Ultra-light axions

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Workshop — New Physics from Galaxy Clustering, CERN

11/22/2022
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Intro/review of ULAs

Deeper dive into modeling uncertainties for ULA observables

Existing CMB and LSS+ [galaxy power spectrum, MW satellite, Ly-α] tests of ULAs

Forecasting — future prospects (HI surveys, S4 lensing, kSZ)
Era of precision cosmology!

Planck Collaboration: Cosmological parameters

In all cases the helium mass fraction for the reionization of hydrogen and simultaneous first reionization of helium. Our baseline results are based on Planck 2018 data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>0.02237 ± 0.000081</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>0.1206 ± 0.0003</td>
</tr>
<tr>
<td>$\sigma_8$</td>
<td>0.803 ± 0.018</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.962 ± 0.013</td>
</tr>
<tr>
<td>$h$</td>
<td>0.677 ± 0.011</td>
</tr>
<tr>
<td>$\Delta^2$</td>
<td>3.341 ± 0.48</td>
</tr>
</tbody>
</table>

The corresponding frequency spectra (expressed as the contribution to the drag). The top group of six rows are the base parameters, which are sampled in the MCMC analysis with flat priors.
40σ detection!

**Angular scale**

- Planck 2018 (MV)
- SPT-SZ 2017 (T, 2500 deg²)
- Planck 2015 (MV)
- ACTPol 2017 (MV, 626 deg²)
- SPTpol 2015 (MV, 100 deg²)

**Source Plane (S)**

- $d_S$

**Lens plane (L)**

- $d_L$
- $d_{LS}$

arXiv: 1807.06210
Galaxy clustering observations

Figure 1. Top: Data points showing inference of the 3D linear matter power spectrum at $z = 0$ from Planck CMB data on the largest scales, SDSS galaxy clustering on intermediate scales, SDSS Ly forest clustering and DES cosmic shear data on the smallest scales. In cases where error bars in the $k$-direction are present, we have used the method of Tegmark & Zaldarriaga (2002) to calculate the central 60% quantile of the region to which each data point is sensitive. In other cases, data points represent the median value of the measurement. The solid black line is the theoretical expectation given the best-fit Planck 2018 $\Lambda$CDM model (this model also enters the computation of the data points themselves). The dotted line for reference shows the theoretical spectrum including non-linear effects.

Bottom: deviation of the data from the Planck best fit $\Lambda$CDM 3D matter power spectrum.

The four cosmological parameters are the scalar spectral index $n_s$, the RMS matter fluctuations amplitude today in linear theory $8$, the matter density today $\Omega_m$, and the expansion rate today $H_0$. The astrophysical parameters (all at $z = 3$) are the normalization temperature of IGM $T_0$, the logarithmic slope of the dependence of the IGM temperature, the effective optical depth of the Ly absorption $\Delta$ and the logarithmic slope $\beta$ of the redshift dependence of $\Delta$.

The central (also dubbed best-guess) simulation is based upon a fiducial model corresponding to the Planck Collaboration et al. (2014) best-fit cosmology. The simulation grid, however, allows us to test other cosmologies.

In Table 1, we list the values of the parameters used in the best-guess simulation, as well as the corresponding MNRAS 000, 1–7 (2015) Chabanier et al. 2019, MNRAS 489, p2247.
What are axions?

New scalar field with global U(1) symmetry!

\[ \mathcal{L}_{CPV} = \frac{\theta g^2}{32\pi^2} G \tilde{G} - \frac{a}{f_a} g^2 G \tilde{G} \]

- Weakly couples to SM gauge fields (via fermions)

- Axion gets mass through non-perturbative QCD effects

\[ m_a \sim \frac{\Lambda^2_{QCD}}{f_a} \]

Relic abundance

\[ m_a < 10^{-2} \text{ eV} \]

Before PQ symmetry breaking, \( \theta \) is generically displaced from vacuum value

EOM:
\[
\ddot{\theta} + 3H \dot{\theta} + m_a^2(T) \dot{\theta} = 0
\]

After \( m_a(T) \gtrsim 3H(T) \), coherent oscillations begin, leading to \( n_a \propto a^{-3} \)

\[ \Omega_{\text{mis}} h^2 = 0.236 \left\langle \theta_i^2 f(\theta_i) \right\rangle \left( \frac{m_a}{6.2 \mu\text{eV}} \right)^{-7/6} \]
In string theory, extra dimensions compactified: Calabi-Yau manifolds

Ultra-light axions (ULAS) in string theory
Ultra-light axions (ULAS) in string theory

In string theory, extra dimensions compactified: Calabi-Yau manifolds

Hundreds of scalars with approx shift symmetry
Ultra-light axions (ULAS) in string theory

- In string theory, extra dimensions compactified: Calabi-Yau manifolds

Hundreds of scalars with approx shift symmetry

Many axions
In string theory, extra dimensions compactified: Calabi-Yau manifolds

Mass acquired non-perturbatively (instantons, D-Branes)

Ultra-light axions (ULAS) in string theory

![Diagram of Calabi-Yau manifolds]

Hundreds of scalars with approx shift symmetry

Many axions

\[ m_a^2 = \frac{\mu^4}{f_a^2} e^{-\text{Volume}} \]
In string theory, extra dimensions compactified: Calabi-Yau manifolds

Mass acquired non-perturbatively (instantons, D-Branes)

Scale of new ultra-violet physics

Ultra-light axions (ULAS) in string theory

Hundreds of scalars with approx shift symmetry

Many axions
In string theory, extra dimensions compactified: Calabi-Yau manifolds

Mass acquired non-perturbatively (instantons, D-Branes)

Ultra-light axions (ULAS) in string theory

Hundreds of scalars with approx shift symmetry

Many axions

Scale of extra dimensions in Planck units

$$m_a^2 = \frac{\mu^4}{f_a^2} e^{-\text{Volume}}$$
In string theory, extra dimensions compactified: Calabi-Yau manifolds

Ultra-light axions (ULAS) in string theory

Axiverse! Arvanitaki+ 2009

\[10^{-33} \quad \text{ma} (\text{in} \ eV) \quad 10^{-18}\]
Cosmology of axions: dark matter and dark energy candidates

\[ \text{Scale of universe} = (1 + z)^{10^3} \]

Density

\[ 10^{-31} \text{ eV} \]

\[ 10^{-10}, 10^{-5}, 10^0 \]

\[ t / \text{Age of Universe} \]
Cosmology of axions: dark matter and dark energy candidates

\[
\text{Density} \sim 10^3 - 10^2 \text{eV} \]

\[
(1 + z)^{-1}
\]

\[
\text{Matter} \rightarrow \text{Axions} \rightarrow \Lambda\text{-like behavior}
\]

\[
\text{Age of Universe}
\]

