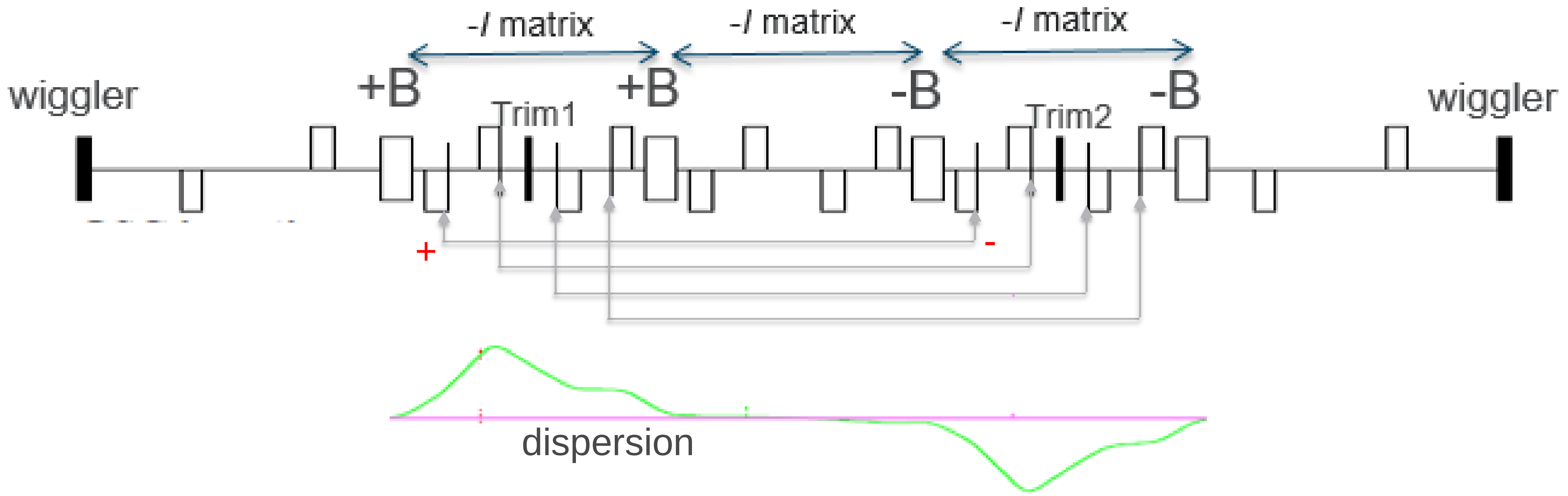
BEAM OPTICS OF COOLING SECTION

Dogleg-like lattice, two positive bends are followed by two negative bends, π phase advance between bends. Small trim magnets produce dispersion bump to control R_{56} (not visible at this scale).



CORRECTION OF SECOND ORDER TIME-OF-FLIGHT ABERRATIONS

Use four sextupole “families”



Betatron phase advance between sextupoles in the pair: $\phi_x = 2\pi$, $\phi_y = \pi$.

This allows to cancel aberrations using pairs of sextupoles with the opposite signs of the field.

Hamiltonian:

$$H(\phi x, \phi y) = (\text{linear terms}) + \sum \left(J_x^{3/2}\,(\mathrm{Cos}[\phi x] + \mathrm{Cos}[3\,\phi x]) + J_x^{1/2}\,J_y\,(\mathrm{Cos}[\phi x] + \mathrm{Cos}[\phi x \pm 2\,\phi y])\right)$$

POSSIBLE APPLICATIONS OF XUV COOLING

1. Produce a beam of relativistic positronium atoms:
   - obtain ultra-cold electrons in one ring
   - obtain ultra-cold positrons in another similar ring
   - merge electrons and positrons in a common long straight section
   - obtain positroniums

