

How can numerical simulations contribute?

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Supercomputing

- More and more organized on European scale (Petaflop Project Pace)
- **DE**ep **C**omputing **I**nitiative within DEISA
→ 2 turbulence projects (2006/2007)
- Resolution of the most intensive gradients in turbulence at $R_\lambda \sim 100$ on a 2048^3
- 800,000 CPUh on 512 CPUs (1.9Tbyte RAM)
- 1 velocity snapshot = 68 Gbyte
- Quasi-Lagrangian analysis in 51^3 around 100 Lagrangian tracers (700 Gbytes)
- UNICORE platform for Grid computing
- Parallel NetCDF for I/O

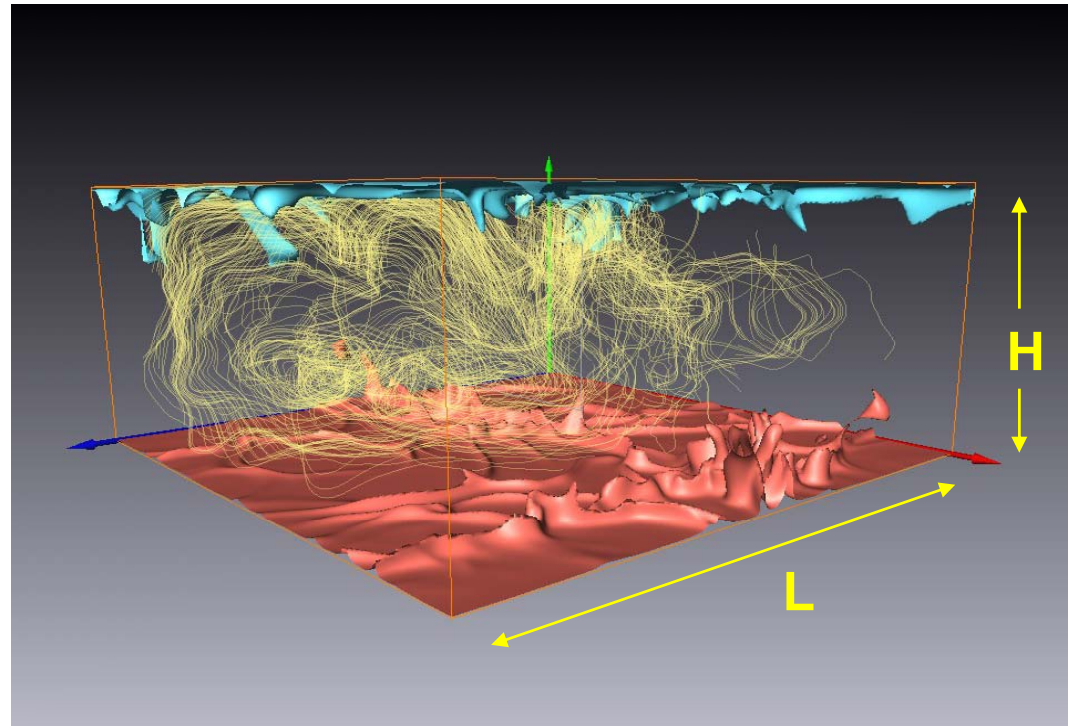


Case 1: Cartesian Cell

Geometry and Parameters

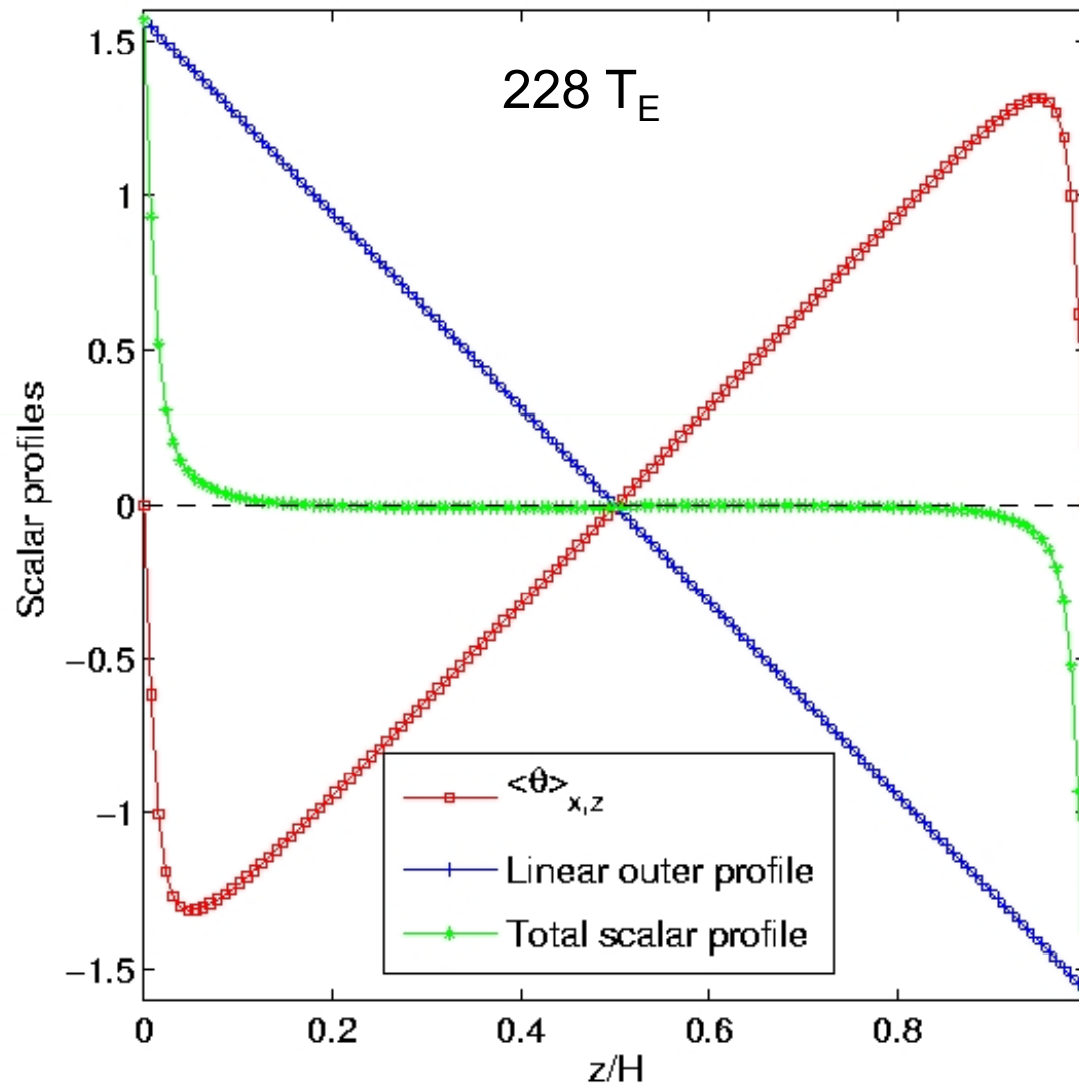
- Cartesian geometry (x,y,z)
- $Ra=10^7-10^9$
- $Pr=0.7$ (later $Pr > 1$)
- Aspect ratio: $\Gamma=L/H=2-32$

