Parameters optimization for EIC Ring Cooler

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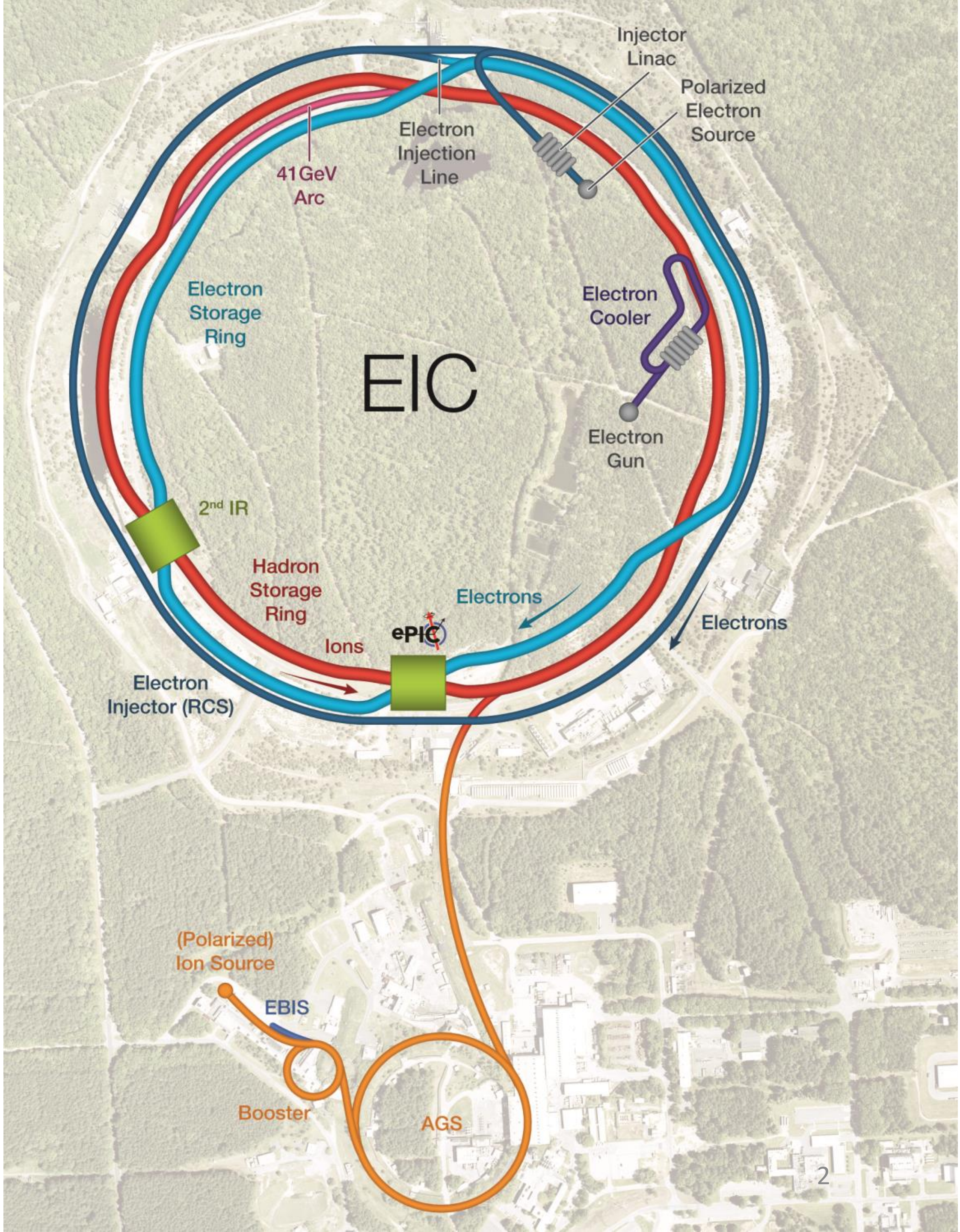
Electron Ion Collider

- The EIC requires both the precooling and the cooling at the top energy to deliver the promised luminosity
- The precooler must provide the operational beam parameters

(*A. Fedotov, MOPAM2R1, this workshop*)

- The high energy cooler must preserve the beam parameters at top energy by counteracting IBS



EIC parameters

The cooler for 275 GeV protons (with $\varepsilon_{x,y} = 11.3, 1\ \mathrm{nm}$; $\sigma_\delta = 6.8 \cdot 10^{-4}$; $\sigma_z = 6\ \mathrm{cm}$) must provide cooling times

$$\tau_x = 2\ \mathrm{hrs}$$
$$\tau_z = 3\ \mathrm{hrs}$$

Parameter		Units	p+	e-	p+	e-	p+	e-	p+	e-	p+	e-
Energy		GeV	275	18	275	10	100	10	100	5	41	5
CM energy		GeV	141		105		63.2		44.7		28.6	
Bunch intensity		10^{10}	19.1	6.2	6.9	17.2	6.9	17.2	4.8	17.2	2.6	13.3
Number of bunches		-	290		1160		1160		1160		1160	
Beam current		A	0.69	0.227	1	2.5	1	2.5	0.69	2.5	0.38	1.93
RMS normalized emittance	H	µm	5.2	845	3.3	391	3.2	391	2.7	196	1.9	196
	V		0.47	71	0.3	26	0.29	26	0.25	18	0.45	34
RMS emittance	H	nm	18	24.0	11.3	20	30	20	26	20	44	20
	V		1.6	2.0	1.0	1.3	2.7	1.3	2.3	1.8	10	3.5
Beta	H	cm	80	59	80	45	63	96	61	78	90	196
	V		7.1	5.7	7.2	5.6	5.7	12	5.5	7.1	7.1	21
IP RMS beam size	H	µm	119		9.5		138		125		198	
	V		11		8.5		12		11		27	
Kx		-	11		11		11		11		7.3	
RMS divergence	H	µrad	150	202	119	211	220	145	206	160	220	101
	V		150	187	119	252	220	105	206	160	380	129
BB parameter	H	10^{-3}	3	93	12	72	12	72	14	100	15	53
	V		3	100	12	100	12	100	14	100	9	42
RMS longitudinal emittance		10^{-3} eV·s	36		36		21		21		11	53/42
RMS bunch length		cm	6	0.9	6	0.7	7	0.7	7	0.7	7.5	0.7
RMS fractional momentum spread		10^{-4}	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8	10.3	6.8
Maximum space charge		-	0.007	Neg	0.004	Neg	0.026	Neg	0.021	Neg	0.05	Neg
Piwinski angle		rad	6.3	2.1	7.9	2.4	6.3	1.8	7	2	4.2	1.1
Longitudinal IBS time		hrs	2		2.9		2.5		3.1		3.8	
Transverse IBS time	H	hrs	2		2		2.0		2.0		3.4	
	V		Lrg		Lrg		4.0		4.0		2.1	
Hourglass factor H		-	0.9		0.9		0.9		0.9		0.9	
Luminosity		$10^{33}\ \mathrm{cm^{-2}s^{-1}}$	1.5		10		4.5		3.7		0.4	

Possible options for high energy cooling

- **EIC baseline design utilizes Coherent Electron Cooling** (with amplification based on the SC-driven microbunching instability [*G. Stupakov, Phys. Rev. Accel. Beams 21, 114402 (2018)*])
- **There are several back-up options based on regular Electron Cooling**:
- Induction Linac based Ring cooler (FNAL)

V. Lebedev et al., JINST 16 T01003 (2021)

- Dual-ring electron accelerator (JLAB)

B. Dhital et al.,Proc. IPAC21, TUXA07 (2021)

F. Lin, MOPPM1R1 (this workshop)

- ERL-based Circulator Ring (JLAB)