10^{-31} \text{ eV}

\[
10^0
\]

\[
10^{-5}
\]

\[
10^{-10}
\]

\[
10^{-3} \quad 10^{-2} \quad 10^{-1}
\]

\[
\text{Time}
\]
Cosmology of axions: dark matter and dark energy candidates

\[ \frac{1}{(1 + z)} \\sim 10^{-3} \]

\[ \frac{1}{10^3} \]

\[ \frac{1}{10^2} \]

\[ \frac{1}{10^1} \]

\[ 10^{-31} \text{ eV} \]

\[ 10^{-5} \]

\[ 10^{-10} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ t/\text{Age of Universe} \]

\[ \text{Density} \]

\[ \text{Matter} \]

\[ \text{Axions} \]

\[ \Lambda \]

\[ \text{matter-like behavior} \]
Cosmology of axions: dark matter and dark energy candidates

Depending on time of this transition — axions can be dark matter or dark energy

Axion mass $m_a$ sets time of this transition

$\begin{align*}
\text{Density} & \quad 10^{-10} \quad 10^{-5} \quad 10^0 \\
\text{Time} & \quad \text{Matter} \quad \text{Axions} \quad \Lambda
\end{align*}$
Cosmology of ultra-light axions: dark matter and dark energy candidates

Scale corresponding to typical galaxy separation today


ULA as dark energy with specific $w(z)$

$ma \lesssim 10^{-27} \text{ eV}$

ULA matter behavior starts too late for struct. formation
Cosmology of ultra-light axions: dark matter and dark energy candidates

Scale corresponding to typical galaxy separation today


ULA as dark matter

\[ m_a \gtrsim 10^{-27} \text{ eV} \]

ULA matter behavior starts in time for struct. formation
ULA cosmology

Structure growth suppressed on small scales

\* Perturbed KG equation

\[ \ddot{\phi} + 3H \dot{\phi} + \left[ m^2 + \frac{k^2}{R^2} \right] \delta \phi \propto \Psi \]

\* Axion deBroglie

\* Astronomical length

\* Kilo-lightyear–giga-lightyear

\* WKB fluid description

\[ c_s^2 \propto \frac{k^2}{(4m_a R^2)} \]

\* Small-scale suppression

\[ \lambda_J \sim 3 \left( \frac{m_a}{10^{-25} \text{ eV}} \right)^{-1/2} \text{ Mpc} \ (1 + z)^{1/4} \]

Hu, Barkana, Gruzinov 2000, Hwang & Noh 2009
Suppressed growth of structure

Structure growth suppressed on small scales
AxionCAMB

Code by Grin et al. 2013, based on CAMB (A. Lewis)
http://github.com/dgrin1/axionCAMB

Included in H recombination
Expansion history

CMB and matter perturbation code including ULAs!

AXIONS!

Einstein equations

NR fluid eqs.

dark matter

gravitational perturbations

baryons ↔ photons

neutrinos

Boltzmann equation

Thomson scattering

ULA of any mass is self-consistently followed from DE to DM regime
See also …

🌟 AxiCLASS (Poulin et al.)
https://github.com/PoulinV/AxiCLASS

🌟 class.FreeSF
https://github.com/lurena-lopez/class.FreeSF

How sound are our approximations? (With Tessa Cookmeyer)
# Effective Fluid approximation (EFA)

<table>
<thead>
<tr>
<th>Background</th>
<th>Perturbations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>early times</strong></td>
<td>$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}<em>0 + m</em>{ax}^2 a^2 \phi_0 = 0$</td>
</tr>
<tr>
<td><strong>late times</strong></td>
<td>$\rho_{ax} = \rho_{osc} \left( \frac{a_{osc}}{a} \right)^3$</td>
</tr>
</tbody>
</table>

$$c_s^2 \equiv \frac{k^2}{4m_{ax}^2 a^2} \frac{1}{1 + \frac{k^2}{4m_{ax}^2 a^2}}$$

Particle number and momentum conserved at transition

$$\begin{align*}
m &= n\mathcal{H}/a \\
a &= a_{osc}
\end{align*}$$
Effective Fluid approximation (EFA)

- WKB approximation (not ‘coarse graining’ ala arXiv: 1904.01016)

- Formal equivalence to work of Ureña-López et al (JCAP July 2016) can be shown (switch at n=100)

\[
\begin{align*}
\frac{d\rho_{ax}}{d \ln a} &= -3 \rho_{ax}, \\
\frac{d \tilde{\theta}}{d \ln a} &= -\frac{k^2}{k_J^2} + e^{-\alpha} \frac{d h_L}{d \ln a} \cos \frac{\tilde{\theta}}{2}, \\
\frac{d \alpha}{d \ln a} &= \frac{1}{2} e^{-\alpha} \frac{d h_L}{d \ln a} \sin \frac{\tilde{\theta}}{2}, \\
\delta_{ax} &= -e^\alpha \sin \frac{\tilde{\theta}}{2}; \\
k u_{ax} &= -\frac{k^2}{2am_{ax}} e^\alpha \cos \frac{\tilde{\theta}}{2}.
\end{align*}
\]

- Are any of these sufficient in the era of precision cosmology?
Error ordering of prescription depends on axion mass

Rough benchmark (Seljak Phys.Rev.D68:083507,2003) for CV-limited parameter estimates violated in some cases:

$$\frac{\Delta C_{\ell}}{C_{\ell}} \gtrsim \frac{3}{\ell} \quad \text{if} \quad \ell \gtrsim \mathcal{O}(10^2) \quad \text{for} \quad r_{\text{ax}} \sim 10^{-1} \quad \text{and} \quad m_{\text{ax}} = 10^{-27} \text{ eV}$$
Matter power spectrum (safe WiggleZ)
Bias results

3σ limit from Planck 2013

- Negligible bias for Planck and S4 (usually)
- Modest bias for S4 (some cases) and CVL (but likely only need to worry if claiming detection)
For a mode $k_{1/2}$ where the matter power spectrum yields half its CDM value, we show our technique’s total error as a function of our switch time parameter $m/H_*$ for various choices of the EFA equation of state $c_s^2$, computed by comparing to a late switch time $m/H_* = 2 \times 10^3$. Using the field sound speed $c_s^2$. 

\[ c_s^2_{\text{efa}} = c_s^2_{\phi} + \frac{5}{4} \frac{H^2}{m^2}. \]

