Large-Scale Structure with Dispersion and Higher Cumulants

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CERN: New Physics from Galaxy Clustering
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(1) Mathias Garny, Dominik Laxhuber, Roman Scoccimarro.

$Perturbation\ theory\ with\ dispersion\ and\ higher\ cumulants:$
$framework\ and\ linear\ theory$

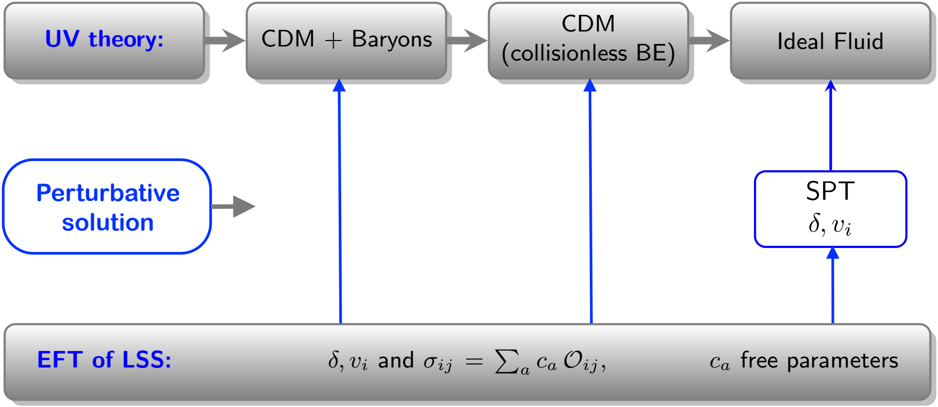
arXiv: 2210.08088, submitted to PRD

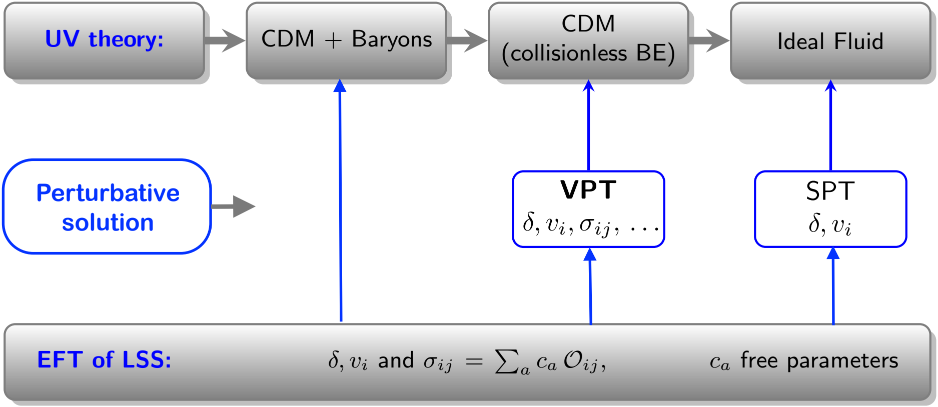
(2) Mathias Garny, Dominik Laxhuber, Roman Scoccimarro.

$Perturbation\ theory\ with\ dispersion\ and\ higher\ cumulants:$
$non-linear\ regime$

arXiv: 2210.08089, submitted to PRD

Introduction


UV theory:
CDM + Baryons
CDM (collisionless BE)
Ideal Fluid
Perturbative solution
SPT
δ, v_i
EFT of LSS:
δ, v_i and $\sigma_{ij} = \sum_a c_a \mathcal{O}_{ij}$,
c_a free parameters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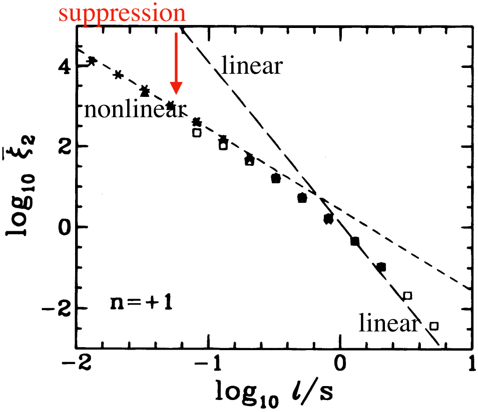
Motivation: NL power vs Linear power $(P_0 \propto k^{n_s},\ n_s = +1)$

- Simulations show for initial conditions with bluer spectra non-linear growth is most suppressed compared to linear.
- Standard perturbation theory (SPT) fails to provide any understanding of this property (UV-divergence for $n_s \geq -1$).