S. Benson et al., ERL19, LINAC20 presentations

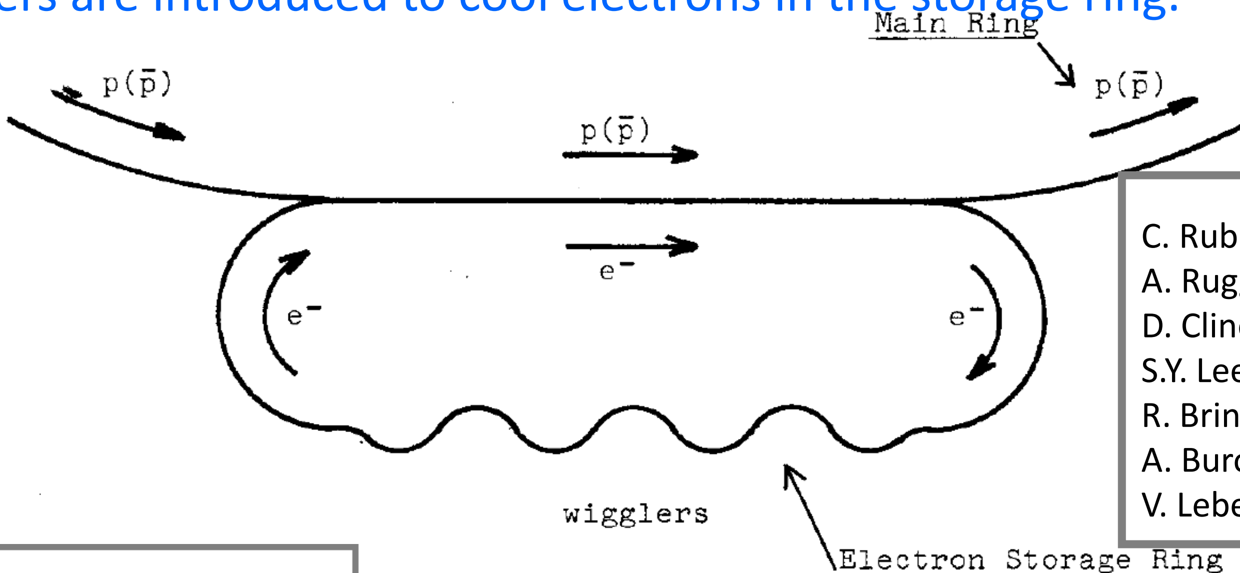
- **Storage Ring Electron Cooler (BNL)**

H. Zhao, J. Kewisch et al., PRAB 24, 043501 (2021)

Storage Ring Electron Cooler Concept

- Concept of using electron storage ring cooler for high-energy applications was considered before for various projects.

Concept: Electrons which circulate in a storage ring cool hadrons. As a result, electrons themselves are being heated by hadrons. Other heating mechanisms, such as IBS, should be also compensated. Because energy of electron is relatively low (150 MeV electrons are needed to cool 275 GeV protons in EIC), damping wigglers are introduced to cool electrons in the storage ring.

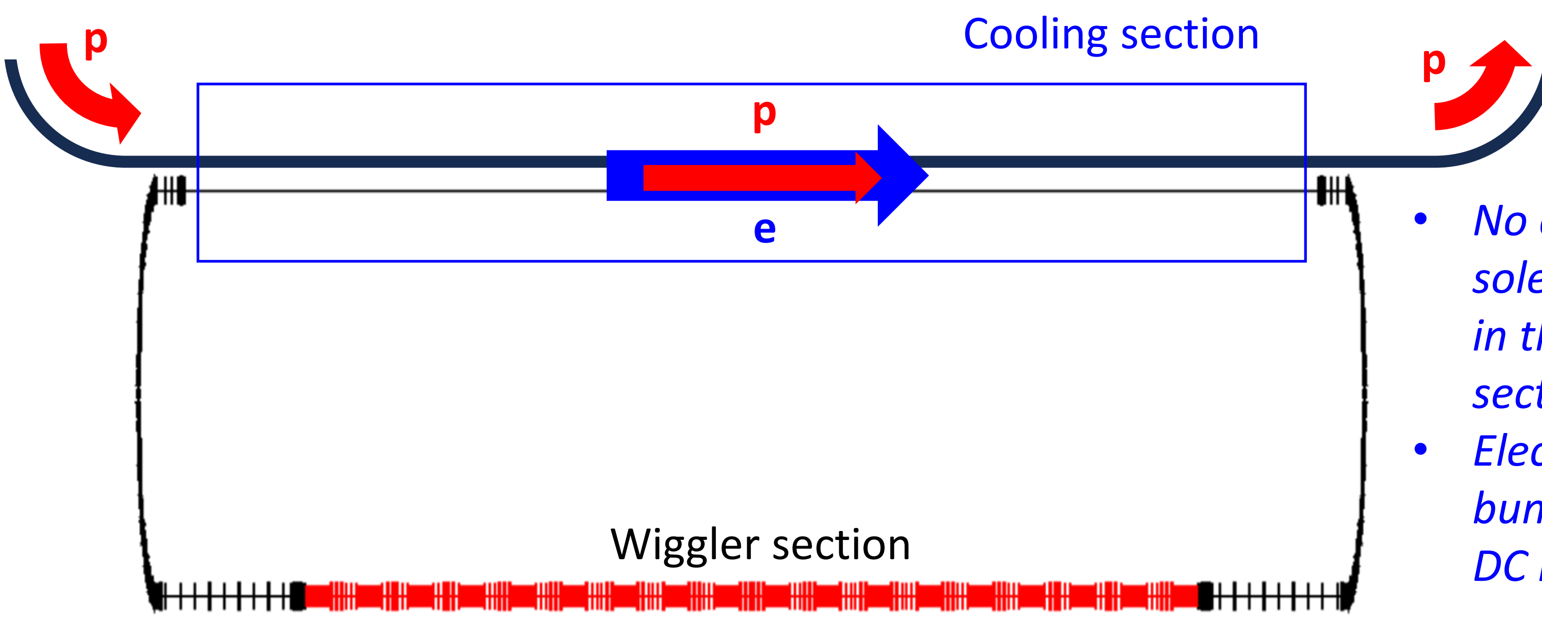


C. Rubbia, LBL-7574, 1978
A. Ruggiero, FN-311 1503, 1978
D. Cline et al., PAC'79, 1979
S.Y. Lee et al., 1997
R. Brinkmann et al., 1998
A. Burov et al., 2000
V. Lebedev, S. Nagaitsev et al., 2021

High-Energy Electron Cooling
A. G. Ruggiero
Fermilab, June 1978

EIC Ring Electron Cooler (EIC REC)

- The EIC REC is a non-magnetized, bunched electron cooler based on an electron storage ring, which utilizes damping wigglers to provide needed radiation damping for the electrons.
- REC must counteract the IBS-driven emittance growth of the proton bunches at 275 GeV ($\boldsymbol{\tau_{IBS(x,z)} = 2, 3}$ **hours**)
- **The bunched, non-magnetized cooling was successfully employed in RHIC runs 2020-2021 to cool colliding Au ions.**

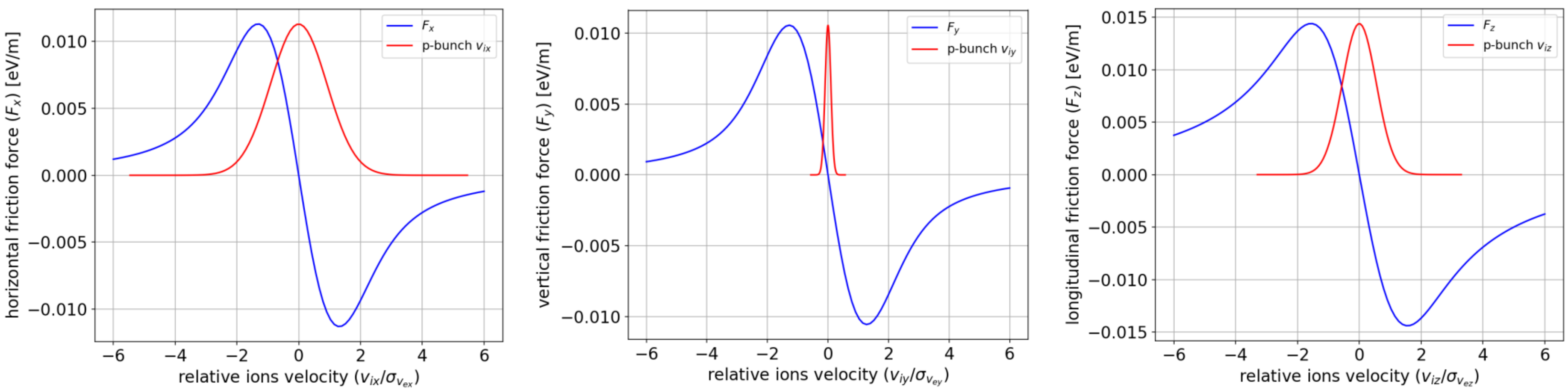


- *No continuous solenoidal field in the cooling section*
- *Electron bunches, not a DC beam*

