A Study of the Wiggler Enhanced Plasma Amplifier for Coherent Electron Cooling

Alexander Zholents, Argonne National Laboratory
Chris Hall, RadiaSoft

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Preamble

The amplifier to be used for Coherent Electron Cooling (CeC)

$$\Delta n(t) = \Delta n_0 \cos(\omega_p(t)t)$$

(**V. Litvinenko , Y. Derbenev, Phys. Rev. Lett., 102:114801, 2009**)

Plasma Cascade Amplifier (PCA)
Obtain a gain in microbunching by creating a runaway instability of plasma oscillations
(**V. Litvinenko *et al*., Phys. Rev. Accel. Beams 24, 014402, 2021**.)

For instability one needs to modulate piecewise $\omega_p(I, \sigma, \gamma)$
(in a direct analogy to a FODO channel described by Hill's equation with piecewise periodic coefficients)

Option 1) use $\sigma(t)$ → Plasma Cascade Amplifier (Litvinenko *et al.*)

Option 2) use $\sigma(t)$ and $\gamma(t)$ → Wiggler Enhanced Plasma Amplifier (WEPA, Zholents, Stupakov)

in the wiggler $\gamma_z = \frac{1}{\sqrt{1-\bar{v}_z^2/c^2}} = \frac{\gamma}{\sqrt{1+K_w^2/2}}$ $\quad K_w = \frac{eB_w\lambda_w}{2\pi m_e c}$

(**G. Stupakov and A. Zholents, 13th Workshop on Beam Cooling and Related Topics, 2021**)

Note: WEPA requires separation of ion and electron beams, PCA does not

Theory

variables

$$\Delta\hat{n}_k(s) = \int_{-\infty}^{\infty} \Delta n(s,z) e^{-ikz} dz$$ Fourier transform of a density perturbation

$$\Delta\eta = \frac{\Delta E}{E}$$ relative energy spread

$$\Delta\hat{\eta}_k(s) = \int_{-\infty}^{\infty} \Delta\eta(s,z) e^{-ikz} dz$$

$$\frac{d}{ds}\mathrm{Im}\,(\mathcal{Z}) = Z_I(k)$$ impedance per unit length

$$\Delta\hat{I}_k = e\Delta\hat{n}_k c$$ Fourier transform of a peak current

equations

$$\frac{d\Delta\hat{\eta}_k}{ds} = ie\frac{\Delta\hat{I}_k c Z_I}{E} = i\frac{r_e c Z_I}{\gamma}\Delta\hat{n}_k$$

$$\frac{d\Delta\hat{n}_k}{ds} = -ik\frac{1}{\gamma_z^2} n_0 \Delta\hat{\eta}_k$$ linearized continuity equation for cold plasma in the wiggler

$$\frac{d^2\Delta\hat{n}_k}{ds^2} = -c^{-2}\omega_p^2(k)\Delta\hat{n}_k$$

plasma frequency

$$\omega_p^2(k) = \frac{I}{I_A\gamma}\frac{kc^3}{\gamma_z^2}(-Z_I(k)) \qquad (r_e n_0 = I/I_A)$$

Theory (2)

$l_w \gg 2\frac{\gamma_z^2}{k}$ subsequent analysis is valid when wiggler is longer than a transient length ~ (1 - 8) cm

$Z_I = Z_{SC} + Z_{Rad}$ in the case of the wiggler impedance (G. Geloni *et al.*, NIM A, 2005)

$\rho(r) = \frac{1}{2\pi\sigma^2 c} e^{-\frac{r^2}{2\sigma^2}}$ electron distribution in transverse coordinate

$$\begin{cases} -Z_{SC}(k) = \dfrac{2k}{\gamma_z^2 c} I_2\left(\dfrac{\sigma}{\Sigma_{SC}(k)}\right) \\ -Z_{Rad}(k) = \dfrac{K_W^2 k}{2\gamma^2 c}\left[\dfrac{\pi}{2} I_3\left(\dfrac{\sigma}{\Sigma_{Rad}(k)}\right) - I_2\left(\dfrac{\sigma}{\Sigma_{Rad}(k)}\right)\right] \end{cases}$$

$$I_2(a) = \frac{1}{2} e^{a^2} \Gamma\left(0, a^2\right)$$

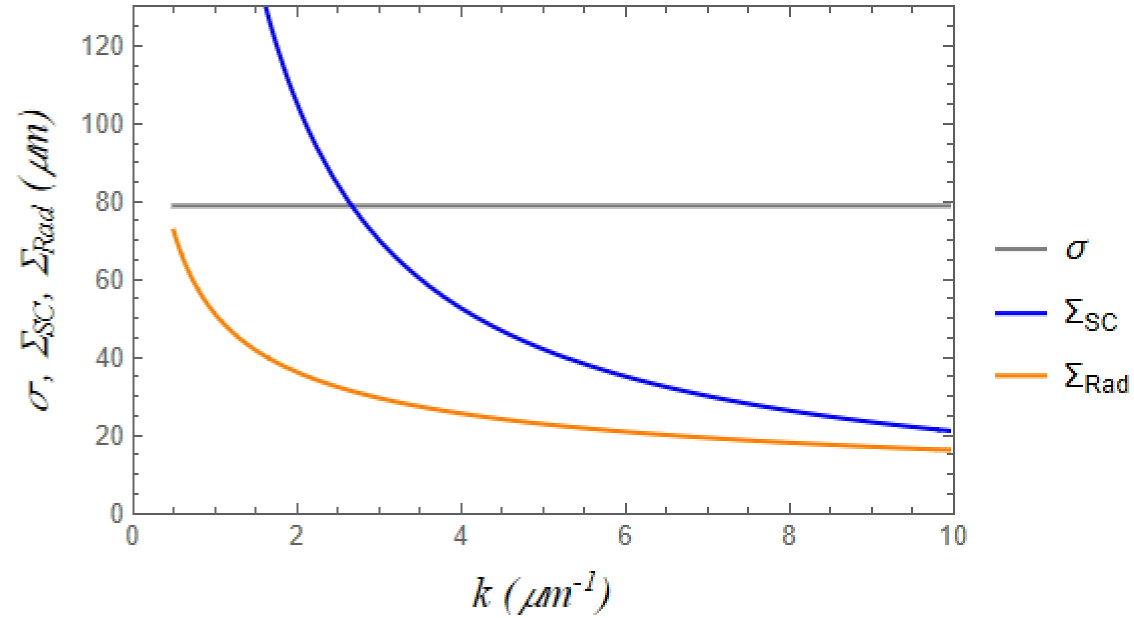
$$I_3(a) = \mathrm{MeijerG}\left[\left((0), \left(-\frac{1}{2}\right)\right), \left((0,0), \left(-\frac{1}{2}\right)\right), a^2\right]$$

$\Gamma(0, x)$ is incomplete gamma function

$\mathrm{MeijerG}\left[\left((0), \left(-\frac{1}{2}\right)\right), \left((0,0), \left(-\frac{1}{2}\right)\right), x\right]$ is the Meijer G-function

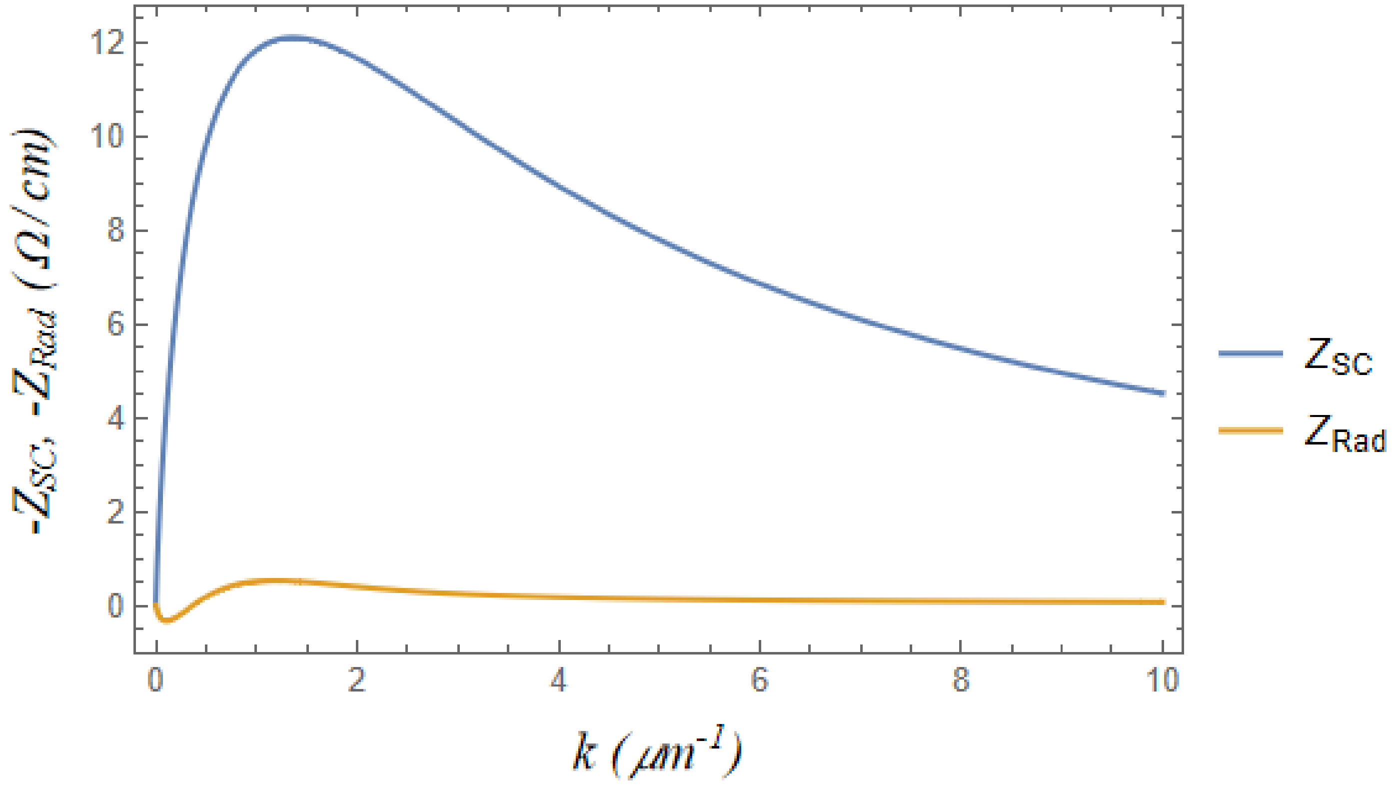
$\Sigma_{SC}(k) = \frac{\gamma_z}{k}$ is space charge diffraction size

$\Sigma_{Rad}(k) = \sqrt{\frac{\lambda_W}{4\pi k}}$ is radiation diffraction size



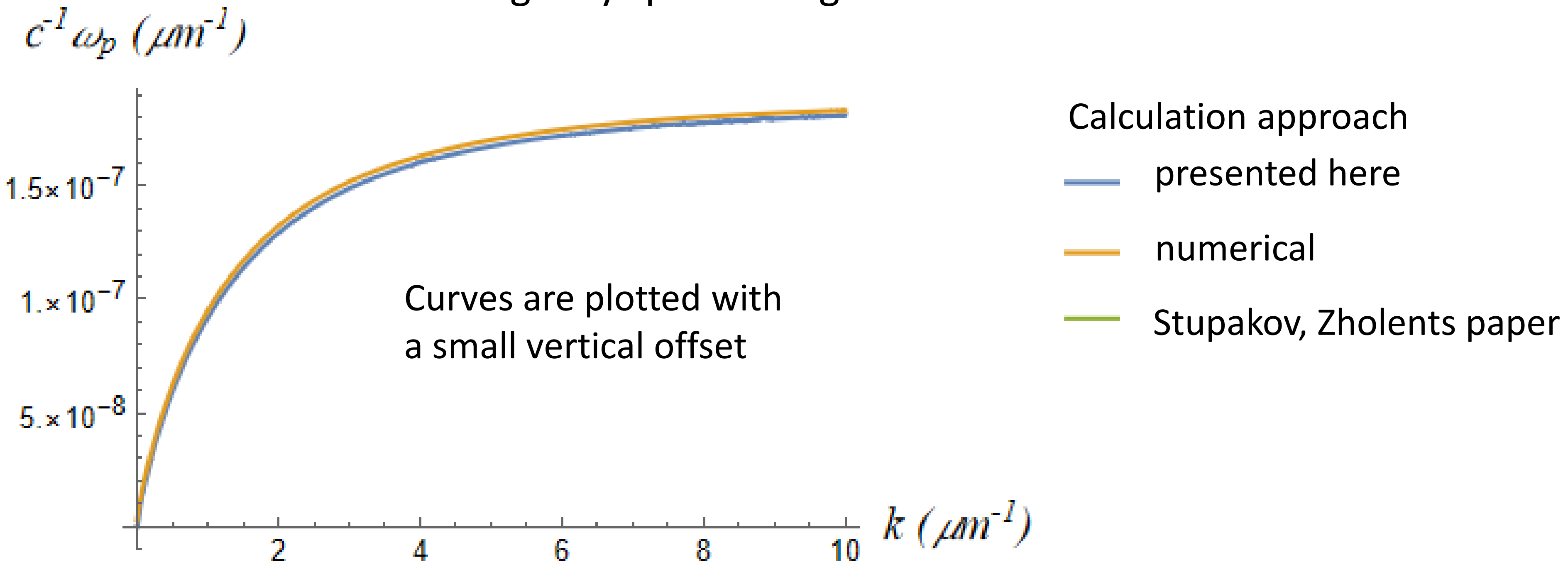
Theory (3)

Impedance as a function of frequency in the case of K_W=1.5, beam energy = 157 MeV, and σ = 80 μm



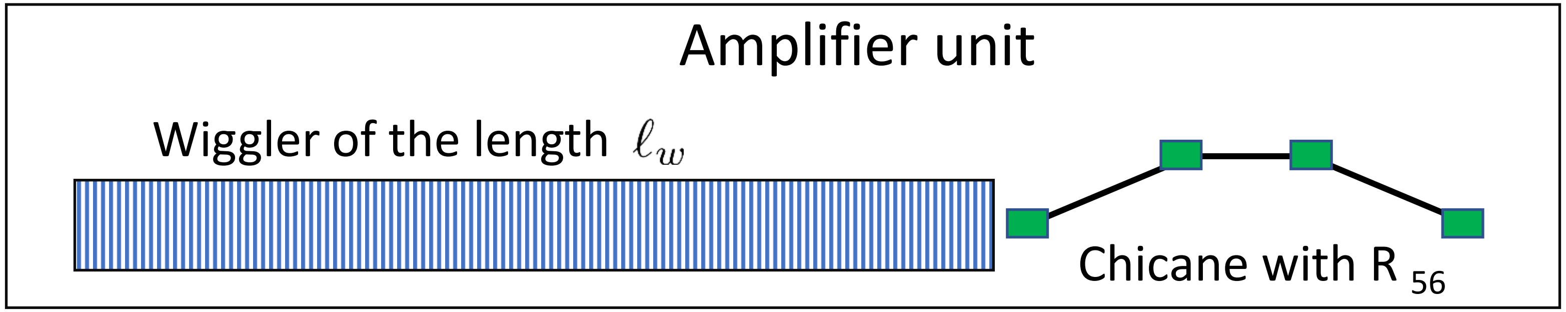
Theory (4)

Plasma frequency in the wiggler
calculated using only space charge effects



three different calculation approaches agree perfectly

Theory (5)



Consider evolution of the vector $\left(\Delta\hat{n}_k, \frac{\gamma_z^2}{\gamma^2}\frac{d\Delta\hat{n}_k}{ds}\right)^T$

Wiggler transport
$$M_W = \begin{pmatrix} \cos(\frac{\omega_p}{c}l_w) & \frac{\gamma^2 c}{\gamma_z^2\omega_p}\sin(\frac{\omega_p}{c}l_w) \\ -\frac{\gamma_z^2\omega_p}{\gamma^2 c}\sin(\frac{\omega_p}{c}l_w) & \cos(\frac{\omega_p}{c}l_w) \end{pmatrix}$$

Chicane transport
$$M_C = \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \gamma^2 R_{56} \\ 0 & 1 \end{pmatrix}$$

Unit transport $M_{\text{unit}} = M_C \cdot M_W$

has two eigenvalues G_1 and G_2. If both $|G_1| = |G_2| = 1$, then the plasma oscillation in the amplifier is stable and there is no gain in the microbunching. The plasma oscillation is unstable when either $|G_1| > 1$ or $|G_2| > 1$

Theory (6)