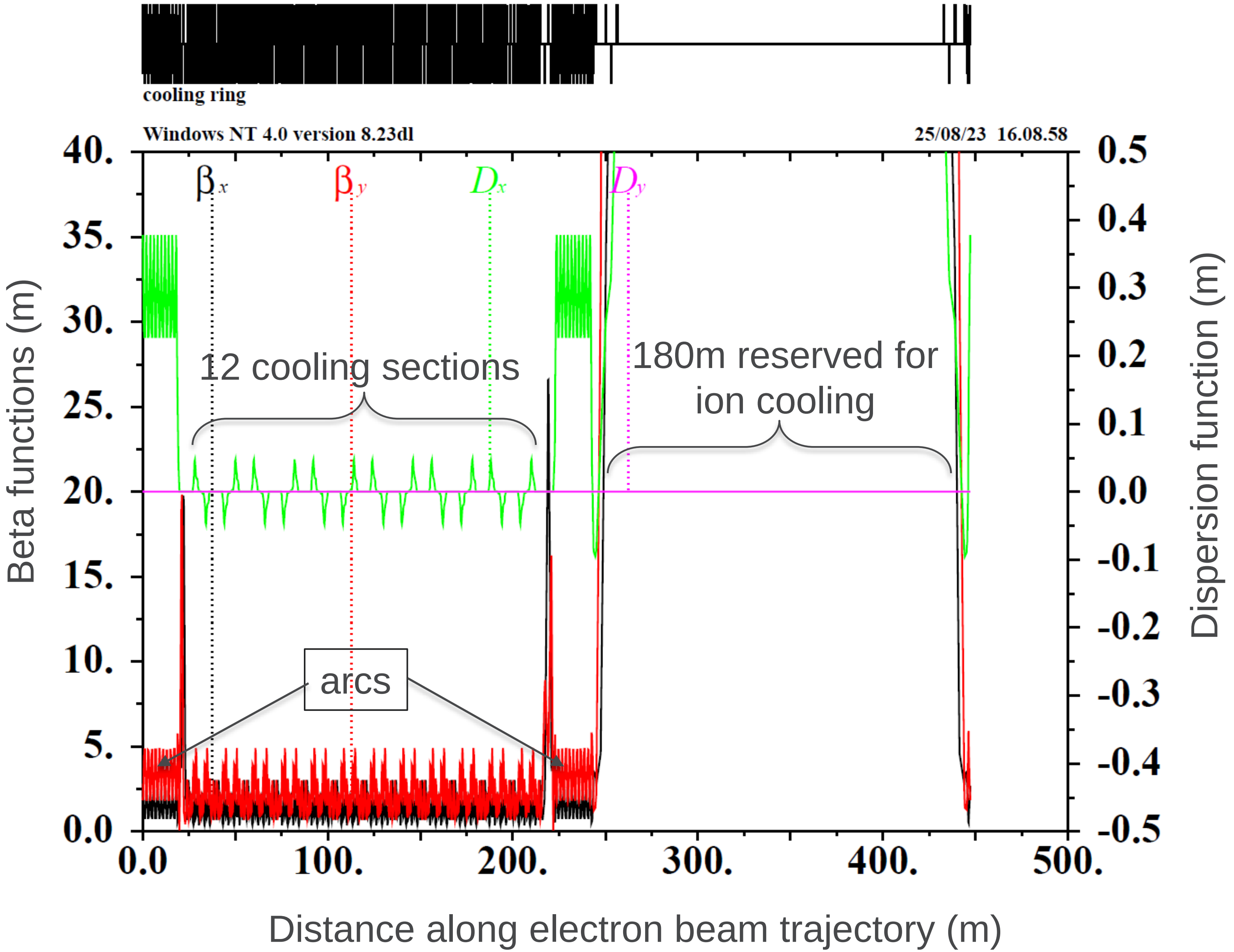
2. Produce a beam of antihydrogen atoms:
   - obtain ultra-cold positrons in one ring
   - obtain antiprotons in another ring
   - merge antiprotons and positrons in a common long straight section for positron cooling of antiprotons
   - obtain antiprotons

3. Prepare “cold” electrons for an electron cooling of ions and protons in the Electron Ion Collider *)

*) Tomorrow, Sergei Seletski will discuss the alternative (main) approach of using the electron storage ring with many wigglers for a strong radiation cooling of electrons