Dynamical friction force

- In the beam frame a heavy ion traveling through a "cloud" of light electrons experiences friction:

$$\vec{F} = \frac{4\pi n_e e^4 Z^2}{m_e} \cdot \int \Lambda_c \frac{\vec{v_i} - \vec{v_e}}{|\vec{v_i} - \vec{v_e}|^3} f_{ve} d^3 v_e$$



Cooling rate for ions with "small relative amplitudes" $v_{i(x,y,z)}/\sigma_{ve(x,y,z)}$:

$$\boldsymbol{\lambda_{x,y}} = \frac{r_e^2 m_e c Z^2 \Lambda_c}{\pi A_i m_p} \cdot \frac{1}{\gamma^2} \cdot \frac{\boldsymbol{N_e}}{\boldsymbol{\varepsilon_{xn}\varepsilon_{yn}\sigma_z\sigma_\delta}} \cdot \frac{\boldsymbol{L_{CS}}}{C_r} \Phi\left(\frac{\sigma_\delta}{\gamma\sigma_\theta}\right)$$

$$\boldsymbol{\lambda_z} = \frac{2 r_e^2 m_e c Z^2 \Lambda_c}{\pi A_i m_p} \cdot \frac{1}{\gamma^2} \cdot \frac{\boldsymbol{N_e}}{\boldsymbol{\varepsilon_{xn}\varepsilon_{yn}\sigma_z\sigma_\delta}} \cdot \frac{\boldsymbol{L_{CS}}}{C_r} \left(1 - \Phi\left(\frac{\sigma_\delta}{\gamma\sigma_\theta}\right)\right)$$

S. Seletskiy, A. Fedotov, BNL-222963-2022-TECH (2022).

Limiting factors

- We would like to increase the cooling by increasing the phase space density of the e-bunch:

$$\boldsymbol{\lambda \propto \frac{N_e}{\varepsilon_{enx}\varepsilon_{eny}\sigma_{ze}\sigma_{\delta e}}}$$

- Yet, there are numerous effects limiting the achievable cooling rate:
- Electrons' space charge: $\Delta\nu_e \propto \frac{N_e}{\varepsilon_e\,\sigma_{ze}}$
- Proton-electron space charge focusing: $\Delta\nu_{pe} \propto \frac{N_i}{\sigma_{zi}\sigma_{i(x,y)}^2}\beta_{eCS}$
- The achievable equilibrium e-bunch parameters are defined by:
  - p-e beam-beam scattering: $\lambda_{BBSz} \propto \frac{N_i}{\sigma_{zi}\sigma_{i(x,y)}^2}\cdot\frac{1}{\sigma_{\delta i}\sigma_{\theta i}^2}\cdot\frac{1}{\sigma_{\delta e}^3\sigma_{\theta e}^2}$
  - e-bunch IBS: $\lambda_{IBSz} \propto \frac{N_e}{\sigma_{ze}\sigma_{e(x,y)}^2}\cdot\frac{1}{\sigma_{\delta e}^2\sigma_{\theta e}}$
  - Radiation damping: $\lambda_{damp} \propto B_{wigg}^2 L_{wigg}$

Total cooling rate

- "Small amplitude" formulas give good cooling times: $\tau_x = 50$ min and $\tau_z = 40$ min
- Total cooling rate is obtained by integrating the friction force over 6D distributions of both beams:

$$\lambda_{x,y,z} = -\frac{4\sqrt{2}}{\pi} C_1 \mathbb{S} \Psi_{x,y,z}$$

$$C_1 = \frac{N_e r_e^2 Z_i^2 m_e c^4 \Lambda_C \eta}{\gamma^2 A_i m_p}$$

If both beams have Maxwell-Boltzmann velocity distributions, then integration over velocities can be reduced to 1D integrals:

$$\Psi_x = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_{\theta x}^2)^{3/2}(1+2p^2 S_{\theta y}^2)^{1/2}(1+2p^2 S_\delta^2)^{1/2}}$$

$$\Psi_y = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_{\theta y}^2)^{3/2}(1+2p^2 S_{\theta x}^2)^{1/2}(1+2p^2 S_\delta^2)^{1/2}}$$

$$\Psi_z = \int_0^\infty \frac{p^2 dp}{(1+2p^2 S_\delta^2)^{3/2}(1+2p^2 S_{\theta x}^2)^{1/2}(1+2p^2 S_{\theta y}^2)^{1/2}}$$

$$S_{\theta x,\theta y} = \gamma\beta c\sqrt{\sigma_{\theta xe,\theta ye}^2 + \sigma_{\theta xi,\theta yi}^2}$$

$$S_\delta = \beta c\sqrt{\sigma_{\delta e}^2 + \sigma_{\delta i}^2}$$

When both beams have 3D Gaussian density distribution:

$$\mathbb{S} = \frac{1}{S_x S_y S_z}$$

$$S_{x,y,z} = \sqrt{\sigma_{xe,ye,ze}^2 + \sigma_{xi,yi,zi}^2}$$

If e-bunch have a flat-top longitudinal distribution, approximated by:

$$f_{ze}(z) = \begin{cases} \frac{1}{L_{ze}}, & -\frac{L_{ze}}{2} \le z \le \frac{L_{ze}}{2} \\ 0, & z < -\frac{L_{ze}}{2} \cup z > \frac{L_{ze}}{2} \end{cases}$$

then:

$$\mathbb{S} = \frac{1}{S_x S_y}\frac{\sqrt{2\pi}}{L_{ze}}\mathrm{erf}\left(\frac{L_{ze}}{2\sqrt{2}\sigma_{zi}}\right)$$

Redistribution of cooling rates

- To redistribute cooling between longitudinal and horizontal directions one needs:
  - Dependence of a longitudinal cooling force on a horizontal coordinate
  - Horizontal ion dispersion in the cooling section (dependence of ions' longitudinal velocity on their horizontal coordinate)
- The redistributed rates can be calculated from:

$$\begin{aligned}\lambda_{x1} &= \lambda_{x0} + k\lambda_{z0}\\ \lambda_{z1} &= \lambda_{z0} - k\lambda_{z0}\\ k &= \frac{D_i^2\sigma_{\delta i}^2 + D_i D_e \sigma_{\delta e}^2}{\sigma_{xi}^2+\sigma_{xe}^2+D_i^2\sigma_{\delta i}^2+D_e^2\sigma_{\delta e}^2}\end{aligned}$$

where λ_{x0} and λ_{z0} are given by formulas from the previous slide with the following substitutions:

$$\begin{aligned}\sigma_{\delta e} &\rightarrow \sigma_{\delta e}\frac{\sigma_{xe}}{\sqrt{\sigma_{xe}^2+D_e^2\sigma_{\delta e}^2}}\\ \sigma_{\delta i} &\rightarrow \sigma_{\delta i}\frac{\sigma_{xi}}{\sqrt{\sigma_{xi}^2+D_i^2\sigma_{\delta i}^2}}\\ S_x &\rightarrow \sqrt{\sigma_{xi}^2+\sigma_{xe}^2+D_i^2\sigma_{\delta i}^2+D_e^2\sigma_{\delta e}^2}\end{aligned}$$

