

# Dark Fifth Forces in the Universe

## Non-linear Cosmology

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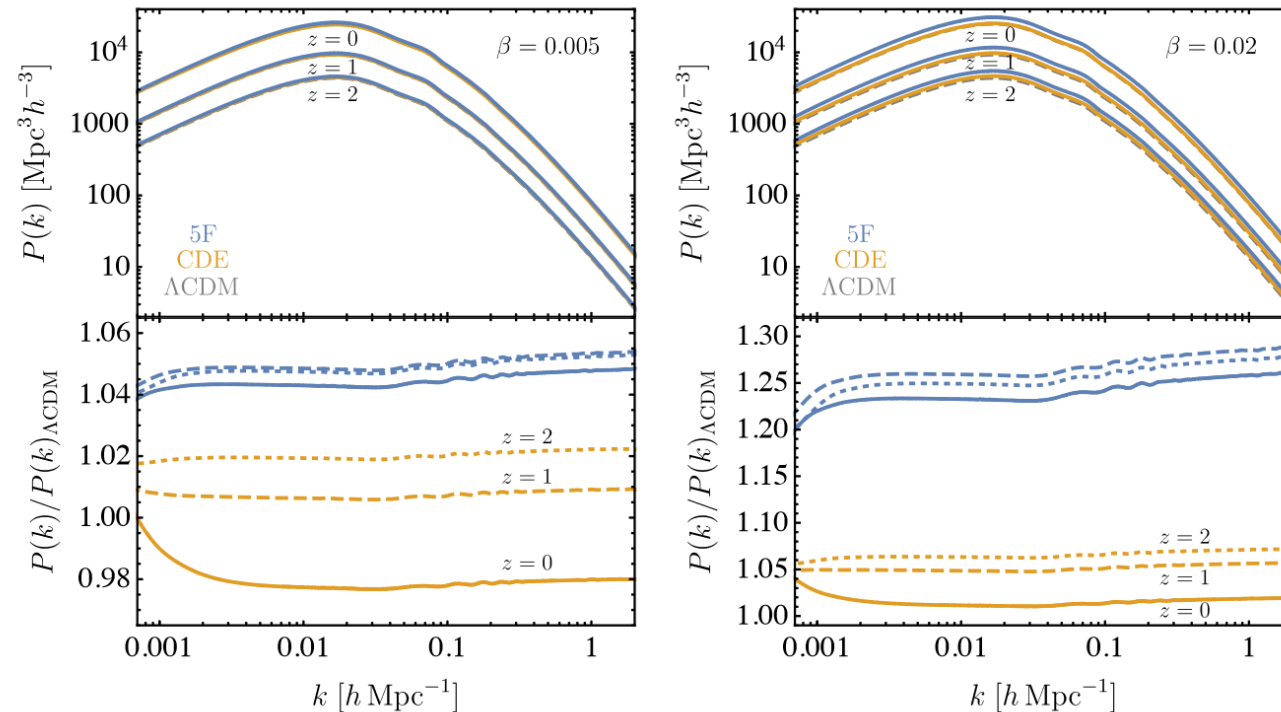
SCUOLA  
NORMALE  
SUPERIORE



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# Why going to Non-linear order?

1. No new phases in the power spectrum. Bounds from full shape.



Archidiacono '22

2. Effects emerging only at non-linear order (e.g. squeezed bispectrum).

# Lagrangian Perturbation Theory

White '14  
Bernardeau '02

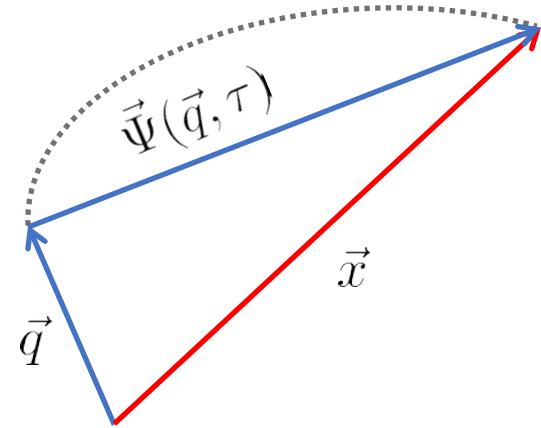
Follow the fluid element:

$$\vec{x} = \vec{q} + \vec{\Psi}(\vec{q}, \tau)$$

Equations for displacements from geodesic equation

$$\tilde{m} \equiv \frac{\partial \log m}{\partial s}$$

$$\begin{cases} \vec{\Psi}''_{\chi} + (\mathcal{H} + \bar{s}' \tilde{m}) \vec{\Psi}'_{\chi} = -\vec{\nabla}_{\chi} \Phi_{\chi} - \tilde{m} \vec{\nabla}_{\chi} s \\ \vec{\Psi}''_b + \mathcal{H} \vec{\Psi}'_b = -\vec{\nabla}_b \Phi_b \end{cases}$$



