Velocity and temperature derivatives in high Reynolds number turbulent flows in the atmospheric surface layer. Part II. Accelerations and related matters.

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We report first results of an experiment, in which explicit information on all velocity derivatives (the nine spatial derivatives, $\partial u_i/\partial x_j$, and the three temporal derivatives, $\partial u_i/\partial t$) along with the three components of velocity fluctuations at a Reynolds number as high as $Re_\lambda \sim 10^4$ is obtained. No use of the Taylor hypothesis was made, and this allowed to obtain a variety of results concerning acceleration and its different Eulerian components along with vorticity, strain and other small-scale quantities. The field experiments were performed at five heights between 0.8 and 10 m above the ground.

The report consists of three parts. The first part, Gulitski *et al.* (2006*a*), is devoted to the description of facilities, methods and some general results. The present second part concerns accelerations and related matters. Finally the third part of our work, Gulitski *et al.* (2006*b*), is devoted to the issues concerning temperature with the emphasis on joint statistics of temperature and velocity derivatives.

1. Introductory notes

As the material derivative of the velocity vector, the fluid particle acceleration field in turbulent flow is among the most natural physical parameters of special interest in turbulence research for a variety of reasons. Problems in which fluid particle acceleration plays a key role are ranging from studies of basic issues as fine-scale intermittency, production of Reynolds stresses, the so-called random Taylor hypothesis and two-phase turbulent flows to applications in turbulent mixing and transport, cloud physics and influence of turbulence on the behavior of insects. In particular, Lagrangian acceleration statistics are at the core of the kinematic theory and modelling of turbulent dispersion, mixing, particulate transport and combustion. Finally, the acceleration gradient tensor is known to govern the topology of quasi-geostrophic stirring (particle dispersion and tracer gradient evolution) and transport properties in nearly two-dimensional and geostrophic turbulence (Bernard *et al.* (1993); Borgas & Sawford (1991); Hua & Klein (1998); La Porta *et al.* (2001); Pope (2002); Shaw (2003); Tsinober *et al.* (2001); Yeung & Pope (1989)).

Hence, it is not surprising that there is a huge interest in a variety of issues associated with fluid particle accelerations in turbulent flows (Aringazin & Mazhitov (2004); Biferale *et al.* (2004); Chen *et al.* (2006); Christensen & Adrian (2001, 2002*a*,*b*); Crawford *et al.* (2005); Hill (2002); Gylfason *et al.* (2004); Hill & Thoroddsen (1997); Hill & Wilczak (1995); Kholmyansky *et al.* (2001*a*); La Porta *et al.* (2001); Lee *et al.* (2004); Lüthi *et al.* (2001); Mann *et al.* (1999); Mordant *et al.* (2003, 2004*a*,*b*,*c*); Ott & Mann (2000); Pinsky *et al.* (2000); Reynolds *et al.* (2005); Sawford *et al.* (2003); Shaw & Oncley (2001);

Tsinober *et al.* (2001); Vedula & Yeung (1999); Voth *et al.* (1998, 2002); Yeung (1997); Yeung & Borgas (2004) and references therein).

The study of basic properties of Lagrangian accelerations was started using direct numerical simulations by Yeung & Pope (1989) and continued by Yeung (1997); Vedula & Yeung (1999) and Tsinober *et al.* (2001). Due to extreme difficulties, the direct measurement of the acceleration of a fluid particle has only recently been achieved in laboratory flows by three-dimensional particle tracking (3DPT), see Crawford *et al.* (2005); La Porta *et al.* (2001); Lüthi *et al.* (2001); Mann *et al.* (1999); Mordant *et al.* (2003, 2004*b*); Ott & Mann (2000); Reynolds *et al.* (2005); Sawford *et al.* (2003), with especially high-precision experiments performed by the group of E. Bodenschatz. These experiments were made in a water flow configuration known as "French washing machine" in which the flow is produced by two counter-rotating discs (as in the classical von Karman flow) and is believed to be highly anisotropic. Among the interesting findings in the direct determination of particle acceleration (both via DNS and 3DPT) is the persistence of the magnitude of accelerations along the fluid particle trajectory as contrasted with variability of its direction. The former has a correlation time of the order of integral Lagrangian time, whereas the latter has a correlation time of the Kolmogorov time only (see Fig. 1).

In the Eulerian setting a successful attempt to measure accelerations was made in a turbulent channel flow by particle-image accelerometry (Christensen & Adrian (2001, 2002*a,b*)) which is an extension of the particle image velocimetry at moderate Reynolds number using air as a working fluid.

Until recently it was believed that it is impossible to employ hot-wire anemometry for

measurements of accelerations:

> *Even in multipoint probe measurements of velocity gradients (e.g., Vukoslavcevic, Wallace, and Balint (1991); Tsinober, Kit, and Dracos (1992)), Taylor's hypothesis is invoked to estimate derivatives along the mean streamwise direction.* Dahm & Southerland (1997).
> *In order to determine* **a** *from Eulerian measurements, it would be necessary to*

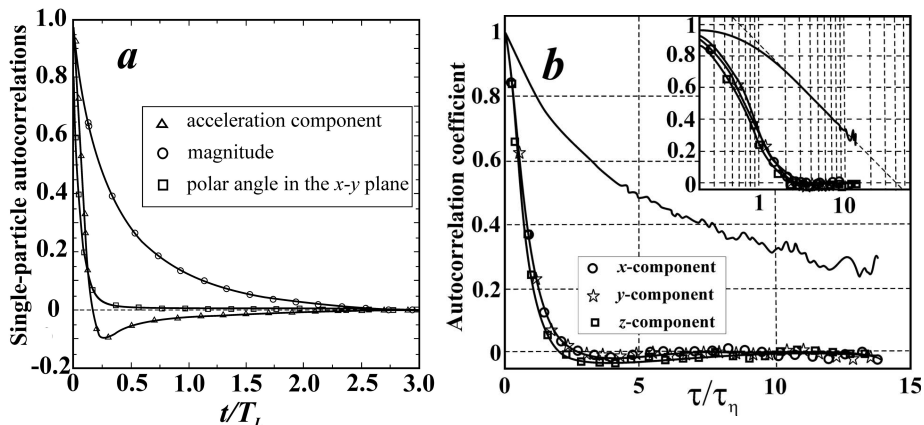FIGURE 1. Autocorrelation functions of the acceleration, **a**, and its components. a — from Yeung (1997) with time scaled by the Lagrangian velocity integral time scale at $Re_\lambda = 140$. b — from Mordant *et al.* (2004b) with time scaled by the Kolmogorov time $\tau_\eta = \sqrt{\nu/\varepsilon}$, the line without symbols is for the acceleration magnitude, inset: semilogarithmic plot.

know $\partial\mathbf{u}/\partial t$ as well as $\mathbf{u}$ and $\nabla\mathbf{u}$ at a point in space (which is possible in DNS but not in experiments). Voth *et al.* (2002)

The only results achievable so far relate to indirect evaluation of acceleration variance, $\langle a^2 \rangle$, using the methods developed by Hill and Wilczak assuming local isotropy (Hill & Thoroddsen (1997); Hill & Wilczak (1995)) and used recently by Gylfason *et al.* (2004). A summarizing figure from this latter paper (Fig. 2) shows various results on the acceleration variance normalized by Kolmogorov scaling, $a_0 = \frac{1}{3}\langle a_k a_k \rangle \epsilon^{-3/2} \nu^{1/2}$. Also shown are additional results from recent particle tracking data, recorded with the 3D-PTV system of Risø National Laboratory (Ott & Mann (2000); Mann *et al.* (2005); Jørgensen *et al.* (2005)) and processed using the method of Lüthi *et al.* (2005) and from some of our recent field experiments to be described below. Though all the results are of the same order, the scatter is not small.