M. Blaskiewicz, BNL-210932-2019-TECH, (2019).

S. Seletskiy, BNL-223860-2023-TECH (2023).

Space charge tune shift

- One can increase cooling by increasing e-bunch's charge while reducing its emittances:

$$\lambda \propto \frac{N_e}{\varepsilon_{x,y}^2 \varepsilon_z} \cdot \frac{L_{CS}}{C_{ring}}$$

- On the other hand, space charge tune shift can be estimated as:

$$\Delta\nu_{ex,ey} = \frac{I_e}{4\pi I_a \gamma^3} \int_0^{C_R} \frac{\beta(s)}{\sigma_e^2(s)} ds = \frac{I_e C_R}{4\pi I_a \gamma^3 \varepsilon_{x,y}}$$

- Depending on e-bunch longitudinal density distribution

Gaussian: $I_e = \frac{N_e e \beta c}{\sqrt{2\pi}\sigma_{ze}}$

$$\lambda_{x,y,z} = -\frac{4\sqrt{2}}{\pi} C_1 \mathbb{S} \Psi_{x,y,z}$$

Flat top: $I_e = \frac{N_e e \beta c}{L_{ze}}$

$$\mathbb{S} = \frac{1}{S_x S_y S_z}$$

$$S_{x,y,z} = \sqrt{\sigma_{xe,ye,ze}^2 + \sigma_{xi,yi,zi}^2}$$

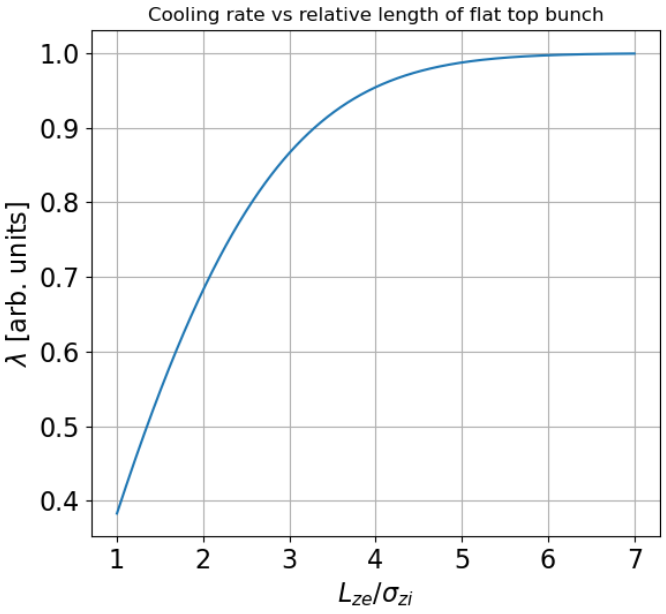
$$\mathbb{S} = \frac{1}{S_x S_y} \frac{\sqrt{2\pi}}{L_{ze}} \mathrm{erf}\left(\frac{L_{ze}}{2\sqrt{2}\sigma_{zi}}\right)$$

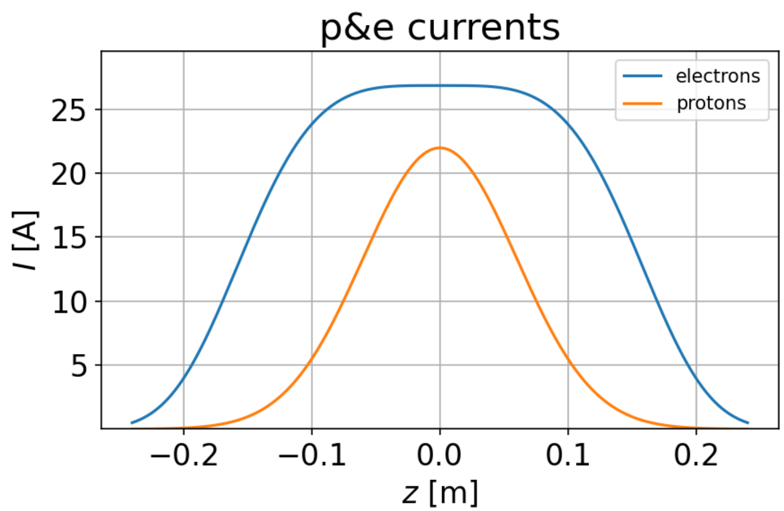
For a fixed $\Delta\nu_e$, one gets a higher cooling rate for the same N_e with a flat top bunch if one makes $L_{ze} > \sqrt{2\pi}\sigma_{ze}$

Optimal longitudinal distribution

- For a flat top (employing double-RF system) e-bunch, for a given $\Delta\nu_e$, the cooling rate dependence on bunch length becomes:

$$\lambda \propto \operatorname{erf}\left(\frac{L_{ze}}{2\sqrt{2}\sigma_{zi}}\right)$$

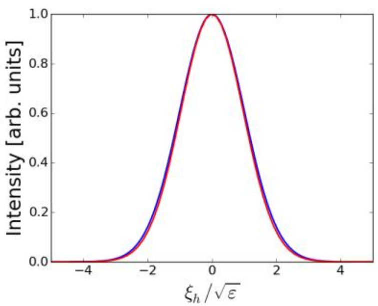




Proton-electron focusing (I)

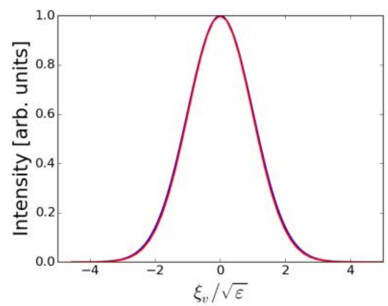
- A focusing of e-bunch by a space charge of co-traveling protons, limits a tolerable transverse size of the electron bunch in the cooling section relative to the proton bunch size
- To determine a tolerable $\Delta\nu_{pe}$, we performed tracking studies which included a realistic focusing from a proton bunch with Gaussian transverse distribution [*S. Seletskiy, A.V. Fedotov, D. Kayran, J. Kewisch, WEPA78, NAPAC2022, Albuquerque, NM, USA (2022)*]

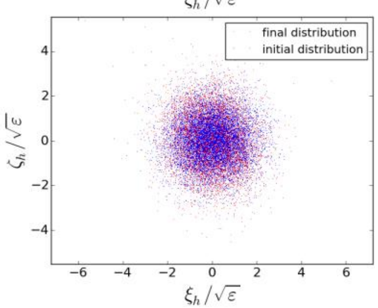
$$\Delta\nu_{pe(x,y)} = \frac{I_p \beta_{eCS(x,y)} L_{CS}}{2\pi I_a \gamma^3 \sigma_{p(x,y)} (\sigma_{px} + \sigma_{py})}$$

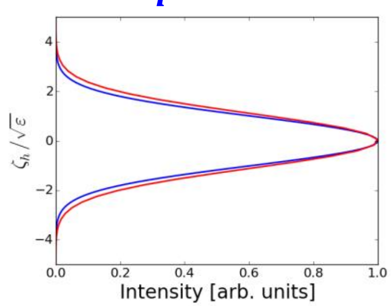


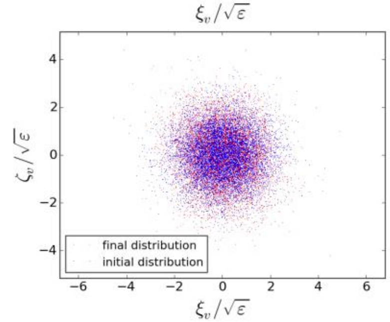
10k particles are tracked for 5k, 10k, and 20k turns

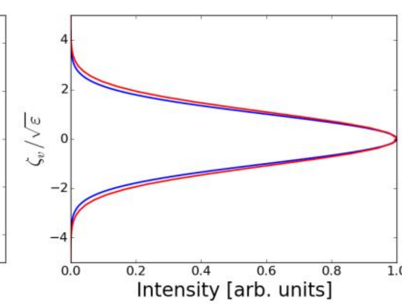
No emittance growth or particles' loss is observed for $\boldsymbol{\Delta\nu_{pe} \leq 0.07}$





