The State of the Art in Hydrodynamic Turbulence: Past Successes and Future Challenges

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Homogeneous and Isotropic Turbulence

- Used to be the main concern of the theoretical physics community
- Tremendous progress in understanding the statistical theory.

Kolmogorov 1941

$$\langle \left([\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{R},t) - \boldsymbol{u}(\boldsymbol{x},t)] \cdot \boldsymbol{R}/R \right)^3 \rangle = -(4/5)\bar{\epsilon}R$$

The long story of anomalous exponents

Linear Advection Models

$$\frac{\partial \phi(\boldsymbol{r},t)}{\partial t} + \boldsymbol{u}(\boldsymbol{r},t)\cdot\boldsymbol{\nabla}\phi(\boldsymbol{r},t) = \kappa\nabla^2\phi(\boldsymbol{r},t)$$

The (non-generic) Kraichnan model: the velocity field is delta-correlated in time, but scaling in space

$$F_{2n}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_{2n}) = \langle \phi(\boldsymbol{r}_1,t)\phi(\boldsymbol{r}_2,t)\cdots\phi(\boldsymbol{r}_{2n},t)\rangle$$

$$\mathrm{Diff\ OP}\quad \mathrm{F_{2n}}(\mathrm{r_1},\mathrm{r_2},\ldots,\mathrm{r_{2n}}) = \mathrm{RHS}$$

G. Falkovich, K. Gawedzki, M. Vergassola, Rev. Mod. Phys. 73, 913 (2001).

Generalization to the generic case

We do no have a differential operator, but rather an integral operator, and zero modes turn to ‘statistically preserved structures’ i.e. eigenfunctions of eigenvalue 1.

Analytic calculations are more difficult, but the lesson is the same: for the decaying problem there are special initial conditions that do not decay, and they carry the anomalous scaling.

A. Celani and M. Vergassola Phys. Rev. Lett. 86, 424 (2001).

I. Arad, L. Biferale, A. Celani, I. Procaccia, and M. Vergassola, Phys. Rev. Lett., 87, 164502 (2001) .

Y. Cohen, A. Pomyalov and I. Procaccia, Phys. Rev. E., 68, 036303 (2003).

Implications for the nonlinear problem

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} + \lambda\boldsymbol{w}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla}p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f},$$

$$\frac{\partial \boldsymbol{w}}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{w} + \lambda\boldsymbol{w}\cdot\boldsymbol{\nabla}\boldsymbol{w} = -\boldsymbol{\nabla}\tilde{p} + \nu\nabla^2\boldsymbol{w} + \tilde{\boldsymbol{f}}.$$

L. Angheluta, R. Benzi, L. Biferale,I, Procaccia and F. Toschi,
Phys. Rev. Lett., 97, 160601 (2006)

The mulitfractal formalism

G. Parisi and U. Frisch and 1985

$$S_p(r) \equiv \langle \left([u(x+R,t) - u(x,t)] \cdot R/R \right)^p \rangle \sim U_L^p \int d\mu(h) \left(\frac{r}{L}\right)^{ph+3-D(h)}$$

To derive this one needs to start with the ``fully unfused" function

$$\mathcal{F}_n(\mathbf{r}_0 \mid X_1 \cdots X_n) = \langle \mathcal{W}_1 \cdots \mathcal{W}_n \rangle$$