- Free-slip boundaries in z
- Equidistant grid
- Periodic side walls



Temperature Profile

$$N_x \times N_y \times N_z = 256 \times 256 \times 129$$



$$\text{Pr}=0.7$$

$$\text{Ra}=1.1 \times 10^7$$

$$\Gamma=2$$

$$R_\lambda=80$$

$$\text{Nu}=27.85 \pm 0.56$$

$$\delta_T = \frac{H}{2\text{Nu}} = 3\Delta$$

$$\frac{\eta_K}{\Delta} \approx 1$$

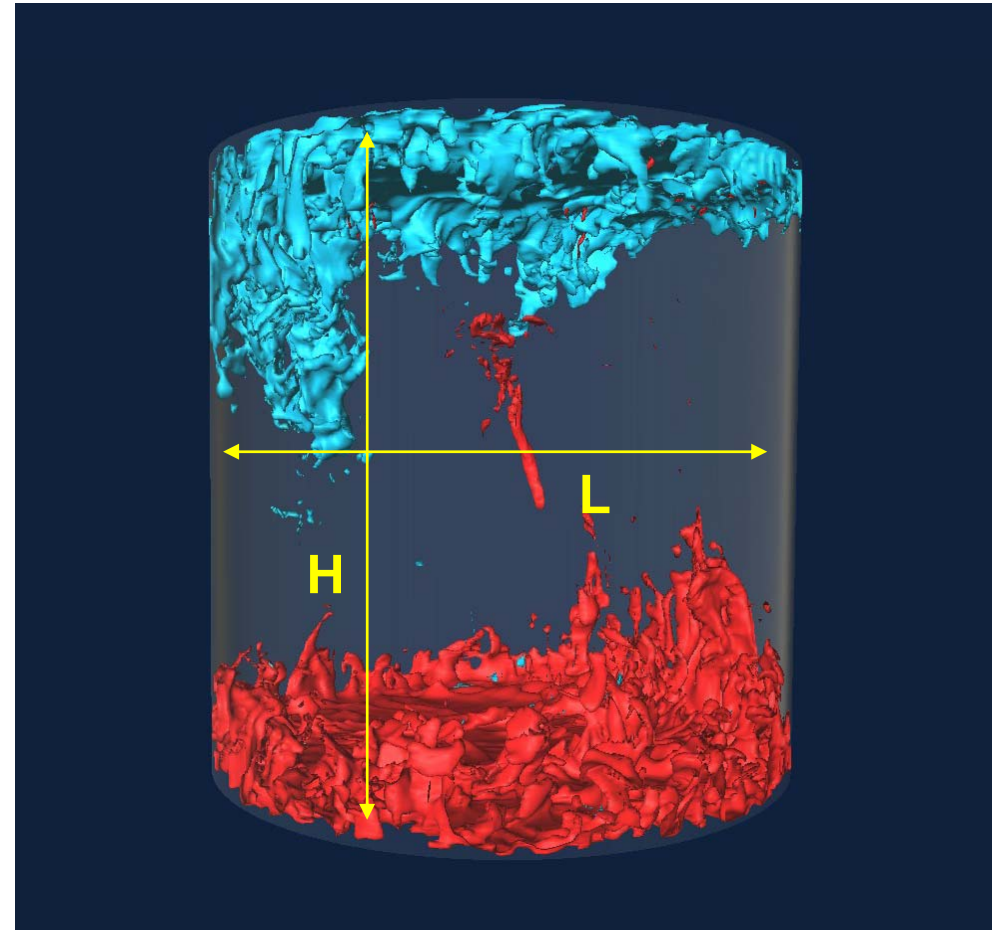
Case 2: Cylindrical Cell

(Verzicco & Orlandi, JCP 1996)

Geometry and Parameters

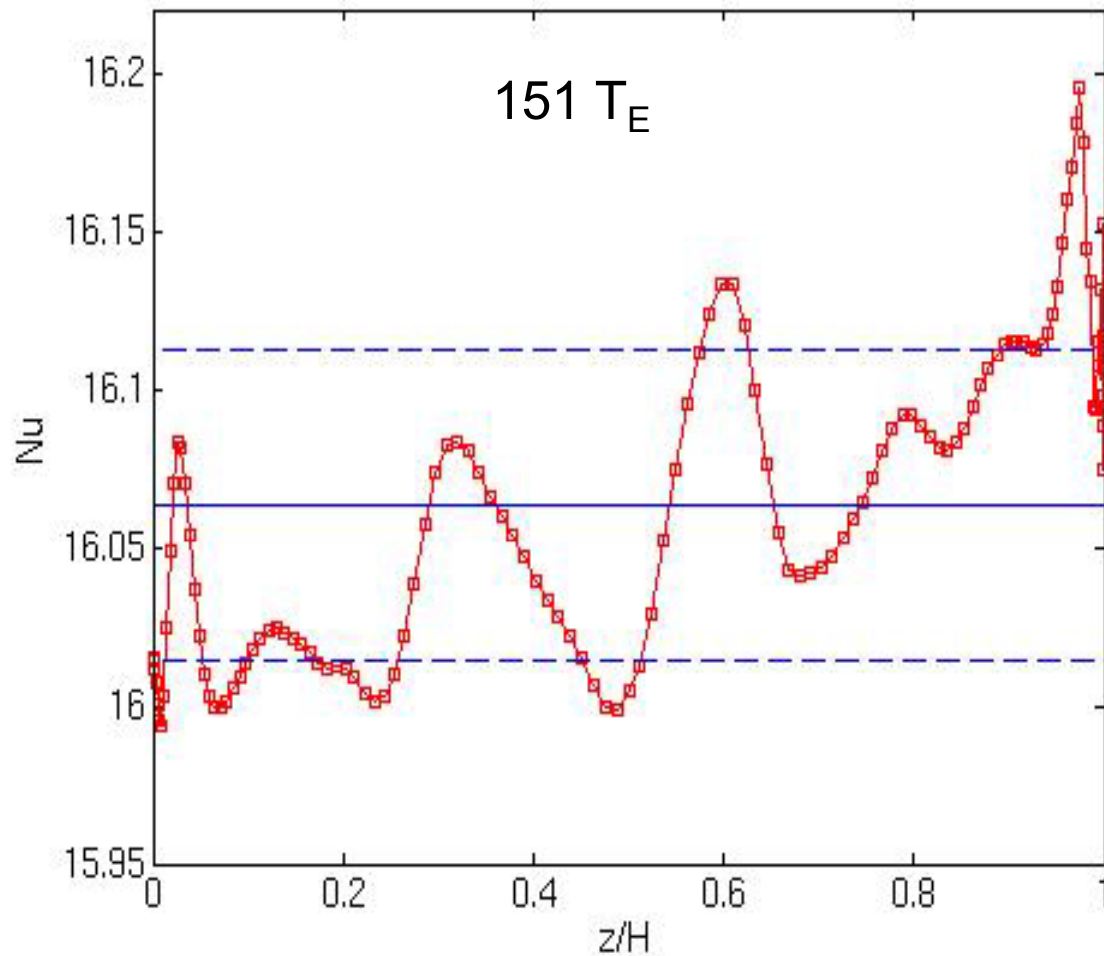
- Cylindrical geometry (r, θ, z)
- $Ra = 10^7 - 10^{12}$
- $Pr = 0.7$ (later $Pr > 1$)
- Aspect ratio: $\Gamma = L/H = 1, 3, 5$

