QUANTIZED VORTICES AND TURBULENCE IN SUPERFLUIDS: EFFECTS OF THE INHOMOGENEITIES, OF THE ANISOTROPY AND STUDY OF FRACTAL DIMENSION OF TURBULENT SUPERFLOW

Maria Stella Mongiovì, David Jou, Brunello Tirozzi, Maria Letizia Bertotti, Bruno Firmani, Sergey Dobrokhotov, Michele Sciacca, Daniela Bianchi, Vittorio Ruggiero

Quantized vortices and superfluid turbulence are the object of experimental and theoretical research, because they provide direct macroscopic manifestations of quantum features. There are various superfluids where quantized vortices and turbulence can be present: He4 and He3 liquid helium, Bose-Einstein condensates, neutron stars, and so on. In superfluid helium 4, three of the most well known situations are turbulence in a rotating cylinder, turbulence in counter-flow (i.e. in the presence of heat flux) and turbulence in co-flow (i.e. in the presence of mass flux).

In the recent years the study of turbulence in superfluids has received new attention, because of its similarity with classical turbulence. Our research group, which includes Italian, Spanish and Russian researchers, is very interested in supporting with theoretical and computational studies the forthcoming CERN initiative in cryogenic experiments. The possible contributes of our research in this project are the following ones:

FORMULATION AND MATHEMATICAL ANALYSES OF MODELS OF TURBULENT SUPERFLUIDS, including

1) Description of anisotropy effects of the container, and of the orientation of vortex lines on the structure and the evolution of the vortex tangle and the vortex distribution in rotating superfluids in the presence of a counterflow or a coflow across the superfluid system. Fractal effects related to the structure of the vortex tangle will be taken into account, if necessary.
2) Influence of nonlocal terms on the evolution of the vortex tangle, in the presence or absence of rotation. We will study the role of the perturbations in the boundaries as sources of vortices, and the transport of these vortices into the bulk of the superfluid. This study of coflow situations, and its relation to an effective viscosity of the turbulent superfluid, will allow us to deal more realistically with the cryogenic microflows of superfluids.
3) Study of inhomogeneous vortex distribution of vortex lines, in the presence or absence of rotation, by taking into account the diffusion of vortex lines towards an homogeneous situation and the role of diffusion in the motion of vortex lines from the walls towards the bulk of the system. We will compare the diffusive behaviour with a drift behaviour determined by centripetal forces on the vortices due to the rotation of the sample, and to the repulsive force amongst vortices.
4) Study of the interaction between particles, superfluid vortex tangle and normal component. Analysis of the influence of the vortex tangle on the Brownian motion and diffusion of the particles.
5) Computation of fractal dimension of turbulent flows in cryogenic Helium, including correlations of the lagrangian variables, by using the perturbation methods of quantum field theory, with regards to the possible explanation of the anomalous exponents

measured in the experiments of cryogenic Helium. We will use the WKB method with suitable averaging. The representation of the global solution by means of the Maslov canonical operator could give interesting new results especially in the experiments where the turbulence of the fluid cannot be described simply in terms of an advection equation for physical observables.

We will study both counterflow situations (vanishing mass flux and nonvanishing heat flux) and coflow situations (nonvanishing mass and vanishing heat flux) and the coupled situation. This would provide a wider basis to compare classical turbulence with quantum turbulence, as classical turbulence is produced by mass flow. In all these cases, we want to compare our theoretical analysis and numerical results with as many experimental data available.

INTEREST OF THE RESEARCH

1) The study of the interaction between the fluxes of particles, the superfluid vortex tangle and the normal component may allow us to better interpret experiments in superfluids helium which uses the Particles Image Velocimetry.
2) From a practical perspective, we mention the use of superfluids for the refrigeration of nanosystems, as small electronic circuits in supercomputers. The study may offer a more detailed basis for a formulation of microfluidics of cryogenic superfluids, because it will outline the role of the walls in very narrow capillaries. We want to follow this aspect, for which the influence of walls and the dynamics of the superfluid on the boundaries may be especially relevant.
3) In astrophysics, superfluid neutron stars provide a challenging physical example for the application of the coupling of rotation (which is very fast in neutron stars) and heat flow. Our studies may be useful to analize some features of the rotation and the structure of such stars. Furthermore, the knowledge of the surface perturbations in a nonsteady situation –for instance, in the acceleration of the rotation rate- may be of interest.
4) From a thermodynamic perspective, our study will allow to incorporate the analysis of quantum turbulence in the framework of non-equilibrium thermodynamics. Furthermore, it will foster the comparison between macroscopic evolution equations for the superfluid and the current microscopic models being studied for the formation and destruction of quantum vortices forming the turbulent tangle. Information theory and maximum-entropy arguments under nonequilibrium constraints will be used for the description of higher-order moments of velocity fluctuations.
5) Compare the several current theoretical approaches among each other and with the experiments on cryogenic Helium. Study the problem of computation of anomalous dimensions in turbulent flows using the Naviers Stokes equation with high Re, or other hydrodynamic equations proposed for turbulent superfluids, and explore the influence of the boundary conditions and initial conditions of the typical experiments of cryogenic Helium.