$$\mathcal{W}_j \equiv \mathcal{W}(X_j)$$

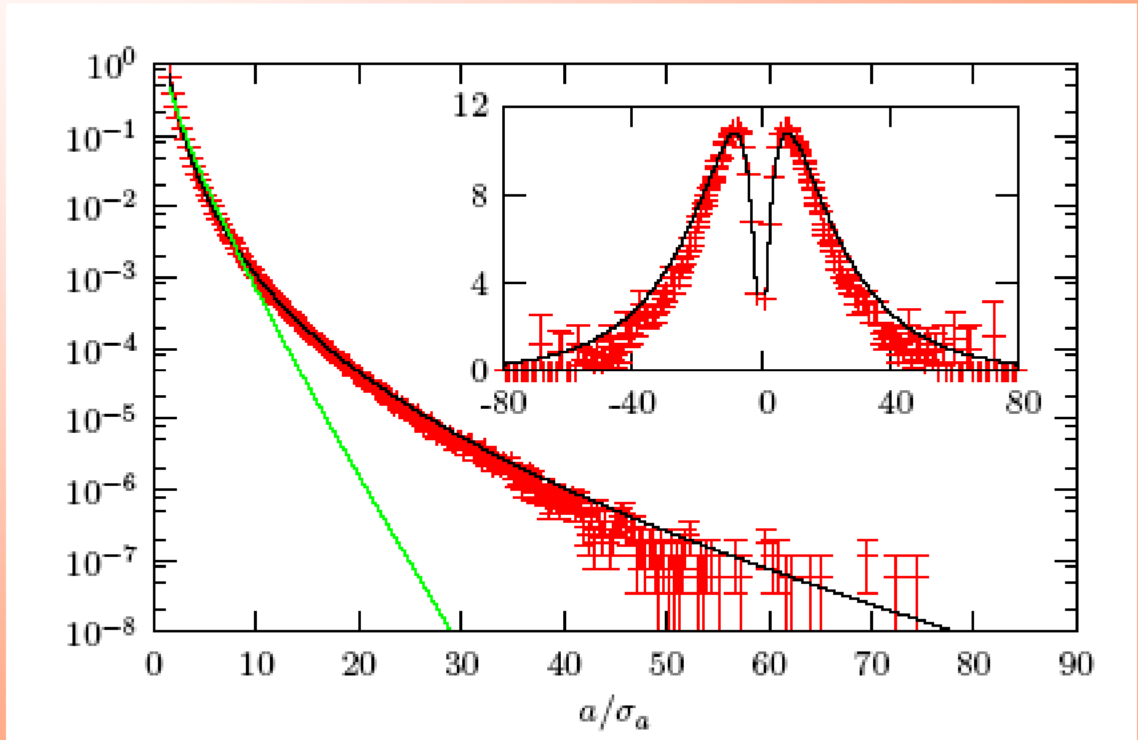
$$X_j \equiv \{\mathbf{r}_j, \mathbf{r}'_j, t_j\}$$

$$\mathcal{F}_n(\mathbf{r}_0 \mid X_1, \ldots, X_n) = U^n \int_{h_{\min}}^{h_{\max}} d\mu(h) \left(\frac{R_n}{L}\right)^{nh + \mathcal{Z}(h)} \tilde{\mathcal{F}}_{n,h}(\mathbf{r}_0 \mid \Xi_1, \Xi_2, \ldots, \Xi_n) \quad (2)$$

V.I. Belinicher, V.S. L'vov, A. Pomyalov and I. Procaccia J. Stat. Phys. 93, 797 (1998).

Fusion rules: V.S. L'vov and I. Procaccia, Phys. Rev. Lett. 76, 2896 (1996).

The greatest success of the multifractal formalism (so far)



L. Biferale and F. Toschi, Journ. of Turb. 6, 1 (20o6).

Anisotropic Turbulence

- All realistic turbulent flows are maintained by anisotropic (and inhomogeneous) forcing.

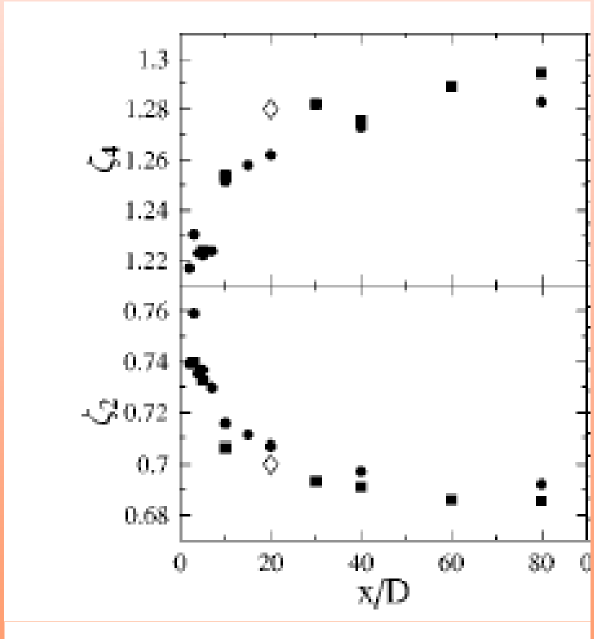
    - This was largely disregarded in data analysis.

Strange results:

Variation of scaling exponents from experiment to experiment...