Eigenvalues

$$\begin{cases} G_1 = \cos(\phi) - \frac{1}{2}\gamma_z^2 R_{56} k_p \sin(\phi) - \sqrt{\left(-\cos(\phi) + \frac{1}{2}\gamma_z^2 R_{56} k_p \sin(\phi)\right)^2 - 1} \\ G_2 = \cos(\phi) - \frac{1}{2}\gamma_z^2 R_{56} k_p \sin(\phi) + \sqrt{\left(-\cos(\phi) + \frac{1}{2}\gamma_z^2 R_{56} k_p \sin(\phi)\right)^2 - 1} \end{cases}$$

$$k_p = \omega_p / c$$
$$\phi = k_p l_w$$

Since it is desirable to have ϕ in a neighborhood of $\pi/2$ and $\gamma_z^2 R_{56} k_p >> 1$, we write approximate G_1 and G_2

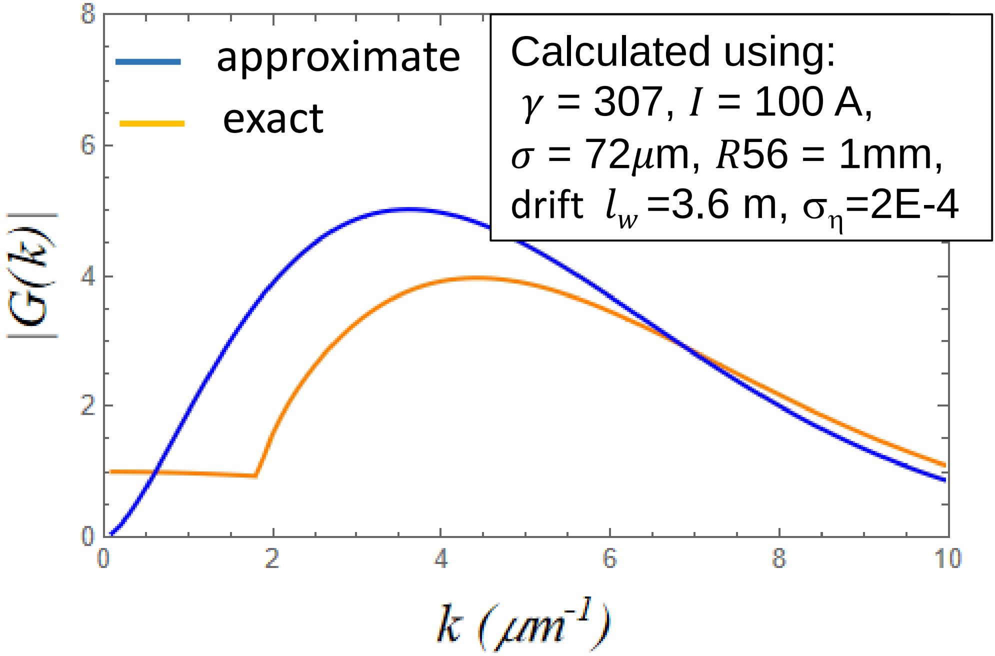
$$G_1 \simeq -\gamma_z^2 R_{56} k_p \sin(k_p l_w)$$
$$G_2 \simeq 0.$$

Using only space charge impedance

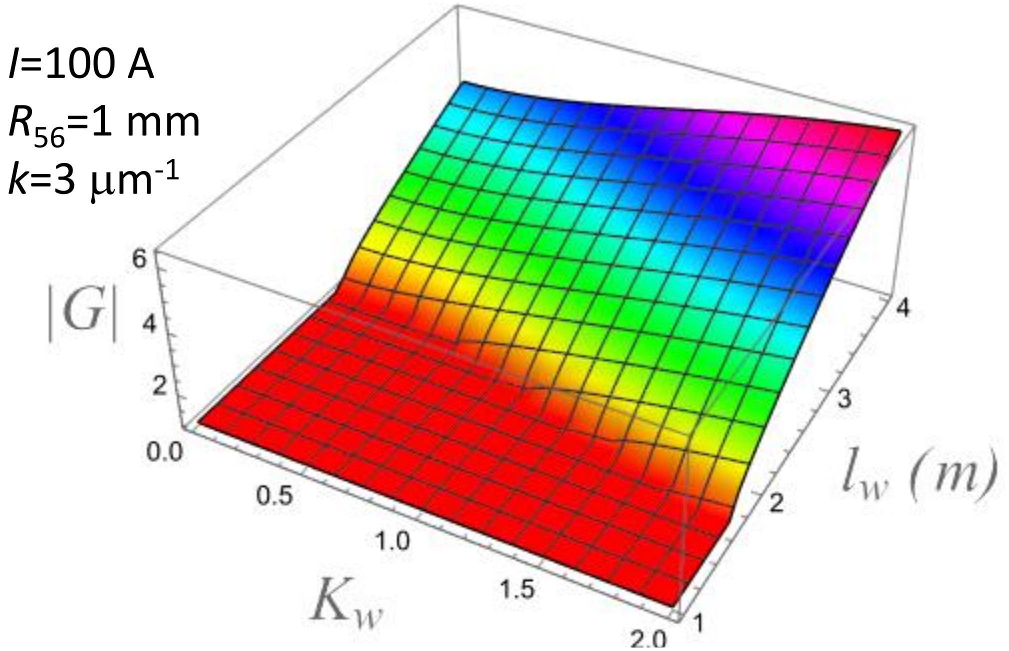
$$|G(k)| \simeq k R_{56} \sqrt{\frac{I}{I_A \gamma} e^{\left(\frac{k\sigma}{\gamma_z}\right)^2} \Gamma\left(0, \left(\frac{k\sigma}{\gamma_z}\right)^2\right)} \sin(k_p l_w) \underbrace{\exp\left[-\frac{1}{2} k^2 \tilde{R}_{56}^2 \sigma_\eta^2\right]}_{\text{debunching}}$$

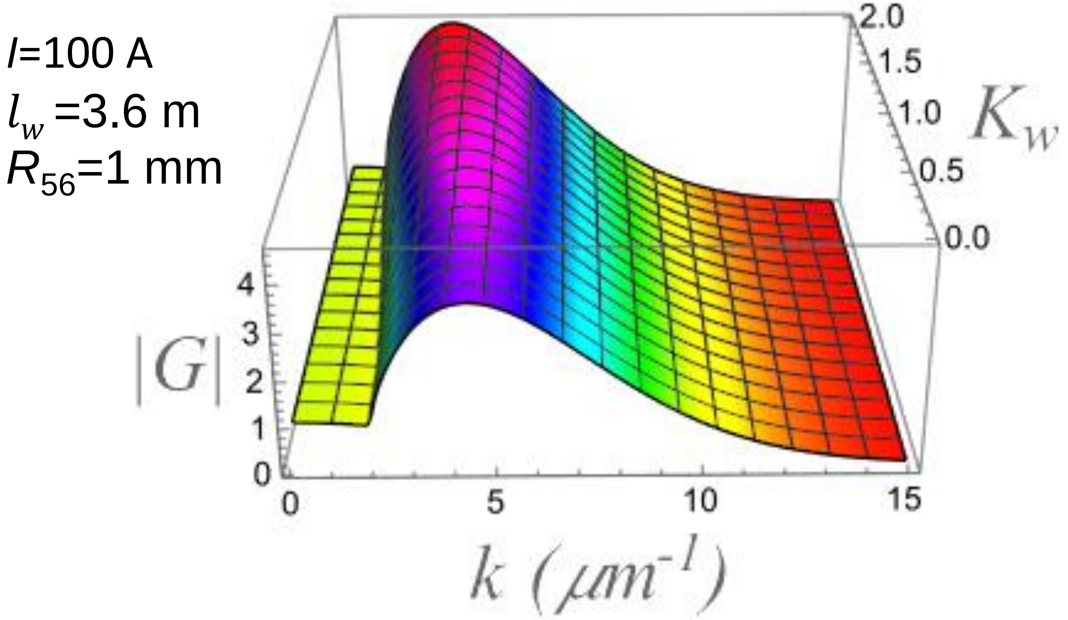
$$\tilde{R}_{56} = \frac{l_w}{\gamma_z^2} + R_{56}$$

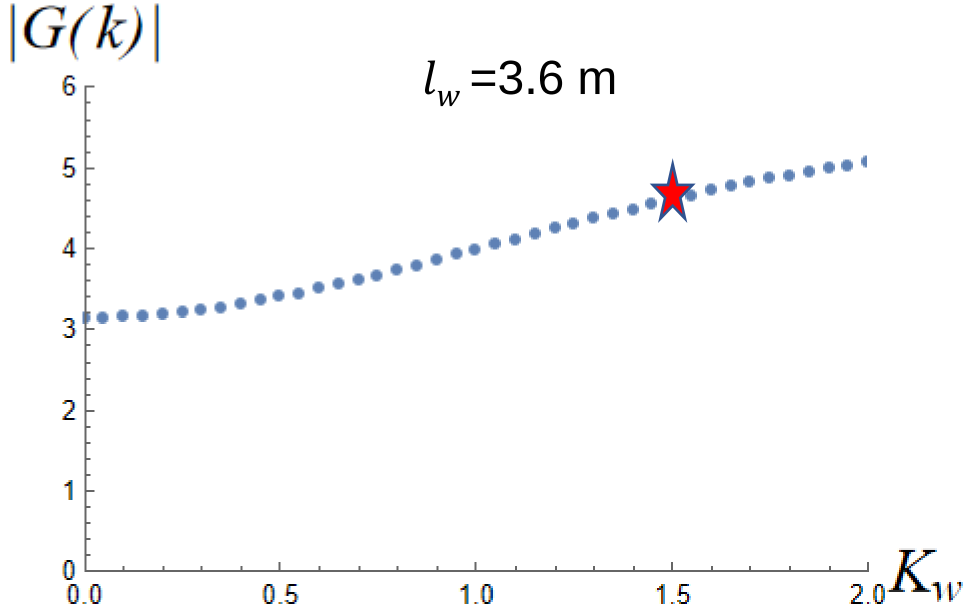
The above formula is valid if using a drift with the length l_w and $\gamma_z \equiv \gamma$, in which case it is identical to the gain from the paper: Stupakov, Baxevanis, Phys. Rev. Acc. and Beams, 2019.