FIG. 9. For a mode $k_{1/2}$ where the matter power spectrum yields half its CDM value, we show our technique’s total error as a function of our switch time parameter $m/H_*$ for various choices of the EFA equation of state $c_s^2$, computed by comparing to a late switch time $m/H_* = 2 \times 10^3$. Using the field sound speed $c_s^2$. 

\[ c_s^2_{\text{efa}} = c_s^2_{\phi} + \frac{5}{4} \frac{H^2}{m^2}. \]
Future work

- Code comparisons!
- CMB Lensing
- Linear matter power spectra (IC for sims, Lyman-α, MW-scale tests)
- Implementing improved EFA
  - AxionCAMB bug fix forthcoming (~% level changes) with forecasts
    - $h$ in background $KG$
    - Neutrino contribution to Hubble
    - RK8 coefficient fix
    - Background temperature norm (CMB)
Table 8. Constraints on the basic six-parameter Planck collaboration: CMB power spectra & likelihood

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>68% limits</th>
<th>Best fit</th>
<th>68% limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$CDM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_a = 0.51$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_\Lambda = 0.18$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planck 2013</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$H_0 = 10^{27}$ eV

Dramatic changes to observables can result
Observables

Figure credit, E. Trott
Data + Analysis

- 240,000 emission line galaxies at $z<1$
- 3.9 m Anglo-Australian Telescope (AAT)
- Planck 2013 temperature anisotropy power spectra (+SPT+ACT)
- Cosmic variance limited to $\ell \sim 1500$
- WiggleZ galaxy survey (linear scales only $k \lesssim 0.2h \ Mpc^{-1}$)

Nested sampling, MCMC, vary $m_a, \Omega_a h^2, \Omega_c h^2, \Omega_b h^2, \Omega_\Lambda, n_s, A_s, \tau_{reion}$
CMB LENSING

A slice of (dark matter) life at $z \sim 1$

Source Plane (S)  Lens plane (L)

\[ \hat{\alpha} = \nabla_{\tilde{\theta}} \left\{ \int \left( \frac{d_{LS}}{d_L d_S} \right) \Phi \left[ d(\eta) \tilde{\theta}, \eta \right] d\eta \right\} \]
below the de Broglie wavelength. This suppresses the axion density in the CMB, and in the evolution of the Hubble rate and other distance standard cosmological model, affecting the Silk damping scale and mode, the initial density perturbations are negligible but they later ground field value across the observable Universe. In the adiabatic isocurvature perturbations in observables. There is a smooth background energy states. For the energy densities of interest in cosmology, the polarisation auto power. The dominant effect comes from the power. The dominant effect visible is the axion Jeans scale, which suppresses power for Figure 2. Adiabatic Power Spectra for DM-like Axions: $\Delta [\ell(\ell+1)]^2 C_\ell^{\phi} / C_\ell$ for different axion masses.

Most of the preceding discussion applies to any scalar or pseudo-scalar field, $\phi$. For the axion field, the following properties hold:

- The axion equation of state changes from $w_a = -1/3$ to $w_a = 0$ as the axion mass increases, which affects the diffusion damping scale, as well as the amplitude of the early $\Lambda$CDM for $m_a < 10^{-11} \text{eV}$. However, the reference to $m_a = 10^{-25} \text{eV}$.

We fix axions to be all of the DM, with $m_a > 10^{-25} \text{eV}$. For larger $m_a$, there is a greater than 10% suppression of lensing power relative to CDM for $m_a < 10^{-25} \text{eV}$. Differences to CDM have not been presented elsewhere before. The dominant effect in the lensing power is caused by the axion Jeans scale, which suppresses structure formation, and thus reduces the total amount of gravitational lensing of the CMB. The structure suppression scale is $10^{25} \text{eV}$. For larger $m_a$, there is a greater than 10% suppression of lensing power relative to CDM for $m_a < 10^{-25} \text{eV}$. Differences to CDM have not been presented elsewhere before. The dominant effect in the lensing power is caused by the axion Jeans scale, which suppresses structure formation, and thus reduces the total amount of gravitational lensing of the CMB. The structure suppression scale is $10^{25} \text{eV}$.

The large suppression of the lensing power is effectively removed at high-$\ell$ by non-linear corrections that can be safely neglected (e.g., Lewis & Challinor 2006). For a discussion of non-linear effects for axions at high-$\ell$ (H15).
Constraints

https://arxiv.org/abs/1708.05681
Figure 14: Joint posterior distributions for an axion with a mass of $10^{32}$ eV for three experimental setups. We note an improvement on the constraint on the axion fraction when breaking the degeneracy with $H_0$ present with the CMB data. The gray shaded area represents the confidence interval for $h$ from the SH0ES measurement \cite{81}.

Figure 15: 68% (dark-colored) and 95% (light-colored) confidence level bounds on the axion density from the CMB data, galaxy clustering and the combined measurements.

The prior favours a higher value of $A_s$ which is slightly degenerate with the axion fraction at that mass as shown in Fig. 16. Another contributing factor is that the CMB prior does not constrain the axion fraction as well as for the axion masses below $10^{25}$ eV. Performing a joint likelihood analysis rather than imposing a prior on the cosmological parameters may allow for stronger constraints for this mass bin and is left for future work. We note however that galaxy clustering measurements alone improve existing constraints on the axion fraction at that mass by over a factor of 4.5 (see Table 3).
LYMAN-α

image courtesy Ned Wright
Figure 6. Here we plot the marginalised posterior distribution of $1/m_\chi$ from both the analyses performed by Iršič et al. (2017a) (green lines, without QP) and ours (red lines, with QP). The vertical lines stand for the $2\sigma$ C.L. limits.
LYMAN-α

Takeshi Kobayashi, Riccardo Murgia, Andrea De Simone, Vid Irsic, and Matteo Viel

Figure 4: The comparison between the constraints on the scalar DM parameter space from the Lyman-α forest data analysis at 2 and 3σ (red regions, see Fig. 1), and the region capable of “solving” the missing satellite problem (cyan region bounded by dashed lines). The green and blue dotted lines refer to models which predict $N_{\text{sub}} = 60$, when choosing $M_{\text{halo}} = 10^{12} M_\odot/h$ and $M_{\text{halo}} = 3 \cdot 10^{12} M_\odot/h$, respectively.

The parameter window for $(m, F)$ of scalar DM where the number of subhalos lies within the “solving” range is shown in Fig. 4 as the cyan shaded area bounded by dashed lines. For reference, when using $M_{\text{halo}} = 1.7 \cdot 10^{12} M_\odot/h$, the aforementioned computation gives the number of subhalos with CDM only as $N_{\text{sub}} = 158$, while $20 \leq N_{\text{sub}} \leq 60$ for the reference warm DM models. (However, as we explained, these absolute values are irrelevant when focusing on the relative suppression of $N_{\text{sub}}$.) The red shaded areas in Fig. 4 represent the 2 and 3σ contours from the Lyman-α forest data analysis, discussed in Section 2. As one can easily see from the plot, there is very little room for simultaneously satisfying these constraints and solving the missing satellite problem.