Colombi, Bouchet & Hernquist (1996)



Framework: Vlasov Perturbation Theory (VPT)

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- In Fourier space the equations of motion can be written in a form **analogous to SPT**,

$$\psi'_{k,a}(\eta) + \Omega_{ab}(k,\eta)\,\psi_{k,b}(\eta) = \int_{pq} \gamma_{abc}(\boldsymbol{p},\boldsymbol{q})\psi_{p,b}(\eta)\psi_{q,c}(\eta)\,, \tag{3}$$

- where now $\psi_{\text{SPT}} = (\delta, \theta), \quad \rightarrow \quad \psi_{\text{VPT}} = (\underbrace{\delta, \theta, g, \delta\epsilon, A}_{S}, \underbrace{w_i, \nu_i}_{V}, \underbrace{t_{ij}}_{T}, \dots)\,.$

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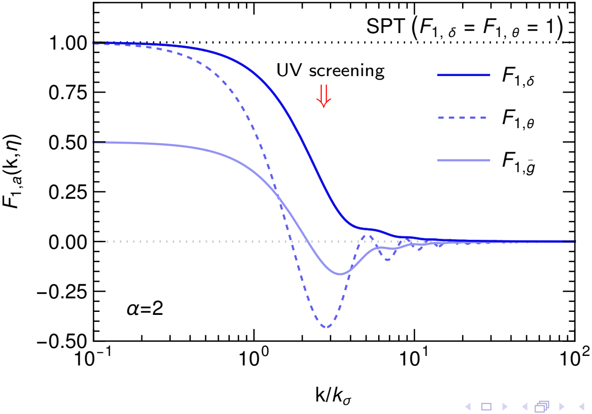
- The linear evolution is governed by time- and scale-dependent matrix $\Omega_{ab}(k,\eta)$ where already the background dispersion $\epsilon(\eta)$ enters.

$\rightarrow$ This sets a new dispersion scale: $\boxed{k_\sigma = 1/\sqrt{\epsilon(\eta)}}$

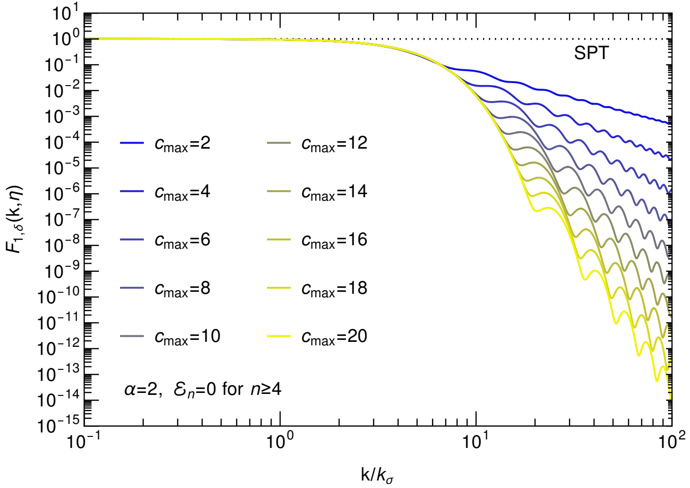
Linear Solutions

VPT: Linear Kernels for δ, θ and $\bar{g}$ (dispersion only)

- Modes that cross the dispersion scale are suppressed (UV screening).
- **VPT** linear modes **know** about small scale structure (e. g. halo formation) through background values of cumulants.
- **SPT:** free linear modes, **no information** on halo formation at small scales.



VPT: Linear Kernel for δ (convergence of hierarchy)

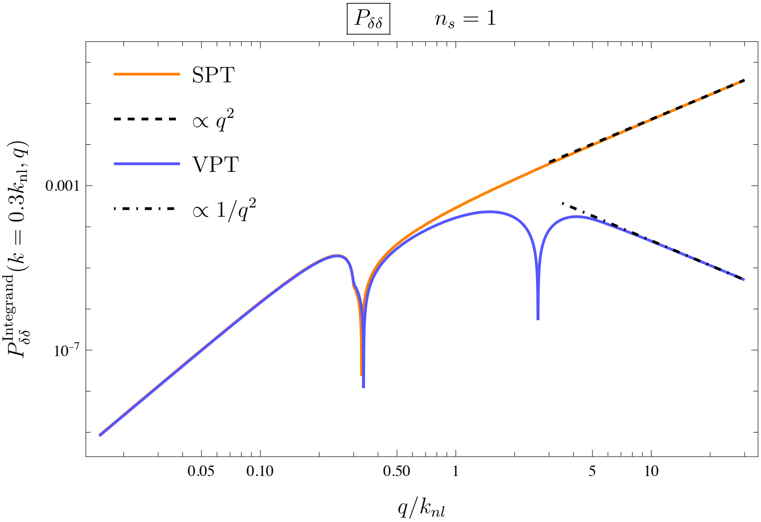


- Suppression is only enhanced when allowing for more complex cumulant truncations $\rightarrow$ UV screening is robust to different hierarchy assumptions.