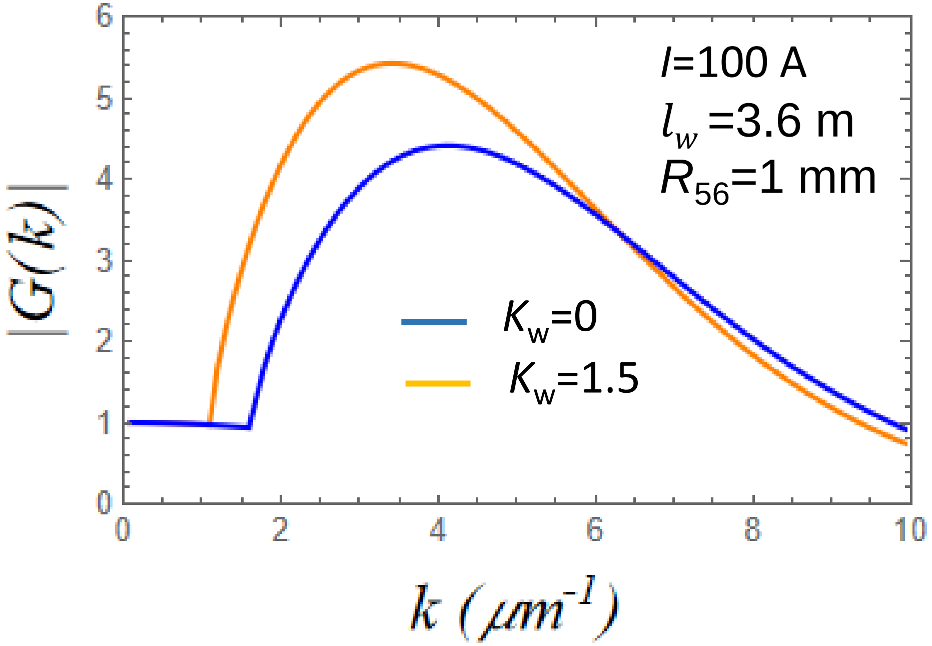


Parametric study of the gain after one amplifier unit

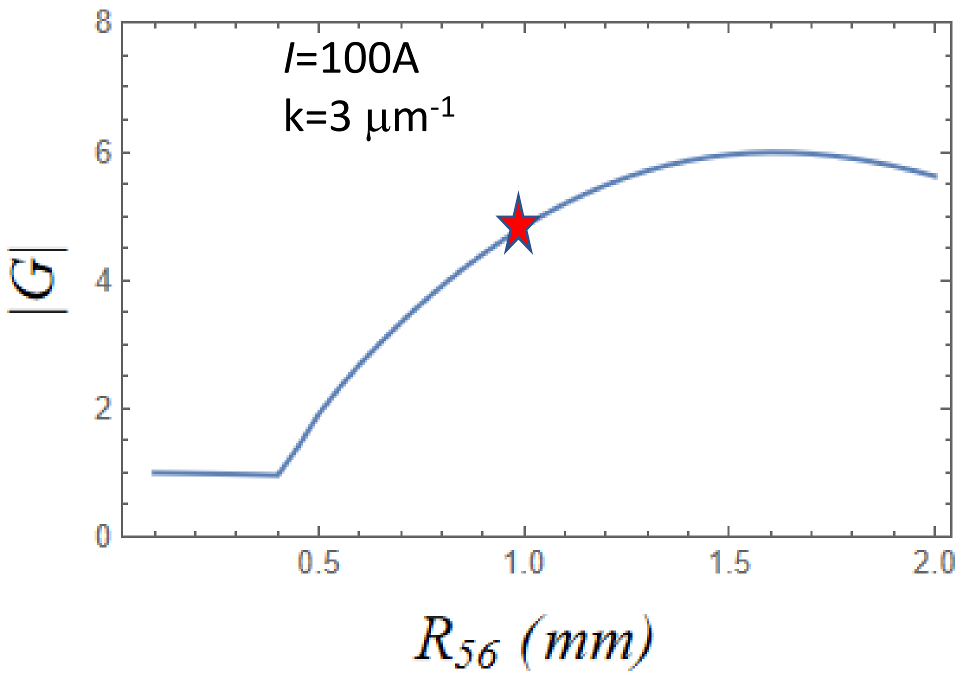


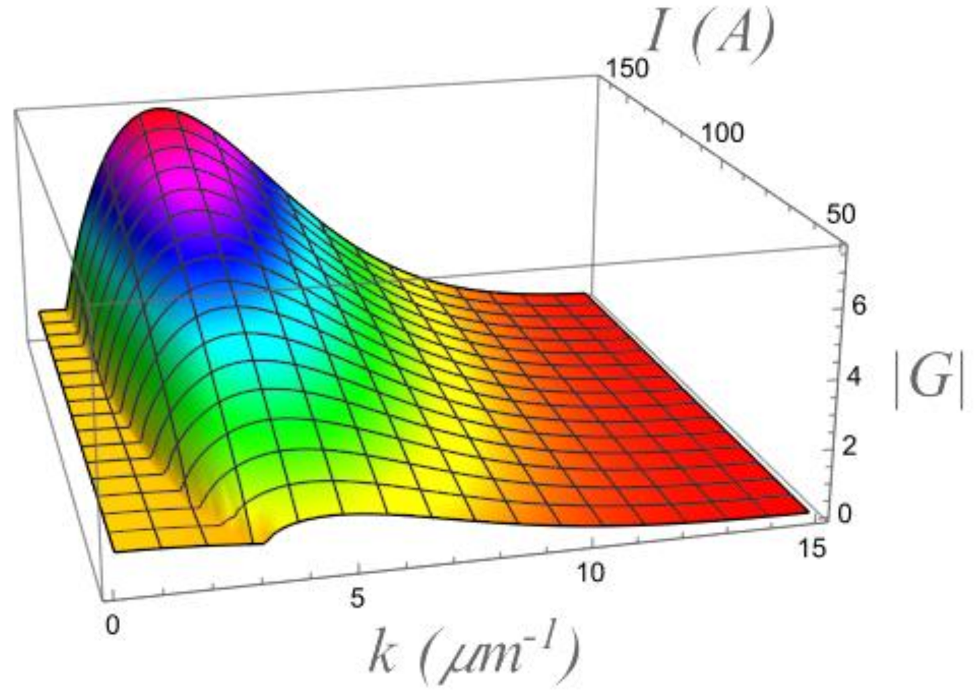




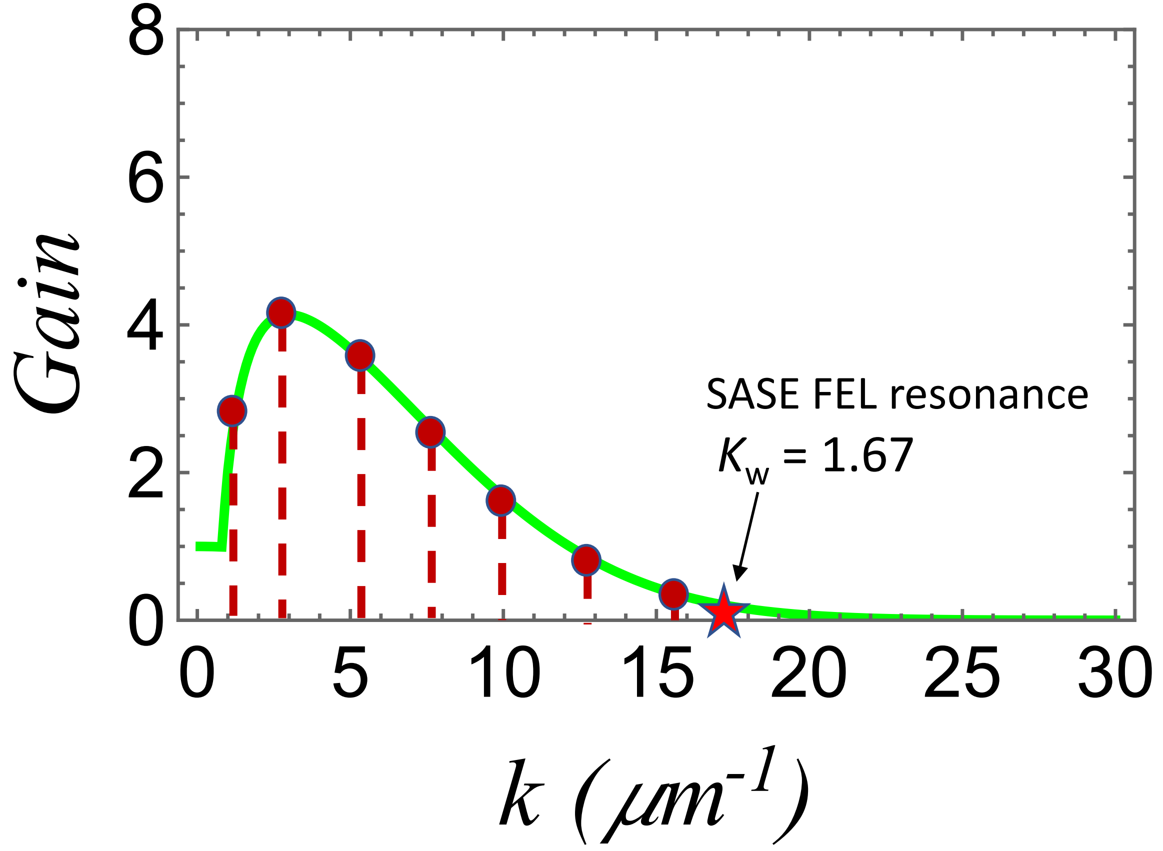


Parametric studies of the gain after one amplifier unit (2)





Goal of simulations



To keep SASE FEL resonance outside of the amplifier bandwidth we have to use $K_w < 1.67$ such as $k_{SASE} > k$

Wiggler-enhanced plasma amplifier was modeled using:

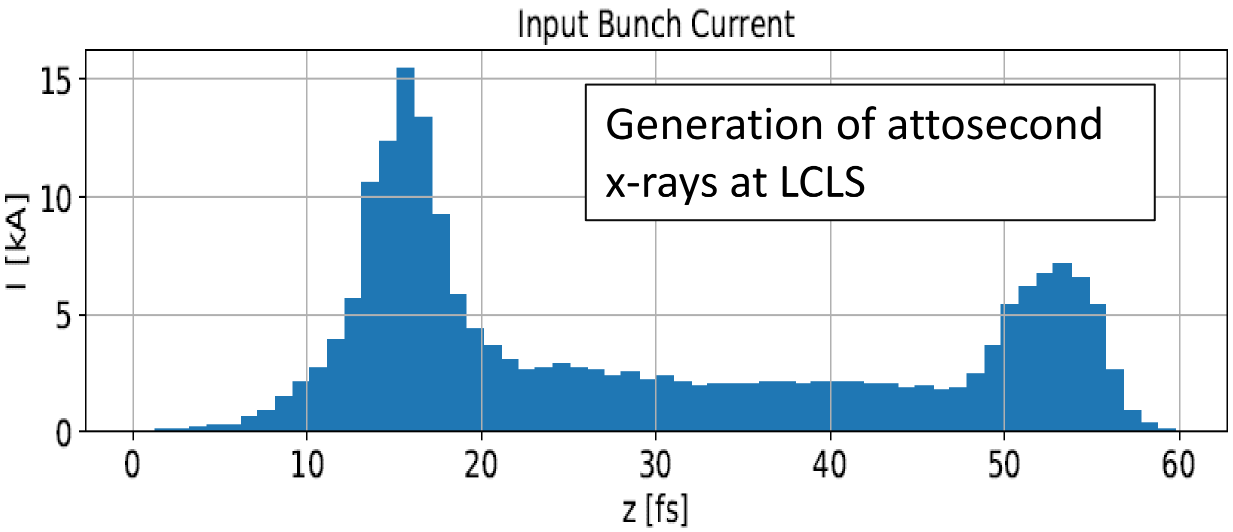
- OPAL-FEL for the wiggler sections (code MITHRA was ported to OPAL for that purpose)
- Elegant for the chicanes

A. Adelmann *et al*., **OPAL** a Versatile Tool for Charged Particle Accelerator Simulations, arXiv:1905.06654, 2019.

A. Fallahi, A. Yahaghi, and F. Kārtner, **MITHRA** 1.0: A Full-Wave Simulation Tool for Free Electron Lasers, Computer Physics Communications, 228, pp.192-208, 2018.

Benchmarking OPAL-FEL with SLAC experiment*)

Regime with large γ, K_W and γ_z



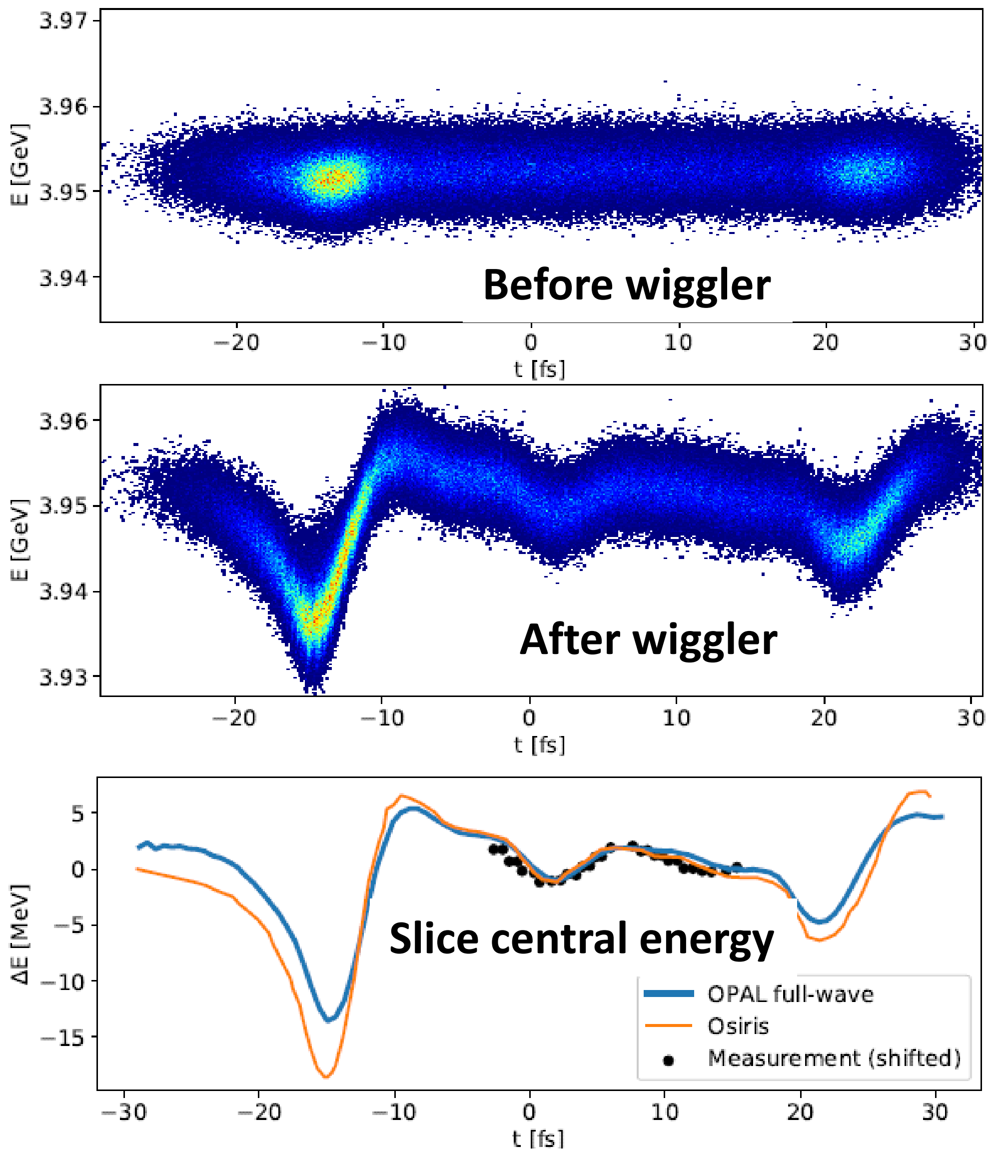
$\sigma_\perp = 70\ \mu m$, $\epsilon = .4$ mm mrad, $Q = 200$ pC, $E = 3.95$ GeV,

$L_w = 2.3$ m, $\lambda_w = 35$ cm, $K_W = 51.5$

$$k_r = 1.7\mu m^{-1}$$

$$\gamma_z = 212$$

MacArthur, *et al.*, PRL, **2019**



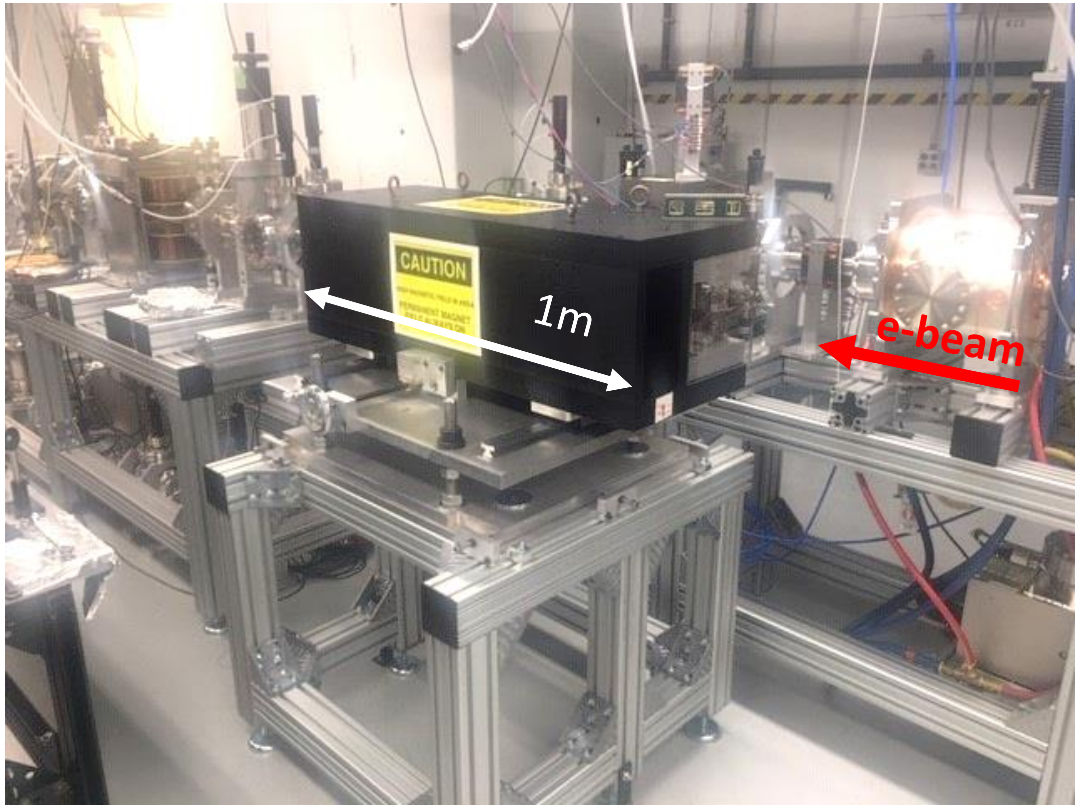
*) Arnau Albà et al., Computer Physics Communications, 2022

Benchmarking OPAL-FEL with ANL experiment *)

Regime with large K_W and small γ, γ_z

*) Arnau Albà et al., Computer Physics Communications, 2022

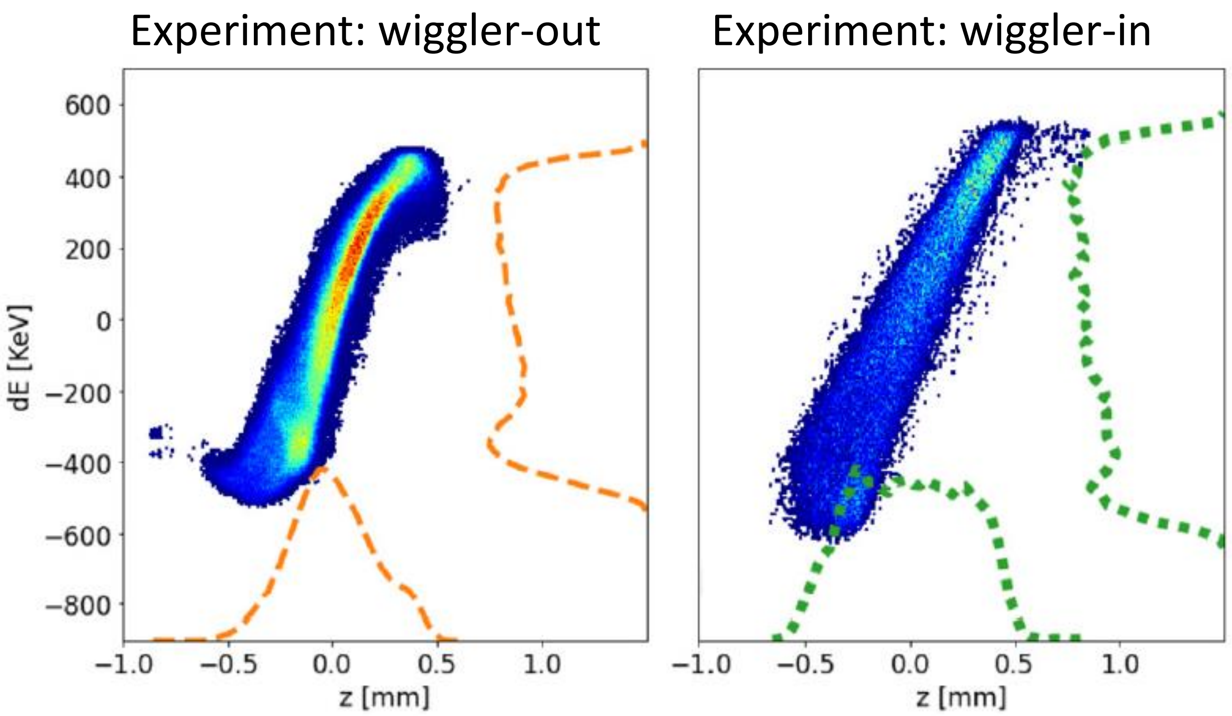
AWA beam energy = 45.4 MeV,
bunch charge = 300 pC, bunch length =0.25 - 0.5 mm