STORAGE RING DESIGN FOR EIC PROJECT

Circumference =447.2 m

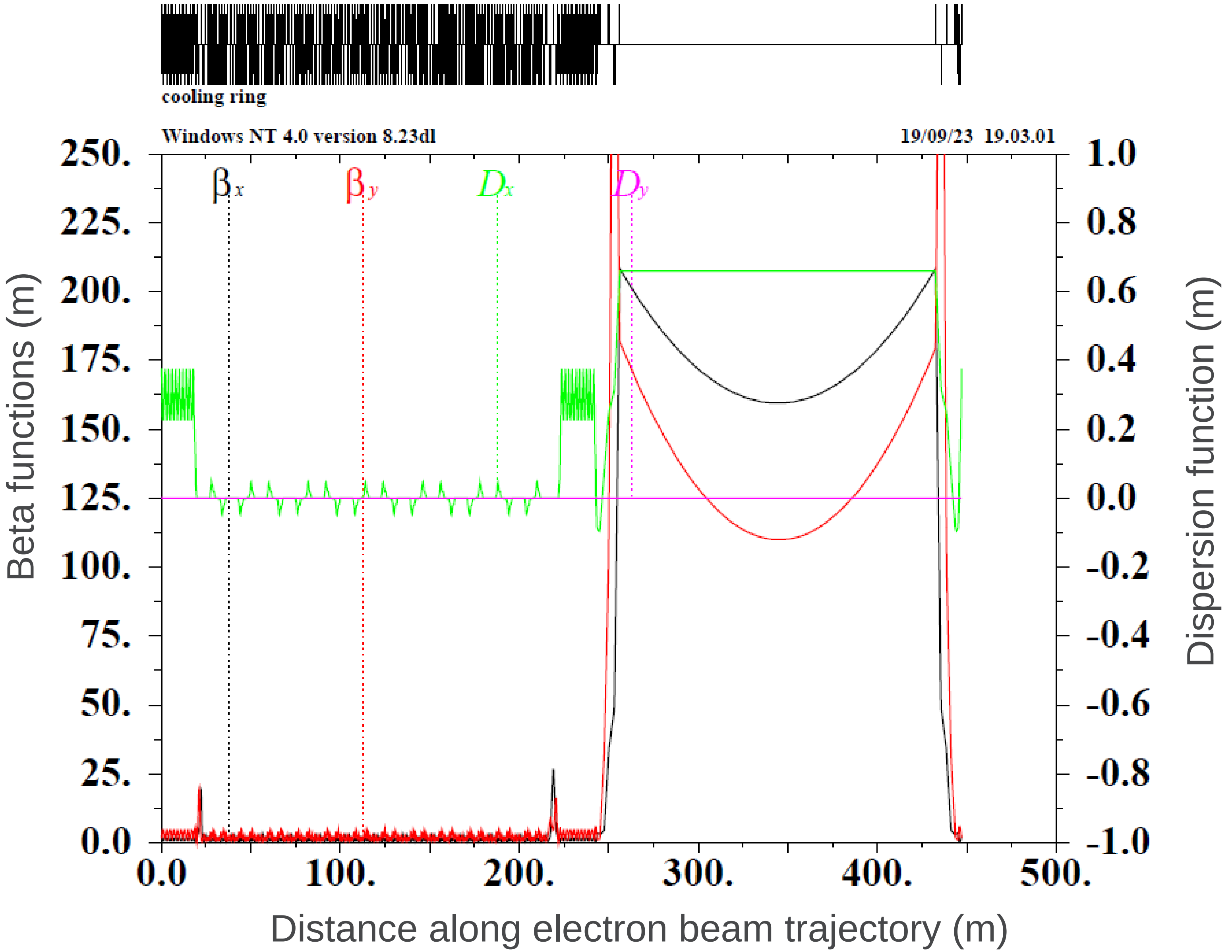


12 cooling sections

180m reserved for ion cooling

arcs

STORAGE RING DESIGN (2)



Twiss functions in the middle of the cooling straight*)

Beta_x = 160 m

Beta_y = 110 m

Dispersion = 0.66 m

*) This set has been chosen by Sergei Seletski for electron cooling of protons

COOLING RATE AND IBS GROWTH RATE

Use wiggler with period of 12.9 cm, peak field of 1.6 T, and 10 periods

Consider 1 cascade with 6 cooling sections (total length ~ 100 m)

Average energy correction (“kick”) per orbit turn due to cooling is 30 eV

“Slice” length ~ 0.13 fs (7.5 PHz bandwidth)

Number of electrons in the “longitudinal slice”: $N_s \simeq 22000$

Diffraction size $\sigma_{dif} \simeq 2$ µm

Number of electrons in the “slice” after transverse slicing: $\hat{N}_s \simeq 2$