However, it appears that it is possible to measure instantaneous accelerations in an Eulerian setting using hot-wire anemometry. We made such a successful attempt (Kholmyansky *et al.* (2001b); Galanti *et al.* (2003)) through an extension of multi-hot-
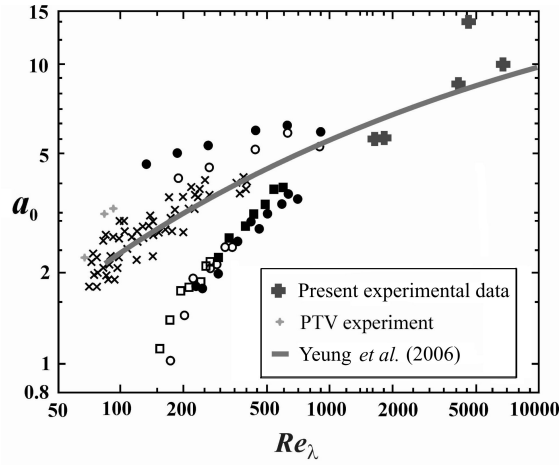
FIGURE 2. Normalized acceleration variance, $a_0 = (1/3)\langle a_k a_k \rangle \epsilon^{-3/2}\nu^{1/2}$, vs Re_λ from Gylfason *et al.* (2004). Points are added from the present experimental data and from the PTV experiments, Lüthi *et al.* (2005). The line is the fit from Yeung *et al.* (2006): $a_0 = \frac{1.3}{R_\lambda^{0.22}} + 0.88 R_\lambda^{0.06}\ln(\frac{R_\lambda}{20})$.

wire technique developed by the group of Professor A. Tsinober (Busen *et al.* (2001); Galanti *et al.* (2003, 2004); Kholmyansky & Tsinober (2000); Kholmyansky *et al.* (2000, 2001*b*,*a*); Tsinober *et al.* (1992, 1997)). The new version of this technique allowed to estimate the spatial derivatives of all three velocity components in the streamwise direction independently of the time derivative, i.e. without invoking the Taylor hypothesis.

This was achieved by constructing a five-array probe (see Fig. 3*a*) with the central array shifted out in the streamwise direction: the so-called NTH-probe ('non-Taylor hypothesis'). We show in Fig. 3*b* one of the earlier results obtained with such a probe in an atmospheric surface layer (Kholmyansky *et al.* (2001*a*)). It is related to the random Taylor hypothesis (Tennekes (1975)) and demonstrates strong anti-alignment between the Eulerian components, $\mathbf{a}_l = \frac{\partial \mathbf{u}}{\partial t}$ (local) and $\mathbf{a}_c = u_k \frac{\partial \mathbf{u}}{\partial x_k} = (\mathbf{u} \cdot \nabla)\mathbf{u}$ (convective), of the fluid particle acceleration, $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$. This effect was also observed in DNS at relatively low Reynolds numbers (Tsinober *et al.* (2001)). The strong anti-alignment of $\mathbf{a}_l$ and $\mathbf{a}_c$ is
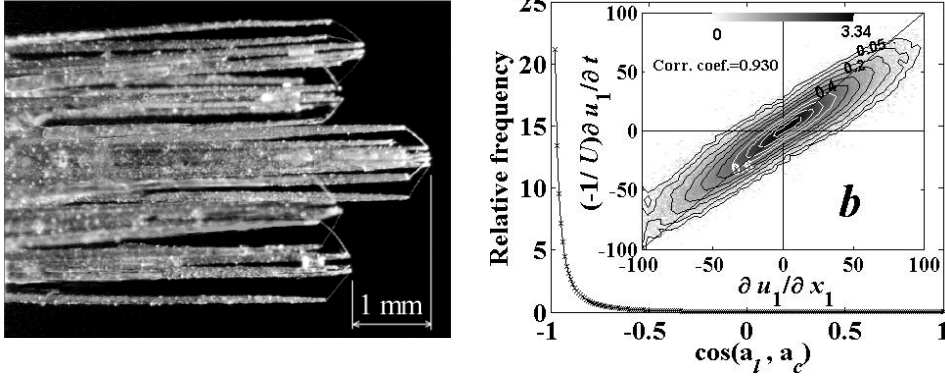
FIGURE 3. a – Tip of the NTH-probe; b – PDF of the cosine between the local, $\mathbf{a}_l$, and convective, $\mathbf{a}_c$, acceleration components. On the inset — joint PDF of the true longitudinal derivative, $\partial u_1/\partial x_1$, and its estimate using Taylor hypothesis, $(-1/U)\partial u_1/\partial t$.

a manifestation of their strong mutual cancellation, so that $\mathbf{a}$ appears to be much smaller than both $\mathbf{a}_l$ and $\mathbf{a}_c$, which represents the main difficulty in reliable determination, e.g. of the variance of $\mathbf{a}$. Thus we have shown already in 2001 (Kholmyansky *et al.* (2001a), also Galanti *et al.* (2003)) that it is possible to use hot-wire techniques for studying the field of fluid particle acceleration in an Eulerian setting.†

We would like to recall why we are using an 'old-fashioned technique' like hot-wire anemometry. The main reason is the possibility of measuring all three components of the velocity fluctuations vector, u_i, and all nine components of the spatial velocity gradients tensor, $\frac{\partial u_i}{\partial x_j}$, and its time derivatives, $\frac{\partial u_i}{\partial t}$. So far, all 'modern' techniques such as PIV, PTV, holography and others are unable to cope with this problem neither in resolution

† The reason that the slope in Fig. 3b is not along the bisector is a combination of errors in estimation of dx_1 (geometry and effective "center of mass") and mean velocity (calibration errors; it is well known that the mean velocity is difficult to obtain from the hot-wire measurements). At the present level of precision we relate the thickness of the ellipse to the calibration errors and do not draw conclusions about the Taylor hypothesis.

nor in the essence, i.e. obtaining the full set — total 15 — of velocity derivatives both in space and time at Reynolds numbers $\sim 10^4$.

1.1. *Main goals and tasks*

The preliminary results mentioned above comprise the basis for setting the main theme and objectives of the present research.

For convenience we, first, recall here some of the notations.

The acceleration is $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$, where $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$ — is the convective acceleration and $\mathbf{a}_l = \partial\mathbf{u}/\partial t$ — is the local acceleration in the frame attached to the ground, $\mathbf{a}_L = \boldsymbol{\omega} \times \mathbf{u}$ — is the Lamb vector, $\mathbf{a}_B = \frac{1}{2}\nabla u^2$ — is the Bernoulli component of acceleration, $\mathbf{a}_{\shortparallel} = (\mathbf{a}\cdot\widehat{\mathbf{u}})\widehat{\mathbf{u}}$ — is the acceleration component parallel to the velocity vector, where $\widehat{\mathbf{u}} = \mathbf{u}/u$ — is the unit vector along velocity vector, and $\mathbf{a}_\perp = \mathbf{a} - \mathbf{a}_{\shortparallel}$ — is the acceleration component normal to the velocity vector.

In addition we consider $\mathbf{a}'_c = \{(\mathbf{u} - \mathbf{U}) \cdot \nabla\}\mathbf{u}$ and $\mathbf{a}'_l = \frac{\partial\mathbf{u}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{u}$ — the convective and local components of acceleration in a frame moving with mean velocity, $\mathbf{U}$, which are different from $\mathbf{a}_l$ and $\mathbf{a}_c$, i.e. while the particle acceleration is a Galilean invariant quantity, it's Eulerian components are not. In a frame moving with constant (mean) velocity, $\mathbf{U}$, the velocity time derivative (i.e. local acceleration, $\mathbf{a}_l$, in a *moving frame* which we denote as $\mathbf{a}'_l$) is

$$\lim_{\tau \to 0} \frac{\mathbf{u}(\mathbf{x} + \mathbf{U}\tau,\ t + \tau) - \mathbf{u}\,(\mathbf{x}, t)}{\tau} = \frac{\partial\mathbf{u}}{\partial t} + (\mathbf{U} \cdot \nabla)\mathbf{u}.$$

The space derivative, e.g. $\frac{\partial}{\partial x_1}$, is

$$\lim_{\xi \to 0} \frac{\mathbf{u}(\mathbf{x} + \xi\mathbf{i}) - \mathbf{u}(\mathbf{x})}{\xi} = \frac{\partial\mathbf{u}}{\partial x_1},$$

but the instantaneous velocity is $\mathbf{u} - \mathbf{U}$. Therefore, the convective derivative (i.e. the convective acceleration, $\mathbf{a}_c$, which we denote as $\mathbf{a}'_c$) becomes $\{(\mathbf{u} - \mathbf{U}) \cdot \nabla\}\mathbf{u}$. In other words

the Eulerian components of acceleration are frame dependent, but the total Lagrangian acceleration — as a true physical quantity — is, of course, frame independent.