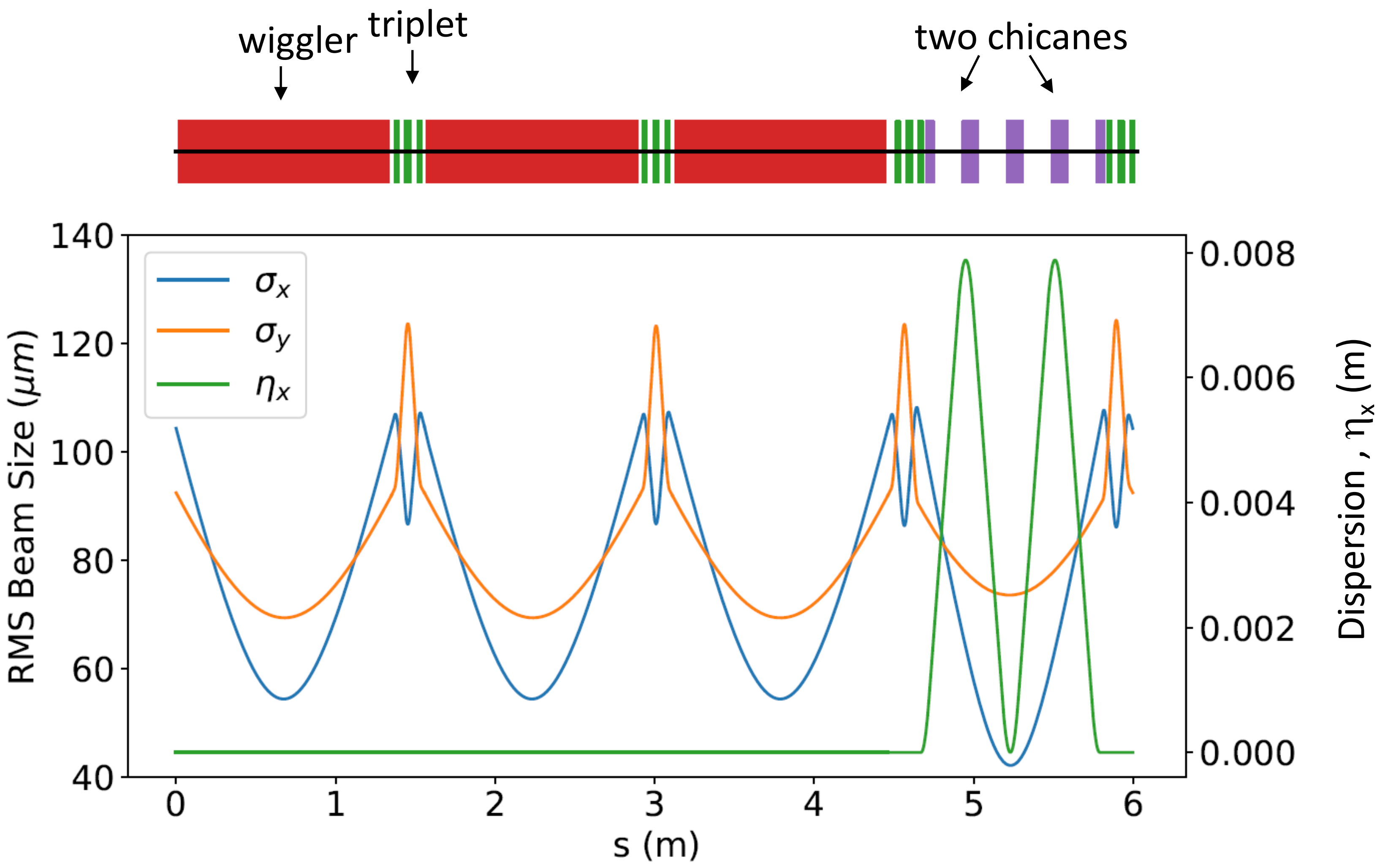
Section of AWA beamline with APS wiggler

Wiggler period = 8.5 cm, K=10.5, γ_z = 11.6, $k_r = 0.01 \mu m^{-1}$

Phase space reconstruction from measurements using magnetic spectrometer and deflecting cavity



Lattice of a single amplifier unit used in simulations



Wiggler:
length = 1.2 m
period = 3.3 cm
K = 1.5
B = 0.49 T

Chicane R_{56} = 1.0 mm
magnet length = 5 cm
magnet field = 0.04 T

Initial electron beam

Core:

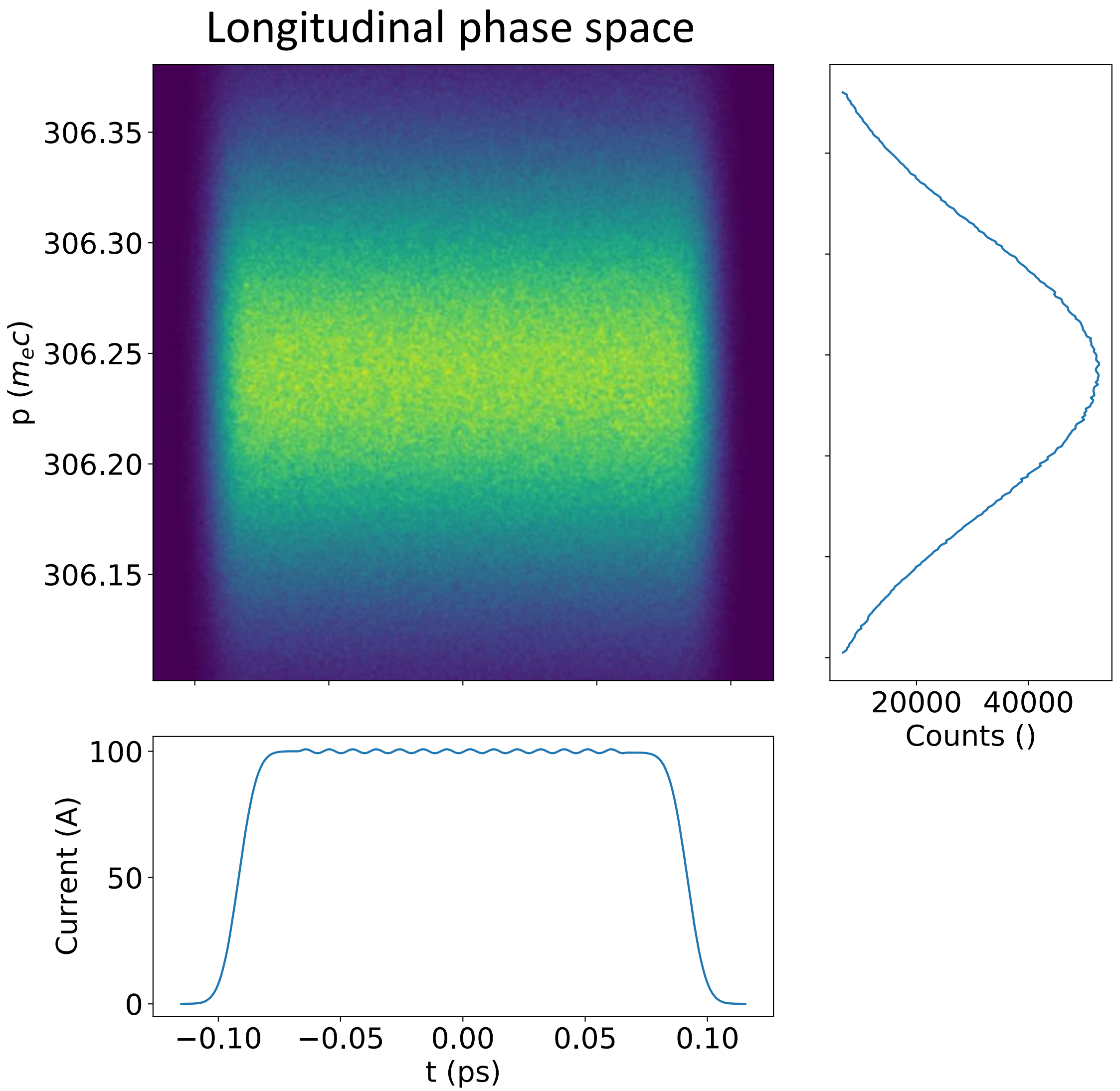
$$n(z) = \Delta n \sin(kz) + n_0$$

$$\delta_m = \frac{\Delta n}{n_0}$$

Tails:

$$n_{\text{tail}}(z) = n(\pm L/2)\text{erf}\left(2\frac{z \pm L/2}{l}\right)$$

- Sampling for distribution in z
    - Calculate cumulative distribution function
    - Numerically invert CDF
    - Generate samples from Halton sequence
    - Apply inverted CDF
- All other distributions are Gaussian



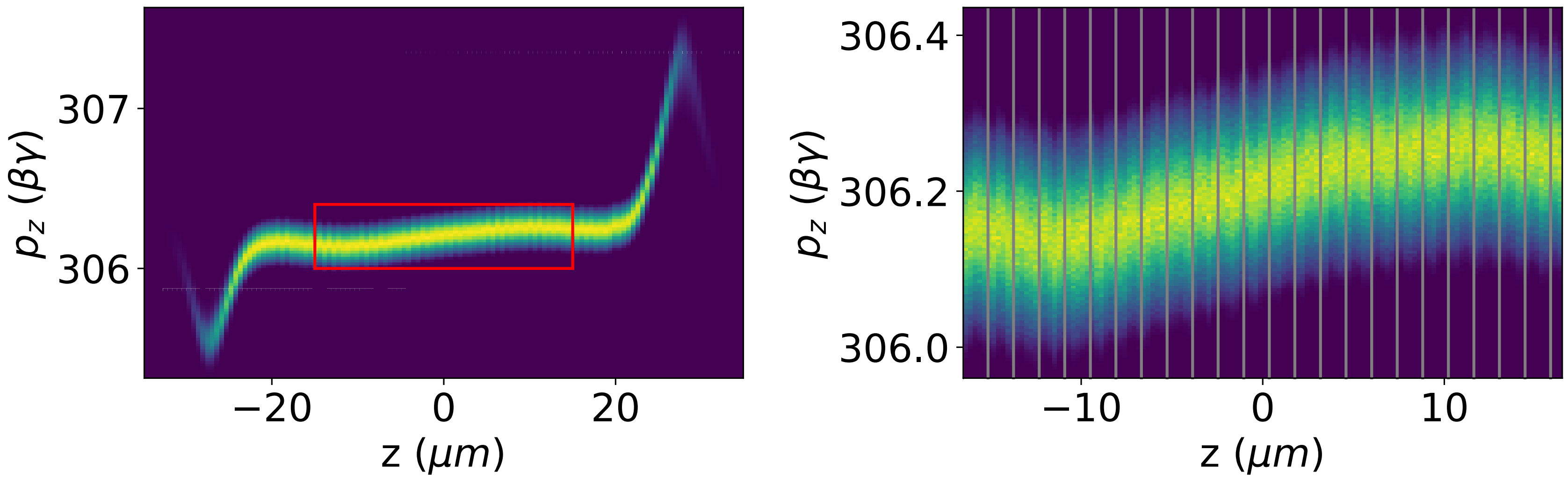
Simulation setup

Define:

- grid resolution,
- energy resolution<8 keV,
- grid domain,
- number of macroparticles,
- seed modulation amplitude.

Δn must be larger than numerical shot noise, and smaller than ~ $0.2n_0$/Gain to avoid gain saturation

Convergence of global beam parameters in many slices in bunch core was used as convergence criteria

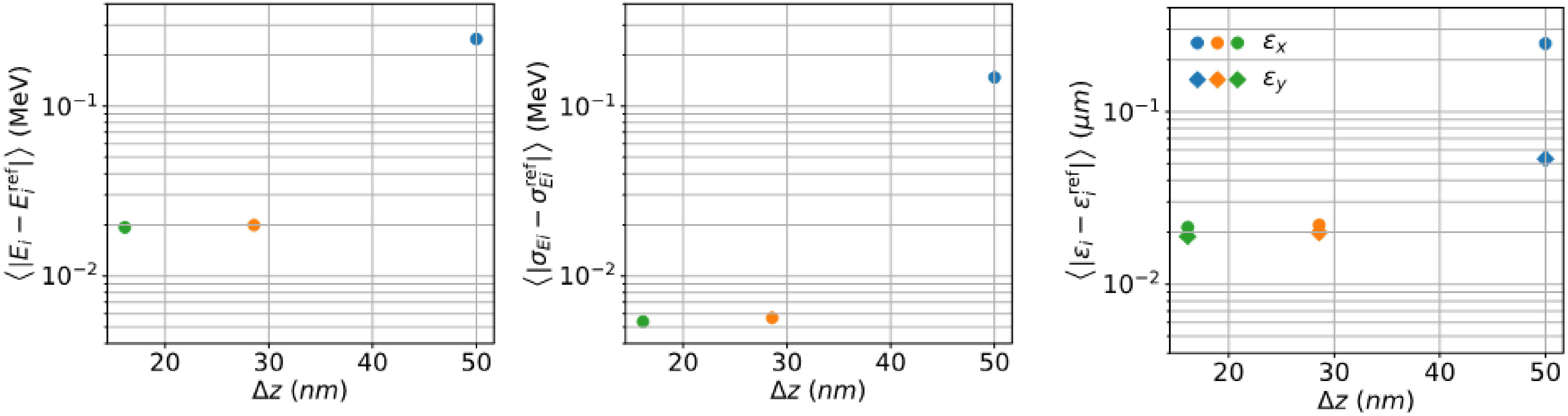


Selection of the solver's grid resolution

Simulation with small grid resolution Δx = Δy= 2.5 μm and Δz = 7.5 nm was used as the reference

Plots show deviations from reference for the beam energy, beam transverse size and emittance.