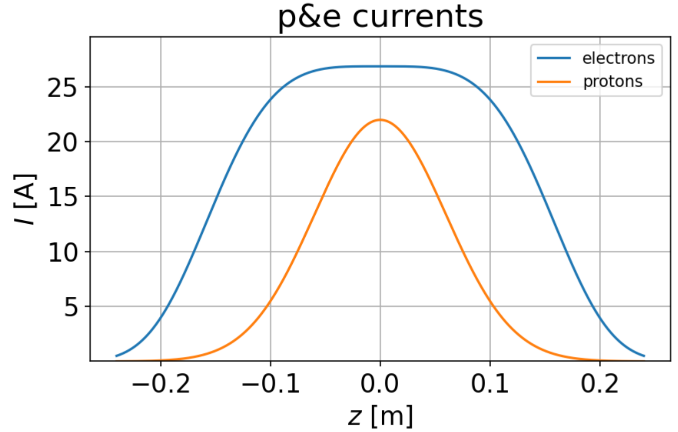






Proton-electron focusing (II)

- An additional effect of the p-e focusing results from a non-uniformity of the longitudinal distribution of a proton bunch.
- Different slices of an e-bunch see different focusing from protons.
- This unavoidable focusing mismatch can cause an "emittance dilution". The diluted equilibrium projected emittance can be estimated as:



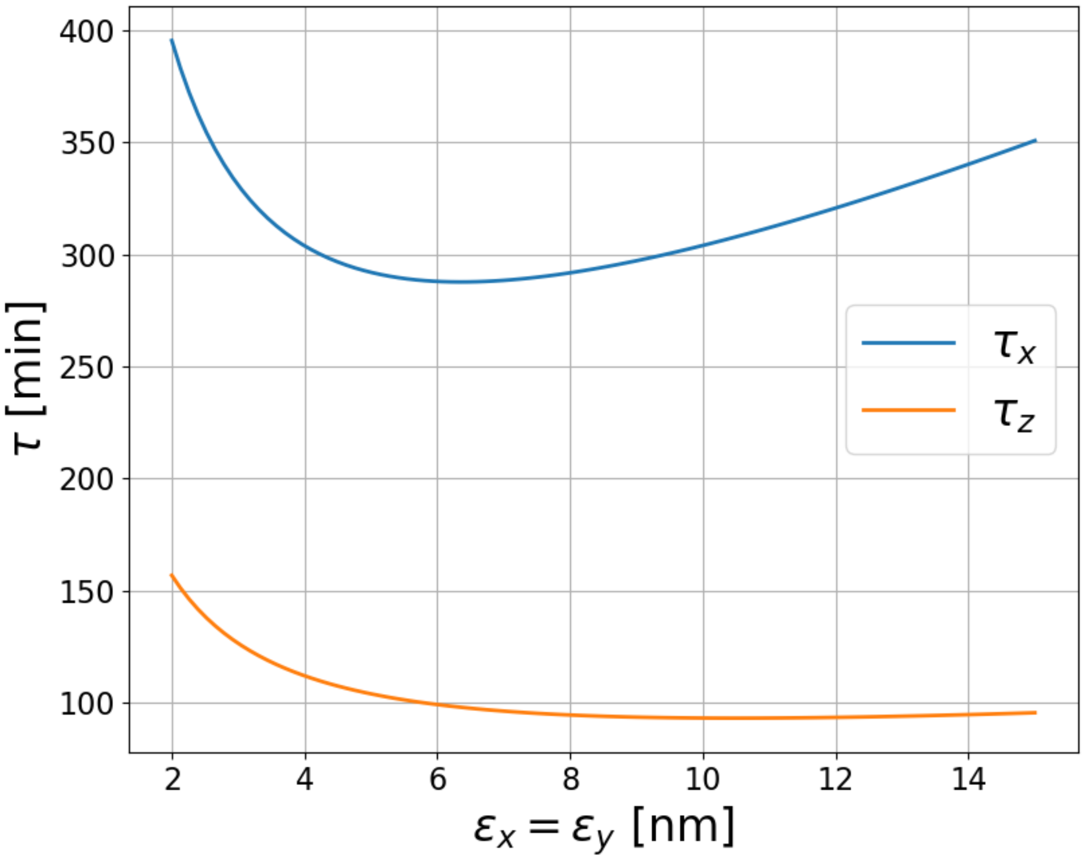
$$\frac{\langle \delta\varepsilon \rangle}{\varepsilon} = 2(\pi \Delta\nu_{pe})^2 \int_{-\infty}^{\infty} f_{ze} \left(e^{-\frac{1}{2}} - e^{-\frac{z^2}{2\sigma_{zi}^2}} \right)^2 dz$$

For $f_{ze}(z) = \begin{cases} \frac{1}{L_{ze}}, & -\frac{L_{ze}}{2} \le z \le \frac{L_{ze}}{2} \\ 0, & z < -\frac{L_{ze}}{2} \cup z > \frac{L_{ze}}{2} \end{cases}$ and $\Delta\nu_{pe} < 0.07$, $\langle \delta\varepsilon/\varepsilon \rangle < 0.015$

S. Seletskiy, S. Nagaitsev, BNL-224133-2023-TECH (2023)

Optimal e-beam parameters (I)

- We keep a constant $\Delta\nu_e = 0.2$ (i.e., now N_e is a function of ε)
- We keep β_e, β_i in the CS such that $\Delta\nu_{pe} < 0.07$
- We assume $\sigma_{pe} = 10^{-3}$