- Scaling exponents depend on the position in the flow..

E. Gaudin, B. Protas, S. Gouion-Durand J. Wojciechowski and J.E Wesfried, Phys. Rev. E **57** R9 (1998).



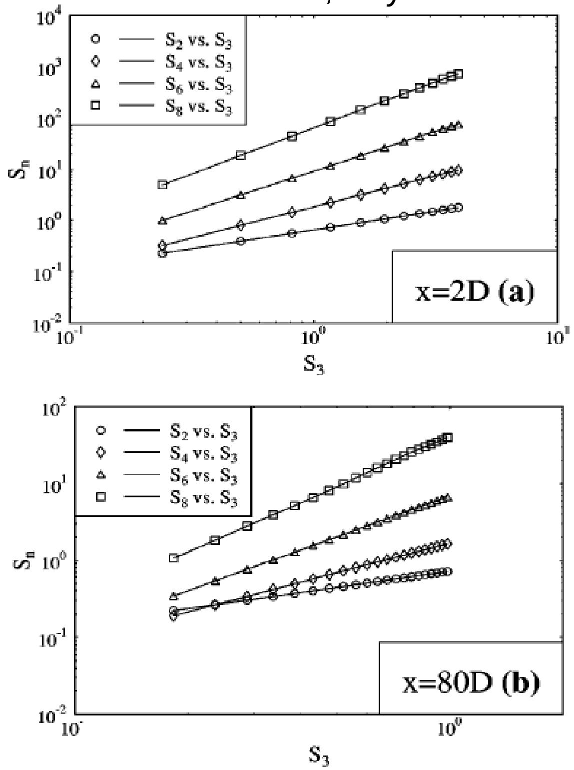


FIG. 4. ESS scaling of the structure functions $S_n(r)$, $n = 2,4,6,8$ at the distance: (a) $2D$ and (b) $80D$ downstream from the obstacle along the axis.

Method of solution

I. Arad, V.S. L'vov and I. Procaccia, Phys. Rev. E, 59, 6753 (1999).

$$S^{(n)}(\boldsymbol{r}) = \langle (\delta u_\ell(\boldsymbol{r}))^n \rangle.$$

Such objects admit simple SO(3) decomposition

$$S^{(n)}(\boldsymbol{r}) = \sum_{j,m} S^{(n)}_{jm}(r) Y_{jm}(\hat{\boldsymbol{r}}).$$

We are interested in particular in the scaling properties of the amplitudes

$$S^{(n)}_{jm}(r) \propto r^{\zeta^{(n)}_j}$$

The main research question is whether the spectrum $\zeta_j^{(n)}$ of scaling exponents is discrete and increasing.

- The Kraichnan model of passive scalar advection

$$\xi_j^{(2)} = \frac{1}{2}\Big(2 - d - \epsilon + \sqrt{(2-d-\epsilon)^2 + \frac{4(d+\epsilon-1)j(d+j-2)}{d-1}}\Big)\,, \quad j \geq 2$$

$$\xi_j^{(n)} = n - \epsilon\Big[\frac{n(n+d)}{2(d+2)} - \frac{(d+1)j(j+d-2)}{2(d+2)(d-1)}\Big] + O(\epsilon^2)$$

- Navier-Stokes turbulence: experiments and simulations

n	$j=0$	$j=2$	$j=4$	$j=6$
	$\zeta_0^{(n)}$ — $n/3$	$\zeta_2^{(n)}$ — $(n+2)/3$	$\zeta_4^{(n)}$ — $(n+4)/3$	$\zeta_6^{(n)}$ — $(n+6)/3$
2	0.70 (2) — 0.66	1.1 (1) — 1.33	1.65 (5) — 2.00	3.2 (2) — 2.66
4	1.28 (4) — 1.33	1.6 (1) — 2.00	2.25 (10) — 2.66	3.1 (2) — 3.33
6	1.81 (6) — 2.00	2.1 (1) — 2.33	2.50 (10) — 3.33	3.3 (2) — 4.00

L. Biferale and I. Procccia, Phys. Rep. 414, 43 (2005)

strong shear without homogeneity

The scaling exponents change altogether

$$\langle[\delta v(r)^3 + \alpha r \cdot S \cdot \delta v(r)^2]^{p/3}\rangle$$
$$\sim \langle\varepsilon(r)^{p/3}\rangle r^{p/3}.$$