Color coding: ● Δx = Δy=40 μm; ● Δx = Δy=10 μm; ● Δx = Δy=5 μm

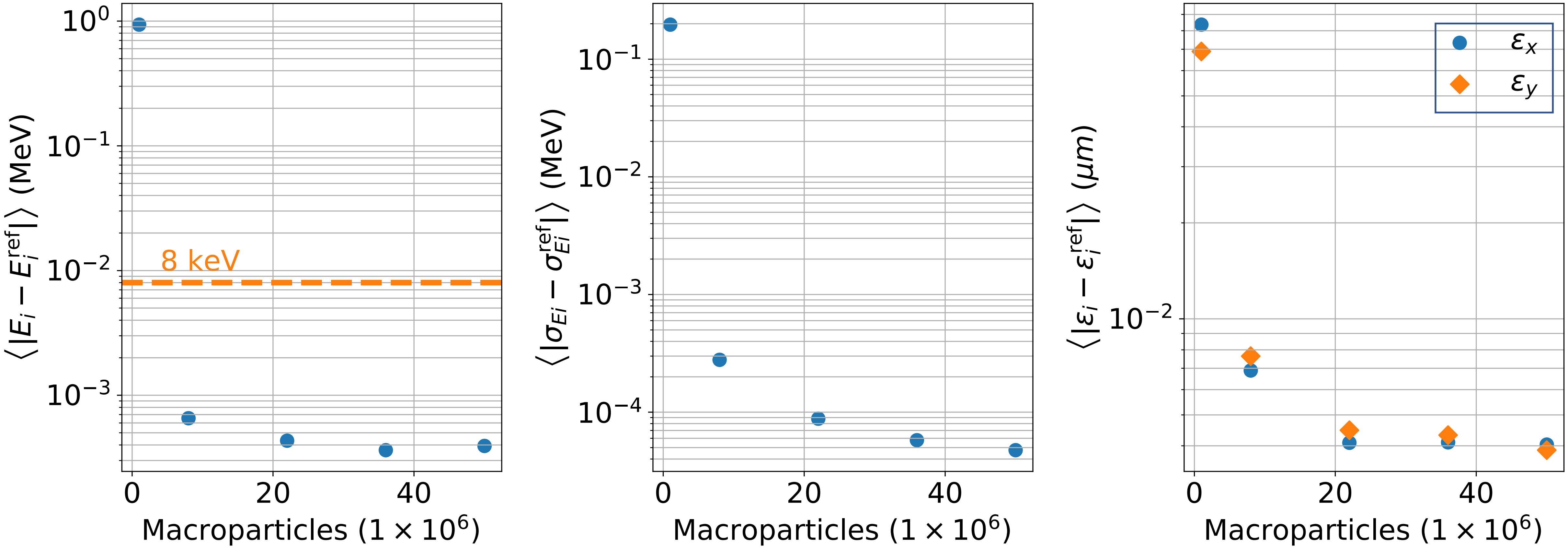


Grid resolution Δx = Δy=10 μm and Δz = 28.6 nm was chosen

Selection of the number of macroparticle

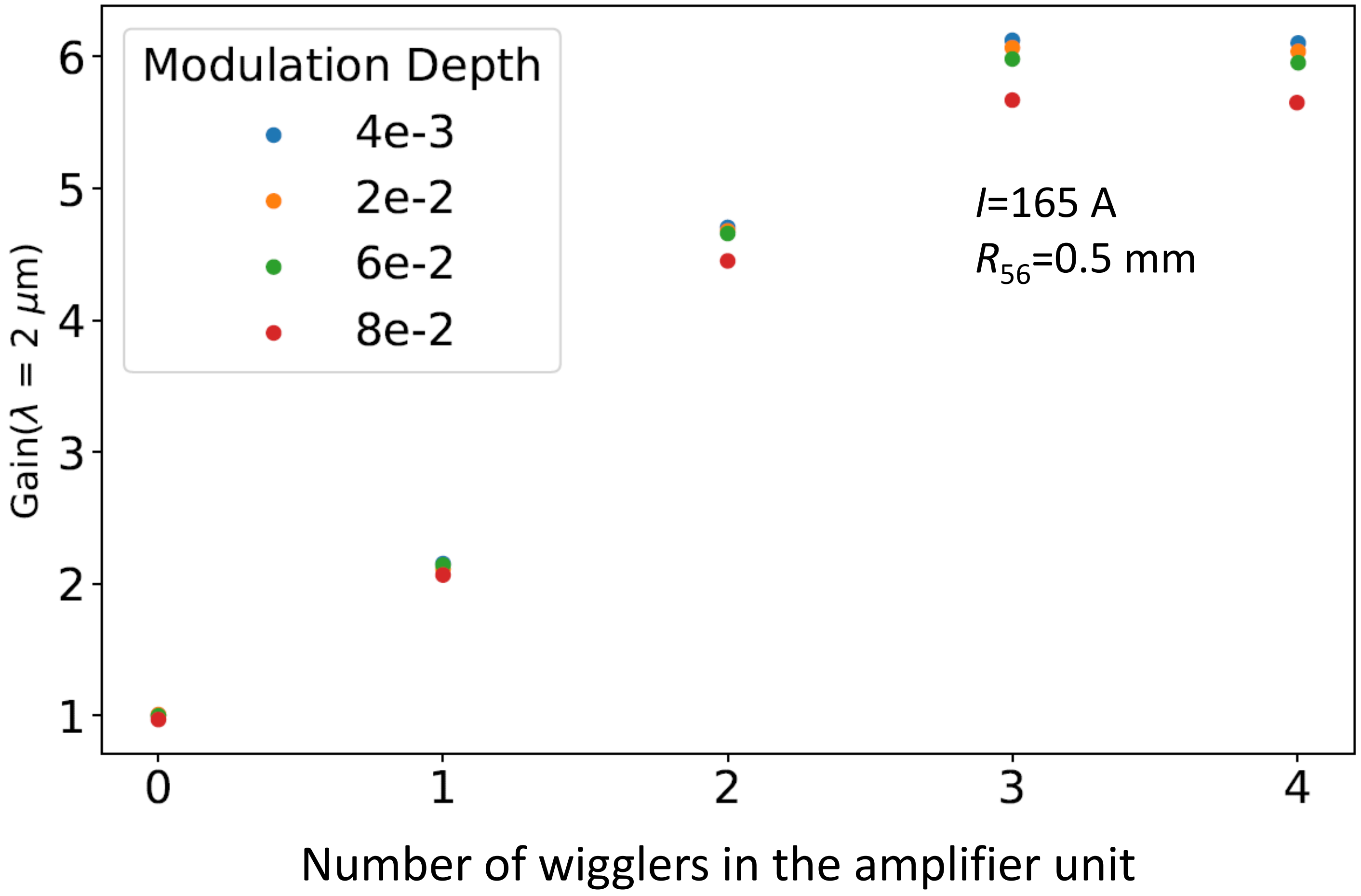
- Calculation using 64x10[6] macroparticles was used as the reference
- Solver grid resolution $\Delta x, \Delta y, \Delta z = 10\,\mu\mathrm{m}, 10\,\mu\mathrm{m}, 28.6\,\mathrm{nm}$ was used

Plots show deviations from reference for the beam energy, beam transverse size and emittance.

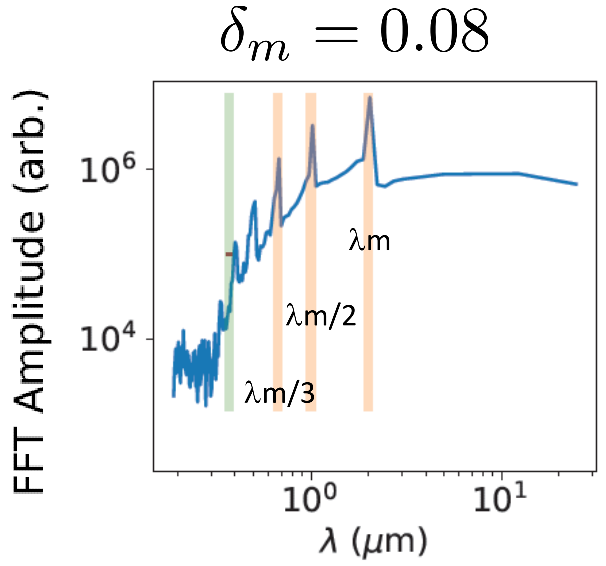


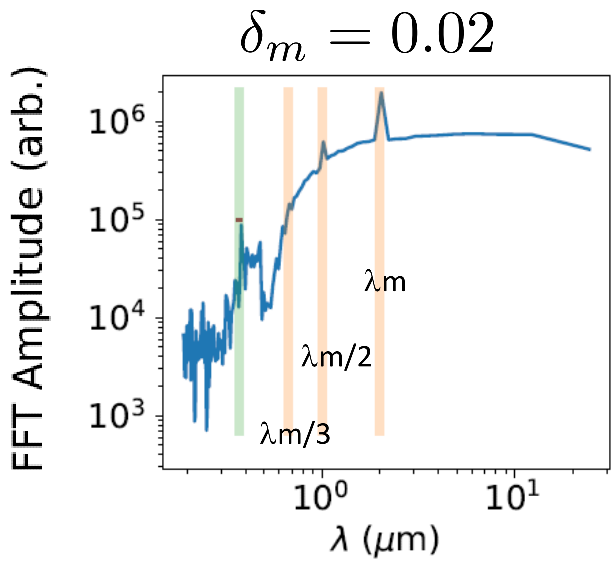
8 x 10[6] macroparticles was selected

Selection of seed modulation amplitude



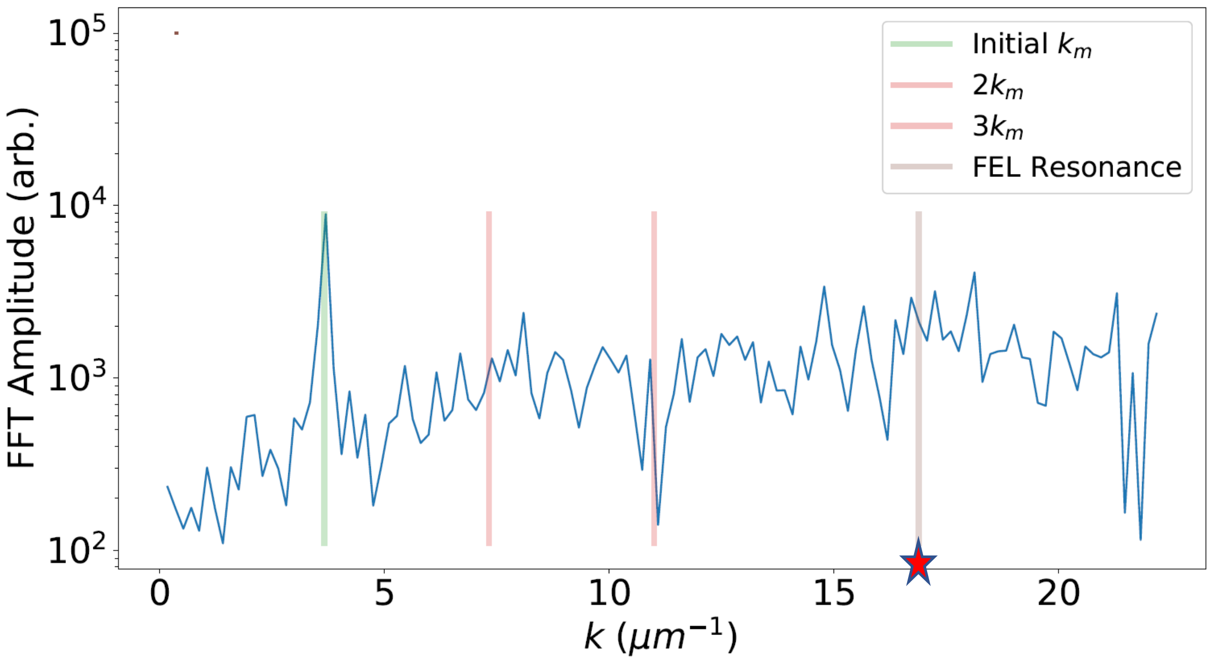
Need to avoid harmonic formation



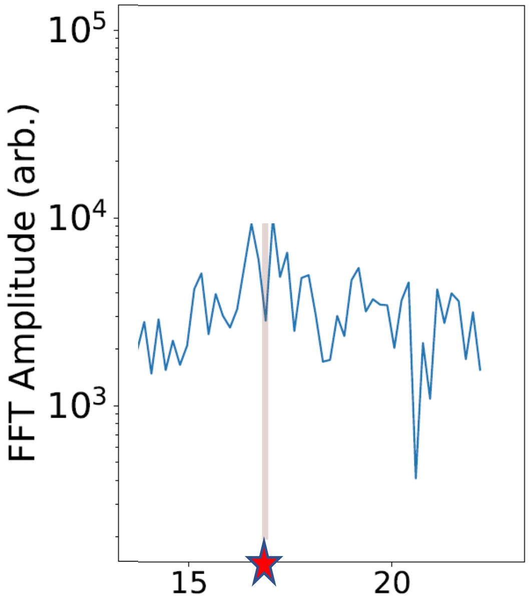


SASE FEL resonance suppression

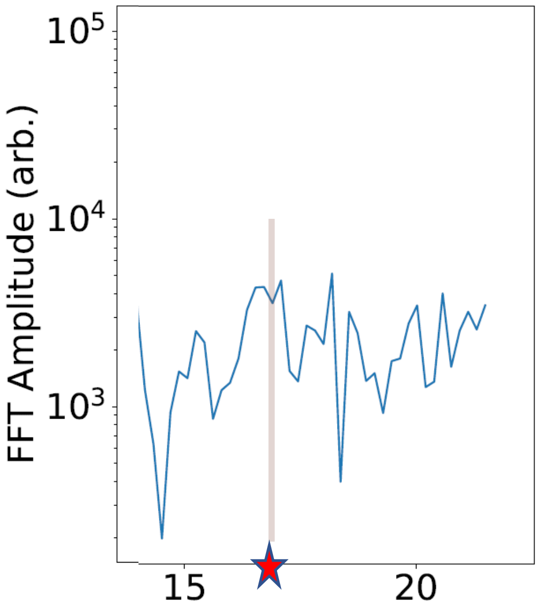
After Wiggler 1



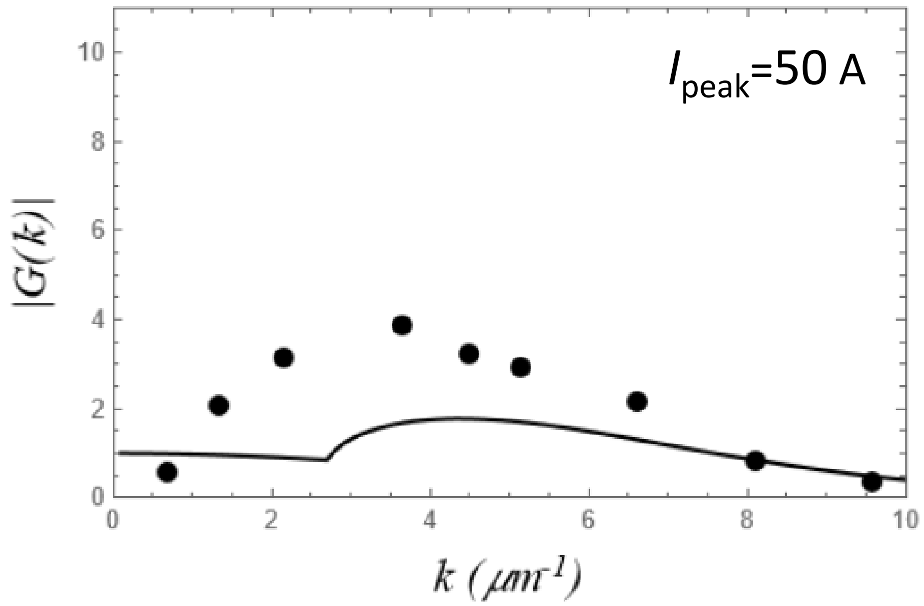
After Wiggler 3
(SASE gain=1.53/per wiggler cell)

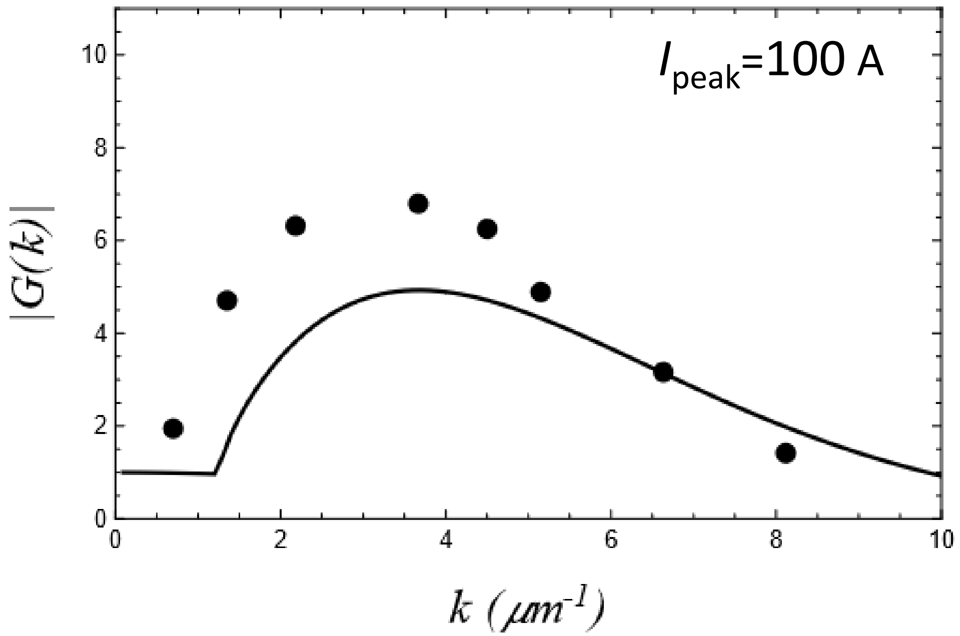


After chicanes



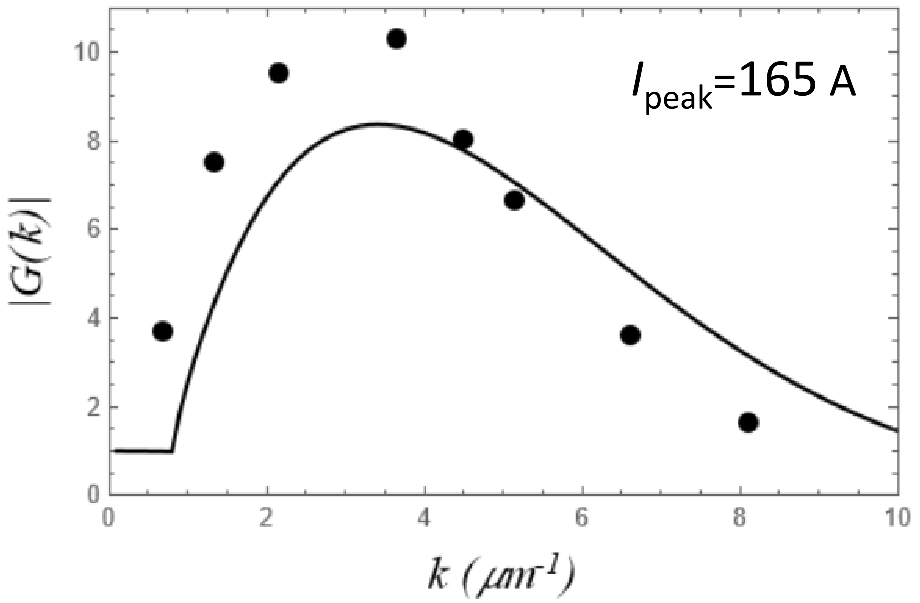
Simulation results for one amplifier unit in comparison with the theory