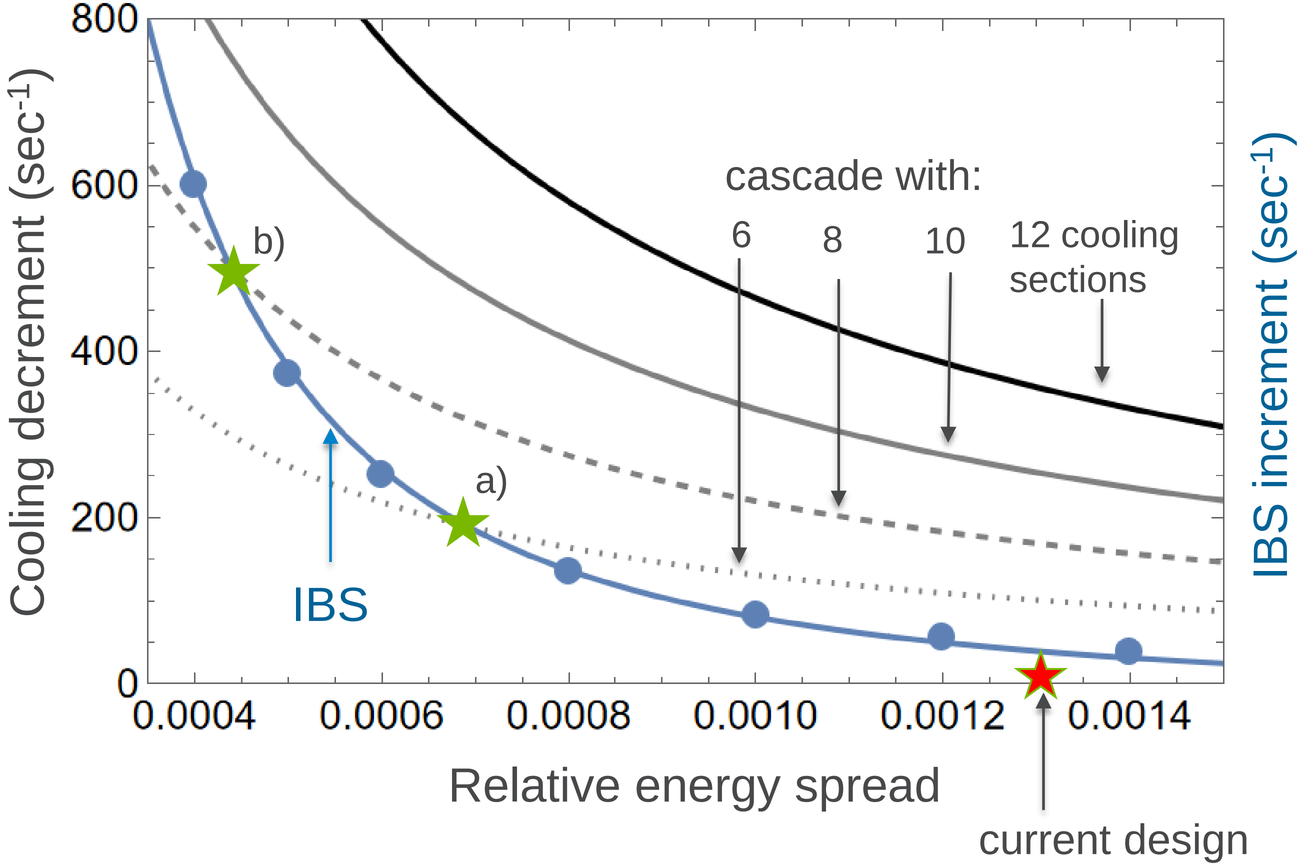
Maxwell’s demon is realized (almost)

Estimated cooling rate is 101 sec^{-1}

Estimated intrabeam scattering growth rate is 39 sec^{-1}

COOLING RATE AND IBS GROWTH RATE (2)

Stochastic cooling in XUV without the amplifier



Beam parameters used for cooling and IBS calculations$^{*)}$

	eRHIC/p	Cooler ring/e	Unit
Energy	270 GeV	147 MeV	
Energy spread	6 E-4	1.3 E-3	
Hor. emittance	11.3	12	nm
Ver. emittance	1.0	6.5	nm
Rms bunch lenth	6	14.5	cm
Bunch intensity	6.9 E10	1.9 E11	
Peak current	23	27	A

*) provided by S. Seletski

a) equilibrium relative energy spread with 6 cooling sections in cascade is 7x10^{-4}

b) equilibrium relative energy spread with 8 cooling sections in cascade is 4.5 x10^{-4}