However, our measurements are in a particular Eulerian frame, i.e. attached to the ground. The consequence is that the Lagrangian acceleration, $\mathbf{a}$, is a sum of two large quantities, $\mathbf{a}_l$ and $\mathbf{a}_c$, due to the contribution of the mean velocity, $\mathbf{U}$. Since there is a strong cancellation between $\mathbf{a}_l$ and $\mathbf{a}_c$ (and consequently $\mathbf{a}$ is much smaller than both $\mathbf{a}_l$ and $\mathbf{a}_c$) there is a strong "amplification" of error when evaluating $\mathbf{a}$. The same observation is true when looking at $\mathbf{a}'_l$ and $\mathbf{a}'_c$ (which are much smaller than $\mathbf{a}_l$ and $\mathbf{a}_c$ due to removal of the contribution from the mean velocity, $\mathbf{U}$): both are much more contaminated by noise (in the sense of relative error) than $\mathbf{a}_l$ and $\mathbf{a}_c$. The problem of the errors that become especially apparent when we deal with a small quantity, obtained as a difference of large ones, was discussed in part I of our paper (Gulitski *et al.* (2006*a*)). It is not easy (if possible) to estimate their impact directly. When results from DNS or other experiments exist, comparison with them can give an indication of that impact.

The main goal is to study the field of Lagrangian accelerations, $\mathbf{a} \equiv \frac{D\mathbf{u}}{Dt}$, and its Eulerian components: the local acceleration, $\mathbf{a}_l = \frac{\partial \mathbf{u}}{\partial t}$, and the convective one, $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$, as it was done for low Reynolds numbers in DNS (Tsinober *et al.* (2001)). This includes the random Taylor (sweeping decorrelation) hypothesis and associated issues of geometrical statistics of accelerations, involving the above mentioned variety of Eulerian components of the total acceleration. In particular, we are interested in mutual (statistical) cancellation between the local acceleration, $\mathbf{a}_l = \frac{\partial \mathbf{u}}{\partial t}$, and the convective one, $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$. Since these quantities are vectors, the degree of this mutual cancellation should be studied both in terms of their magnitude and the geometry of vector alignments. Among other things this cancellation should be reflected in that the total acceleration, $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$, is much smaller than its local and convective components, $\mathbf{a}_l$

and $\mathbf{a}_c$, in large negative correlation between $\mathbf{a}_l$ and $\mathbf{a}_c$, and in strong (anti-)alignment of $\mathbf{a}_l$ and $\mathbf{a}_c$.

A natural by-product will be the direct check of the Taylor hypothesis using the new technique mentioned above. We would like to stress that only a technique allowing to measure simultaneously and independently the spatial and temporal velocity derivatives enables one to address all these issues. It is essential to note that in principle neither the random Taylor hypothesis nor the conventional one can be *strictly* valid.

Other aspects include one- and two-point statistics, conditional statistics of accelerations and its Eulerian components and Reynolds number effects.

2. Acceleration variance.

2.1. *General notes.*

The acceleration variance, $\langle a_k a_k \rangle$, is a key quantity in a number of issues. Following the above cited papers, we use the Kolmogorov normalized version of acceleration variance, $a_0 = (1/3) \langle a_k a_k \rangle \epsilon^{-3/2} \nu^{1/2}$ (Yaglom (1949); Obukhov & Yaglom (1951); Monin & Yaglom (1975)). Its value and scaling with the Reynolds number are essential for stochastic Lagrangian models and for Lagrangian probability density function models of turbulent diffusion if these models are to incorporate finite Reynolds number effects.

As mentioned, at the present state of the art, the acceleration variance, $\langle a_k a_k \rangle$, and consequently a_0 are not directly obtainable from Eulerian measurements due to extremely strong cancellation between $\mathbf{a}_l$ and $\mathbf{a}_c$. As found by Tsinober *et al.* (2001) from DNS simulations, the magnitude of the correlation coefficient between $\mathbf{a}_l$ and $\mathbf{a}_c$ is increasing with Re_λ and is already about -0.97 at $Re_\lambda = 240$. Therefore, one can expect that this correlation coefficient will be extremely close to minus unity at larger Re_λ. Consequently $\mathbf{a}_l + \mathbf{a}_c$ will become very much smaller than both $\mathbf{a}_l$ and $\mathbf{a}_c$. Already at $Re_\lambda = 240$ the

RMS of $\mathbf{a}$ is less than 0.1 of both $\mathbf{a}_l$ and $\mathbf{a}_c$. Hence $\mathbf{a}$, obtained as a sum of $\mathbf{a}_l$ and $\mathbf{a}_c$ at large Re_λ, is "drowning" in noise. It is noteworthy that PTV experiments at low Re_λ (Lüthi *et al.* (2005)) exhibit the same problem. Namely, the RMS of the fluid particle acceleration obtained as a sum of $\mathbf{a}_l$ and $\mathbf{a}_c$ is two times larger than that obtained directly.

Hill & Wilczak (1995) found a relation determining the acceleration variance via the fourth-order structure functions of the velocity field

$$\langle a_k a_k \rangle = \chi - \frac{35}{2}\nu \left\langle (\partial u_1/\partial x_1)^3 \right\rangle, \qquad (2.1)$$

with

$$\chi = 4 \int_0^\infty r^{-3}[S_{u_1}^{(4)}(r) + S_{u_2}^{(4)}(r) - 6S_{u_1 u_2}^{(4)}(r)]dr, \qquad (2.2)$$

$$S_{u_1}^{(4)}(r) = \left\langle (\Delta u_1)^4 \right\rangle; \; S_{u_2}^{(4)}(r) = \left\langle (\Delta u_2)^4 \right\rangle; \; S_{u_1 u_2}^{(4)}(r) = \left\langle (\Delta u_1)^2 (\Delta u_2)^2 \right\rangle;$$

$$\Delta u_i = u_i(x+r) - u_i(x).$$

This expression was obtained with the only assumption of local isotropy, so that Δu_3 can be used in (2.2) instead of Δu_2. Hill & Wilczak (1995) argued also that a sufficiently precise result can be obtained from

$$\chi = 4H\chi \int_0^\infty r^{-3} S_{u_1}^{(4)}(r)dr, \qquad (2.3)$$

with $H\chi = $ const for sufficiently large Reynolds numbers. Vedula & Yeung (1999) determined $H\chi$ from DNS and showed that it approaches a constant value of approximately 0.65 for Re_λ greater than about 200. Also note that assuming isotropy we obtain

$$-\frac{35}{2}\nu \left\langle (\partial u_1/\partial x_1)^3 \right\rangle = \nu \left\langle \omega_i \omega_j s_{ij} \right\rangle.$$

This method was implemented in previous attempts to determine acceleration variance using hot-wire anemometry by Hill & Thoroddsen (1997) and Gylfason *et al.* (2004).

Our Fig. 2 shows a number of points obtained from our data by the above method. The main feature is that there seems to be no saturation in the Re-dependence of a_0. There are two possibilities. First, this may be a genuine effect. In that case the scaling proposed by Yaglom (1949) is not "perfect" and the acceleration variance is larger than that proposed by Yaglom. Second, the trend seen in Fig. 2 is due to the imperfections of the indirect method. The issue seems to be open and requires further much more precise measurements.

3. One-point statistics

3.1. *PDFs*

We proceed with showing in Figs. 4, 5, and 6 the PDFs related to accelerations. All the PDFs exhibit strong deviation from Gaussian distributions. It is noteworthy that this deviation cannot be considered as a pure manifestation of intermittency for the following reason. Acceleration is a non-linear function of velocity and its derivatives. Therefore, even for a purely Gaussian velocity field one cannot expect the PDFs, associated with accelerations, (as any nonlinear functions/functionals of velocity field) to be Gaussian (Tsinober (2001)). Indeed, it was shown by Tsinober *et al.* (2001), using the data from DNS, that this is really the case: the PDFs of accelerations of a Gaussian velocity field are strongly non-Gaussian, but somewhat less than those for a real velocity field, resulting from the Navier–Stokes equations.