Non-Linear Regime

One-Loop Power Spectrum Integrand $(P_0 \propto k^{n_s},\, n_s = +1)$

$$P_{\delta\delta}^{\text{Integrand}}(k,q,\eta)|_{\text{SPT}} \propto q^{n_s+1}\,, \quad P_{\delta\delta}^{\text{Integrand}}(k,q,\eta)|_{\text{VPT}} \propto q^{-2}\,, \quad P_{\delta\delta}(k,\eta) = \int \mathrm{d}\ln q\, P_{\delta\delta}^{\text{Integrand}}(k,q,\eta).$$



One-Loop Power Spectrum Integrand $(n_s = -1, \ldots, +2)$

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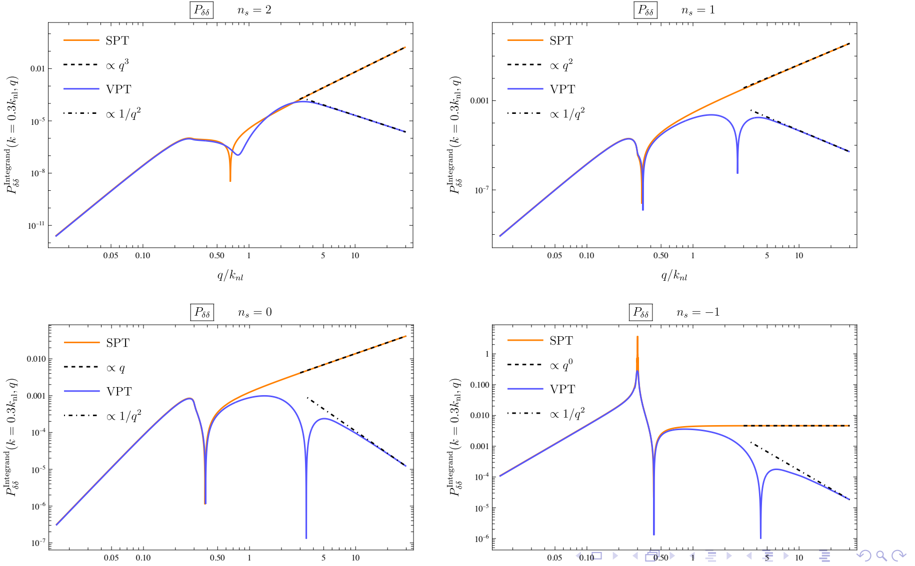
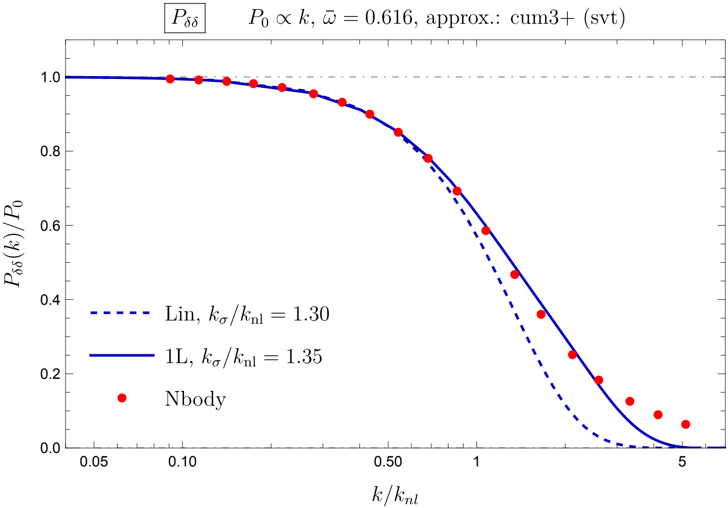
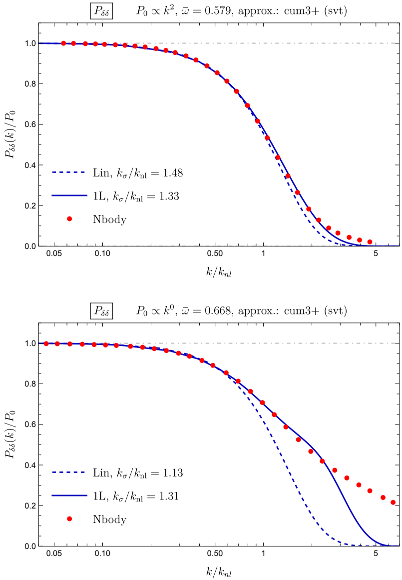