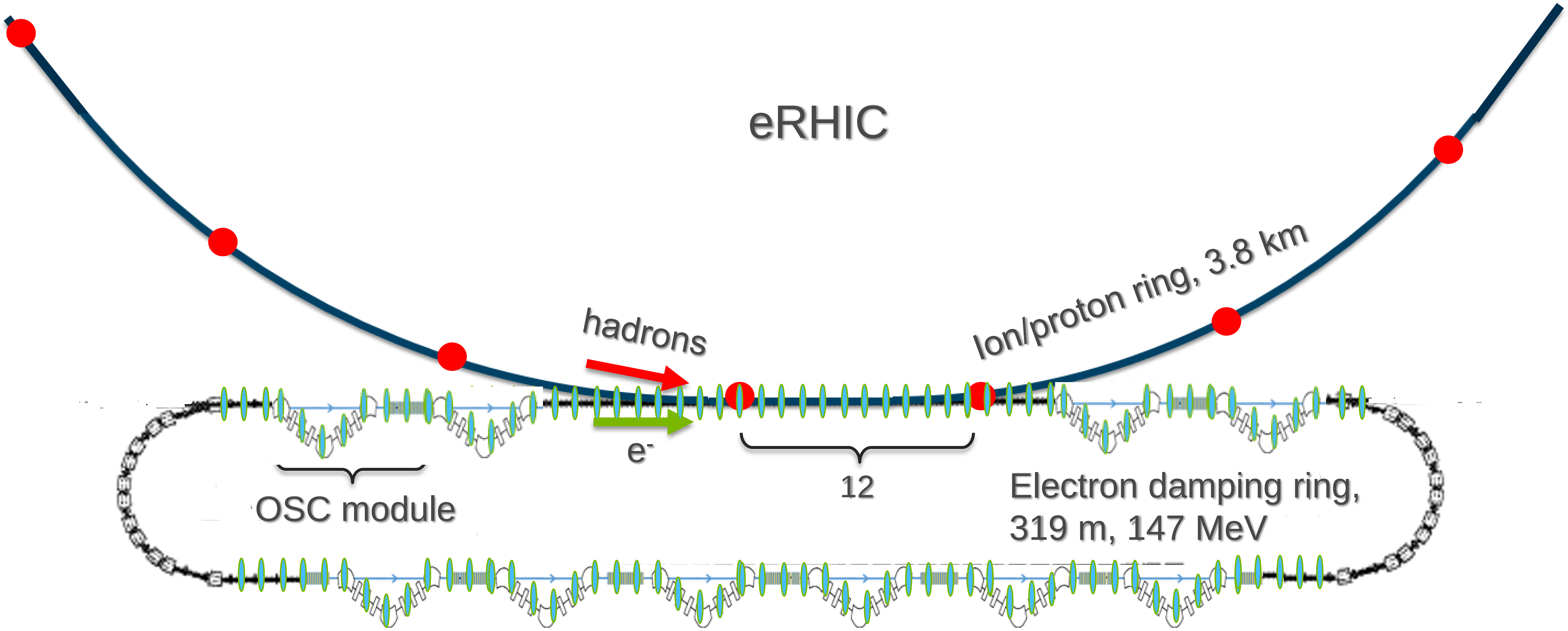
CONCLUSION

Key Takeaways

- Stochastic cooling of electrons and positrons in XUV is feasible, in principle.
- It provides a viable alternative to radiation cooling of electrons for electron cooling in EIC project. Design shows a good margin in cooling capacity above the required performance.
- However, achieving an order of a magnitude better stability of the pathlength through cooling section(s) than it was demonstrated in OSC experiment at IOTA is questionable.

BACK UP SLIDES

CURRENT IDEA FOR COOLING THE COOLER OF HADRONS



- Number of ion/proton bunches = 1320
- Ion/proton bunch rep. rate = 103 MHz
- Proton beam average current = 1 A
- Number of electron bunches = 1321
- Electron bunch rep. rate = 1.24 GHz
- Electron beam average current = 1.9 A

Each electron bunch makes 12 orbit turns in the damping ring between two subsequent interactions with ion/proton bunch!

SCALING WITH ENERGY

Consider the case when the peak magnetic field in the wiggler and relative beam energy spread in the ring are independent of beam energy.

$$\rho \propto \gamma,\ \beta_x \propto \gamma^{-1},\ \lambda_c \propto \gamma^{-2} \text{ and } \sigma_\delta = \text{Const}$$

Since $E_x \propto \gamma^3$, then $\frac{d\gamma(\tau)}{dt} = \frac{e}{m_e c^2} E_x \cdot \beta_x \propto \gamma^2$

Cooling rate $\propto \gamma$

Intrabeam scattering growth rate $\propto \gamma^{-3}$

Compare to a wiggler dominated storage ring (under the same condition of a fixed wiggler's peak magnetic field)

$$\sigma_\delta \propto \sqrt{\gamma}$$

Cooling rate $\propto \gamma$

BEAM OPTICS OF COOLING SECTION

Dispersion due to trim magnets. Note 10 times smaller scale for the dispersion.