Poisson's equations

$$\begin{cases} \nabla_x^2 \Phi = \frac{3}{2} \mathcal{H}^2 \Omega_m (f_{\chi} \delta_{\chi} + (1 - f_{\chi}) \delta_b) \equiv \frac{3}{2} \mathcal{H}^2 \Omega_m \delta_m \\ \nabla_x^2 s = \frac{3}{2} \mathcal{H}^2 \Omega_m \beta f_{\chi} \tilde{m} \delta_{\chi} \end{cases}$$

Energy conservation

$$\rho(\vec{x}) d^3x = \bar{\rho} (1 + \delta(\vec{x})) d^3x = \bar{\rho} d^3q \quad \longrightarrow \quad \delta(\vec{x}) = \left( \det \frac{\partial \vec{x}}{\partial \vec{q}} \right)^{-1} - 1 = -\vec{\nabla} \vec{\Psi} + \frac{1}{2} \left( (\vec{\nabla} \vec{\Psi})^2 + \partial_i \Psi_j \partial_j \Psi_i \right) + \mathcal{O}(\Psi^3)$$

# Lagrangian Perturbation Theory

Convenient variables:

$$\begin{cases} \vec{\Psi}_m = f_\chi \vec{\Psi}_\chi + (1 - f_\chi) \vec{\Psi}_b \\ \vec{\Psi}_r = \vec{\Psi}_\chi - \vec{\Psi}_b \end{cases} \longrightarrow \begin{cases} \vec{\Psi}_r'' + \mathcal{H} \vec{\Psi}_r' = -(\vec{\Psi}_r \cdot \vec{\nabla}) \vec{\nabla} \Phi - \tilde{m} \vec{\nabla}_\chi s - \bar{s}' \tilde{m} \vec{\Psi}_m' \propto \beta \\ \vec{\Psi}_m'' + \mathcal{H} \vec{\Psi}_m' = -\vec{\nabla}_m \Phi_m - f_\chi \tilde{m} \vec{\nabla}_\chi s - f_\chi \bar{s}' \tilde{m} \vec{\Psi}_m' \end{cases}$$

Solve order by order in Fourier space:

$$\vec{\Psi}(\vec{k}, \tau) = \vec{\Psi}^{(1)}(\vec{k}, \tau) + \vec{\Psi}^{(2)}(\vec{k}, \tau) + \dots$$

Structure of the solutions up to second order:

$$\vec{\Psi}_{m,r}^{(1)}(\vec{k}, \tau) = \frac{i\vec{k}}{k^2} \delta_0(\vec{k}) D_{1m,1r}(\tau)$$

$$\vec{\Psi}_m^{(2)}(\vec{k}, \tau) = \frac{i\vec{k}}{2k^2} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} (2\pi)^3 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \delta_0(\vec{k}_1) \delta_0(\vec{k}_2) \overbrace{\left(1 - (\hat{k}_1 \cdot \hat{k}_2)^2\right)}^{\gamma(\vec{k}_1, \vec{k}_2)} D_{2m}(\tau) \quad \alpha(\vec{k}_1, \vec{k}_2)$$

$$\vec{\Psi}_r^{(2)}(\vec{k}, \tau) = \frac{i\vec{k}}{2k^2} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} (2\pi)^3 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2) \delta_0(\vec{k}_1) \delta_0(\vec{k}_2) \left[ \left(1 - (\hat{k}_1 \cdot \hat{k}_2)^2\right) D_{2r}^{np}(\tau) + \overbrace{\left(1 + \frac{\hat{k}_1 \cdot \hat{k}_2}{2} \left(\frac{k_2}{k_1} + \frac{k_1}{k_2}\right)\right)}^{\alpha(\vec{k}_1, \vec{k}_2)} D_{2r}^p(\tau) \right]$$

# Growth Factors and Green's Functions

General equations for growth factors:

$$\begin{cases} L_{\tau m} D_{nm} \equiv \left( \partial_\tau^2 + \frac{2}{\tau} \partial_\tau - \frac{6}{\tau^2} \right) D_{nm} = S_m(\tau) \equiv S_0(D_{(n-1)m}^{(0)}, \dots, D_{1m}^{(0)}) + \beta \tilde{m}^2 f_\chi^2 S_1(D_{nm}^{(0)}, \dots, D_{1m}^{(0)}) \\ L_{\tau r} D_{nr} \equiv \left( \partial_\tau^2 + \frac{2}{\tau} \partial_\tau \right) D_{nr} = \beta \tilde{m}^2 f_\chi^2 S_r(D_{nm}^{(0)}, \dots, D_{1m}^{(0)}) \end{cases}$$