VPT predictions vs Simulations

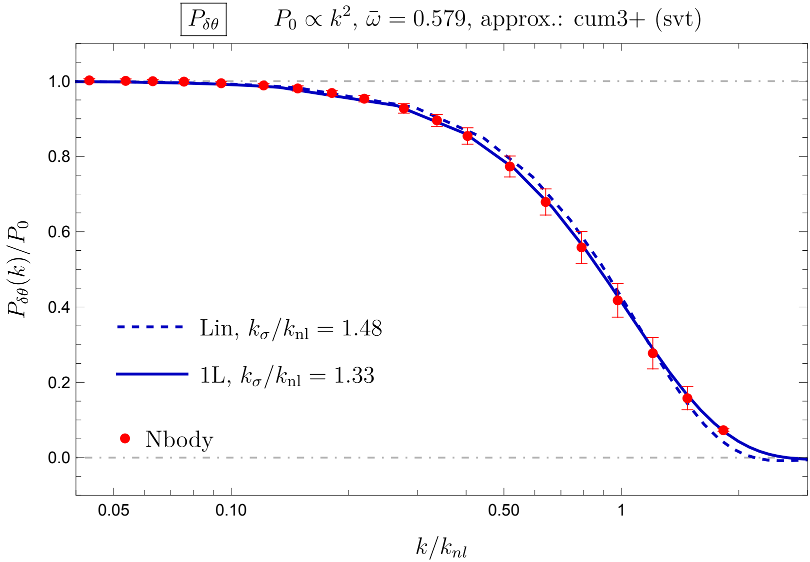
One-Loop Power Spectrum: $P_{\delta\delta} / P_0$ (here: k_σ matched, $n_s = 1$)



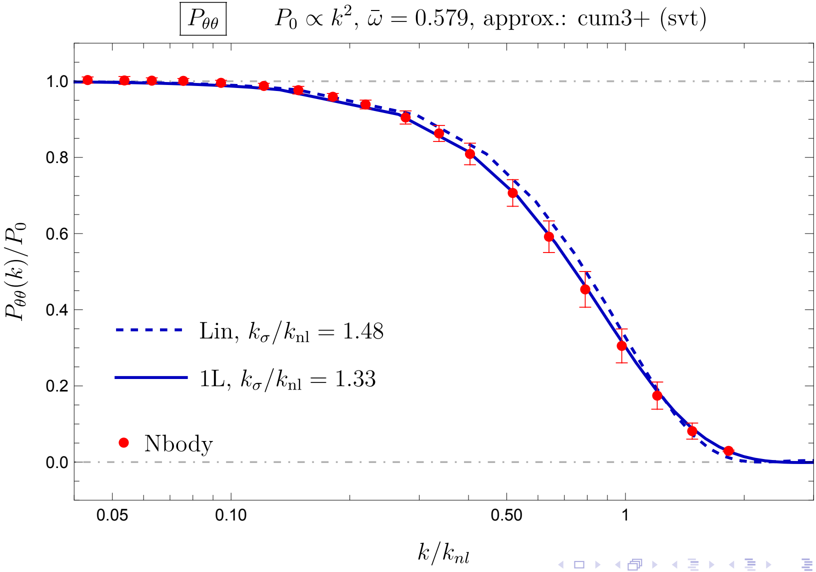
One-Loop Power Spectrum: $P_{\delta\delta} \,/\, P_0$ (here: k_σ matched, $n_s = -1, .., +2$)



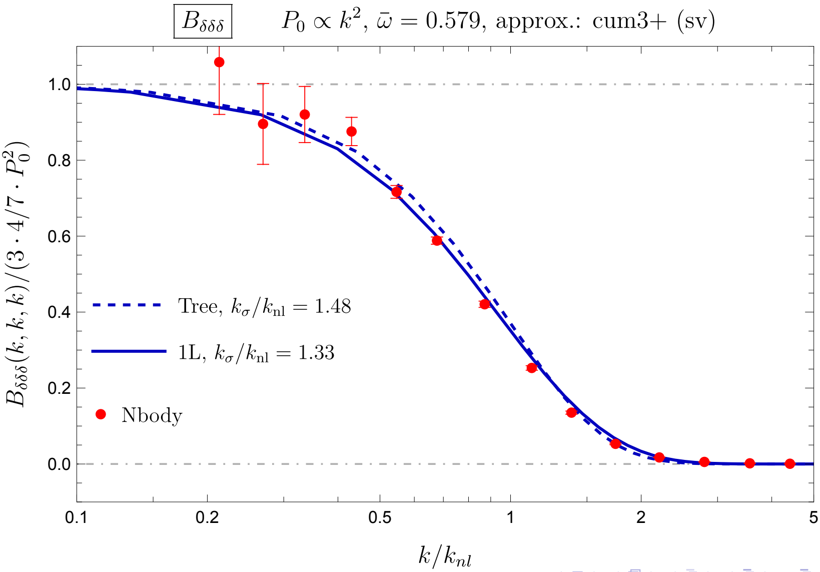
One-Loop Power Spectrum: $P_{\delta\theta}$ ($n_s = 2$, no free parameters)