Let us also discuss the effects of the observational uncertainties in the MW halo mass; for instance, a recent comprehensive dynamical analysis of redshifts and distances of 64 dwarf galaxies around the MW has led to $M_{\text{halo}} = 2.8 \cdot 10^{12} M_\odot$. The detailed value of $M_{\text{halo}}$ does matter when focusing instead on the absolute value of $N_{\text{sub}}$, and past studies such as \cite{61} have pointed out the degeneracy between the MW halo mass and the DM parameters required for relaxing the missing satellite problem.

In order to take into account these issues, we have iterated the same analysis with different input values for $M_{\text{halo}}$, and compared the corresponding predictions to a fixed satellite number $N_{\text{sub}} = 60$; this value is chosen as a sum of the 11 MW classical satellites and the 15 ultra-faint satellites from SDSS, with the latter value multiplied by a numerical factor which accounts for the limited sky coverage. The analysis reveals that the constraints from both the Lyman-α forest and the satellite problem are in tension, and that a solution to the missing satellite problem requires a scalar DM mass in the range $m_\alpha \geq 1 - 40 \times 10^{-22} \text{ eV}$.

\begin{equation}
    m_\alpha \geq 1 - 40 \times 10^{-22} \text{ eV}
\end{equation}
Axion dark matter mass [log(eV)]

-22  -21  -20  -19  -18

-22 Rogers and Peiris 2021
CMB/reionization

-20.7 Subhalos

-20.7 Ly-αf (previous work)

-18 BHSR

Ly-αf (this work)
Astrophysical attraction (small-scale challenges to ΛCDM)

Marsh, Physics Reports Volume 643, 1 July 2016, Pages 1-79

Safarzadeh & Spergel: The Astrophysical Journal, Volume 893, Number 1

\[ \dot{a}_x = k v_a \]

\[ \dot{v}_a = H v_a + k c_s^2 s_a \]

Dimensionless Power Spectrum Axion Sensitivity

Axion Constraint z-Bin Test

- low-z
- high-z

8 m class telescope — all sky survey every two weeks!

Images courtesy of stsci.org
DES constraints from MW satellites

We obtain $M_0 < 1.4 \times 10^8 \text{M}_{\odot}$ at 95% confidence from our fiducial FDM fit. Applying linear MW-host mass scaling yields $M_0 < 1.8 \times 10^8 \text{M}_{\odot}$ at 95% confidence, or $m_\phi > 2.9 \times 10^{-21} \text{eV}$. This translates to an upper limit on the de Broglie wavelength of $\lambda_{DB}/\text{h} \sim 0.5 \text{kpc}$, roughly corresponding to the sizes of the smallest MW satellite galaxies. Thus, the FDM model invoked to reconcile the apparent mismatch between the predicted and observed inner dark matter density profiles of dwarf galaxies \cite{29}, and to fit the internal dynamics of low-surface-brightness \cite{79,80} and ultradiffuse \cite{81} galaxies, is strongly disfavored by MW satellite abundances.

To connect to particle models of FDM, we plot this limit in the well-motivated parameter space of ultralight axion mass versus axion-photon coupling in Fig. 3. For the range of axion-photon couplings that we consider, this mixing has a negligible effect on structure formation. We reiterate that our constraint was derived assuming a light scalar field without self-interactions; this assumption may be violated in specific ultralight axion models. Although our analysis and Lyman-\(\alpha\) forest studies exclude a similar region of parameter space \cite{39,40}, our work probes structure on complementary physical scales with distinct theoretical and observational systematics.

Discussion.

In this Letter, we used a state-of-the-art model of the MW satellite galaxy population to place stringent and robust limits on three fundamental DM paradigms: WDM, IDM, and FDM. Although some of these alternative DM models gained popularity by solving apparent "small-scale structure" challenges facing CDM, recent...
Galaxy-galaxy lensing

Complications to non-linear scale observables

- Sub-halo mass function
- $c(M)$ for inner halo profile
- Scale-dependent bias (are axions of low mass in halos?)
- Redshift-dependent bias
Extra-galactic HI surveys: $10^6 M_\odot \rightarrow 10^{11} M_\odot$

Figure/survey params from T. Westmeier
1. ALFALFA (Arecibo), done—
   * 30,000 extragalactic HI line sources out to $z \sim 0.06$,
2. Wallaby (SKA pathfinder, 36 X 12 m)
   * 500,000 sources expected

Simulations:
1) Specify survey volume — draw from mass function
2) Use semi-analytic $M_{\text{halo}} \rightarrow M_{\text{HI}}$ conversion
3) Random LOS, geometric, realization
4) Mock observation
Extra-galactic HI surveys

$10^{-24}$ eV

$10^{-20}$ eV
Extra-galactic HI surveys
1) Abundance matching + halo modeling to self-consistently model 2-pt function—
2) Compare with Illustris TNG THESAN sims (DM and hydro!)
3) Close QSO pairs
4) Explore ULA, late phase transitions, ULA interactions, anharmonic corrections
ULAs and the kinetic SZ effect

• **Bulk flow contribution is kinetic (or kinematic) SZ effect:**

\[
\frac{\Delta T_{\text{ksz}}}{T_{\text{CMB}}} = -\sigma_T \int dl \left( \hat{r} \cdot \mathbf{v}_e \right) \simeq -\tau \frac{v_{\text{los}}}{c}
\]

\[
\Delta T_{\text{ksz}} \approx 10 \, \mu\text{K}
\]

Sunyaev ++ (1980)
• Second-order contributions to power spectra from reionization

\[ T \propto \bar{n}_e \nu \delta \]

\[ \frac{dv}{dt} \propto -\nabla \Phi \propto \delta \rightarrow \nu \propto \delta \]

\[ T \propto \bar{n}_e \delta^2 \]

\[ C_l^{TT} \propto P^2(k) \]
\( C_\ell = \int \frac{dx}{x^2} P_\perp \left( \frac{\ell + \frac{1}{2}}{x}, a \right) g^2(x) \).

\[ P_\perp(k, a) = \frac{a^2 H^2(a)}{8\pi^2} \int_0^\infty dy \int_{-1}^1 dx P_0(k \sqrt{1 - 2xy + y^2}) P_0(ky) \frac{G^2(k \sqrt{1 - 2xy + y^2}, a)}{G_0^2(k \sqrt{1 - 2xy + y^2})} \frac{G^2(ky, a)}{G_0^2(ky)} \]

\[ G = \frac{\delta_m(k, a)}{\delta_m(k, a = 1)} \]
Byproducts of Dark Energy: Observational Limits and Theoretical Possibilities
• Challenging to robustly detect ULA effect, even with future generation experiments.