The PDF of the centered acceleration modulus $a - \langle a \rangle$ (Fig. 6*a*) deserve special mentioning. It is strongly positively skewed. This behavior is closely related to the two-point statistics of acceleration moduli.
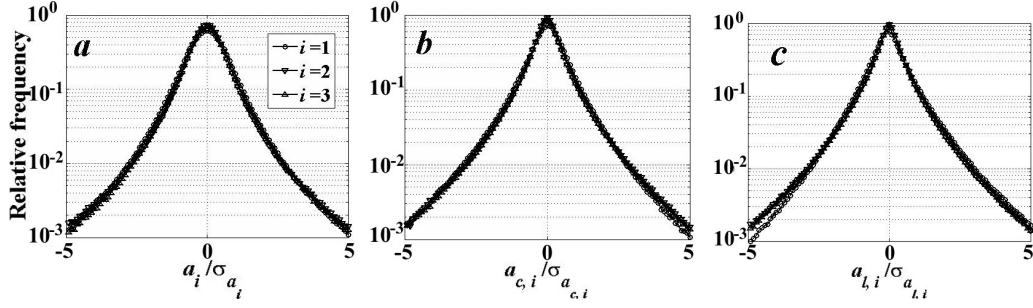
FIGURE 4. PDFs of the a_i; $a_{c,i}$, $a_{l,i}$, $i = 1, 2, 3$.
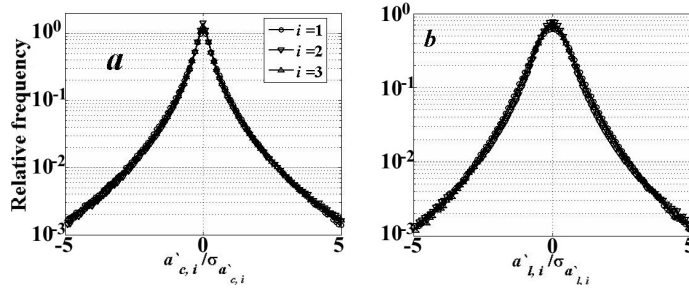
FIGURE 5. PDFs of the $a'_{c,i}$, $a'_{l,i}$, $i = 1, 2, 3$.
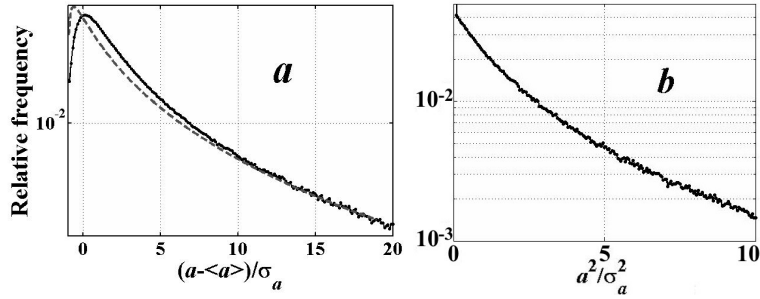
FIGURE 6. a – PDF of $a - \langle a \rangle$ (solid line). For comparison shown also PDF for log-normal distribution with variance 1, shifted to the left by $\langle a \rangle / \sigma_a$ (dotted line). b – PDF of $a^2 = a_k a_k$.

3.2. *Geometrical statistics*

By the term geometrical statistics we denote alignments between a variety of vectors. In case of accelerations and its different components this is quite rich and instructive.

Variance	$\mathbf{a}$	$\mathbf{a}_l$	$\mathbf{a}_c$	$\mathbf{a}_l'$	$\mathbf{a}_c'$
	9900	39900	60100	9100	2600

TABLE 1. Variances in $\mathrm{m\,s^{-2}}$ of $\mathbf{a}$, $\mathbf{a}_l$, $\mathbf{a}_c$, $\mathbf{a}_l'$, $\mathbf{a}_c'$.

3.2.1. *Random Taylor Hypothesis*

These issues take their origin from Taylor (1935) and are known as the Taylor hypothesis. Here we demonstrate one of the basic alignments associated with the random Taylor hypothesis (or the sweeping decorrelation hypothesis) which concerns the relations between the instantaneous Lagrangian acceleration and its (Eulerian) 'components' $\mathbf{a}_l = \partial \mathbf{u}/\partial t$ and $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$. It was suggested by Tennekes (1975) that

> *in turbulence with high Reynolds numbers... the dissipative eddies flow past an Eulerian observer in a time much shorter than the time scale which characterizes their own dynamics. This suggests that Taylor's 'frozen-turbulence' approximation should be valid for the analysis of the consequences of large-scale advection of the turbulent microstructure.*

The outcome is that *i)* there should be a strong cancellation effect between $\mathbf{a}_l$ and $\mathbf{a}_c$, so that $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c$ is much smaller than both $\mathbf{a}_l$ and $\mathbf{a}_c$ (e.g. comparing their variances), and *ii)* $\mathbf{a}_l$ and $\mathbf{a}_c$ should be strongly anti-aligned. Both effects were convincingly demonstrated by Tsinober *et al.* (2001) with DNS data for relatively small $Re_\lambda \lesssim 400$ and with preliminary experiments, Kholmyansky *et al.* (2001a). In our experiments we observed these effects at $Re_\lambda \sim 10^4$.

The results are shown in Table 1 with variances of $\mathbf{a}$, $\mathbf{a}_l$, $\mathbf{a}_c$, $\mathbf{a}_l'$, $\mathbf{a}_c'$ and in Fig. 7 with an example of the PDFs of $\cos(\mathbf{a}_l, \mathbf{a}_c)$, $\cos(\mathbf{a}_l', \mathbf{a}_c')$. Both clearly exhibit the two aspects just mentioned above. It is noteworthy that the anti-alignment is observed also for the acceleration components associated with the frame moving with the mean velocity, i.e.
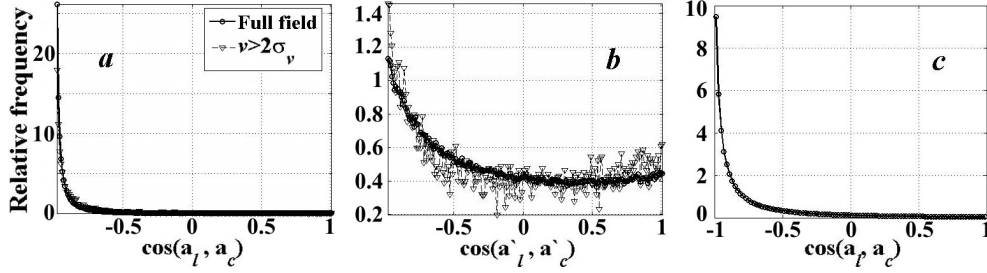
FIGURE 7. PDFs of $\cos(\mathbf{a}_l, \mathbf{a}_c)$ in the frame attached to the ground (a) and in the frame moving with the mean velocity (b) from field experiment; PDF of $\cos(\mathbf{a}_l, \mathbf{a}_c)$ from PTV experiment (c).

$\cos(\mathbf{a}'_l, \mathbf{a}'_c)$, though in a much weaker form, most probably due to the large relative error in these quantities.

Another view on the above is shown in Fig. 8 with the joint PDFs of Cartesian components of local and convective accelerations (only those for x_1-component as the x_2 and x_3 components exhibit very similar behavior) for both acceleration components in the frame attached to the ground and the frame moving with the mean velocity, where the correlation coefficients between $\mathbf{a}_l$, $\mathbf{a}_c$ and $\mathbf{a}'_l$, $\mathbf{a}'_c$ are shown in the upper right corner of each figure. It is seen that the correlation between acceleration components in the frame, attached to the ground, is pretty high, whereas the correlation in the frame, moving with the mean velocity, is rather weak due to large relative error for these quantities.

