# Longitudinal Problem Set

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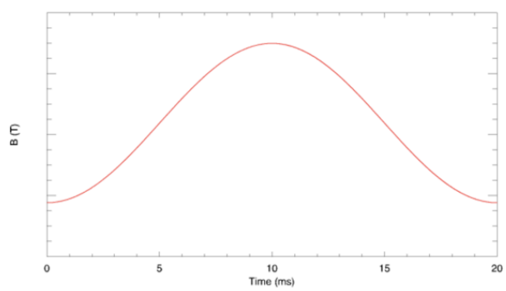
Due in $9^{th}$ November 2022

(a) ISIS Synchrotron




(b) ISIS dipole field as a function of time

**Problem 1.1.** ISIS is a proton synchrotron that operates from 70 to 800 MeV on a 50 Hz sinusoidal main magnet field (see figure b) above). Given the main dipole field at 70 MeV is 0.17639 T calculate: the magnetic rigidity at that energy, the bending radius $\rho$ and the magnetic rigidity and dipole field at top energy (800 MeV).

**Problem 1.2.** If 26.8% of the circumference is taken up with dipoles what is the mean radius $R$ of the synchrotron? Calculate the revolution frequency at 0 ms (70 MeV) and 10 ms (extraction at 800 MeV).

**Problem 1.3.** Calculate the following parameters at 0, 5 and 10 ms

a. Momentum

b. Kinetic Energy

c. Relativistic parameters $\beta$, $\gamma$

d. What does $\gamma_t$ have to be for ISIS to remain below transition throughout acceleration?

**Problem 1.4.** What is the minimum RF voltage required as a function of time (0 - 10 ms) to accelerate a proton at ISIS? Why do we need more?

**Problem 1.5.** Given a mean dispersion of 1 m, what is the $\gamma_t$? What transition kinetic energy does that correspond to? Calculate the slip factor $\eta$ at 0, 5 and 10 ms.

From the longitudinal equation of motion on slide 29 one can derive the symplectic mapping equations:

$$\Delta E_{n+1} = \Delta E_n + V_1(\sin\phi_n - \sin\phi_s) \quad (1)$$

$$\phi_{n+1} = \phi_n - \frac{2\pi h\eta}{E_0\beta^2\gamma}\Delta E_{n+1} \quad (2)$$

from the $n^{\text{th}}$ to the $(n+1)^{\text{th}}$ turn where $V_1$ is the peak gap voltage per turn and $E_0$ is the rest energy. These can be used to follow a particle's trajectory in the longitudinal phase space $(\Delta E, \phi)$ turn by turn.

**Problem 2.1.** Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates (0, $\phi$) in the range $0 < \phi < \pi$ and calculates and applies the mapping equations over $n$ turns. Assume a **constant energy** ($t = 0$ ms, 70 MeV, $\phi_s = 0$) and use the parameters you calculated in problems 1.3 and 1.5. (Other key parameters: $h = 2$, $V_1 = 19$ kV per turn and a sensible number for $n$.) Track four more trajectories with start co-ordinates $(0.5\text{ MeV}, \pi)$, $(-0.5\text{ MeV}, -\pi)$, $(1\text{ MeV}, \pi)$, $(-1\text{ MeV}, -\pi)$ and plot out all the longitudinal phase space trajectories on one graph.

**Problem 2.2.** For the above problem (2.1), pick out the three different types of trajectory and briefly describe the motion in each. Compare to the analogous situation of a simple pendulum.

**Problem 2.3.** Now assume we're half-way through acceleration ($t = 5$ ms) and use the parameters associated with this time-point from problems 1.3 and 1.5. Again, make the assumption that $\eta$, $E_0$, $\beta$ and $\gamma$ are constant over the $n$ turns you are tracking over. (Other key parameters: $h = 2$, $V_1 = 150$ kV per turn and a sensible number for $n$). You'll need to calculate $\phi_s$.

Assume several initial particle co-ordinates (0, $\phi$), this time in the range $-\pi < \phi < \pi$ and ensure you track a particle with start co-ordinates $(0,\ \pi - \phi_s - 0.02)$ to approximately trace out the separatrix. Plot out all the particles' longitudinal phase space trajectories over $n$ turns on one graph.

**Problem 2.4.** Calculate the RF bucket height, $\Delta E_{\text{max}}$ (energy acceptance) using equation 36 on slide 45 for the cases in 2.1 and 2.3. Compare with the bucket height from your separatrix particle trajectories. (Note that $E_s$ in the equation is the total beam energy: rest mass and kinetic energy.)

**Hand in a copy of your program(s) with its (their) output. Marks are available for the code itself! Do get in touch if you have any questions or are having any issues.**

**OPTIONAL PROBLEM...for those who are interested**

**Problem 3.1.** What if we now introduce a second harmonic RF system at twice the frequency of the first ($h = 4$) such that we now have the energy gain per turn

$$\Delta E_{\text{turn}} = e\left(V_1 \sin\phi_s + V_2 \sin(2\phi_s + \theta)\right), \quad (3)$$

where $V_2$ is the second harmonic RF voltage and $\theta$ is the phase difference between the two RF systems. How does this change our mapping equations? Try putting this into your tracking code with $V_2 = V_1$ for the case $\phi_s = 0$. How does the RF bucket change with

a. $\theta = \pi$?

b. $\theta = 0$?