Introduce the Green's functions:

$$\begin{cases} L_\tau G(\tau, \tau') = \delta(\tau - \tau') \\ G(\tau - \tau') = 0 \text{ if } \tau < \tau' \end{cases} \longrightarrow D(\tau) = \int^\tau d\tau' G(\tau, \tau') S(\tau')$$

We only need the Green's function in  $\Lambda$ CDM:

Log correction to  $D_{1m}$

$$G_m(\tau, \tau') = \frac{1}{5} \left( \frac{\tau^2}{\tau'} - \frac{\tau'^4}{\tau^3} \right), \quad G_r(\tau, \tau') = \tau' - \frac{\tau^2}{\tau'}$$

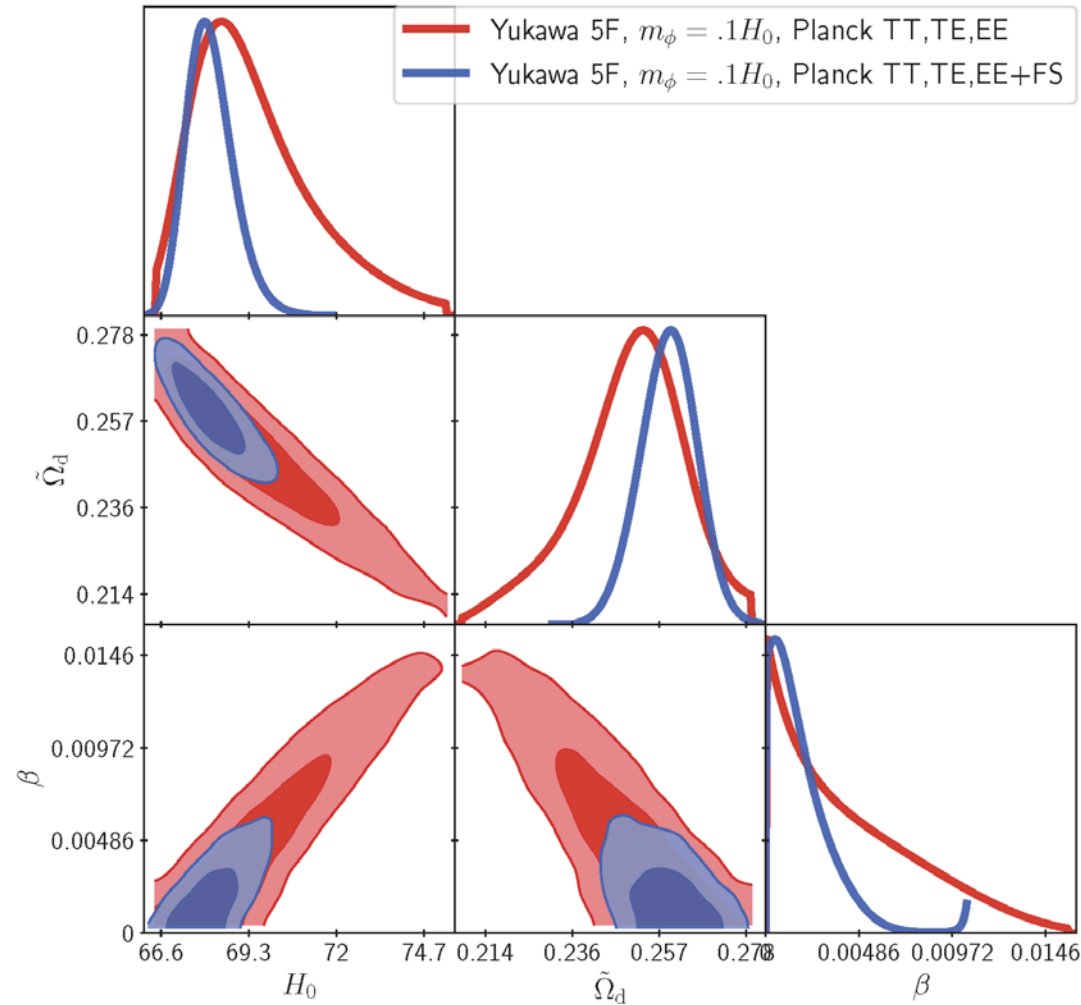
Non-decaying relative growth factor

$$D_{1m}(\tau) = D_{1m}^{(0)}(\tau) (1 + 2\beta f_\chi \tilde{m}^2 \log \tau)$$

$$D_{nr}(\tau) \propto \beta f_\chi^2 \tilde{m}^2 (D_{1m}^{(0)}(\tau))^n$$

# Power Spectrum: Full Shape Analysis

Spatial kernels for  $\vec{\Psi}_m$  identical to  $\Lambda$ CDM  $\longrightarrow$  Full Shape affected only by  $D_{1m}$