Our main focus here is on the original random Taylor hypothesis concerning the local and convective accelerations. However, it appears that the random Taylor hypothesis is valid not only for the velocity field, but for a variety of other quantities such as velocity derivatives (vorticity and strain), temperature and its gradient, and solenoidal passive vectors. In other words, the random Taylor hypothesis has a universal nature, which is manifested in a strong tendency for cancellation between the local temporal derivative, $\partial Q/\partial t$, and the advective derivative, $u_k \partial Q/\partial x_k$, of whatever quantity, Q. Thus the full
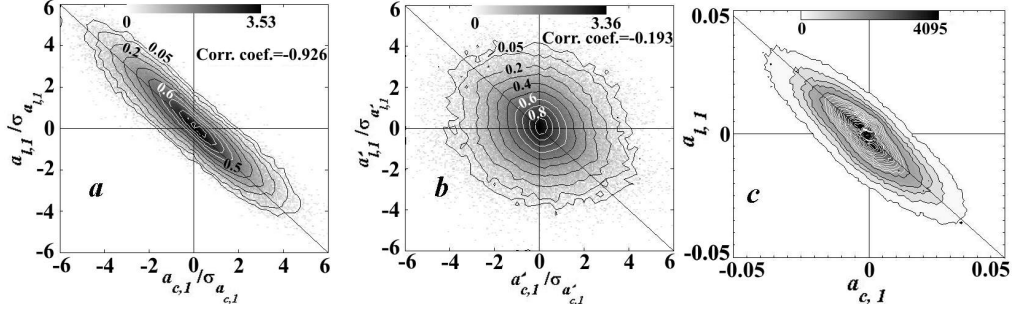
FIGURE 8. Joint PDFs of x_1-components of $\mathbf{a}_l$ and $\mathbf{a}_c$ in the frame attached to the ground (a) and in the frame moving with the mean velocity (b) from field experiment; joint PDF from PTV experiment (c).

material (Lagrangian) derivative, $DQ/Dt = \partial Q/\partial t + u_k \partial Q/\partial x_k$, is much smaller (at least an order of magnitude) than its Eulerian components. Evidence regarding this issue is given by Galanti *et al.* (2003).

3.2.2. *Alignments with the eigenframe, $\boldsymbol{\lambda}_k$, of the rate of strain tensor, s_{ij}.*

The alignments are shown in Figs. 9–12. The first important feature of all the alignments with the eigenframe, $\boldsymbol{\lambda}_k$, of the rate of strain tensor, s_{ij}, is that they all are similar for the quantities, associated with the frame attached to the ground, and for the ones in the frame moving with the mean velocity; in several cases they are very similar. This is a clear indication that the quantities, associated with the frame moving with the mean velocity, are captured correctly in spite of much larger error in their estimates as explained above.

Apart from the above important technical aspect we provide comments on the nature of the alignments shown as follows.

One of the common features is that all the vectors tend to be normal to the intermediate eigenvector, $\boldsymbol{\lambda}_2$, of the rate of strain tensor and tend to be aligned with both the pure stretching eigenvector, $\boldsymbol{\lambda}_1$, and the pure compression eigenvector, $\boldsymbol{\lambda}_3$. This is consistent with the intuition, based on the fact that vorticity is strongly aligned with $\boldsymbol{\lambda}_2$ and the
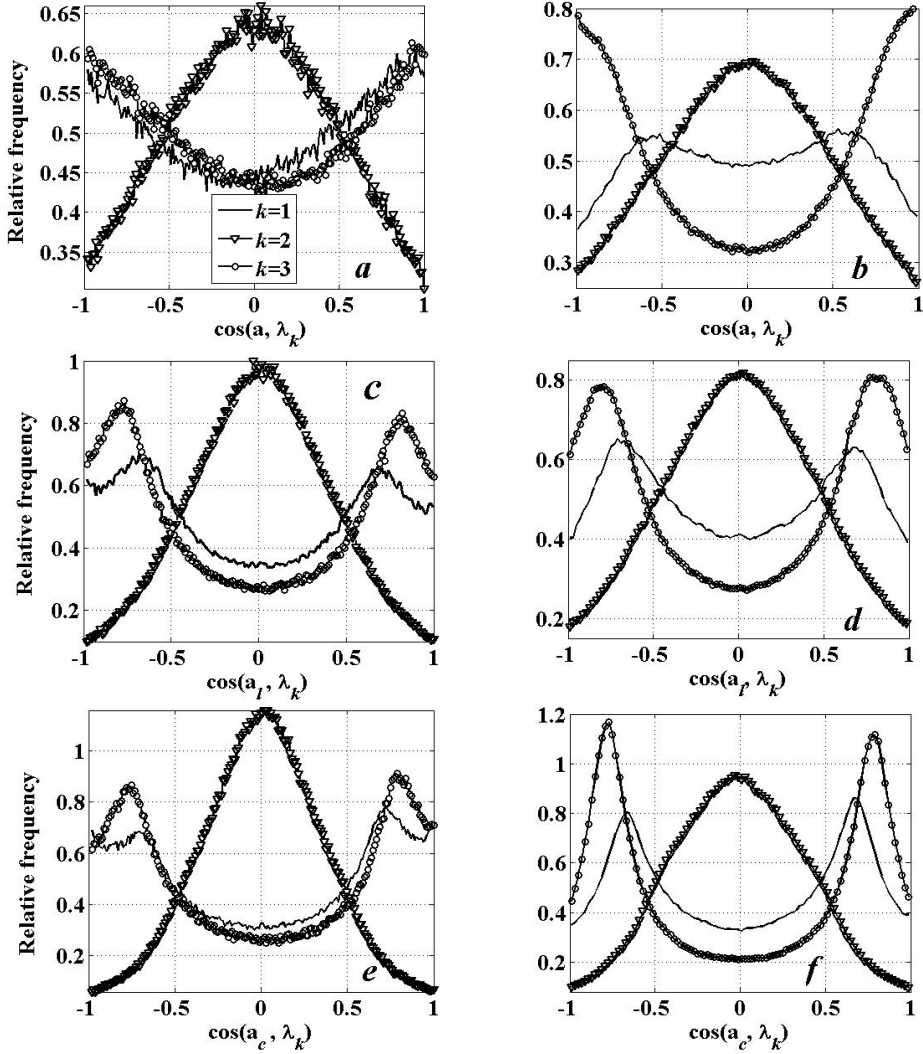
FIGURE 9. PDFs of the cosine of the angle between the acceleration components, $\mathbf{a}$, $\mathbf{a}_l$, $\mathbf{a}_c$, and the eigenframe, $\boldsymbol{\lambda}_k$, of the rate of strain tensor, s_{ij}, obtained at $Re_\lambda \sim 10^4$ in our field experiment (*left*) and at $Re_\lambda \sim 10^2$ in PTV experiment (*right*) by Lüthi *et al.* (2005).

expectation that fluid particle acceleration is mostly normal to vorticity and consequently aligned with the pure stretching and compressing eigen-directions of the rate of strain tensor. This indeed is observed, as seen from Fig. 13.

It is noteworthy that the alignments, obtained from hot-wire measurements, are similar
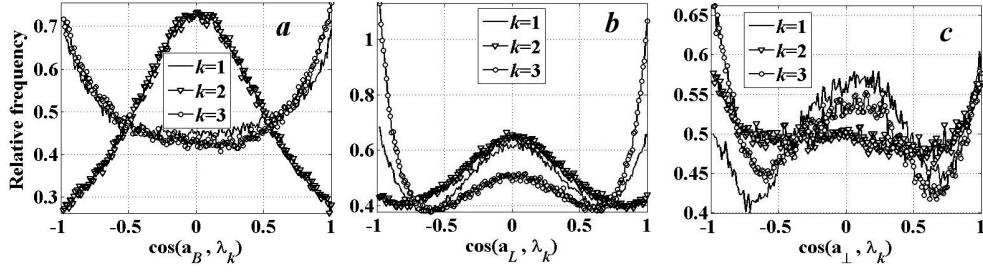
FIGURE 10. Alignments of the acceleration components, $\mathbf{a}_B$, $\mathbf{a}_L$, $\mathbf{a}_\perp$, with the eigenframe, $\boldsymbol{\lambda}_k$, of the rate of strain tensor, s_{ij}, associated with frame attached to the ground.
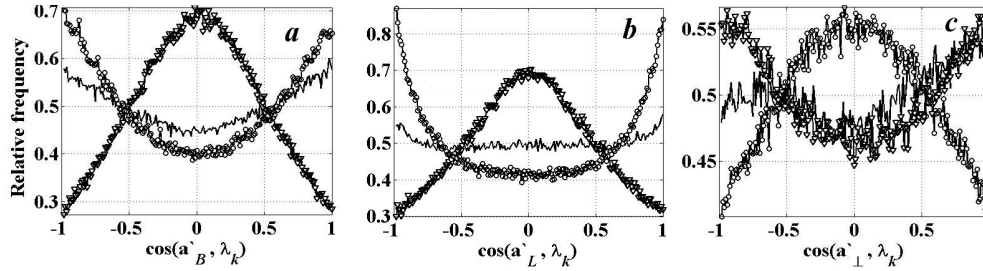
FIGURE 11. Same as in Fig. 10, associated with frame moving with the mean velocity.
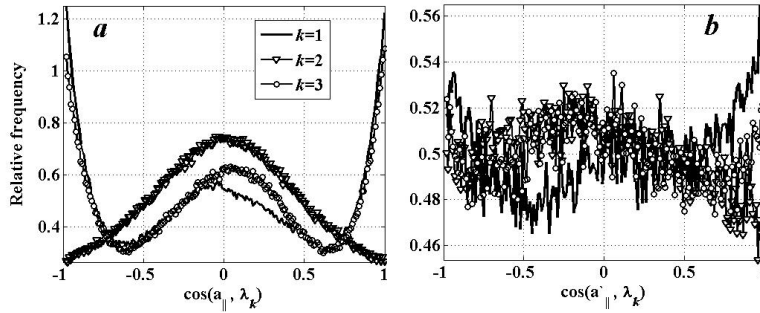