• Need to include impact of patchy reionization (in progress) (with V. Goyal)

• SOLUTION — LOOK NEAR HEAVIEST COSMIC STRUCTURES
Temperature change towards cluster due to local flow:

\[
\frac{\Delta T}{T_{\text{CMB}}} (\hat{n}_i) = -\tau_{e,i} \frac{\hat{r}_i \cdot \mathbf{v}_i}{c}.
\]

\[\Delta T_{\text{kSZ}} \approx 10 \, \mu\text{K}\]

Estimate assuming homogeneous optical depth:

\[
\hat{t}_{\text{ksz}}(r) = -\frac{\sum_{i<j,r} [T(\hat{n}_i) - T(\hat{n}_j)] c_{ij}}{\sum_{i<j,r} c_{ij}^2} = \bar{\tau}_e \frac{v_{12}(r)}{c} T_{\text{CMB}}
\]
$v_{12}(r) \propto \int d\ln k \langle b(k, M, a) \rangle \Delta^2(k, a) \frac{d\ln \delta(k, a)}{dt} W(kr)$
**Halo bias**

\[ v_{12}(r) \propto \int d\ln k \left\langle b(k, M, a) \right\rangle \Delta^2(k, a) \frac{d\ln \delta(k, a)}{dt} W(kr) \]
KSZ — EFFECT OF ULAS

Typical fluct. on scale $k$

$$v_{12}(r) \propto \int d\ln k \langle b(k, M, a) \rangle \Delta^2(k, a) \frac{d\ln \delta(k, a)}{dt} W(kr)$$
KSZ — EFFECT OF ULAS

\[ \nu_{12}(r) \propto \int d\ln k \langle b(k, M, a) \rangle \Delta^2(k, a) \frac{d\ln \delta(k, a)}{dt} W(kr) \]
- Cluster flows *enhanced* in ULA models
- Rarer (more biased) peaks of density
- More biased

\[
m_{\text{ax}} = 10^{-26} \text{ eV}, \quad \Omega_{\text{ax}} = 0.4\Omega_{\text{DM}}
\]
**KSZ — HALO MODEL DETAILS**

- **Pair conservation**
  \[ \langle v_{12} \rangle \propto \frac{d\xi}{d \ln a} \]


- **Bias via halo model**

\[
\langle \xi_h \rangle_m = \frac{1}{2\pi^2} \int k^2 dk j_0(kr) \frac{G^2(k, a)}{G_0^2(k)} P_{0, \text{lin}}(k) B^2(k, a).
\]

\[
\langle \frac{d\xi_h}{d \ln a} \rangle_m = \frac{3}{\pi^2 r^3} \int_0^r dr' r'^2 \int k^2 dk j_0(kr')
\times \left[ \frac{\ln \frac{G^2(k, a)}{G_0^2(k)} P_{0, \text{lin}}(k) B(k, a) N(k, a)}{\frac{d\ln G}{d \ln a} \frac{G_0^2(k)}{G_0^2(k)}} \right].
\]

\[
B(k, a) = \frac{1}{\bar{n}(a)} \int_{M_{\text{min}}}^{M_{\text{max}}} \text{dm} \ n(m, a) b(m, a) \widetilde{W} [kR(m)].
\]

- **First calculation of bias factor from first principles, building on Bhattacharya and Mueller results (but different in detail!)—likely implications for standard neutrino physics and constraints from kSZ**
As mentioned in the body of the paper, if the scale dependence is weak, our approximation is exact. For small axion

\[ \langle \xi_h \rangle_m \]

we have the mean

\[ b_h^q(k, a) = \frac{\int_{M_{\min}}^{M_{\max}} dm \, m \, n(m, a) b^q(m, a) \tilde{W}^2 [k R(m)]}{\int_{M_{\min}}^{M_{\max}} dm \, m \, n(m, a) \tilde{W}^2 [k R(m)]} \]

\[ B(k, a) = \frac{1}{\bar{n}(a)} \int_{M_{\min}}^{M_{\max}} dm \, n(m, a) b(m, a) \tilde{W} [k R(m)] . \]
The assumption of scale independent growth is violated. This leads to additional contributing
malisms yielding Eqs. (reproduced in Table 2).

\[ F_{\mu\nu} = \sum_i \sum_{j,k} \frac{\partial v(r_j, z_i)}{\partial p_\mu} C^{-1}(r_j, r_k, z_i) \frac{\partial v(r_k, z_i)}{\partial p_\nu} \]

Survey | Parameters | Survey Stage
--- | --- | ---
CMB | \( \Delta T_{\text{instr}} \) (\( \mu \)K arcmin) | II | III | IV
Galaxy | \( z_{\text{min}} \) | 0.1 | 0.1 | 0.1
 | \( z_{\text{max}} \) | 0.4 | 0.4 | 0.6
 | No. of \( z \) bins, \( N_z \) | 3 | 3 | 5
 | \( M_{\text{min}} \) (10^{14} M_\odot) | 1 | 1 | 0.6
Overlap | Area (1000 sq. deg.) | 4 | 6 | 10

For symmetry reasons we will thus use an eective sky coverage by 

\( \Delta \Omega = \Omega_{\text{limit}} - \Omega_{\text{DM}} \).

\( \sigma_{\eta_a} \) vs. \( m_a \) for Stage II, III, and IV. The left panel shows the 1\( \sigma \) and 2\( \sigma \) limits on \( \eta_a \) as a function of \( m_a \) for various survey stages. The right panel shows the marginalized constraints on \( \eta_a \) for the same survey stages, along with the $\chi^2$-derived constraints.
\[ \frac{\Delta T}{T_{\text{CMB}}} \langle \hat{n}_i \rangle = -\tau_{e,i} \frac{\hat{r}_i \cdot \mathbf{v}_i}{c}. \]

\[ B \rightarrow b(z)B \]
Galaxy bias: $\mathcal{B} \rightarrow b(z)\mathcal{B}$
**Systematic Uncertainties**

Figure 11. Forecasted detection sensitivity in $\eta = \Omega_{\text{a}}/\Omega_{\text{DM}}$ as a function of $m_a$ for an SIV survey as defined by Ref. [54], for different minimum cluster masses. As above regions above the dotted lines (or shaded areas) would be detectable at 2 ($or$ 1).