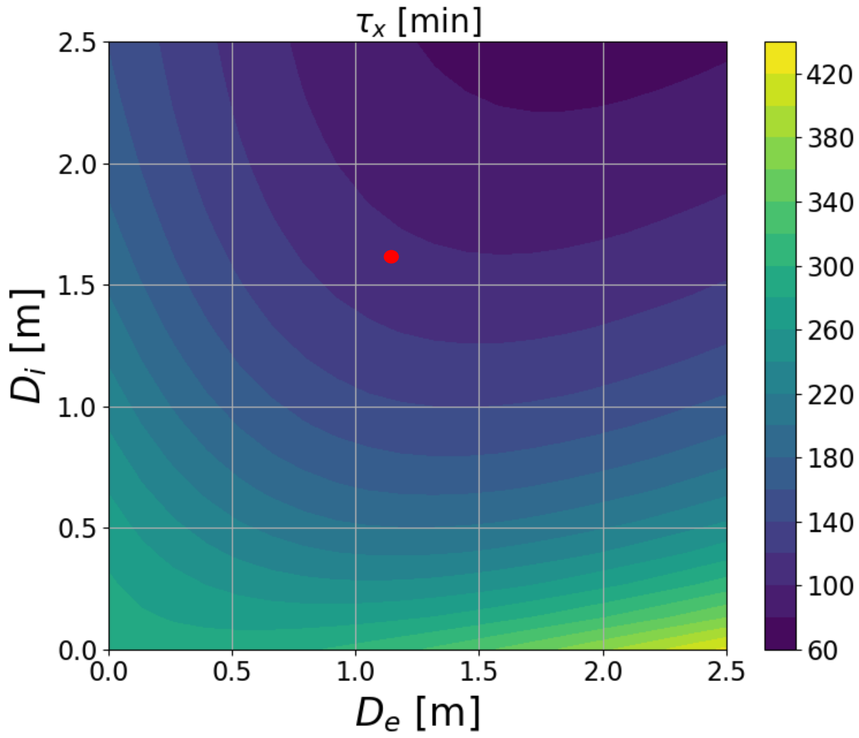


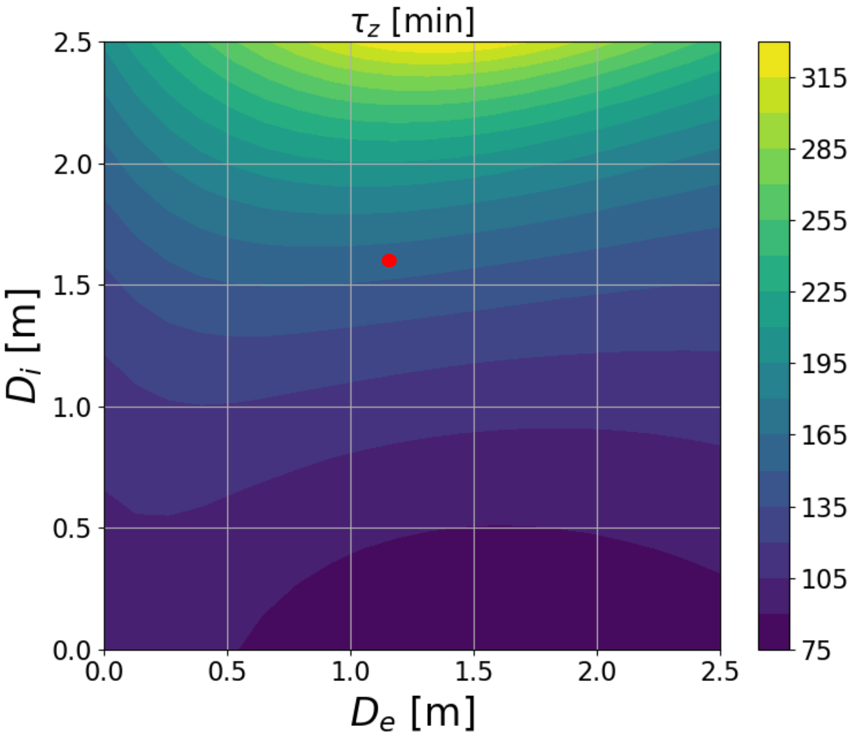
Optimal $\varepsilon = 6$ nm
(the smaller is ε,
the smaller is N_e)

protons parameters:
$\varepsilon_{x,y} = 11., 1.$ nm
$N_p = 6.9 \cdot 10^{10}$
$\sigma_{\delta p} = 6 \cdot 10^{-4}$
$\sigma_{zp} = 6$ cm

Optimal e-beam parameters (II)

- $z - x$ redistribution at $D_{ex} = 1.2$ m and $D_{ix} = 1.6$ m





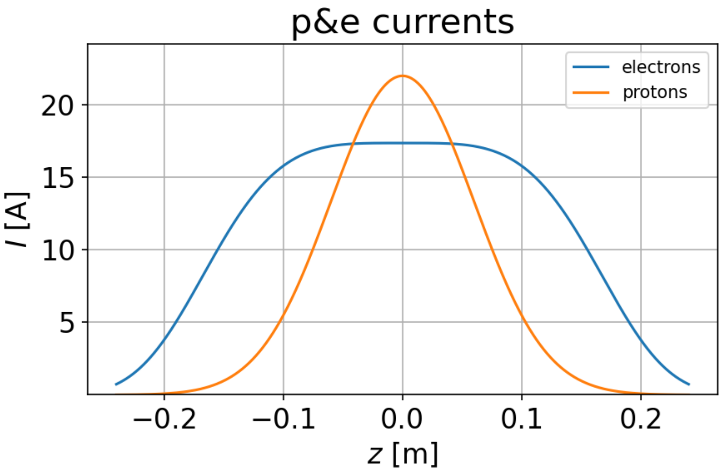
Optimal e-beam parameters (III)

- With $D_{ex} = 1.2$ m, $D_{ix} = 1.6$ m :

$\tau_x = 1.9$ hrs
$\tau_z = 2.5$ hrs
($\tau_y = 3.4$ hrs)

Optimal parameters:

$\varepsilon_{x,y} = 6$ nm
$N_e = 1.3 \cdot 10^{11}$
$\sigma_{\delta e} = 10^{-3}$
$L_{ze} = 36$ cm
$D_{ex} = 1.2$ m
$D_{ix} = 1.6$ m



Cooling rates area calculated for $\beta_{e(x,y)} = 150$ m; $\beta_{p(x,y)} = 250, 1600$ m.

Equilibrium parameters

- Electrons equilibrium emittance and energy spread are defined by a balance of the intra-beam scattering (IBS) rate, beam-beam scattering (BBS) rate, quantum excitation (small effect), and a rate of radiation damping:

$$\frac{d\varepsilon}{dt} = \left(-\lambda_{damp} + \lambda_{IBS} + \lambda_{BBS}\right)\varepsilon + C_q$$

$$C_q = \lambda_{damp}\,\varepsilon_{nat}$$

- To determine the equilibrium parameters for each of our lattice options we perform a turn-by-turn tracking with the dedicated simulation code [*H. Zhao, J. Kewisch, M. Blaskiewicz, A. Fedotov, Phys. Rev. Accel. Beams 24, 043501 (2021)*]
- Damping rate and quantum excitation depend on the ring lattice only.

$$\lambda_{damp} \propto B_{wigg}^2 L_{wigg}$$

- Yet, IBS and BBS depend on beam parameters dynamically.

Intra-beam and beam-beam scattering

- For the IBS the Bjorken-Mtingwa model with horizontal and vertical dispersion is used. The code uses fast algorithm for IBS calculation in the absence of x-y coupling [*S. Nagaitsev, Phys.Rev. STAccel.Beams 8, 064403 (2005)*]

$$\lambda_{IBSz} \propto \frac{N_e}{\sigma_{ze}\sigma_{e(x,y)}^2} \cdot \frac{1}{\sigma_{\delta e}^2 \sigma_{\theta e}} \cdot f_{IBS}(\sigma_{\delta e}, \sigma_{\theta e})$$

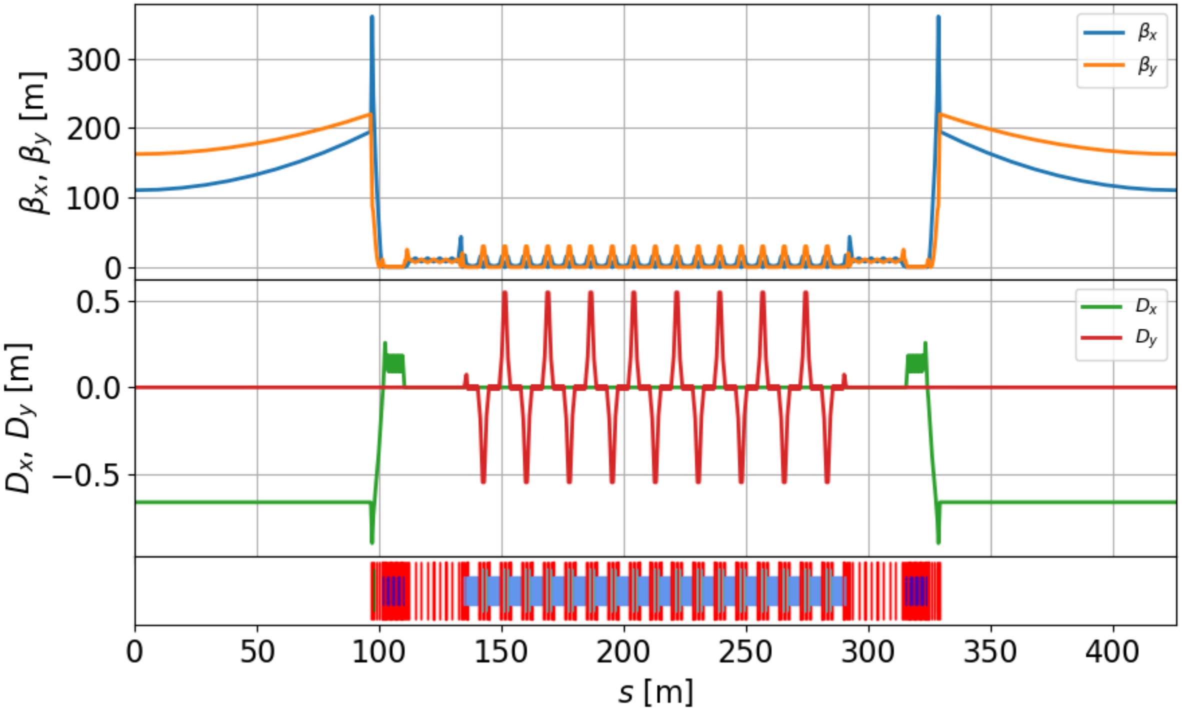
- For the BBS the model for heating of electrons due to collisions with hadrons was developed [*H. Zhao and M. Blaskiewicz, doi.org/10.18429/JACoW-NAPAC2019-TUPLM24, (2019)*]