FIGURE 12. Alignments of the acceleration component, $\mathbf{a}_{\shortparallel}$, with the eigenframe, $\boldsymbol{\lambda}_k$, of the rate of strain tensor, s_{ij}, associated with frame attached to the ground (*a*) and frame moving with the mean velocity (*b*).

to those observed in low Reynolds experiments using three-dimensional particle tracking velocimetry by Lüthi *et al.* (2005). Examples of such alignments are shown in Fig. 9.
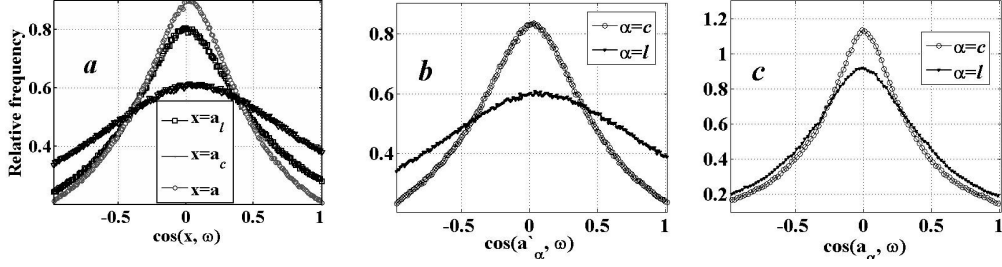
FIGURE 13. PDFs of the cosine of the angle between acceleration, $\mathbf{a}$, and vorticity, $\boldsymbol{\omega}$, in the frame attached to the ground (a) and in the frame moving with the mean velocity (b) from field experiment; similar PDF from PTV experiment (c).

3.2.3. *Additional significant alignments*

As mentioned, there are various physically meaningful Eulerian components of acceleration: $\mathbf{a} = \mathbf{a}_l + \mathbf{a}_c = \mathbf{a}_l + \mathbf{a}_L + \mathbf{a}_B = \mathbf{a}_{\shortparallel} + \mathbf{a}_\perp$, where $\mathbf{a}_c = (\mathbf{u} \cdot \nabla)\mathbf{u}$ — is the convective acceleration and $\mathbf{a}_l = \partial \mathbf{u}/\partial t$ — is the local acceleration in the frame attached to the ground, $\mathbf{a}_L = \boldsymbol{\omega} \times \mathbf{u}$ — is the Lamb vector, $\mathbf{a}_B = \frac{1}{2}\nabla \mathbf{u}^2$ — is the Bernoulli component of acceleration, $\mathbf{a}_{\shortparallel} = (\mathbf{a} \cdot \widehat{\mathbf{u}})\widehat{\mathbf{u}}$ — is the acceleration component parallel to the velocity vector, where $\widehat{\mathbf{u}} = \mathbf{u}/u$ — is the unit vector along the velocity vector, and $\mathbf{a}_\perp = \mathbf{a} - \mathbf{a}_{\shortparallel}$ — is the acceleration component normal to the velocity vector; similar quantities are defined in the frame moving with the mean velocity. The availability of these quantities allows to address a rich variety of alignments.

First, here again, we point to the similarity between the alignments involving quantities associated with the frame attached to the ground and quantities in the frame moving with the mean velocity. We show those pairs of figures in which strong alignments are observed.

Second, it is of special interest that the alignments shown above are similar also to those observed in low Reynolds number experiments using three-dimensional particle
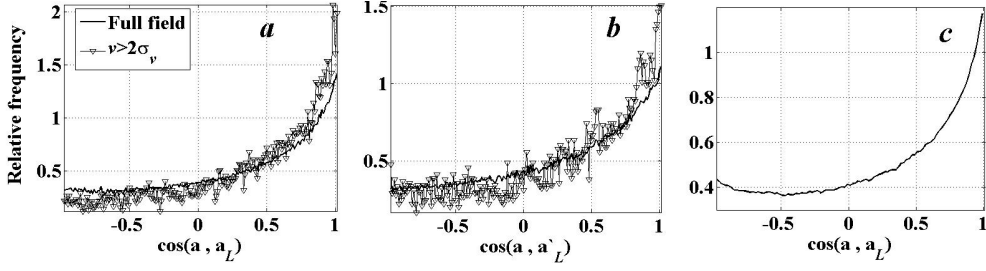
FIGURE 14. Same as in Fig. 13 for $\cos(\mathbf{a},\ \mathbf{a}_L)$.
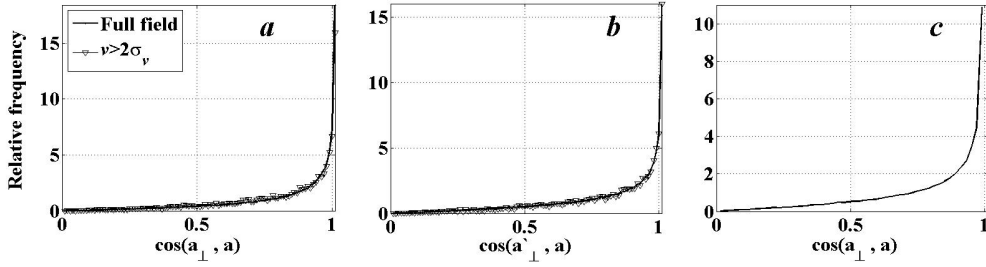
FIGURE 15. Same as in Fig. 13 for $\cos(\mathbf{a},\ \mathbf{a}_\perp)$.
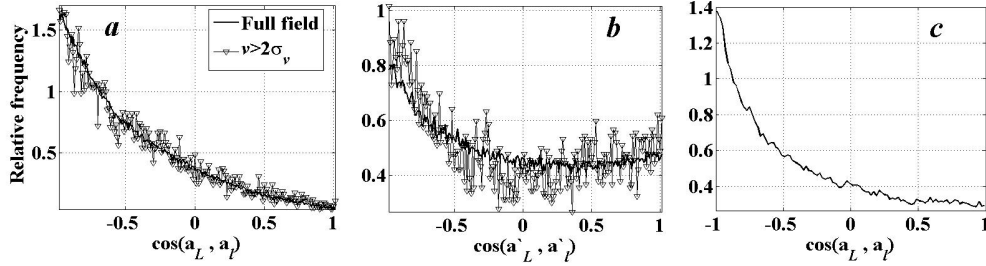
FIGURE 16. PDFs of $\cos(\mathbf{a}_L, \mathbf{a}_l)$ in the frame attached to the ground (*a*) and in the frame moving with the mean velocity (*b*), field experiment; similar PDF from PTV experiment (*c*).

tracking velocimetry in experiments by Lüthi *et al.* (2005). Examples of such alignments are shown in Figs. 14–20.