F. Toschi, E. Leveque and G. Ruiz-Chavarria, PRL 85, 1436 (2000).

Implications both for exponents and for LES

E. Leveque, F. Toschi, L. Shao and J.-P. Bertoglio J. Fluid Mech. 570 (2007) 491

Wall-Bounded Turbulence

In a channel

$$\mathcal{R}e \equiv \frac{L\sqrt{p'L}}{\nu_0}, \qquad y^+ \equiv \frac{y\mathcal{R}e}{L}, \qquad V^+ \equiv \frac{V}{\sqrt{p'L}} .$$

$$V^+(y^+) = \frac{1}{\kappa_K} \ln y^+ + B, \text{ for } 30 \le y^+ \ll Re .$$

$\kappa_K = 0.44 \pm 0.03$ and the intercept $B \approx 6.13$

$$K \equiv \left\langle |\boldsymbol{u}(\boldsymbol{r},t)|^2 \right\rangle /2, \qquad W \equiv -\langle u_x(\boldsymbol{r},t) u_y(\boldsymbol{r},t)\rangle$$

In a constant shear flow $W/K \equiv W^+/K^+ = c_N^2, \qquad c_N \approx 0.53 .$

$$\kappa_K = \left(c_N \sqrt{C_2}\right)^3 .$$

T.S. Lo, V.S. L'vov, A. Pomyalov and I. Procaccia, Euro. Phys. Lett. 72, 943(2005)

Fluid flow with a flat plate

Question: what is the boundary between turbulent and laminar fluid?

Experimental answer: (Roddam Narasimha): $y \sim x^{0.8}$

V.S. L'vov, I. Procaccia and O. Rudenko, in preparationn $y \sim \frac{x}{(\log x)^2}$

Interesting relation between temporally developing turbulent boundary layer, and between stationary boundary layer

Free jet

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

The riddle of the week (month, decade…) : what is the angle and why?

Rough pipes of diameter D roughened by sand grains of diameter r

Nikuradze (1933), G. Gioia and P. Chkraborty PRL 96, 044502 (2006)
N. Goldenfeld, PRL 96, 044503 (2006)

$$f = \tau / \rho U^2$$

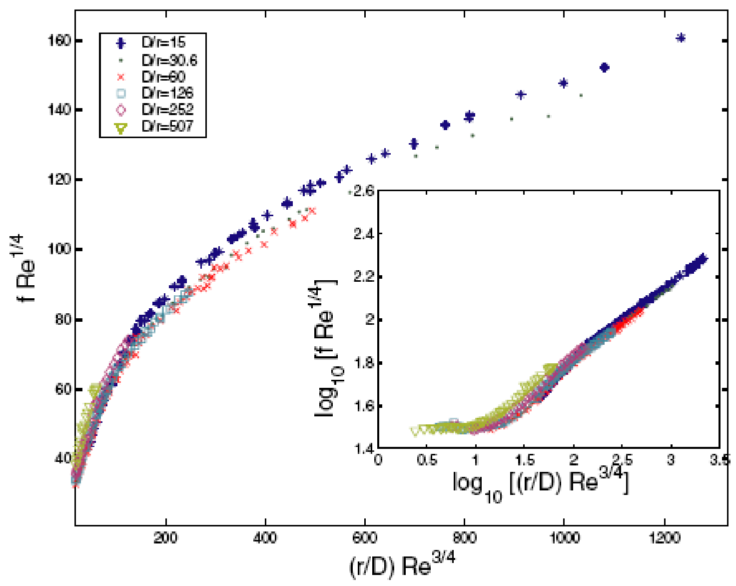
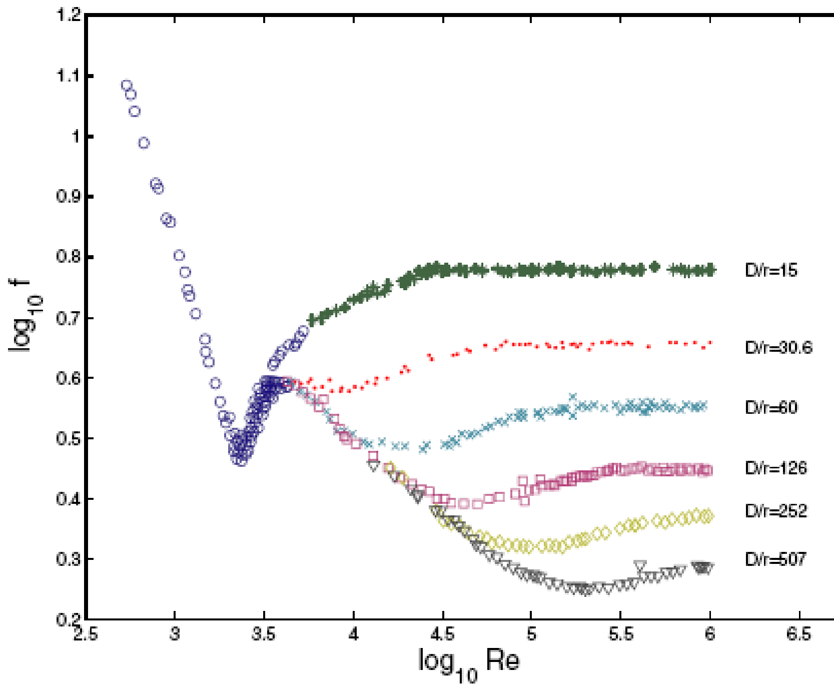