Here, we do not marginalize over uncertainties in the bias. Doing so degrades the constraints obtained with lower minimum masses more strongly, partially eliminating any gains made by including lower mass clusters. The main improvement is the ability to probe higher axion masses.

We have different dependencies on unknown bias factors, specifically scaling as $\sim b^2$ and $\sim b$, respectively, and it is thus likely that these distinct data sets will prove complementary by breaking each others' degeneracies. Weak lensing is likely to be comparably sensitive to this new physics, but manifests distinct systematics (e.g. galaxy alignment, image point-spread function measurement errors) [205], making combined probes necessary to robustly detect new physics.

At the moment, there are constraints to ULA DM from the absorption spectra of high-$z$ quasars, known as the Lyman-$\alpha$ forest [206–210], imposing a limit of $\eta_{\text{axion}} \lesssim 0.2$ for $m_a \lesssim 10^{-21}$ eV. Future Lyman-$\alpha$ measurements could reach an order of magnitude lower sensitivity to the absorption optical depth [13], and while a ULA-specific forecast does not yet exist, it could be that this offers an additional factor of $\sim 10$ improvement in sensitivity $\eta_{\text{axion}}$ for $m_a \lesssim 10^{-24}$ eV, competitive with the pairwise kSZ sensitivity level forecast in our work.

Thinking further ahead into the future, intensity mapping efforts with the cosmological 21-cm and other lines could offer novel probes of the linear density field. Efforts like HIRAX [211] and the Square Kilometer Array (SKA) [212] could offer a full additional order-of-magnitude improvement in sensitivity $\eta_{\text{axion}}$ for masses as high as $m_a \sim 10^{-24}$ eV [140], but must progress to a robust 21-cm fluctuation detection before being useful as a fundamental physics probe.

**V. Conclusions**

The next decade of cosmological observations will yield nearly cosmic-variance limited measurements of CMB polarization, as well as deep spectroscopic surveys of $\sim 10^7$ galaxies that facilitate ever more precise maps of cosmological large-scale structure. These measurements will improve our understanding of reionization, cluster thermodynamics, radio point sources, galaxy formation, and fundamental physics [11]. Increasingly, cosmological data will be used not only to probe the dark-sector energy budget but also its particle content. Ultralight axions could exist over many decades in mass and are a well-motivated candidate to compose some or all of the dark matter. Going beyond WMAP and Planck measurements, much of the sensitivity of upcoming CMB experiments to dark-sector particle physics will be driven by secondary anisotropies, such as gravitational lensing and the kinetic Sunyaev-Zel'dovich effect [11].
Figure 11. Forecasted detection sensitivity in $\eta_{\text{limit}} = \Omega_{\text{axion}}/\Omega_{DM}$ as a function of $m_a$ for an SIV survey as defined by Ref. [54], for different minimum cluster masses. As above regions above the dotted lines (or shaded areas) would be detectable at 2$\sigma$ (or 1$\sigma$).

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V. CONCLUSIONS

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In this section we will show that the bispectrum estimator is equivalent to the kSZ template formalism from [10]. This equivalence establishes some interesting properties of the pair sum estimator which are not obvious in different implementations of the estimator. One may also wonder how best to evaluate some intermediate results in the derivation will be used in later sections of the paper.

Summarizing previous sections, we have now shown that the optimal estimator for kSZ tomography is:  

\[ \hat{E} = \sum_{ij} \frac{x_{ij} \cdot r}{|x_{ij}|} W'(|x_{ij}|) \tilde{T}(\theta_j) \]

\[ = \frac{1}{2} \sum_{ij} \frac{x_{ij} \cdot \hat{r}}{|x_{ij}|} W'(|x_{ij}|)(\tilde{T}(\theta_j) - \tilde{T}(\theta_i)) \quad \text{(where } x_{ij} = x_j - x_i) \]

\[ \hat{E} = \frac{K_s}{\chi^2 F_{BB}} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k_L}{(2\pi)^3} \frac{d\theta}{(2\pi)^3} \frac{P_{gg}(k_L)P_{ge}(k_s)}{P_{gg}^0(k_L)P_{ge}^0(k_s)} C_l^{T,T,T} \delta_d(k_L)\hat{\delta}_d(k_s) T(l) (2\pi)^3 \delta^3 \left( k_L + k_s + \frac{1}{\chi^*} \right) \]

where

- \( K_s \)
- \( F_{BB} \)
- \( P_{gg}(k_L) \)
- \( P_{ge}(k_s) \)
- \( C_l^{T,T,T} \)
- \( \delta_d(k_L) \)
- \( \hat{\delta}_d(k_s) \)
- \( T(l) \)
- \( \delta^3 \)
- \( k_L + k_s + \frac{1}{\chi^*} \)
Further increase sensitivity and also probe up to $m \lesssim 10^{-21}$ eV.

Adding information from secondary CMB anisotropies in the kinetic Sunyaev-Zeldovich effect could be examined to clarify the relationship between UDM field and fluid descriptions.

From observations like CMB-HD, we advocate that predictions for linear theory observables should be re-calibrated allowing the CMB to distinguish between particle CDM (e.g. WIMPs) and wave-like UDM in the most constrained regime relevant to challenges coming from observations at the Milky Way-scale.

Weak lensing shear from the Dark Energy Survey (+DES) sets the strongest bounds on the UDM energy density for $m \gtrsim 10^{-24}$ eV. A future intensity mapping survey from the Square Kilometre Array (SKA-IM) could probe and $m \lesssim 10^{-22}$ eV.

Electromagnetic gravitational potential oscillations for PTA residuals are sensitive to gravitational potential oscillations for $m < 10^{-30}$ eV.

Cosmic energy density $\Omega_\phi$ can be constrained by a combination of the high-redshift UV luminosity function and galaxy surveys. A joint analysis of these probes sets the strongest bounds on the UDM energy density for $m \lesssim 10^{-24}$ eV.

The kinetic Sunyaev-Zeldovich mean pairwise velocity (kSZ) measurement in CMB-S4 and DESI will also probe heavier UDM, while the Ostriker-Vishniac (kSZ-OV) signal in a future CMB-HD experiment could probe a sub-dominant contribution of UDM up to $m \lesssim 10^{-22}$ eV.

Take-home message(s):

- The ultralight scalars behave like dark energy and are constrained by a combination of CMB and galaxy surveys.
- Proposed small-scale physical probes, for cosmic energy density can exclude particle CDM (e.g. WIMPs) and wave-like UDM.
- The blue line shows the forecast sensitivity for CMB-S4 and DESI will also probe heavier UDM, while the Ostriker-Vishniac (kSZ-OV) signal in a future CMB-HD experiment could probe a sub-dominant contribution of UDM up to $m \lesssim 10^{-22}$ eV.