4. Two-point statistics

The auto-correlations of vectors $\mathbf{a}$, $\mathbf{a}_c$, $\mathbf{a}_l$, $\mathbf{a}_c'$ and $\mathbf{a}_l'$ (such as $\langle \mathbf{a}_{x_i}(x+r) \cdot \mathbf{a}_{x_i}(x) \rangle$) and their Cartesian components (such as $\langle \mathbf{a}_{x_1}(x+r) \cdot \mathbf{a}_{x_1}(x) \rangle$) and the modulus of $\mathbf{a}$ are shown in Figs. 21, 22 along with mean values of the cosine of the angle between
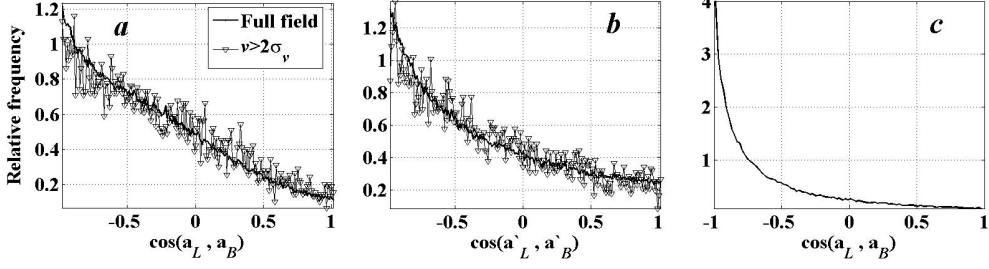
FIGURE 17. Same as in Fig. 16 for $\cos(\mathbf{a}_L, \mathbf{a}_B)$.
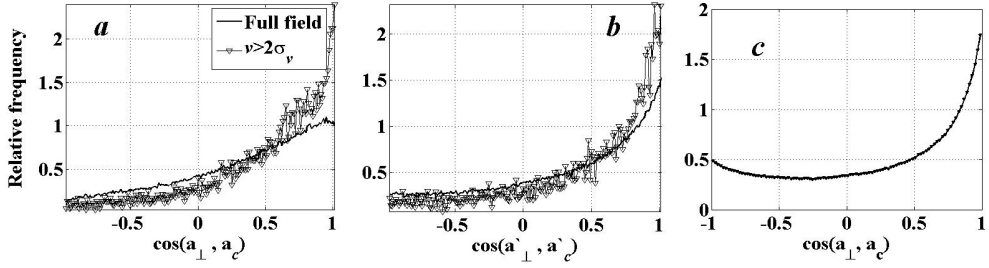
FIGURE 18. Same as in Fig. 16 for $\cos(\mathbf{a}_\perp, \mathbf{a}_c)$.
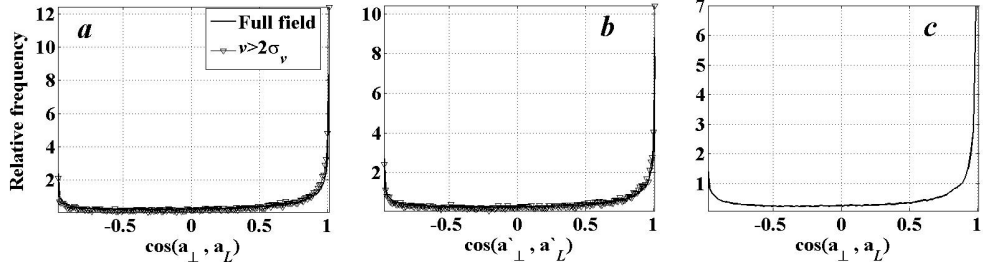
FIGURE 19. PDFs of $\cos(\mathbf{a}_\perp, \mathbf{a}_L)$ in the frame attached to the ground (a) and in the frame moving with the mean velocity (b), field experiment; similar PDF from PTV experiment (c).
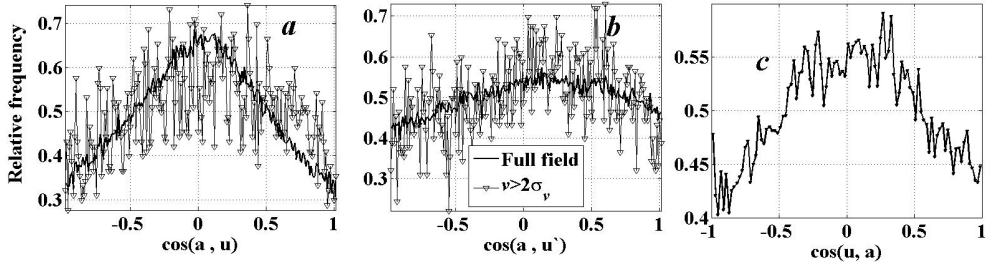
FIGURE 20. PDFs of the cosine of the angle between the acceleration, $\mathbf{a}$, and velocity, $\mathbf{u}$, in the frame attached to the ground (a) and in the frame moving with the mean velocity (b) from field experiment; similar PDF from PTV experiment, (c).
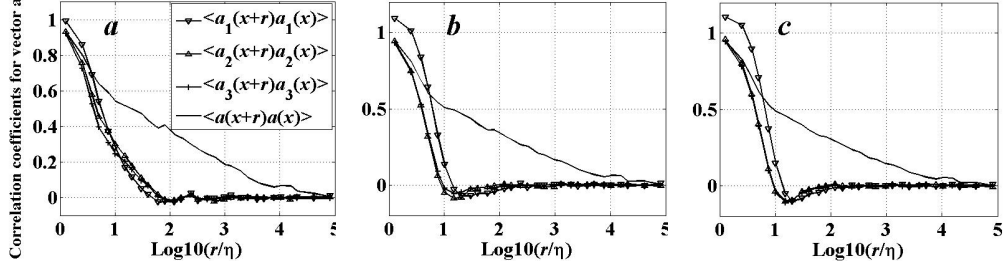
FIGURE 21. The auto-correlations of vectors **a** (*a*), **a**$_c$ (*b*), **a**$_l$ (*c*) and their Cartesian components from field experiment.
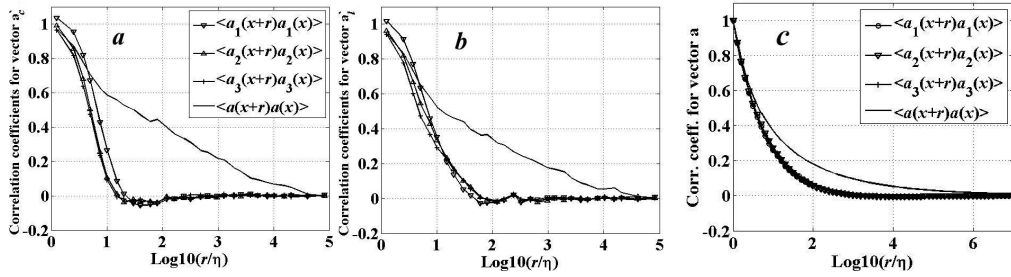
FIGURE 22. The auto-correlations of vectors **a**$'_c$ (*a*), **a**$'_l$ (*b*) from field experiment. The auto-correlations of vectors **a** (*c*) from PTV experiment.
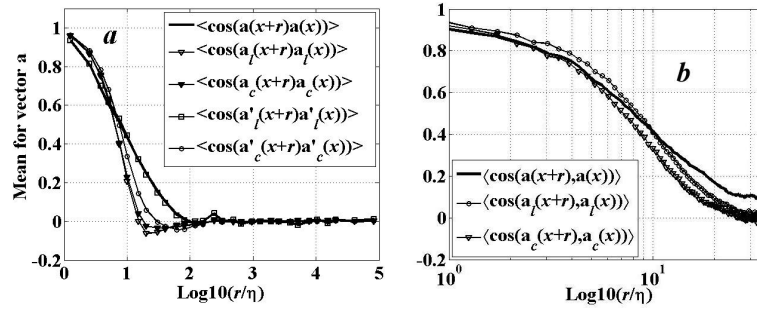

FIGURE 23. Mean values of the cosine of the angle between corresponding vectors at two positions, $\langle \cos\{\mathbf{a}(x+r),\ \mathbf{a}(x)\}\rangle$ from field experiment, (*a*) and from PTV experiment (*b*).

corresponding vectors at two positions $\langle \cos\{\mathbf{a}(x+r),\ \mathbf{a}(x)\}\rangle$, Fig. 23. Here the values of the distance, r, between the two points with sequential numbers i and $i+n$ is calculated as $r = n \cdot \Delta t \langle u_1 \rangle$, where $\Delta t = 10^{-4}$ s – is the sampling time.