FIG. 2 (color online). Friction factor for turbulent flow in a rough pipe, as reported by Nikuradze [11], scaled according to the text. Inset: the same plot on a logarithmic scale.

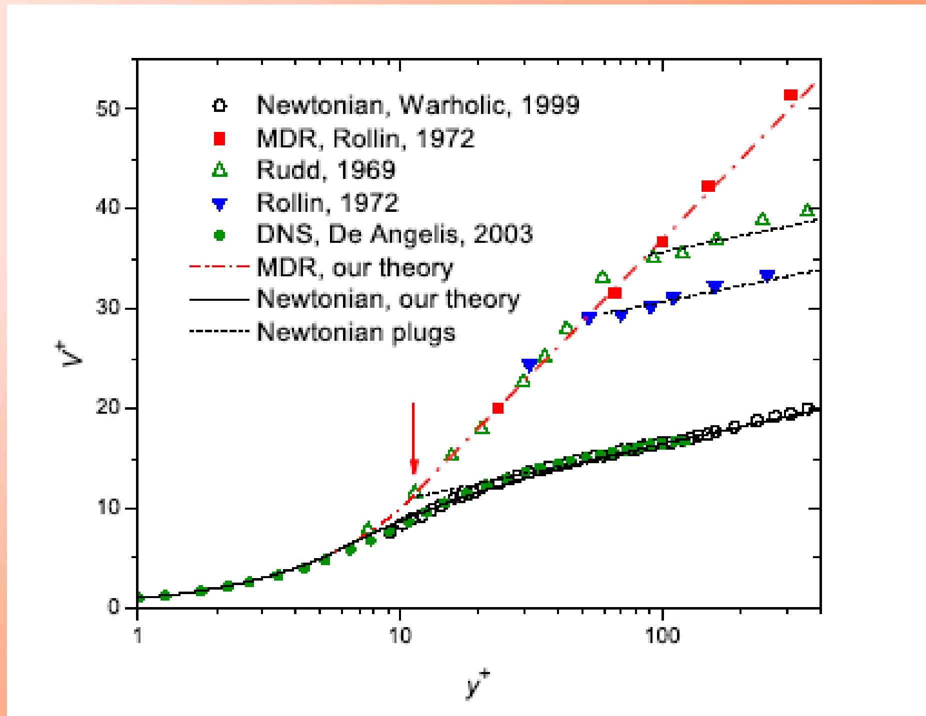


Turbulence with Additives

- Small concentrations of polymers

$$V^+(y^+) = \kappa_V^{-1} \ln y^+ + B_V$$

$$\kappa_V^{-1} \approx 11.7 \text{ and } B_V \approx -17.$$



I. Procaccia, V.S. L'vov and R. Benzi, Rev. Mod. Phys. In press

- Small concentrations of bubbles

My Take home message:

Experimental Physicists in Turbulence:

Stop pretending that you have isotropic homogeneous turbulence

(We are bored with yes Kolmogorov, no Kolmogorov)

Start studying seriously wall bounded turbulence, there are many challenges to make you famous