- In order to prepare for future ambitious large-scale surveys, CMB-S4 should improve UDM sensitivity to $m \lesssim 10^{-24}$ eV.

- Consequently, CMB-S4 should improve UDM sensitivity to $m \lesssim 10^{-24}$ eV.
Take-home message(s)

From 2203.14195, Snowmass Ultra-light Dark Matter white paper

I’ll be hiring a postdoc this year - please consider applying!
Hubble parameter

\[ H_I = ??? \]

Determined by energy scale

ULAs as an Inflationary Probe

Radius of Our Observable Universe

Inflation

47 billion

CMB

Now

Earth

Atomic

Time
ULAs as an Inflationary Probe

The amplitude and spectral index are given by:

\[ r = 16 \varepsilon \approx 0.17 \left( \frac{2.1 \times 10^{-9}}{A_s} \right) \left( \frac{H_I}{10^{14} \text{ GeV}} \right)^2 \]
**Axions and Isocurvature**

- Quantum zero-point fluctuations in axion field

\[ \sqrt{\left\langle a^2 \right\rangle} = \frac{H_I}{2\pi} \]

- Subdominant species seed isocurvature fluctuations

Some schematics from Wands, Enqvist, Lyth, Takahashi (2012-2015)
Axions and Isocurvature

- Quantum zero-point fluctuations in axion field

\[ \sqrt{\langle a^2 \rangle} = \frac{H_I}{2\pi} \]

- Subdominant species seed isocurvature fluctuations

Neutrinos

CDM

Photons

Baryons
CMB power is suppressed on small scales (large dramatic e-

tistical potential wells set up by axions, and so axions

inged by using a modified version of

mode, as well as the more general suppression of small-

from the more familiar pure CDM isocurvature. This

of axion isocurvature lead to sharply di-

ever, the radically di-

map over to constraints to

detection would be evidence that the additional degree of

Stepping beyond the axiverse paradigm, an isocurvature

We will present constraints in a forthcoming paper [23].

thermore, an accompanying isocurvature signal would be

scale of inflation using the concordance of

novel and truly unambiguous way to measure the energy

that there are sources of observable tensor modes possible

even with low-scale inflation [25] these regions provide a

flationary origin of these modes, and thus on

versa, thus providing a non-trivial cross-check on the in-

ference of (0

↵

normalisation di-

FIG. 3: CMB axion isocurvature power spectrum, with adi-

Spectra from A

`

`C

`/2p 

[∝ K2]

increasing ma

104 =( 0.01)2

107

106

105

104

103

102

101

100

101

102

103

104

`(` +1)C`/2p 

Increasing $m_a$

$\ell(\ell+1)C_\ell/2\pi \mu K^2$

Multipole $\ell$

$10^{-4} = (0.01)^2$

$m_\alpha = 10^{-32} \text{ eV}$

$m_\alpha = 10^{-29} \text{ eV}$

$m_\alpha = 10^{-28} \text{ eV}$

$m_\alpha = 10^{-20} \text{ eV}$

Spectra from AXIONCAMB using initial conditions obtained in DG+ (2017)
Planck 2013 TT

Figure 37. The 2013 Planck CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low-values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.

Table 8. Constraints on the basic six-parameter $\Lambda$CDM model using Planck data. The top section contains constraints on the six primary parameters included directly in the estimation process, and the bottom section contains constraints on derived parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit</th>
<th>68% limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_b h^2$</td>
<td>$0.022068$</td>
<td>$0.02207$ $\pm 0.00033$</td>
</tr>
<tr>
<td>$\Omega_c h^2$</td>
<td>$0.1196$</td>
<td>$0.1199$ $\pm 0.0031$</td>
</tr>
<tr>
<td>$100\theta_{MC}$</td>
<td>$1.04122$</td>
<td>$1.04132$ $\pm 0.00068$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$0.097$</td>
<td>$0.0980$ $\pm 0.00063$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>$0.9624$</td>
<td>$0.9616$ $\pm 0.0094$</td>
</tr>
<tr>
<td>$10^9 A_s$</td>
<td>$3.098$</td>
<td>$3.103$ $\pm 0.072$</td>
</tr>
<tr>
<td>$\Omega_\Lambda h^2$</td>
<td>$0.6825$</td>
<td>$0.686$ $\pm 0.020$</td>
</tr>
<tr>
<td>$\Omega_m h^2$</td>
<td>$0.3175$</td>
<td>$0.314$ $\pm 0.020$</td>
</tr>
<tr>
<td>$\Omega_\gamma$</td>
<td>$0.8344$</td>
<td>$0.834$ $\pm 0.027$</td>
</tr>
<tr>
<td>$z_{eq}$</td>
<td>$11.35$</td>
<td>$11.4$ $\pm 2$</td>
</tr>
<tr>
<td>$H_0$</td>
<td>$67.4$</td>
<td>$67.04$ $\pm 1.1$</td>
</tr>
<tr>
<td>$10^9 A_s$</td>
<td>$2.215$</td>
<td>$2.23$ $\pm 0.16$</td>
</tr>
<tr>
<td>$\Omega_\Lambda h^2$</td>
<td>$0.14300$</td>
<td>$0.1423$ $\pm 0.0029$</td>
</tr>
<tr>
<td>$\Omega_m h^2$</td>
<td>$0.14305$</td>
<td>$0.1426$ $\pm 0.0025$</td>
</tr>
<tr>
<td>Age/Gyr</td>
<td>$13.819$</td>
<td>$13.813$ $\pm 0.058$</td>
</tr>
<tr>
<td>$z_\Lambda$</td>
<td>$10.9048$</td>
<td>$10.9037$ $\pm 0.65$</td>
</tr>
<tr>
<td>$\theta_{MC}$</td>
<td>$1.04139$</td>
<td>$1.04148$ $\pm 0.00066$</td>
</tr>
<tr>
<td>$z_{eq}$</td>
<td>$3.386$</td>
<td>$3.391$ $\pm 0.69$</td>
</tr>
</tbody>
</table>

\[\alpha \equiv \frac{P_{S_{c\gamma}}(k)}{P_{S_{c\gamma}}(k) + P_{R}(k)} \leq 0.039\]
ULAS AND ISOCURVATURE FLUCTUATIONS

Planck 2013 TT

![Planck 2013 TT plot](image)