REFERENCES

[1] D.Jou, G.Lebon, M.S.Mongiovì: “Second sound, superfluid turbulence and intermittent effects in liquid helium II", Phys. Rev. B, (2002), **66,** 224509 (9 pages).

[2] D.Jou, M.S.Mongiovì: "Phenomenological description of counterflow superfluid turbulence in rotating containers", Phys. Rev. B 69, 094513 (2004).
[3] M.S.Mongiovì, D.Jou: "Superfluid turbulence in rotating containers: phenomenological description of the influence of the wall", Phys. Rev. B 72, 104515 (2005).
[4] M.S.Mongiovì, D.Jou: "Generalization of Vinen's equation including transition to superfluid turbulence", J. Phys: Condensed Matt 17, 4423 (2005).
[5] M.S.Mongiovì, D.Jou: "Nonlocal effects in superfluid turbulence: application to the low-density to high-density transition and to vortex decay", Phys. Rev. B 71, 094507 (2005).
[6] M.S.Mongiovì, D.Jou: "Nonlocal effects in superfluid turbulence: Application to the low-density-to high-density-state transition and to vortex decay", Phys. Rev. B, 63, 12501, (2005).
[7] D.Jou, Mongiovì M.S.: "Non-Equilibrium Thermodynamics of Unstead Superfluid Turbulence, in Counterflow in Rotating Situations", Phys. Rev. B 72, 144517 (2005).
[8] D.Jou, Mongiovì M.S.: "Description and evolution of anisotropy in superfluid vortex tangles with counterflow and rotation". Phys. Rev. B, (2006), 74, 054509.
[9] D.Jou, Mongiovì M.S.: "Nonequilibrium effective temperature of superfluid vortex tangles". Physics Letters A. (2006), 359, 183-186.
[10] M.S.Mongiovì, D.Jou: "Thermodynamical derivation of a hydrodynamical model of inhomogeneous superfluid turbulence", Phys. Rev. B, (2007), 75, 024507 (14 pages).
[11] M.S.Mongiovì, D.Jou : M.Sciacca., "Energy and Temperature of Superfluid Turbulent Vortex Tangles". Phys. Rev. B, (2007), in press.
[12] D.Jou, M.S.Mongiovì, M.Sciacca.: "Vortex density waves in a hydrodynamical model of superfluid turbulence". Phys. Letters A, (2007), in press. doi: 10.1016/j.physleta.2007.03.078.
[13] M.L.Bertotti, S.V.Bolotin, "Chaotic trajectories for natural systems on a toros", Discrete and Continuous Dynamical Systems, (2003), Series A, vol. 9, n.5, 1343-1357.
[14] V.Ruggiero, P.Lanucara, M.P.Busico, A.Caserta, B.Firmani, "Numerical modelling of the ground motion; a parallel approach for finite element methods", Applied and Industrial Mathematics in Italy, 487-495, Ser. Adv. Math. Appl. Sci., vol. 69, World Sci. Publ., Hackensacle, N.Y. (2005).
[15] A.Caserta, B.Firmani, V.Ruggiero, P.Lanucara, M.P.Busico: "Finite elements method: a new approach for parallel computing", XXIX European Seismological Commission: General Assembly. 25 Settembre - 7 Ottobre 2004. Trieste (Italy).
[16] A.Caserta, B.Firmani, C.Giuffrida, V.Ruggiero, P.Lanucara: "Input data web interface for numerical simulations ofseismic wave propagation". XXIX European Seismological Commission: General Assembly. 25 Settembre - 7 Ottobre 2004. Trieste (Italy).
[17] A.Caserta, B.Firmani, V.Ruggiero, P.Lanucara, M.P.Busico: "Numerical Modelling of the ground motion: a parallel approach for finite element methods". VII congresso nazionale SIMAI, Venezia Settembre 2004, pp.487-495, World Scientific 705-731).
[18] S.Dobrokhotov, B.Tirozzi: "On an hamiltonian property of the truncated Hugoniòt-Maslov chain for trajectories of mesoscale vortices", Doklady Mathematics, Vol. 65, N. 3, pag. 453-458, 2002 Communicated by Academic. M. Maslov.
[19] S.Dobrokhotov, E.Semenov, B.Tirozzi: "On the computation of the integrals of the Hugoniot-Maslov chain for singular vortex of the shallow water equation", Theoretical and Mathematical Physics, 139(1), 500-512, 2004.
[20] S.Dobrokhotov, B.Tirozzi, A.I.Shafarevich: "The Cauchy-Riemann conditions and localized solutions of the linearized shallow water equation in shallow water theory", Prikl. Matematika i Mekanika, v. 69, N3, pp 804-809, (Engl Transl. Applied Mathematics and Mechanics 2005).