The main features are similar to those obtained for Lagrangian correlations both in DNS (Yeung (1997)) and experimentally (Mordant *et al.* (2004*a*)). The first is that all the

vectors and their components are correlated over much shorter distances — typically 10 Kolmogorov lengths — than their moduli. The latter are correlated over large distances of order 10^4 Kolmogorov lengths, which is comparable with the integral scale just like for Lagrangian correlations both in DNS (Yeung (1997)) and experiments (Mordant *et al.* (2004*a*)). This behavior is observed also in PTV experiments by Lüthi *et al.* (2005) as seen from Fig. 22 (*c*).

A similar feature is found for the cosines of the corresponding angles, Fig. 23, though the difference is much smaller. An interesting aspect is that the above feature is the same both for acceleration components in the frame attached to the ground and in the frame moving with the mean velocity (Fig. 22*a,b*).

The behavior of second-order structure functions of Cartesian components of $\mathbf{a}$, $\mathbf{a}_c$, $\mathbf{a}_l$, $\mathbf{a}'_c$ and $\mathbf{a}'_l$ is consistent with the results on correlations shown above.

5. Conditional statistics, non-locality

Fluid particle accelerations are considered as small-scale quantities. The long-range correlations of the modula of accelerations reported above and those by previous authors can be considered as a kind of manifestation of non-locality. This issue is addressed in the present section.

We have seen in the first paper of this series Gulitski *et al.* (2006*a*), that the statistics of derivatives (i.e. small-scale quantities) is not independent of large-scale quantities, as is usually assumed in a variety of theoretical approaches. Similar results were obtained by Sawford *et al.* (2003); Mordant *et al.* (2004*a*); Crawford *et al.* (2005) for Lagrangian statistics of accelerations. Our results exhibit the same feature. Namely, the conditional statistics of accelerations is strongly dependent on velocity, Fig. 24. This can be considered as an additional clear indication of direct coupling between large and small scales
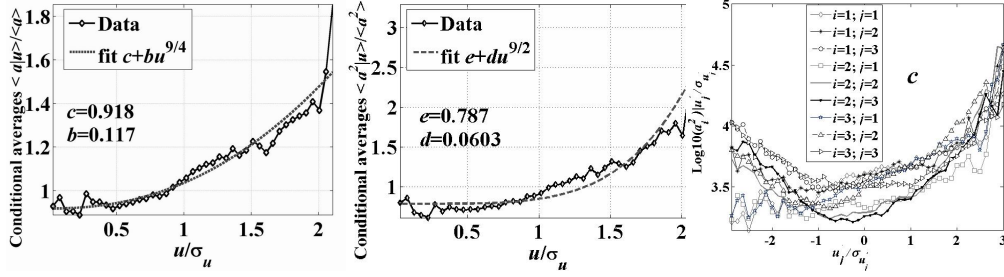
FIGURE 24. Conditional averages of a (a) and a^2 (b) on the magnitude of the vector of velocity fluctuations, u, and of a_i^2 on u_j, where $i, j = 1, 2, 3$ (c). The fits are as implied by simple arguments of Aringazin & Mazhitov (2004) and used later by Crawford *et al.* (2005). Note that in Crawford *et al.* (2005) $e \sim 2$ and $d \sim 10^{-2}$ and is Reynolds dependent.

and non-locality of turbulent flows. This means that it would be too optimistic to expect quantitative universality of the acceleration statistics, e.g. such as those shown in Fig. 24*a,b* among others. Hence the fits shown in these figures (implied by simple arguments of Aringazin & Mazhitov (2004) and used later by Crawford *et al.* (2005)) cannot be considered much more than just fits. Nevertheless, they reflect the *qualitative* trends correctly and are the same in Crawford *et al.* (2005) and in Fig. 24*a,b*. However, they are very different *quantitatively*, as should be expected, since the large scale properties (and Reynolds numbers) of the flow studied by Crawford *et al.* (2005) (French washing machine) are qualitatively and essentially different from those in the atmospheric surface layer addressed in this study. Therefore, it does not make much sense (if at all) to look for *quantitative* comparison between the two.

Before concluding we would like to remind that our observations are made in Euler frames. Nevertheless, we observed the same tendency of acceleration moduli to be correlated over large distances order of 10^4 Kolmogorov lengths, which is comparable with the integral scale, just as for Lagrangian correlations.

6. Conclusions and possible future directions

From the technical point the main achievement is the possibility to employ the multi-hot-wire technique without invoking the Taylor hypothesis and thereby to access the fluid particle acceleration and a variety of its Eulerian components. This required the design and manufacturing of a special probe and introduction of modifications in the existing system.

The new technique allowed to obtain a number of results not accessible until now. Our results prove the feasibility of correct measurements of the streamwise derivatives without invoking the Taylor hypothesis, thus allowing to address many important issues associated with accelerations and related matters. These concern a number of properties of fluid particle accelerations and its Eulerian components such as variances, geometrical statistics of accelerations, random Taylor (sweeping decorrelation) hypothesis, conditional statistics and non-locality. In particular it was shown that the random Taylor hypothesis is valid both in the system of coordinates attached to the ground and in the system moving with the mean velocity. In the latter case the results can be regarded as qualitative only, due to large relative error. The strong cancellation effects between the local and convective accelerations do not allow to obtain reliably, e.g. the variance of the Lagrangian acceleration, thus posing a challenge for drastic improvement of the quality of measurements and requiring non-trivial investment.

The obtained results conform with and confirm one of the main conclusions of Kholmyansky & Tsinober (2000); Kholmyansky *et al.* (2001*b,a*) that the basic physics of turbulent flow at high Reynolds number $Re_\lambda \sim 10^4$, at least qualitatively, is the same as at moderate Reynolds numbers, $Re_\lambda \sim 10^2$. This appears to be true not only for such basic processes as enstrophy and strain production, geometrical statistics, the role of concentrated vor-

ticity and strain, reduction of nonlinearity and non-locality, but also with respect to a variety of issues concerning accelerations and their Eulerian components.

It is important to emphasize, that our claim that

the basic physics of turbulent flow at high Reynolds number $Re_\lambda \sim 10^4$, at least qualitatively, is the same as at moderate Reynolds numbers, $Re_\lambda \sim 10^2$

does not mean that what is called "Reynolds number dependence" is unimportant. An immediate example comes from the indirect evaluation of the acceleration variance. It gives clear indications that — if scaled as proposed by Yaglom (1949) — it exhibits a clear Re-dependence and does not saturate at least up to $Re_\lambda \sim 10^4$. Another well-known example is the behavior of flatness of individual velocity derivatives and similar quantities based on vorticity and/or strain.

Reynolds dependence is of extreme importance in a great variety of purely engineering problems and other applications. It is important in basic issues of asymptotic behavior and limiting state(s) of turbulent flows as $Re \to \infty$. It remains to classify and distinguish between Reynolds-dependent and Reynolds-independent quantities/phenomena in turbulence. At this stage we hold the opinion, that at a qualitative level the basic physics of turbulence is Reynolds number independent.

One of the main challenges for future efforts in technical aspects is the reduction of the relative error for the acceleration components in the system moving with the mean velocity. This requires substantial improvement of the system such as *i)* further miniaturization of the probe design both with respect to its individual arrays as well as the whole probe in order to minimize the influence of velocity gradients across the individual arrays and the whole probe, and *ii)* further improvement of the calibration system. It is possible to build a probe of the overall scale only a bit larger than 1 mm. It can be done with the available technology on the basis of arrays as shown in Fig. 25.
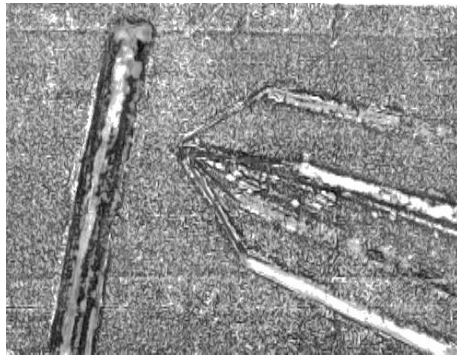
FIGURE 25. An array (0.35 mm at the tip) built with a Pt-10% Rho wire, 1.25 mkm in diameter. On the left — a human hair (< 70 mkm in diameter) is shown.

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