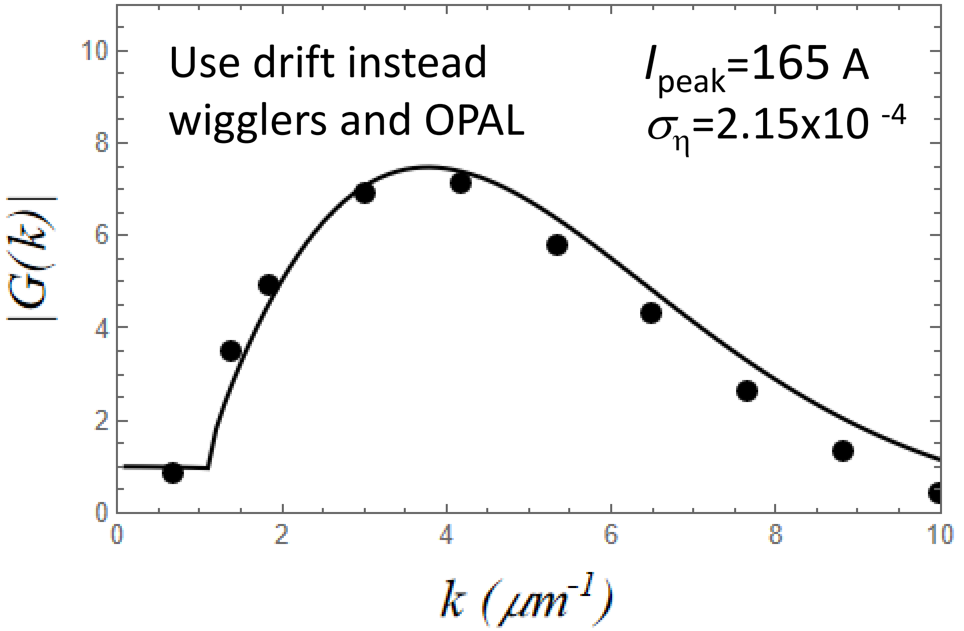




R_{56} = 1 mm
σ_η = 2.1x10 $^{-4}$

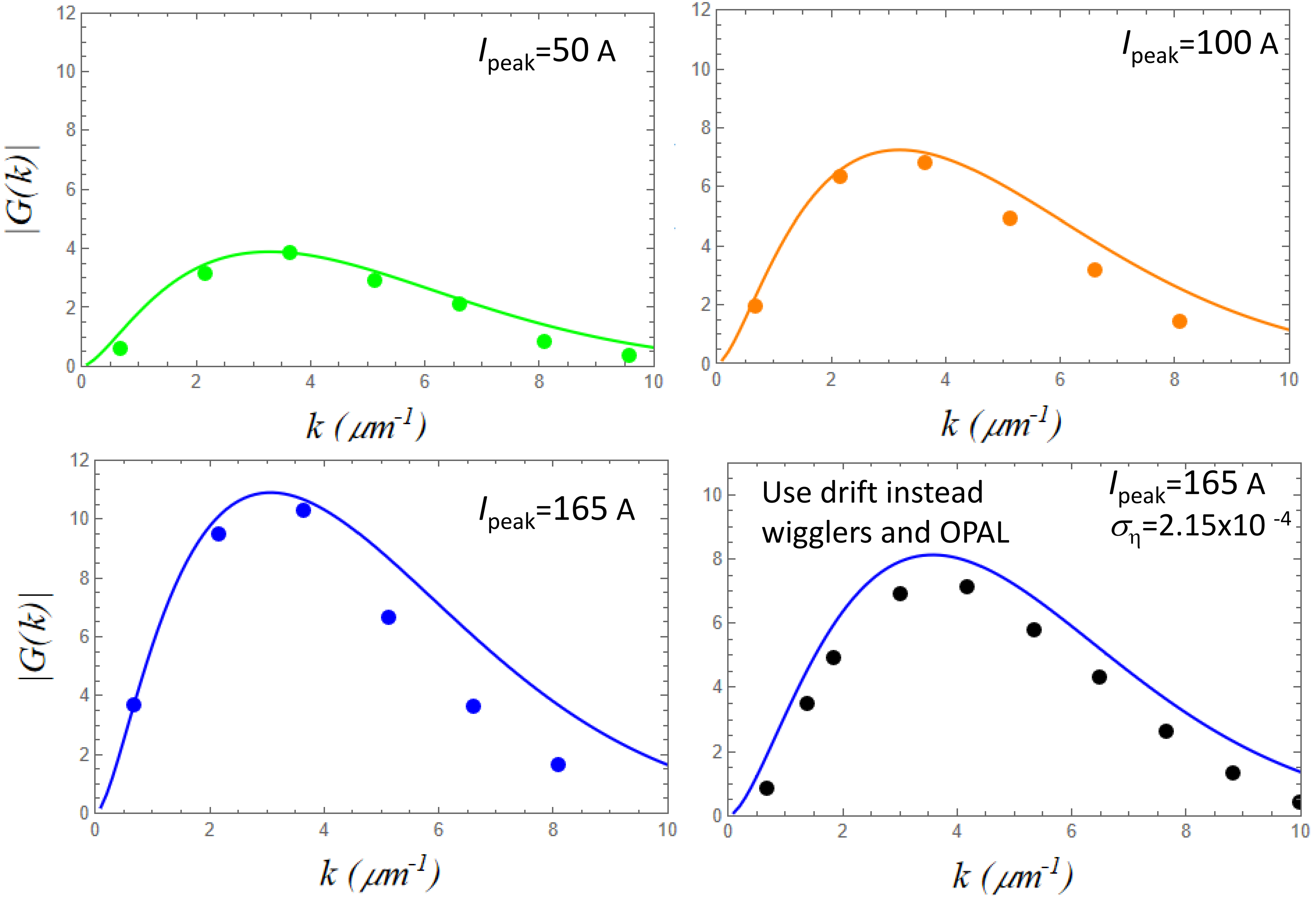
Dots – simulations
Solid lines – theory





The agreement between simulations and theory is better without wigglers

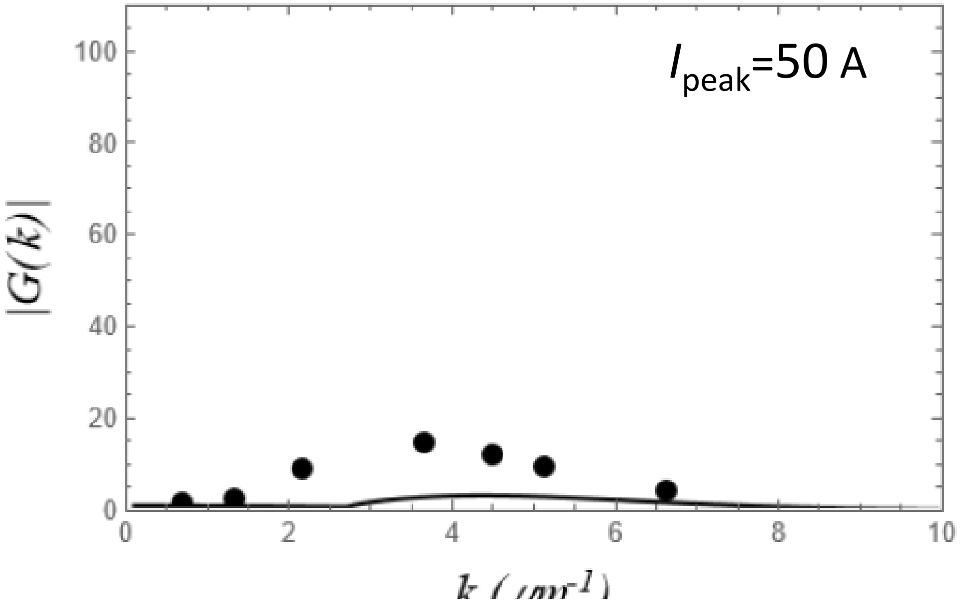
Simulations in comparison with approximate theory

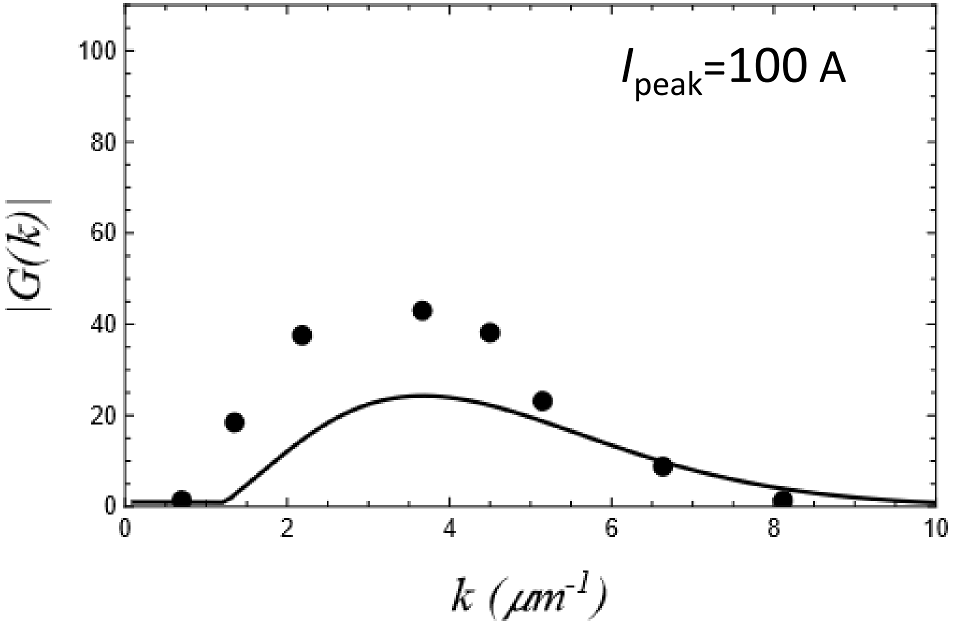


Dots – simulations
Solid lines – approximate theory

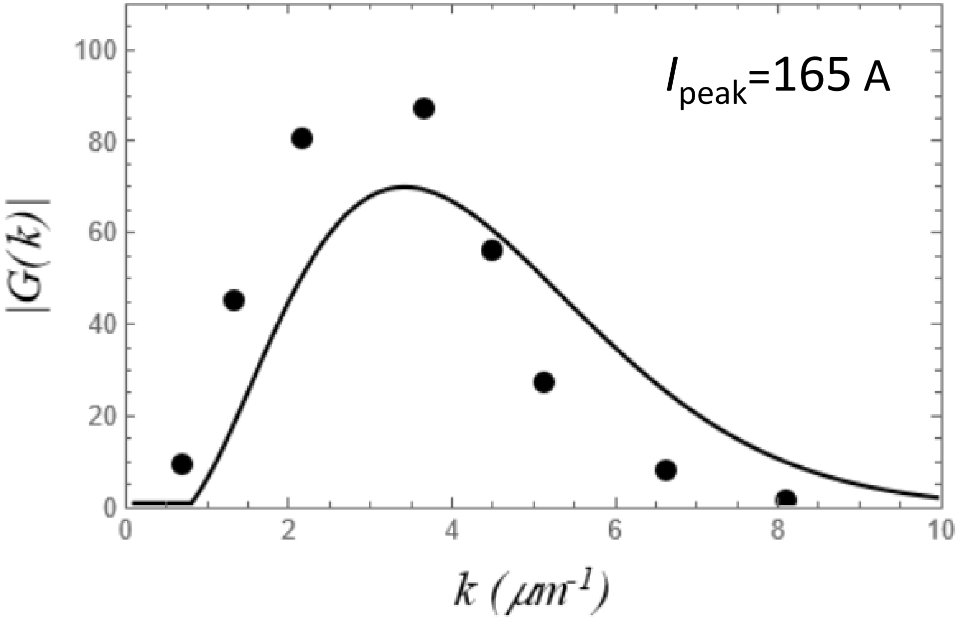
The agreement between simulations and approximate theory is better, but not in the case without wigglers

Simulation results for two amplifier units



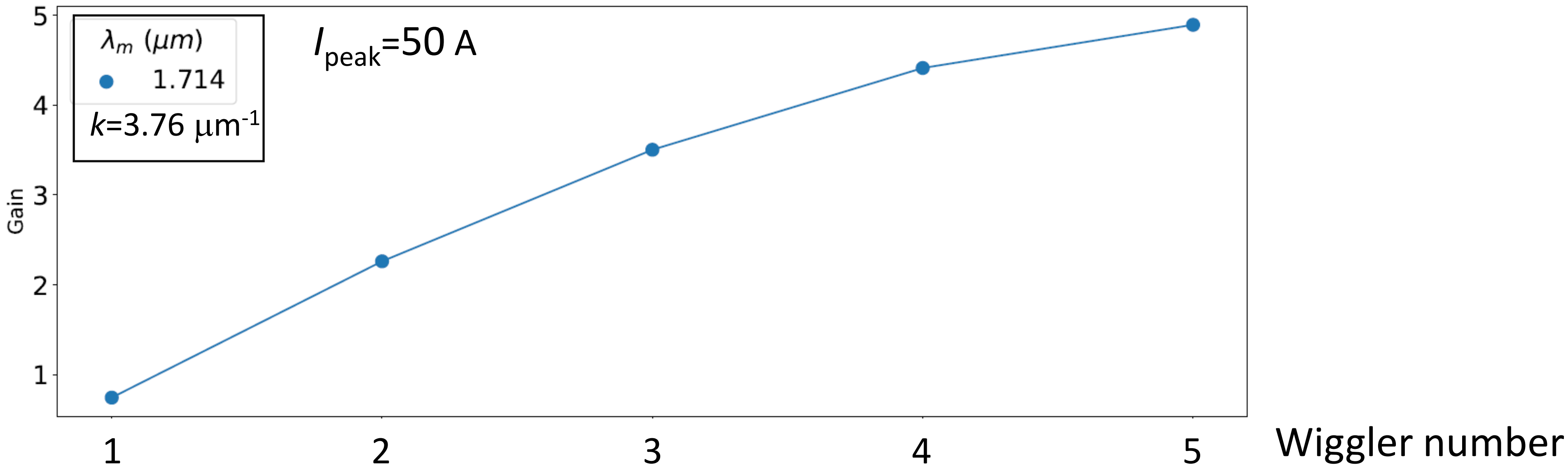


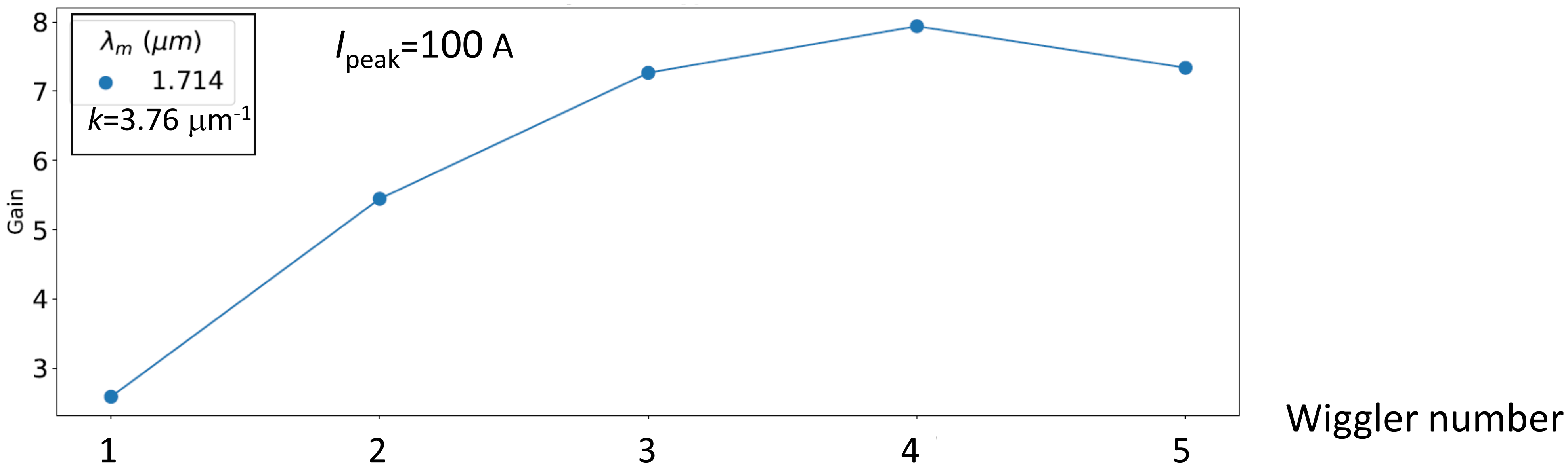
R_{56} = 1 mm
σ_η = 2.1x10 $^{-4}$



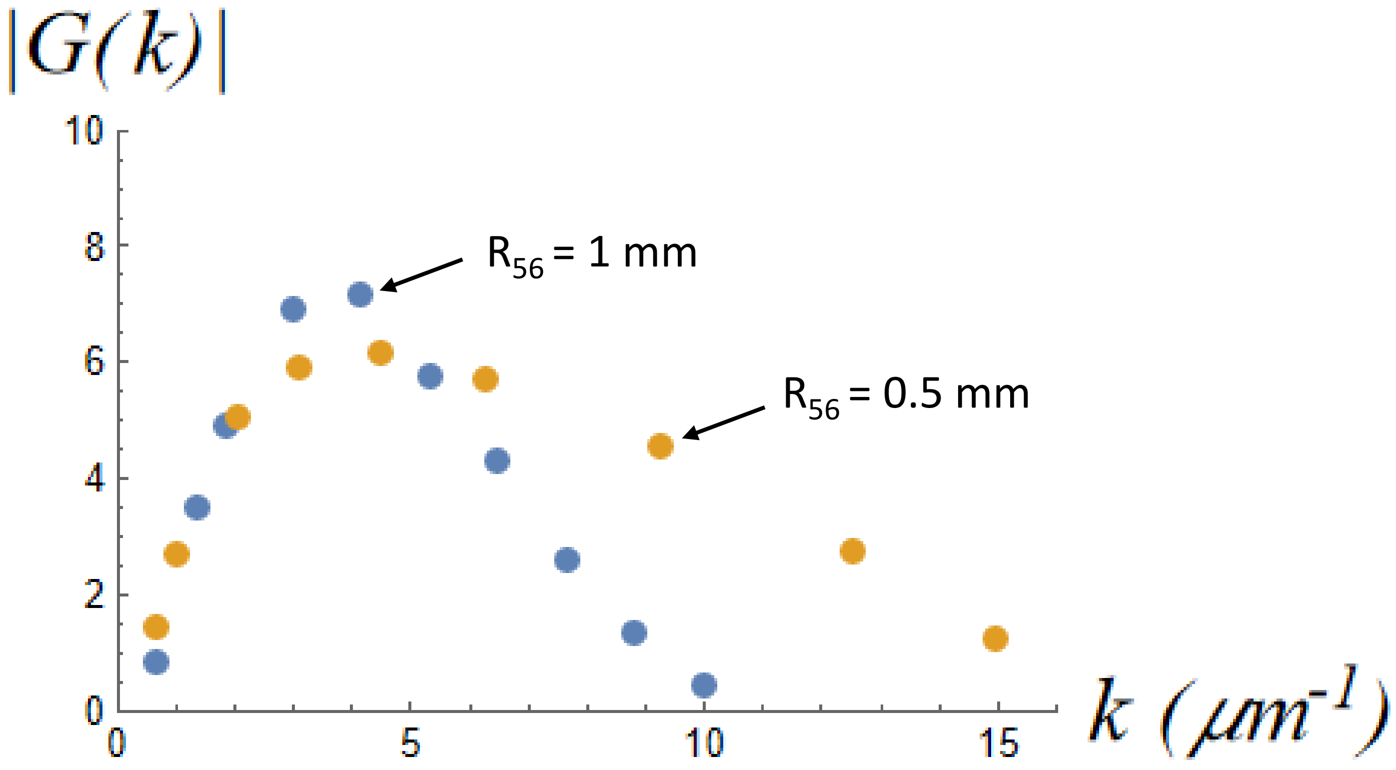
Dots – simulations
Solid lines – exact theory