- No-slip boundaries for flow
- Non-equidistant grid
- Temperature fixed at bottom/top
- Adiabatic side walls



Nusselt Number Convergence

$$N_\theta \times N_r \times N_z = 257 \times 165 \times 128$$



$$\text{Pr}=0.7$$

$$\text{Ra}=10^7$$

$$\Gamma=3$$

$$R_\lambda=(\text{Ra}/\text{Pr})^{1/4} \sim 100$$

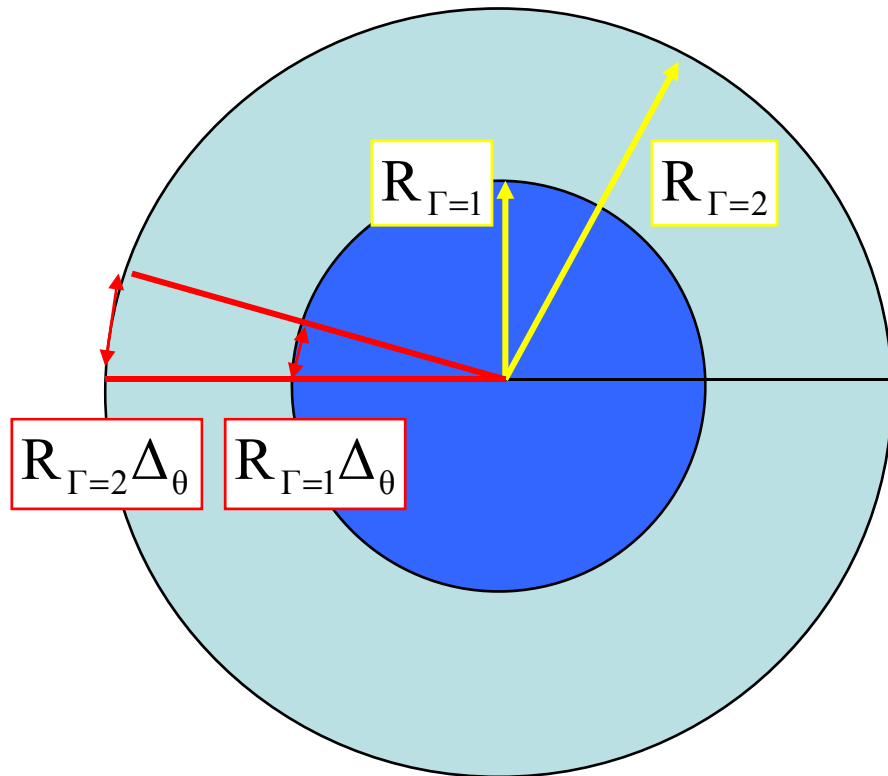
$$\text{Nu}=16.06 \pm 0.05$$

$$\delta_T = \frac{H}{2 \text{Nu}} \approx 0.03$$

$$\frac{\eta_K}{\tilde{\Delta}} \geq \frac{3}{4}$$

$$\tilde{\Delta} = \sqrt[3]{\max(\Delta_r) R \Delta_\theta \max(\Delta_z)}$$

Towards Large Γ



Example:

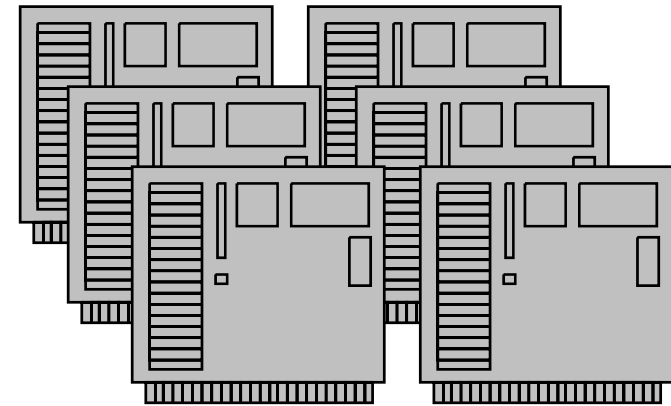
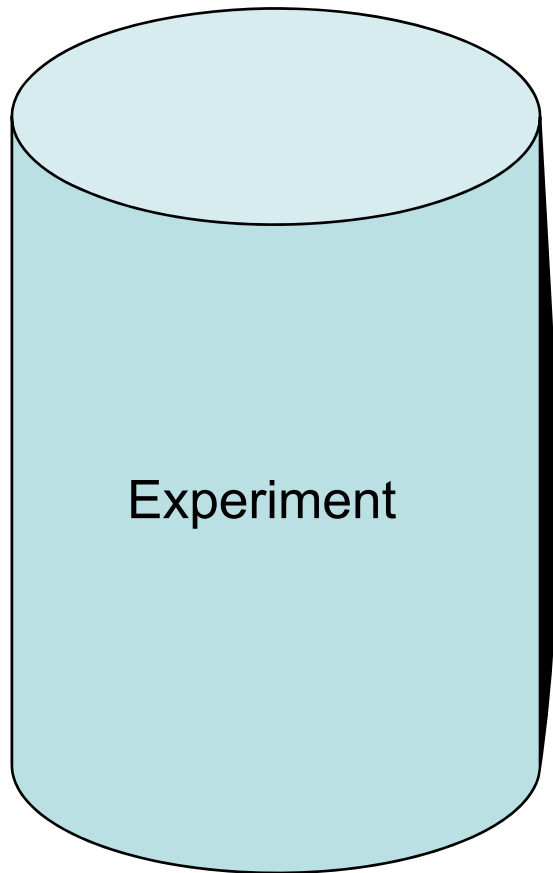
Sustain resolution & Double Γ

$$\tilde{\Delta}_{\Gamma=2} = \sqrt[3]{\max(\Delta_r) \underbrace{\Gamma R_{\Gamma=1}}_{R_{\Gamma=2}} \frac{\Delta_\theta}{\underbrace{\Gamma}_{\Delta_{\theta,\Gamma=2}}} \max(\Delta_z)}$$

$$\tilde{\Delta}_{\Gamma=2} = \tilde{\Delta}_{\Gamma=1} \rightarrow \Gamma N_\theta \times \Gamma N_r \times N_z$$

Number of lateral grid points grows with Γ^2 if the strong resolution constraints are sustained!

Contributions (I)



Direct participation

Parallel data processing
Database for later use

Indirect participation

Associated
suprcomputing projects
on classical/quantum
turbulence

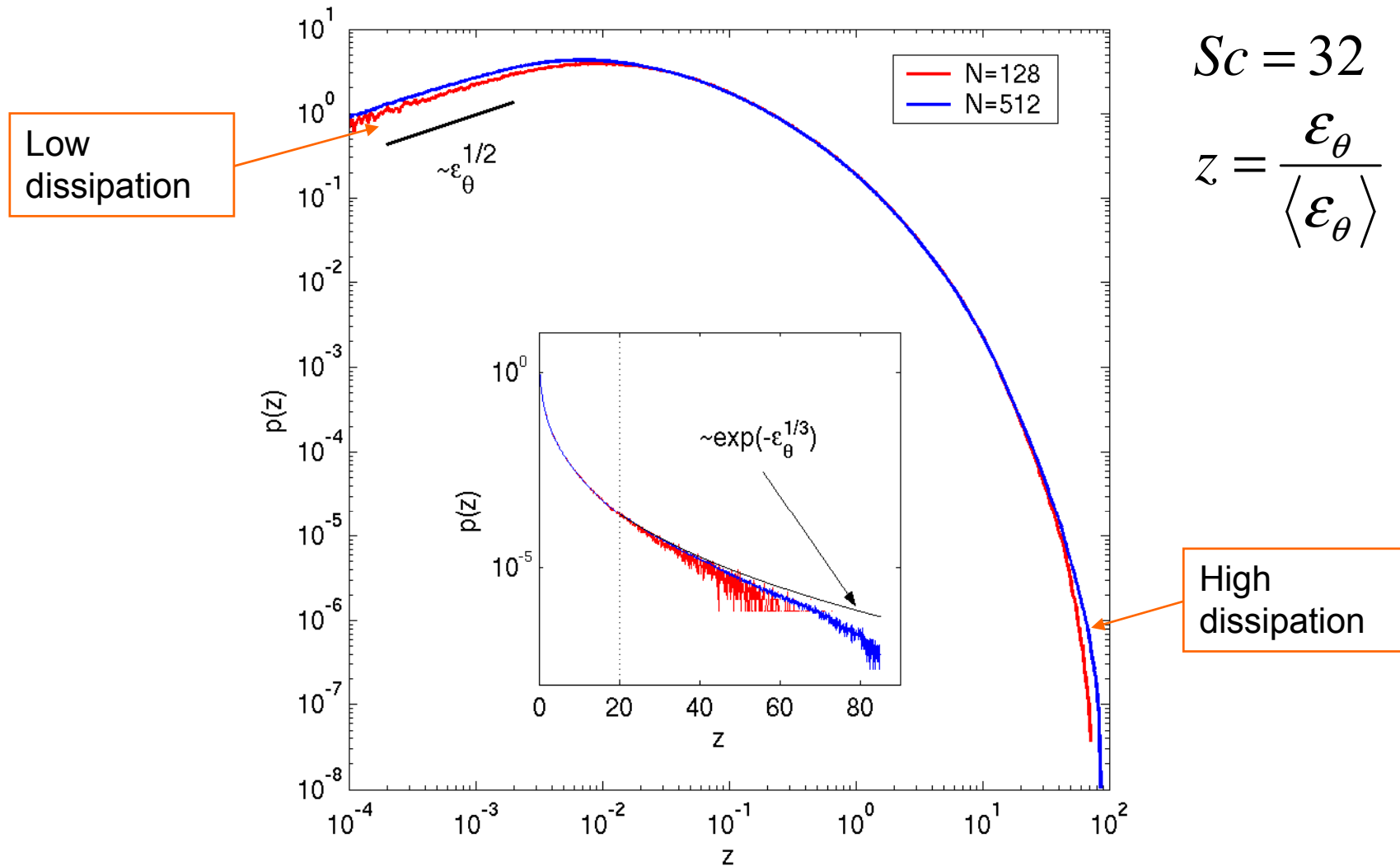
Contributions (II)