QCD axion

\[
\frac{\Omega_a}{\Omega_a + \Omega_c} \lesssim 10^{-12} \left( \frac{10^{14} \text{GeV}}{H_I} \right)^{7/2}
\]

ULAs

\[
\frac{\Omega_a}{\Omega_a + \Omega_c} \lesssim 10^{-3} \left( \frac{10^{14} \text{GeV}}{H_I} \right)
\]


Using the Full Power of the Cosmic Microwave Background to Probe Axion Dark Matter

Figure 9. Derived inflationary parameters: the left and right panels show scatter plots from MCMC chains for all samples that satisfy either \( b_{iso} > 0.01 \) or \( r(d) > 0.01 \) respectively. The isocurvature is only non-trivial for the DM-like axions with \( m_{a} = (10^{24}, 10^{25}) \) eV, and the lightest DE-like axions \( m_{a} = 10^{33} \) eV. In the intermediate regime isocurvature is negligible and constraints are driven by limits to the tensor amplitude. This leads to constraints on the allowed value of the tensor-to-scalar ratio \( r(d) \) as shown in Figure 10, and is also shown through a 2D contour plot in Figure 11. Note that the points are not explicitly weighted by the posterior (as in e.g. Figure A1), but the point density of the chains is representative of the goodness of fit.

Figure 10. Constraints on the derived tensor amplitude: At low axion mass the isocurvature mode is negligible due to the bound on \( W_{ah}^2 \) in the mixed DM model. The derived bound \( r(d) < 0.09 \) at 95% C.L. for \( m_{a} = 10^{26} \) eV is driven by the tensor contribution to the \( T \) and \( E \) modes. As the axion mass, and thus energy density, increase, the axion isocurvature contribution becomes more important, tightening the derived constraint to \( r(d) \approx 0.04 \) and \( r(d) \approx 0.01 \) for \( m_{a} = 10^{25} \) eV and \( m_{a} = 10^{24} \) eV respectively.

We recall that the initial field value is set by the decay constant as \( f \approx q_{i} f_{a} (\text{Eq. 8}) \). Since \( |q_{i}| \ll p \) we have that \( |f_{i}| \ll p f_{a} \). The condition for production of isocurvature perturbations is approximately \( f_{a} \gtrsim H_{I} / 2p \), up to thermal corrections to the potential. If \( |f_{i}| \approx H_{I} \) then \( f_{a} \approx H_{I} \) and spontaneous symmetry breaking for production of isocurvature perturbations is approximately

\[ \frac{\Omega_{a} h^{2}}{\Omega_{a} h^{2}} \]

\[ 10^{-24} \text{eV} \]

\[ 10^{-25} \text{eV} \]

\[ 10^{-26} \text{eV} \]
We forecast the errors on axion isocurvature for the base line CMB-S4 experiment with a 1 $\mu$K-arcmin noise level and a 1 arcminute beam: the isocurvature limit will be improved by a factor of approximately five compared to Planck, allowing for detection of axion-type isocurvature at $2 \Delta A_I/A_s < 0.008$.

The axion isocurvature amplitude is:

$$A_I = \sqrt{\varepsilon_a} \xi d \left( \frac{H_I}{M_{pl}} \right)^2 \left( \frac{i}{M_{pl}} \right)^2.$$  

Equation (5.3)

The initial axion displacement, $\xi$, fixes the axion relic abundance such that $\varepsilon_a = \varepsilon_a(\xi, m_a)$ [470, 471, 472, 473, 474, 475]. Thus, if the relic density and mass can be measured by independent means, a measurement of the axion isocurvature amplitude can be used to measure the energy scale of inflation.

If the QCD axion is all of the DM, axion direct detection experiments can be used in conjunction with CMB-S4 to probe $H_I$ in the range $2.5 \times 10^6 - 4 \times 10^9$ (QCD axion + direct detection) (5.4)

This is demonstrated in Fig. 33 (left panel) for the case of ADMX [476] (in operation), and CASPEr [477] (proposed), where we have used the standard formulae relating the QCD axion mass and relic abundance to the decay constant (e.g. Ref. [467]).

Combining axion DM direct detection with CMB-S4 isocurvature measurements allows a unique probe of low-scale inflation, inaccessible to searches for tensor modes.

Figure 33. Axion dark matter isocurvature. Red bands show the isocurvature amplitude consistent with Planck and detectable with CMB-S4. Left Panel: The QCD axion: measuring the energy scale of inflation with CMB-S4+axion direct detection. Here we restrict axions to be all of the DM. The purple regions show the range of $f_a$ accessible to axion direct detection experiments. Combining ADMX [476] (in operation), CASPEr [477] (proposed), and CMB-S4 it is possible to measure $4 \times 10^5 - 4 \times 10^9$. Right Panel: ALPs - a combination measurement using CMB-S4 alone. Assuming 1% of the total DM resides in an ultralight axion, the mass and axion density can be determined to high significance using, for example, the lensing power. The isocurvature amplitude can also be determined, allowing for an independent determination of $H_I$ in the same regime as is accessible from tensor modes (purple band).
We forecast the errors on axion isocurvature for the base line CMB-S4 experiment with a $1\,\mu\text{K}$-arcmin noise level and a 1 arcminute beam: the isocurvature limit will be improved by a factor of approximately five compared to Planck, allowing for detection of axion-type isocurvature at $2\sigma$ significance in the region $0.008 < A^I/A_s < 0.038$.

The axion isocurvature amplitude is:

$$A^I = \sqrt{\pi a d \Omega_{\text{fid}}^2 \left( \frac{H_I}{M_{\text{pl}}} \right)^2 \left( \frac{\Delta}{M_{\text{pl}}} \right)^2}.$$  (5.3)

The initial axion displacement, $i$, fixes the axion relic abundance such that $\Omega_a^i = \Omega_{\text{dm}}^i$. Thus, if the relic density and mass can be measured by independent means, a measurement of the axion isocurvature amplitude can be used to measure the energy scale of inflation, $H_I$.

If the QCD axion is all of the DM, axion direct detection experiments can be used in conjunction with CMB-S4 to probe $H_I$ in the range $2.5 \times 10^6 \text{GeV}/H_I$. This is demonstrated in Fig. 33 (left panel) for the case of ADMX [476] (in operation), and CASPEr [477] (proposed), where we have used the standard formulae relating the QCD axion mass and relic abundance to the decay constant (e.g. Ref. [467]).

Combining axion DM direct detection with CMB-S4 isocurvature measurements allows a unique probe of low-scale inflation, inaccessible to searches for tensor modes.

The observational/experimental horizon for axion dark matter/dark energy tests is bright! Potential trouble for GUT-scale inflation.