Gain versus a number of wigglers in the amplifier unit (2)





Amplifier bandwidth is mostly affected by the energy spread and a little bit by CSR in chicanes



Credits

Useful discussions with Gennady Stupakov are acknowledged

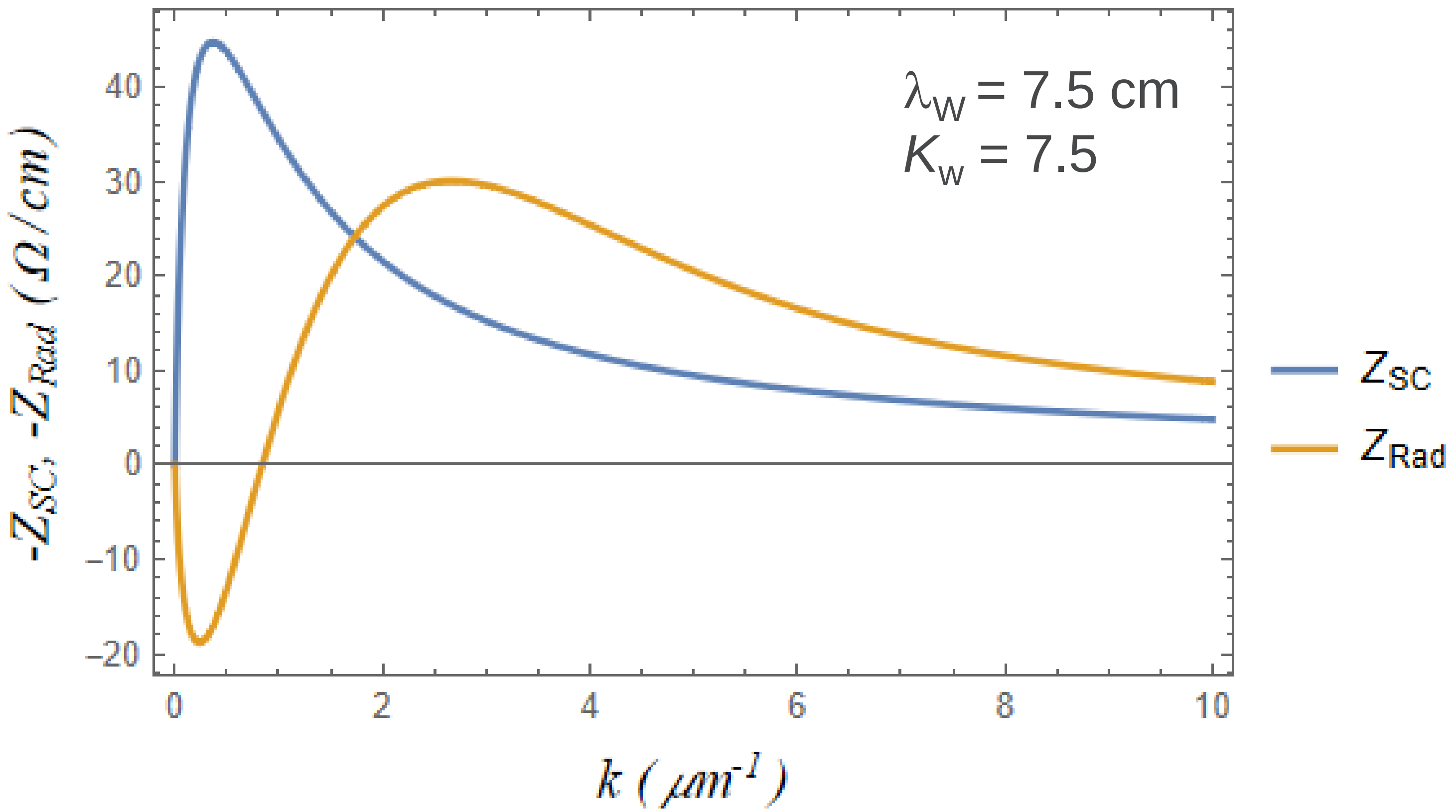
Conclusion

- WEPA is a viable candidate for the CeC amplifier. It offers a competitive gain-bandwidth product and uses smaller real estate comparing to a drift-based microbunching amplifier.
- Care was taken to control for nonphysical numerical effects and limitations in the simulation.
- Simulations and analytical calculations qualitatively agree but show some important differences.
- Similar simulations and calculations performed for the microbunching amplifier with drifts replacing wigglers show better agreement.

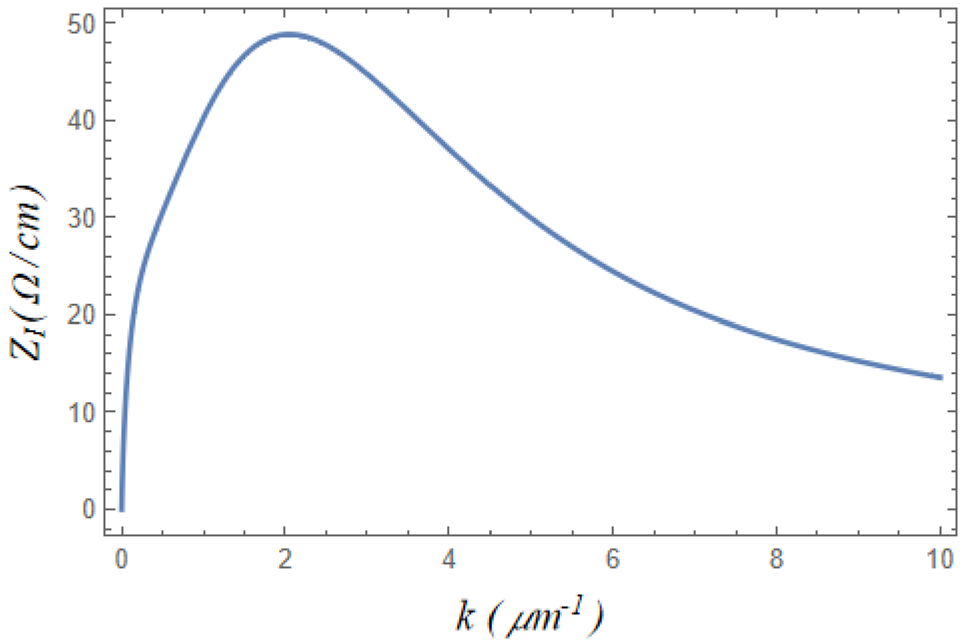
Backup slides

Case of a very large wiggler parameter

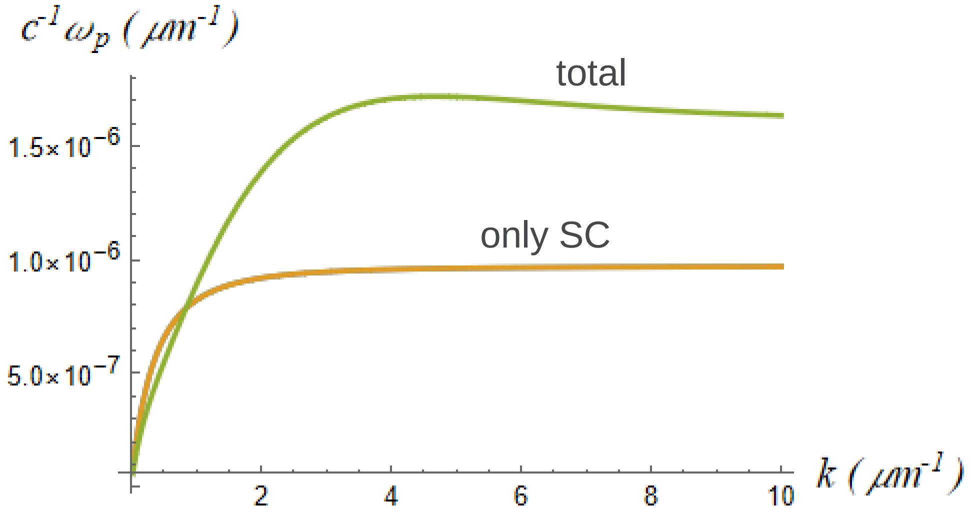
$K_w > 7.5$ such as $k_{SASE} < 0.5\ \mu m^{-1}$



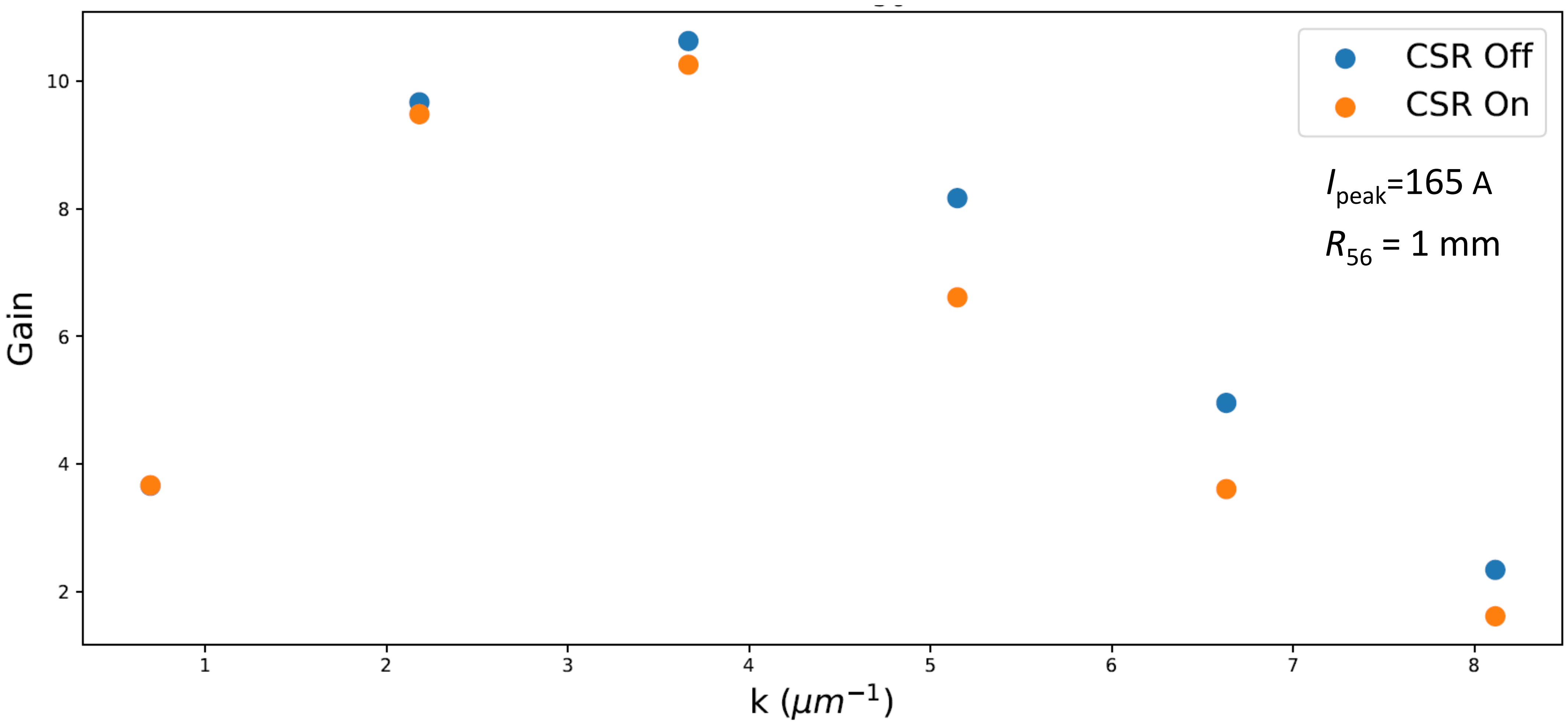
Total reactive impedance



Plasma frequency in wiggler



Chicane CSR Impact on Gain



MITHRA 1.0

Fallahi, Yahaghi, Kärtner, Computer Physics Communications, **2018**

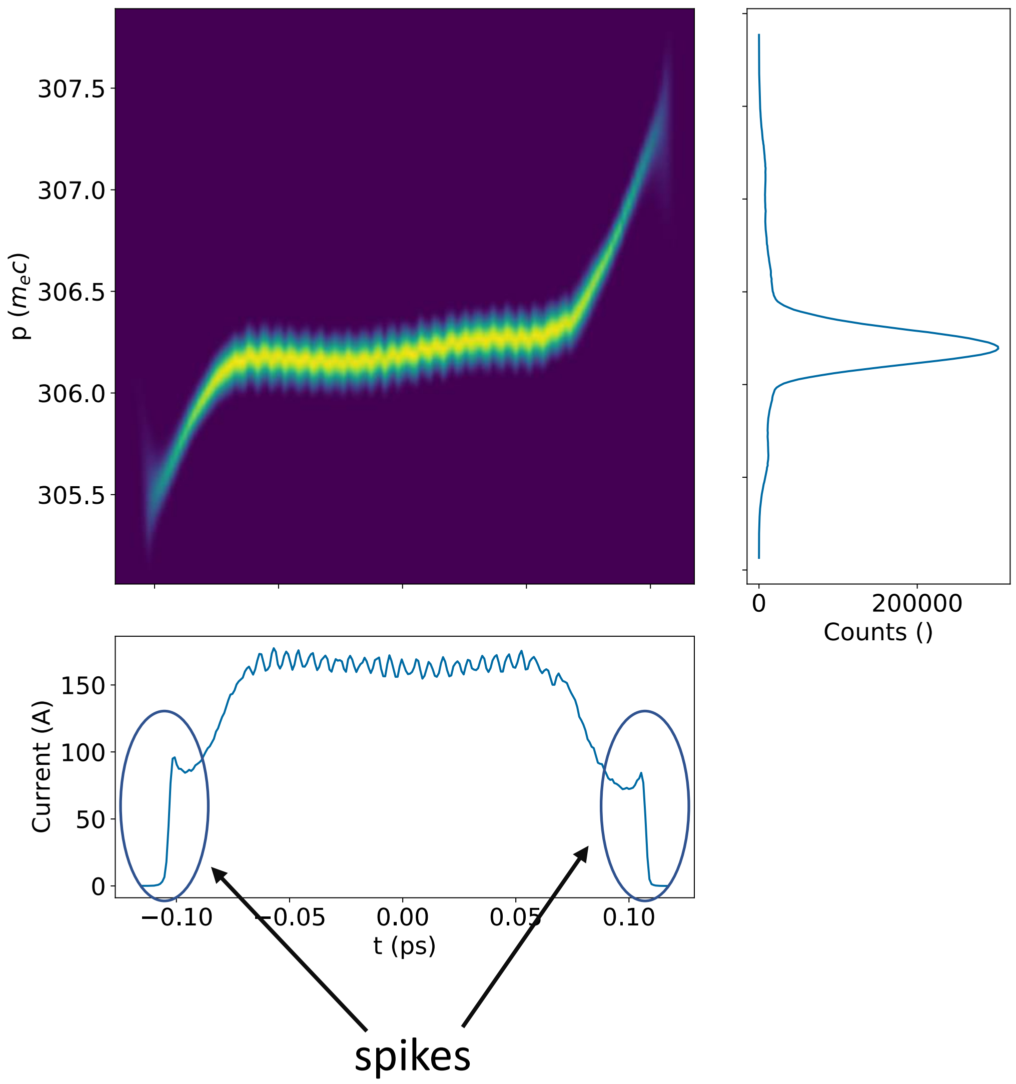
Program summary
Program Title: MITHRA
Program Files doi: http://dx.doi.org/10.17632/9f5k4zbtkg.1
Licensing provisions: GNU General Public License 3
Programming language: C++
Nature of problem: Full-wave simulation of the free electron laser radiation is accomplished by the code. MITHRA transforms the particle positions and momenta to the bunch rest frame using the Lorentz transformation. Electrons entering the undulator start radiating due to the induced wiggling motion. **The back-effect of the radiation on the bunch results in the modulation of the electron position, which in turn generates a coherent radiation.** This process as the main principle behind the operation of free electron lasers is simulated using Maxwell equations, electron motion equations and relativity principles.
Solution method: Non-standard Finite Difference Time Domain (NSFDTD) combined with Particle-in-Cell (PIC) is implemented in the Lorentz-boosted framework to calculate the FEL radiation. Parallelization is done using both multi-threading (open-MP) and message passing interface (MPI) to maximize the computation efficiency.