In the cooling section:

$$\lambda_{BBSz} \propto \frac{N_i}{\sigma_{zi}\sigma_{i(x,y)}^2} \cdot \frac{1}{\sigma_{\delta i}\sigma_{\theta i}^2} \cdot \frac{1}{\sigma_{\delta e}^3 \sigma_{\theta e}^2} \cdot f_{BBS}(\sigma_{\delta e}, \sigma_{\theta e}, \sigma_{\delta i}, \sigma_{\theta i})$$

REC lattice example

J. Kewisch et al., WE4P24, Proc. FLS2023, Luzern, Switzerland, (2023)



- We developed a lattice with $B_{wigg} = 2.4$ T and overall $L_{wigg} = 76$ m
- In the CS $D_{ex} = 0.66$ m
- $\lambda_{damp(x,y,z)} = 29, 29, 57\ [\text{sec}^{-1}]$
- $\lambda_{BBS(x,y,z)} = 1, 0, 5\ [\text{sec}^{-1}]$
- $\lambda_{IBS(x,y,z)} = 28, 29, 50\ [\text{sec}^{-1}]$

We obtained for the e-bunch

$N_e = 1.9 \cdot 10^{11}$

$L_{ze} = 34$ cm

$\varepsilon_{e(x,y)} = 6, 13$ nm

$\sigma_{\delta e} = 1.3 \cdot 10^{-3}$

With $D_{ix} = 1.35$ m:

$\boldsymbol{\tau_x}$ **= 2 hrs**

$\boldsymbol{\tau_z}$ **= 3 hrs**

($\tau_y = 6$ hrs)

Currently achieved parameters

Table 1: Beam parameters in the REC cooling section

	electrons	protons
relativistic γ	293	
number of particles per bunch	$1.9 \cdot 10^{11}$	$6.9 \cdot 10^{10}$
geometric emittance (x, y) [nm]	6, 13	11.3, 1
β-function (x, y) [m]	160, 110	200, 1000
rms relative momentum spread	$1.3 \cdot 10^{-3}$	$6 \cdot 10^{-4}$
rms bunch length (Gaussian p-bunch) [cm]		6
FWHM bunch length (flat top e-bunch) [cm]	34	
horizontal dispersion [m]	0.66	1.35
cooling time (x,y,z) [hrs]		**2, 6, 3**

Table 2: The REC storage ring parameters

ring circumference [m]	426
cooling section length [m]	180
number of damping wigglers	18
damping wiggler length [m]	4.2
damping wiggler field [T]	2.4
wiggler gap [cm]	2
wiggler period [cm]	20
number of bunches	140
charge per bunch [nC]	30
peak current [A] (flat top e-bunch)	27
average current [A]	3
space charge tune shift (x,y)	0.2, 0.1
p-e focusing tune shift (x,y)	0.06, 0.06

Possibilities

- **Stronger wigglers** (increase B_{wigg} to 3.1 T)
  - This would provide $\lambda_{damp(z)} \approx 100\ \mathrm{s}^{-1}$.
  - Assuming that the main effect is on $\sigma_{\delta e}$, and fixing all other parameters, we roughly estimate $\sigma_{\delta e} \approx 10^{-3}$
  - Without any other major changes, one gets $\tau_x = 1.8$ hrs and $\tau_z = 2.9$ hrs.
- **e-bunch magnetization** (*V. Lebedev et al.,"CDR: A ring-based electron cooling for EIC", JINST 16 T01003 (2021)*)
  - Alleviates space charge effects
- **Optical stochastic cooling of electrons** *(A. Zholents, Stochastic Cooling of Electrons and Positrons With EUV Light, TUPPM1R1, this workshop)*
  - Stronger damping while using weaker wigglers

Summary

- The Ring Electron Cooler is based on proven and well-established technique of Electron Cooling
- The REC utilizes non-magnetized, bunched electron cooling to counteract the IBS-driven emittance growth of 275 GeV protons.
- The present analysis shows that the REC can provide the required cooling times

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Backup slides

Total cooling rate (special case)

If beams' angular spreads are such that $\sigma^2_{\theta xe} + \sigma^2_{\theta xi} = \sigma^2_{\theta ye} + \sigma^2_{\theta yi}$

$$\lambda_{x,y} = -\frac{C_1}{\pi S_\theta^2 S_\delta}\mathbb{S}\Phi\left(\frac{S_\delta}{\gamma S_\theta}\right)$$

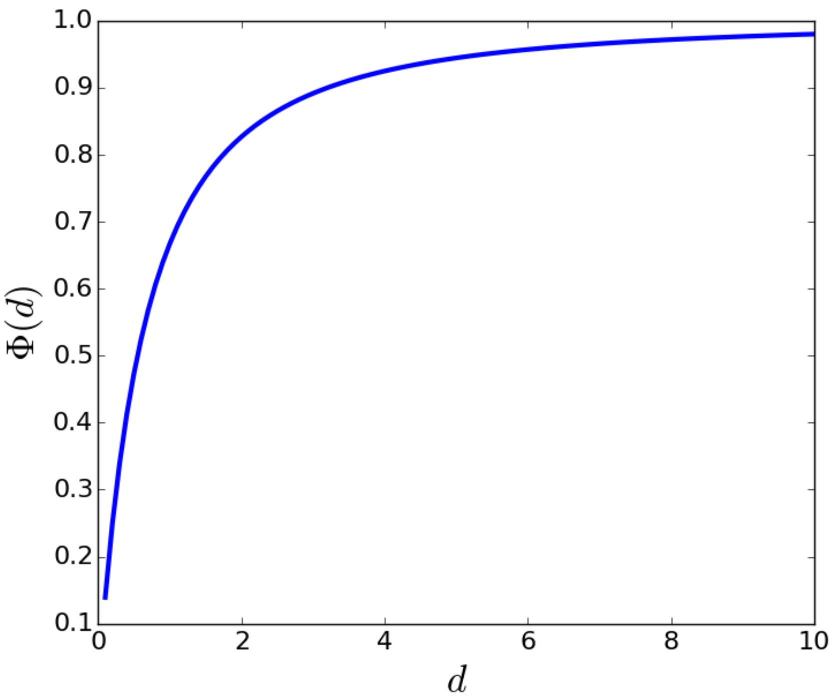
$$\lambda_z = -\frac{2C_1}{\pi S_\theta^2 S_\delta}\mathbb{S}\left[1-\Phi\left(\frac{S_\delta}{\gamma S_\theta}\right)\right]$$

$$\Phi(d) = \begin{cases} \frac{d}{1-d^2}\left(\frac{\arccos(d)}{\sqrt{1-d^2}}-d\right), & d<1 \\ 2/3, & d=1 \\ \frac{d}{d^2-1}\left(\frac{\log(d-\sqrt{d^2-1})}{\sqrt{d^2-1}}+d\right), & d>1 \end{cases}$$

$$C_1 = \frac{N_e r_e^2 Z_i^2 m_e c^4 \Lambda_C \eta}{\gamma^2 A_i m_p}$$

$$S_{\theta x,\theta y} = \gamma\beta c\sqrt{\sigma^2_{\theta xe,\theta ye} + \sigma^2_{\theta xi,\theta yi}}$$

$$S_\delta = \beta c\sqrt{\sigma^2_{\delta e} + \sigma^2_{\delta i}}$$



Redistribution (special case)