[21] B.Tirozzi, S.Puca, S. Pittalis, A.Bruschi, S.Morucci, E.Ferraro, S.Corsini: "Neural Networks and Sea Time Series Reconstruction and Extreme event analysis", (book) Collana MSSET, Birkhauser Boston, 2005.
[22] S.Yu. Dobrokhotov,S. Ya. Sekersh-Zenkevich, B. Tirozzi, T. Ya. Tudorovski "Asymptotic theory of tsunami waves: geometrical aspects and the generalized Maslov representation", Publications of Research Institute for Mathematical Sciences, ISSN 0034-5318, Kyoto, 2006.
[23] S.Yu. Dobrokhotov, S. Ya. Sekersh-Zenkevich, B. Tirozzi, T. Ya. Tudorovski, "Description of the Diffusion of a Tsunami Wave by means of the Maslov Canonical Operator", Dokladi Ak Nauk 517.9, Communicated by the Academician V. P. Maslov, 2005.
[24] S.Yu. Dobrokhotov, S.Sekerzh-Zenkovich, B. Tirozzi, B. Volkov "Explicit asymptotics for tsunami waves in the framework of the piston model", Russian Journal of Earth Sciences, Vol. 8, ES4003, doi:10.2205/2006ES000215 pp 1-12, 2006.
[25] S. Yu. Dobrokhotov, S. O. Synitsyn, B. Tirozzi. "Asymptotics of localized solutions of the one-dimensional wave equation with variable velocity. I. The Cauchy Problem", Russian Journal of Mathematical Physics, Vol. 14, N. 1, p. 28-56, 2007.
[26] S. Reutskiy, B. Tirozzi, "Numerical solutions of Shallow Water equation and typhoon's propagation", Russian J. of Mathematical Physics

ADDRESSES

Maria Stella Mongiovì,
Dipartimento di Metodi e Modelli Matematici,
Università di Palermo, Facoltà di Ingegneria,
Viale delle Scienze, 90128, Palermo, Italy;
e-mail: mongiovi@unipa.it

David Jou,
Departament de Fisica,
Universitat Autonoma de Barcelona,
08193 Bellaterra, Catalonia, Spain;
e-mail: david.jou@uab.es

Maria Letizia Bertotti,
Dipartimento di Metodi e Modelli Matematici,
Università di Palermo, Facoltà di Ingegneria,
Viale delle Scienze, 90128, Palermo, Italy;
e-mail: bertotti@unipa.it

Bruno Firmani,
Dipartimento di Ingegneria Meccanica e Strutturale,
Università di Trento,
Via Mesiano 77, 38050, Trento, Italy;
e-mail: bruno.firmani@ing.unitn.it

Michele Sciacca,
Dipartimento di Metodi e Modelli Matematici,
Università di Palermo, Facoltà di Ingegneria,
Viale delle Scienze, 90128, Palermo, Italy;

e-mail: msciacca@unipa.it

Sergey Dobrokhotov,
Institute for Problems in Mechanics,
Russian Academy of Sciences,
101-1, prosp.Vernadskogo, Moscow 119526 Russia;
e-mail : dobr@ipmnet.ru

Daniela Bianchi,
Department of Physics,
University of Rome “La Sapienza”
P. A. Moro 5, 00185 Roma, Italy;
e-mail : danielabianchi12@virgilio.it

Brunello Tirozzi,
Department of Physics,
University of Rome “La Sapienza”
P. A. Moro 5, 00185 Roma, Italy;
e-mail : b.tirozzi@libero.it

Vittorio Ruggiero,
CASPUR (Consorzio interuniversitario per le Applicazioni di Supercalcolo per Università e Ricerca)
Via dei Tizii 6, 00185 Roma, Italy;
e-mail: ruggiero@caspur.it