Unlike **OPAL**’s static solver, which can stretch and rotate the grid as necessary to tightly enclose the bunch at each time step, FDTD requires the grid to be fixed in space because each grid point is updated using its previous values, and thus it needs to enclose the whole domain where there will be EM fields of interest during the simulation. In the case of an undulator this means that the computational domain should have the same length as the undulator, usually within a range Lu=1–100 m, but should also have a grid spacing small enough to resolve the bunch of length σz=0.1–1 mm and the resonance frequency of the radi-ation λr=0.01–100 μm, which represents an order of $\sim 10^6$–10^{10} cells just for the longitudinal axis, extremely large even for the current state-of-the-art HPC clusters.

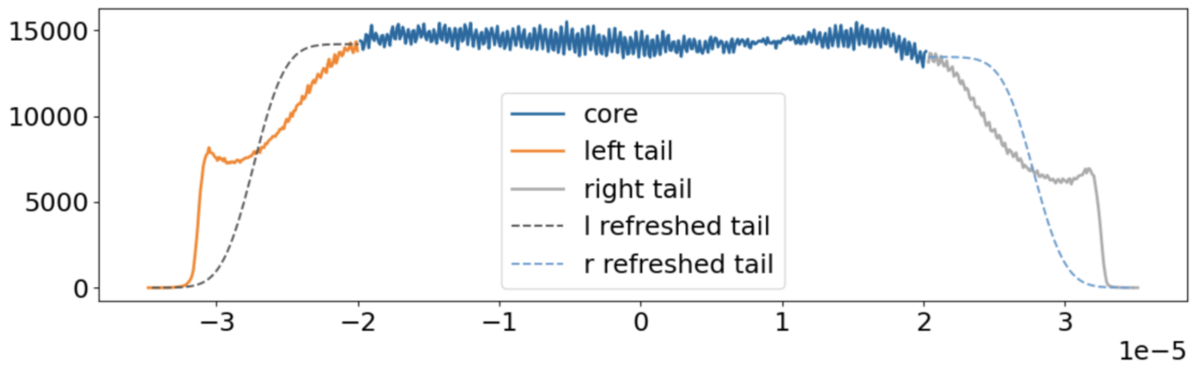
Simulation Parameters

Parameter	**Value**	**Unit**
Beam energy, $\mathcal{E}_0$	157	MeV
Peak current	50, 100, 165	A
Relative energy spread, $\sigma_{\mathcal{E}}$	2×10^{-4}	μm
Normalized emittance, ϵ_x/ϵ_y	2.2, 2.2	μm
Average beta-function in the wiggler, β_x/β_y	0.75, 0.84	m
rms beam size, σ_x/σ_y	72, 77	μm
Domain, L_x, L_y, L_z	1700, 1700, 90	μm
Grid, Δ_x, Δ_y, Δ_z	10, 10, 0.0285714	μm
Macroparticles, N_p	8e6	
Number of grid points	~ $90\text{x}10^6$	

Mitigating impact of finite bunch length

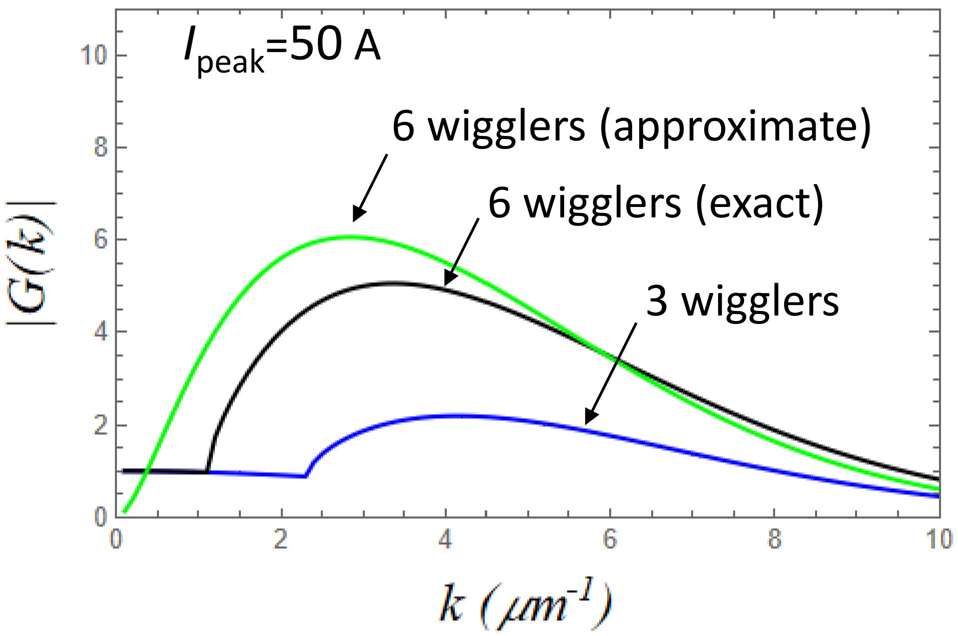


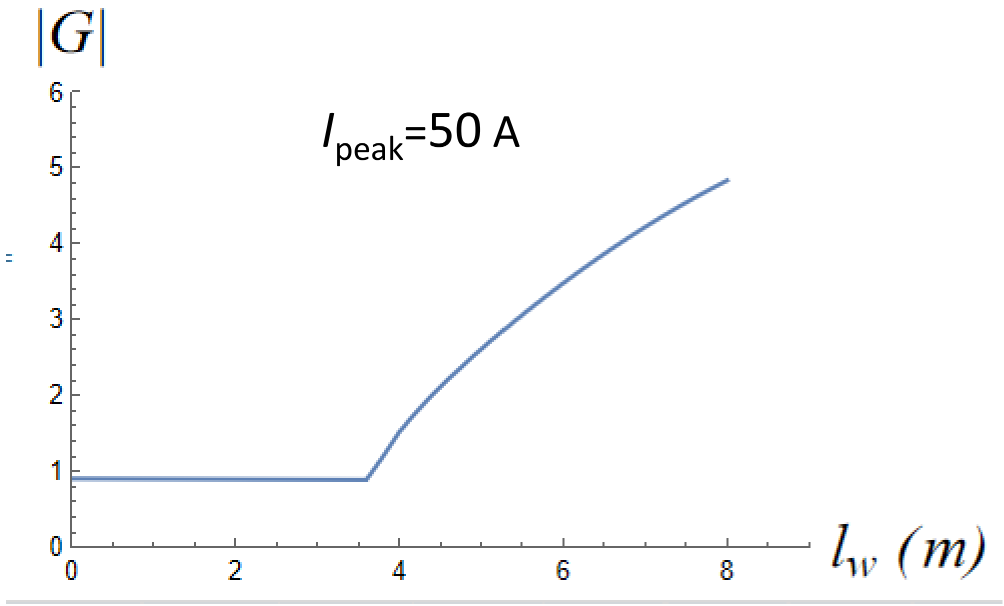
Regenerate tails after each wiggler to mitigate impact of spikes



(simulation domain is much shorter than bunch length)

Gain versus a number of wigglers in the amplifier unit



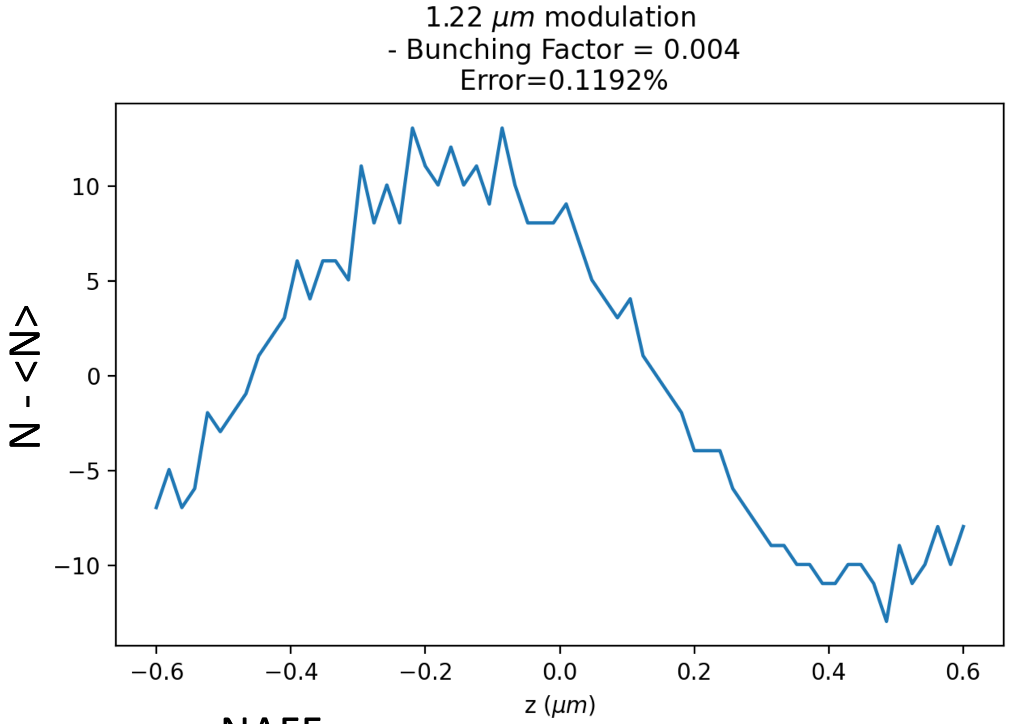


Gain determination using NAFF vs Bunching Factor

bunching factor $b(k) = \frac{1}{N} \sum_j e^{ik\zeta_j}$

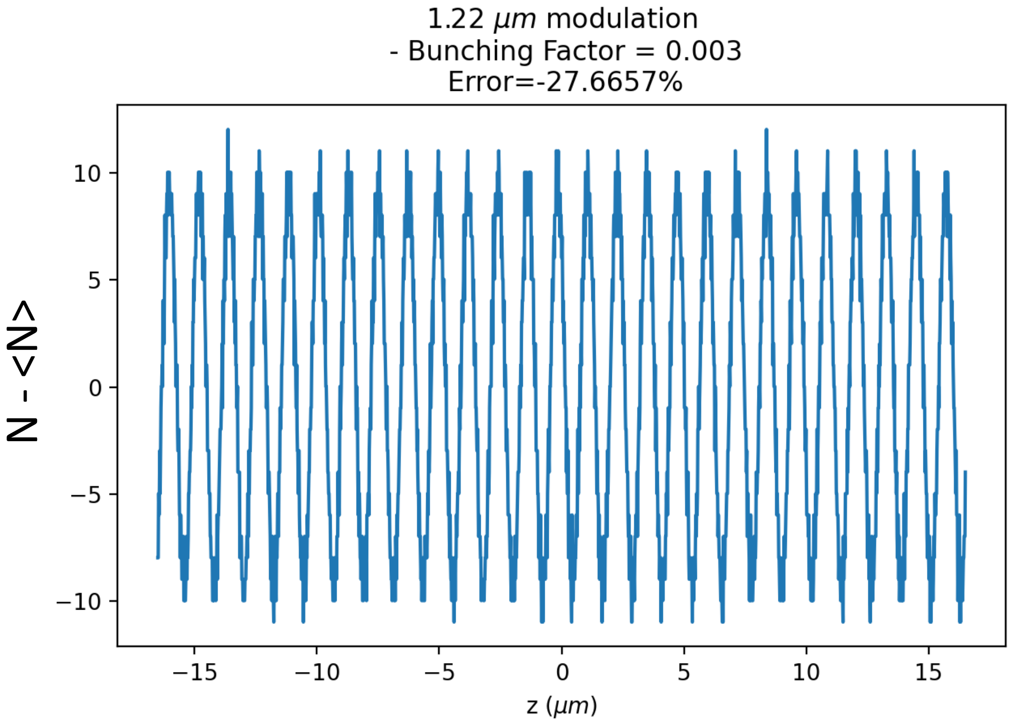
Examples using seed modulation = 0.004

Exactly 1 period of data

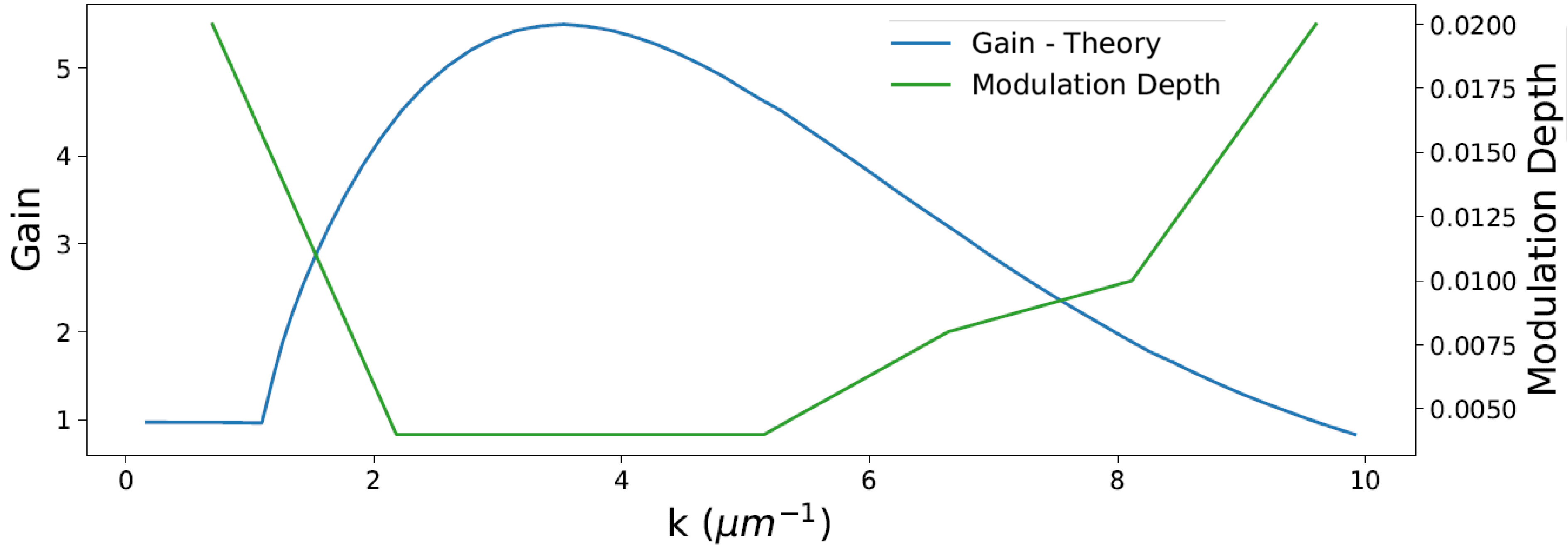


NAFF:
λ = 1.22 μm
Amplitude / Avg(counts) = 0.003979
Error = -0.5 %

Bunch core



NAFF:
λ = 1.22 μm
Amplitude / Avg(counts) = 0.003999
Error = -0.01 %



Gain - Theory
Modulation Depth
Gain
1
2
3
4
5
0
2
4
6
8
10
k (μm^{-1})
0.0050
0.0075
0.0100
0.0125
0.0150
0.0175
0.0200
Modulation Depth