- Resolution of high-Ra boundary layers
- Transition from laminar to turbulent boundary layer at $Ra > 10^{11}$
- What are the differences to „classical“ boundary layers?
- Detailed Lagrangian study of breakdown of wind for aspect ratios $\Gamma > 1$
- Ultimate regime of convection?

- M. Emran (Ilmenau)
R. Verzicco (Bari)

Resolution in High-Sc Mixing

(Schumacher, Sreenivasan & Yeung, JFM 2005)



Resolution Criterion

(Grötzbach, JCP 1983; Schumacher, Sreenivasan & Yeung, JFM 2005)

- Maximum wavenumber (or finest scale) resolved $k_{\max} \approx \pi/\Delta$
- Resolution constraint for $Pr \leq 1$

$$k_{\max} \eta_K \geq \alpha \quad (\alpha > 1)$$

Recent experiences from pseudospectral DNS: $\alpha > 3$

- Kolmogorov length

$$\eta_K = \frac{v^{3/4}}{\langle \varepsilon \rangle^{1/4}} = H \left(\frac{Pr^2}{Nu Ra} \right)^{1/4}$$

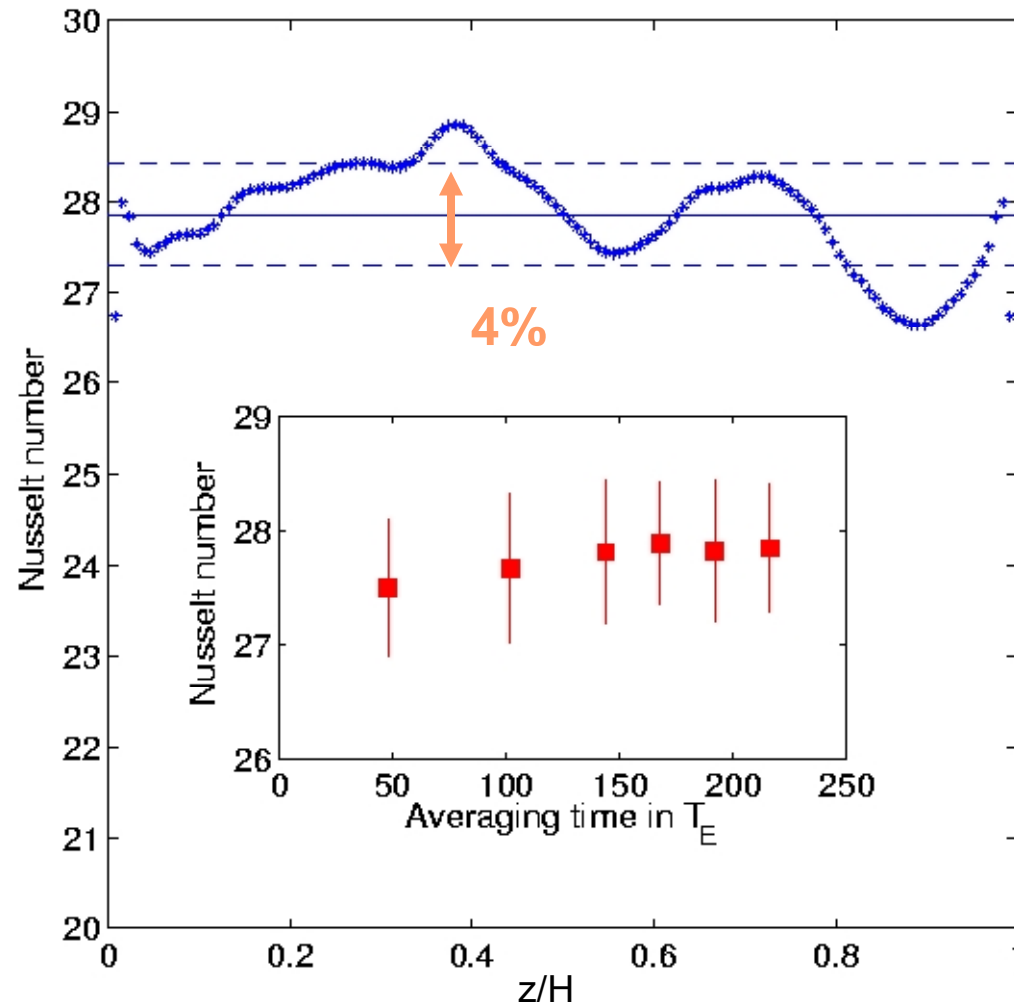
- Grid spacing

$$\Delta \leq \frac{H\pi}{\alpha} \left(\frac{Pr^2}{Nu Ra} \right)^{1/4}$$

or

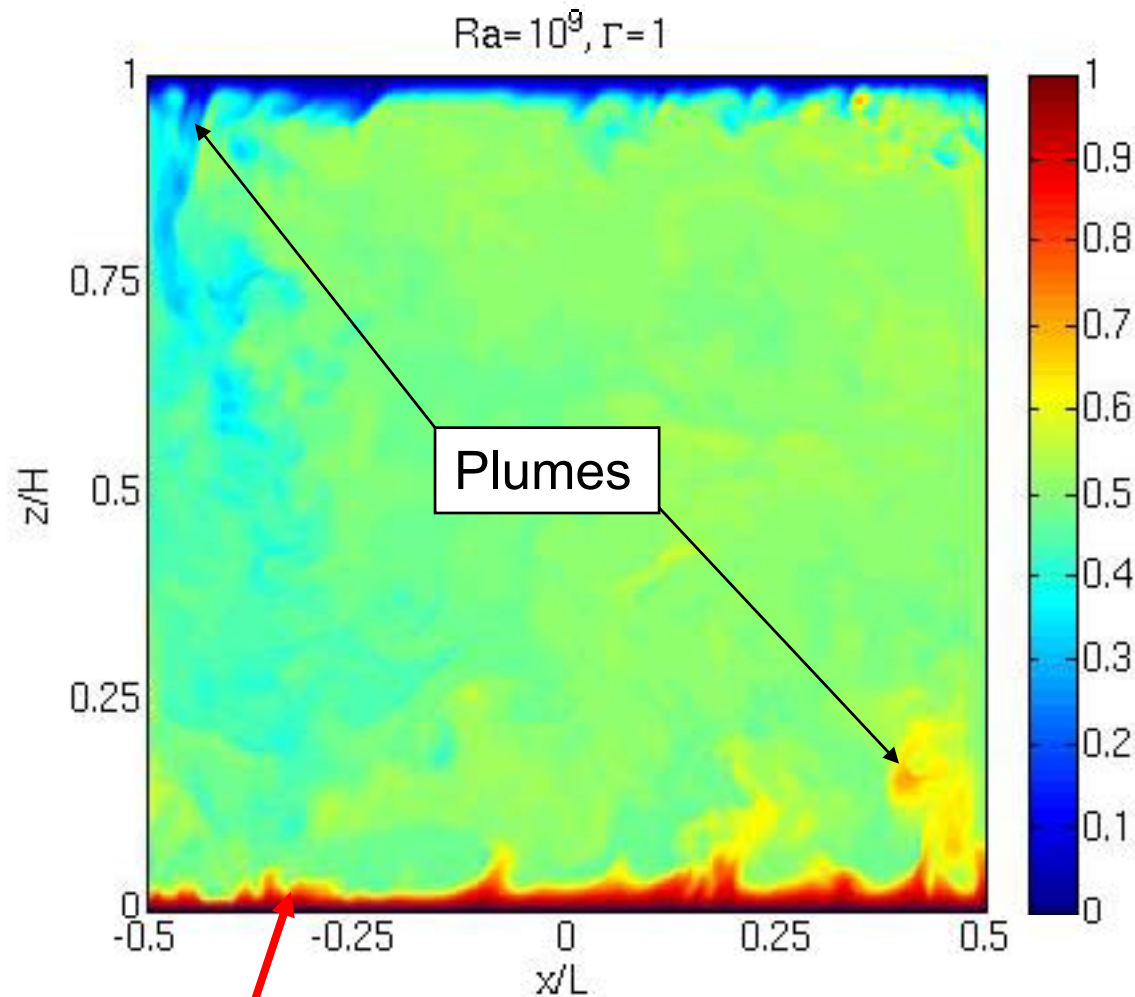
$$\frac{\eta_K}{\Delta} \geq \frac{\alpha}{\pi}$$

Nusselt Number Convergence



Very slow convergence !