$$
\begin{aligned}
\lambda_x &= -P\left(c_{x0} + c_{z0}\frac{D_i^2\sigma_{\delta i}^2 + D_e D_i \sigma_{\delta e}^2}{D_i^2\sigma_{\delta i}^2 + \sigma_{xi}^2 + D_e^2\sigma_{\delta e}^2 + \sigma_{xe}^2}\right) \\
\lambda_z &= -P\left(c_{z0} - c_{z0}\frac{D_i^2\sigma_{\delta i}^2 + D_e D_i \sigma_{\delta e}^2}{D_i^2\sigma_{\delta i}^2 + \sigma_{xi}^2 + D_e^2\sigma_{\delta e}^2 + \sigma_{xe}^2}\right) \\
P &= \frac{\sqrt{D_e^2\sigma_{\delta e}^2 + \sigma_{xe}^2}}{\sigma_{xe}\sqrt{D_i^2\sigma_{\delta i}^2 + \sigma_{xi}^2 + D_e^2\sigma_{\delta e}^2 + \sigma_{xe}^2}\sqrt{\sigma_{yi}^2 + \sigma_{ye}^2}\sqrt{\sigma_{zi}^2 + \sigma_{ze}^2}} \\
c_0 &= \frac{N_e r_e^2 Z_i^2 c \Lambda_C \eta}{\pi \gamma^4 \beta^3}\frac{m_e}{A_i m_p}\frac{1}{\sigma_{\theta e}^2 \sigma_{\delta e}} \\
c_{x0} &= c_0 \cdot \Phi\left(\frac{\sigma_{\delta e}}{\gamma \sigma_{\theta e}}\frac{\sigma_{xe}}{\sqrt{\sigma_{xe}^2 + D_e^2\sigma_{\delta e}^2}}\right) \\
c_{z0} &= 2c_0 \cdot \left[1 - \Phi\left(\frac{\sigma_{\delta e}}{\gamma \sigma_{\theta e}}\frac{\sigma_{xe}}{\sqrt{\sigma_{xe}^2 + D_e^2\sigma_{\delta e}^2}}\right)\right] \\
\Phi(d) &= \begin{cases} \frac{d}{1-d^2}\left(\frac{\arccos(d)}{\sqrt{1-d^2}} - d\right), & d < 1 \\ 2/3, & d = 1 \\ \frac{d}{d^2-1}\left(\frac{\log(d-\sqrt{d^2-1})}{\sqrt{d^2-1}} + d\right), & d > 1 \end{cases}
\end{aligned}
$$

BBS rate

$$\lambda_{x,y,z} = C_{x,y,z}\Psi_{x,y,z}$$

$$\begin{aligned}
C_x &= \frac{r_e^2 Z_i^2 c^4 L_C n_i \eta_e}{2\pi^2\gamma\Delta_{ix}\Delta_{iy}\Delta_{iz}\Delta_{ex}^3\Delta_{ey}\Delta_{ez}} \\
C_y &= \frac{r_e^2 Z_i^2 c^4 L_C n_i \eta_e}{2\pi^2\gamma\Delta_{ix}\Delta_{iy}\Delta_{iz}\Delta_{ex}\Delta_{ey}^3\Delta_{ez}} \\
C_z &= \frac{r_e^2 Z_i^2 c^4 L_C n_i \eta_e}{2\pi^2\gamma\Delta_{ix}\Delta_{iy}\Delta_{iz}\Delta_{ex}\Delta_{ey}\Delta_{ez}^3}
\end{aligned}$$

$$\Delta_{x,y} = \gamma\beta c\theta_{x,y} \qquad \Delta_z = \beta c\sigma_p$$

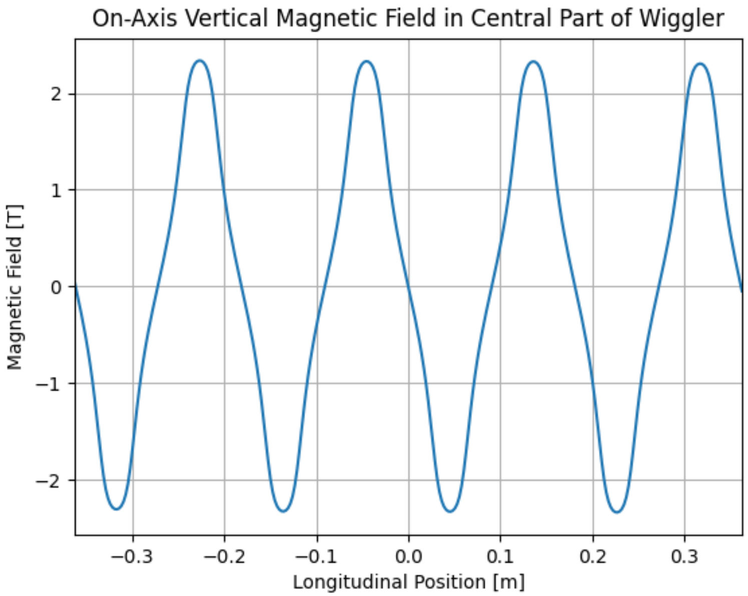
$$\begin{aligned}
\Psi_x &= \int d^3u \left(\frac{u^2-u_x^2}{\Delta_{ex}^2u^3} I2_x I0_y I0_z - \frac{u_x u_y}{\Delta_{ey}^2 u^3} I1_x I1_y I0_z - \frac{u_x u_z}{\Delta_{ez}^2 u^3} I1_x I0_y I1_z\right) \\
\Psi_y &= \int d^3u \left(\frac{u^2-u_y^2}{\Delta_{ey}^2u^3} I2_y I0_z I0_x - \frac{u_y u_z}{\Delta_{ez}^2 u^3} I1_y I1_z I0_x - \frac{u_y u_x}{\Delta_{ex}^2 u^3} I1_y I0_z I1_x\right) \\
\Psi_z &= \int d^3u \left(\frac{u^2-u_z^2}{\Delta_{ez}^2u^3} I2_z I0_x I0_y - \frac{u_z u_x}{\Delta_{ex}^2 u^3} I1_z I1_x I0_y - \frac{u_z u_y}{\Delta_{ey}^2 u^3} I1_z I0_x I1_y\right)
\end{aligned}$$

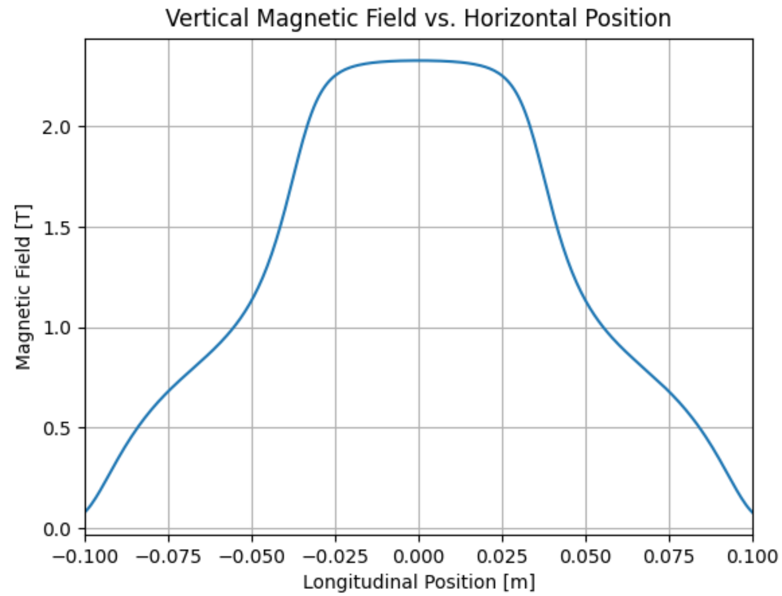
$$u^2 = u_x^2 + u_y^2 + u_z^2,\ In_m = In\left(\frac{1}{2\Delta_{em}^2}, \frac{1}{2\Delta_{im}^2}, u_m\right)$$

$$\begin{aligned}
I0(a,b,c) &= \sqrt{\frac{\pi}{a+b}}\exp\left(-\frac{ab}{a+b}c^2\right) \\
I1(a,b,c) &= -I0(a,b,c)\cdot\frac{bc}{a+b} \\
I2(a,b,c) &= I0(a,b,c)\cdot\left[\frac{1}{2(a+b)} + \frac{b^2c^2}{(a+b)^2}\right]
\end{aligned}$$

Wiggler's field map

The field-map for the "test" wiggler with 181 mm period and 20 mm gap was calculated in Radia





Courtesy of Oleg Chubar