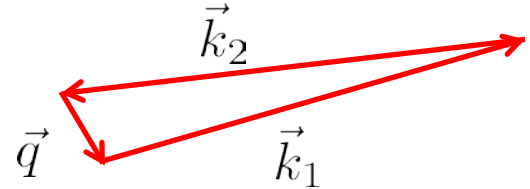


# EP violations: squeezed bispectrum

Peloso '13  
Creminelli '13  
Lewandowski '19

Equal-time squeezed bispectrum:

$$\lim_{q \rightarrow 0} \langle \delta(\vec{q}, \tau) \delta(\vec{k}_1, \tau) \delta(\vec{k}_2, \tau) \rangle$$



1. Equivalence principle holds
2. Adiabatic initial conditions



Existence of free-falling frame for the long mode



Consistency relations:  $\lim_{q \rightarrow 0} \langle \delta(\vec{q}, \tau) \delta(\vec{k}_1, \tau) \delta(\vec{k}_2, \tau) \rangle' = O_a(q, k) P(q) P(k)$

Analytic function of  $q$

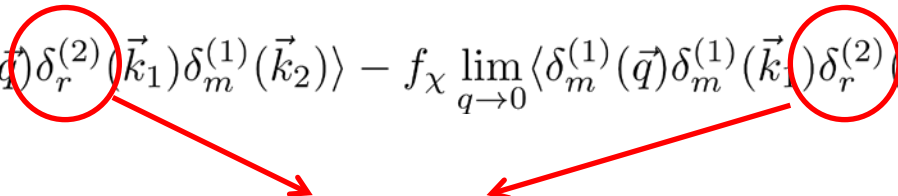
Poles are generated in  $q$ :  $\lim_{q \rightarrow 0} \langle \delta(\vec{q}, \tau) \delta(\vec{k}_1, \tau) \delta(\vec{k}_2, \tau) \rangle' \propto \frac{\vec{q} \cdot \vec{k}}{q^2} P(q) P(k)$

Violation



# EP violations: squeezed bispectrum

Fifth forces induce violation of the EP in the growth of  $\delta_r$

$$\lim_{q \rightarrow 0} \langle \delta_\chi(\vec{q}) \delta_\chi(\vec{k}_1) \delta_b(\vec{k}_2) \rangle = (1 - f_\chi) \lim_{q \rightarrow 0} \langle \delta_m^{(1)}(\vec{q}) \delta_r^{(2)}(\vec{k}_1) \delta_m^{(1)}(\vec{k}_2) \rangle - f_\chi \lim_{q \rightarrow 0} \langle \delta_m^{(1)}(\vec{q}) \delta_m^{(1)}(\vec{k}_1) \delta_r^{(2)}(\vec{k}_2) \rangle$$


Only term with the correct spatial structure

$$\sim \alpha(\vec{k}_1, \vec{k}_2) = 1 + \frac{\hat{k}_1 \cdot \hat{k}_2}{2} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right)$$

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$$\lim_{q \rightarrow 0} \langle \delta_\chi(\vec{q}) \delta_\chi(\vec{k}_1) \delta_b(\vec{k}_2) \rangle' = \lim_{q \rightarrow 0} \frac{D_{2r}^p}{D_{1m}^2} P_m(q) P_m(k) (f_\chi \alpha(q, k_1) - (1 - f_\chi) \alpha(q, k_2)) = \underbrace{\frac{\beta f_\chi \tilde{m}^2}{2} P_m(q) P_m(k)}_{\text{Smoking gun!}} \frac{\vec{q} \cdot \vec{k}}{q^2}$$

Smoking gun!

To be compared with  $\Delta \langle \delta_m(\vec{k}_1) \delta_m(\vec{k}_2) \delta_m(\vec{k}_3) \rangle \propto \beta \log \tau$

# Outlook

- Precise full-shape analysis: include BAO reconstruction
- Bispectrum:  $O(\beta \log \tau)$  deviations from  $\Lambda$ CDM vs  $O(\beta)$  EP violation
- Bispectrum: non-adiabatic initial conditions?

Thanks for the attention