Temperature Field (Pr=0.7)



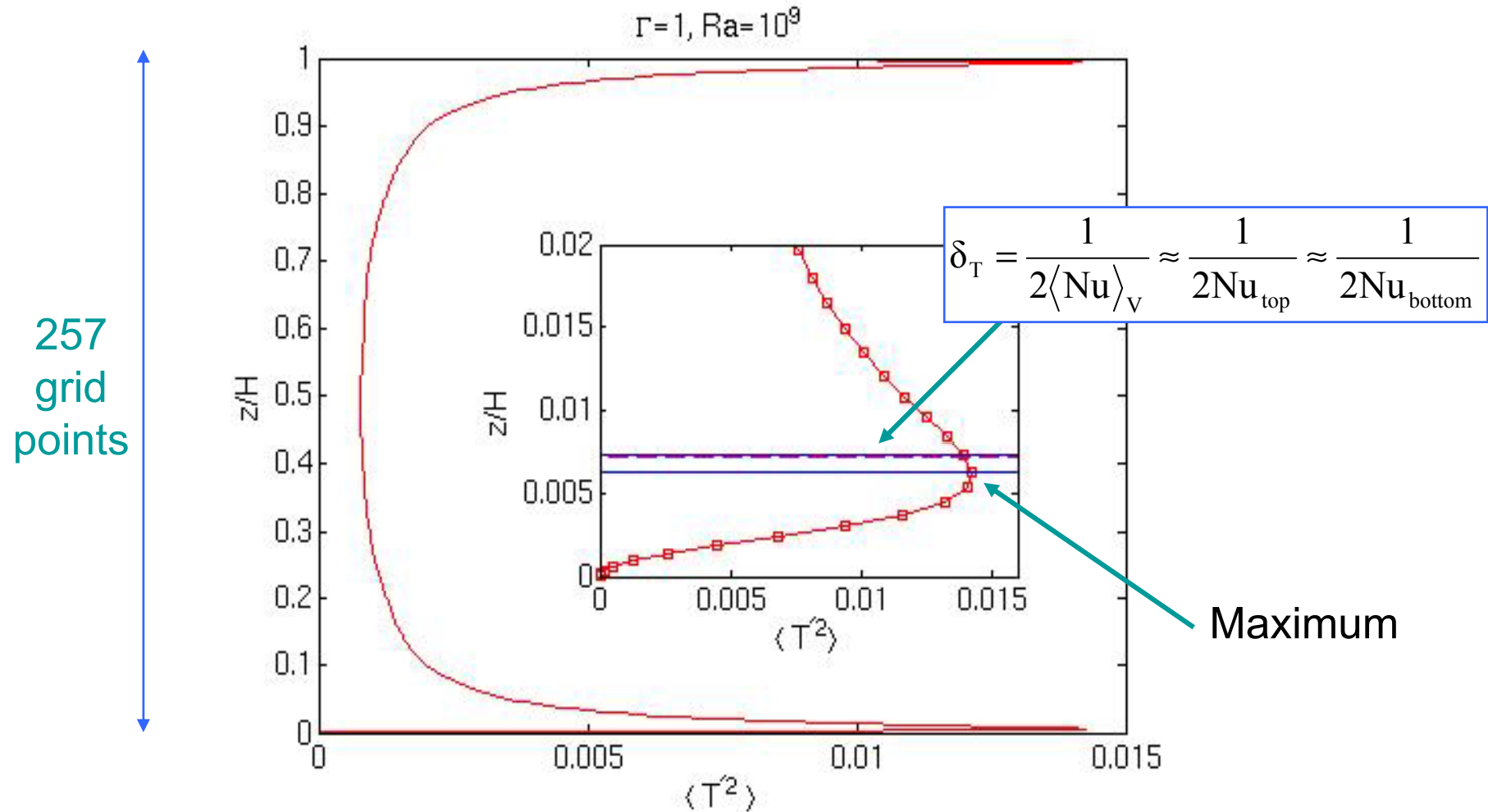
$$\frac{\eta_K}{\tilde{\Delta}} \geq \frac{3}{4}$$

$$\delta_T = \frac{1}{2 \text{Nu}} \approx 0.0077$$

$$N_\theta \times N_r \times N_z = 257 \times 193 \times 257$$

$$\tilde{\Delta} = \sqrt[3]{\max(\Delta_r) R \Delta_\theta \max(\Delta_z)}$$

Temperature Fluctuation Profile



Thermal boundary layer is resolved with 13 grid cells

Boundary layer thickness deviates from peak mean square by 17.5%