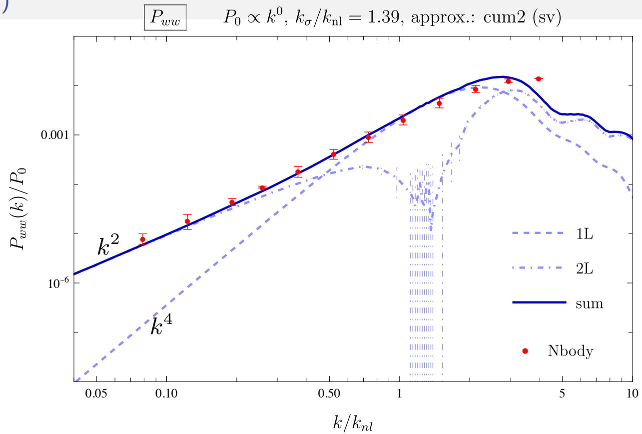
One-Loop Power Spectrum: $P_{\theta\theta}$ ($n_s = 2$, no free parameters)



equilateral Bispectrum: $B_{\delta\delta\delta}$ ($n_s = 2$, no free parameters)



Two-Loop Vorticity Power Spectrum: P_{ww} ($n_s = 0$, no free parameters)



- Going to two-loop reveals the general scaling $P_{w_i w_i}(k,\eta) \propto k^2$ for $k \to 0$, i. e. we found in general $n_w = 2$ for $k \to 0$.
- The one-loop contribution is accidentally suppressed as k^4.
- This transition at low-k is confirmed by N-body simulations.

Conclusion

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(iv) **Generation of vorticity** as well as vector and tensor modes of the dispersion tensor at **second order PT**, P_{ww} scales as k^2 for $k \rightarrow 0$ in general which is confirmed by N-body measurements.

$\Rightarrow$ An explorative study of the Two-Loop showed only minor differences up to the non-linear scale.

$\Rightarrow$ Applications to ΛCDM and redshift-space distortions are planned in future.

Thank you!

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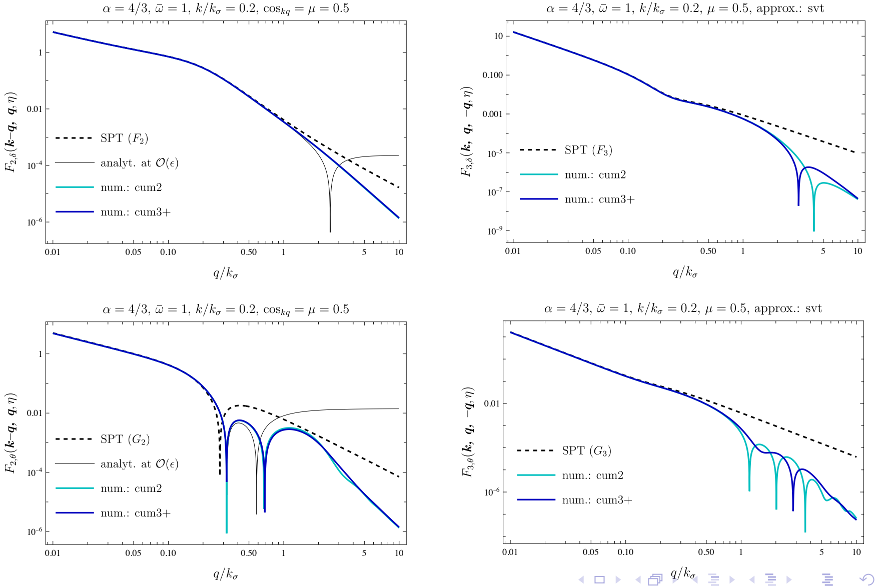
VPT: Determination of Background dispersion $\epsilon(\eta)$

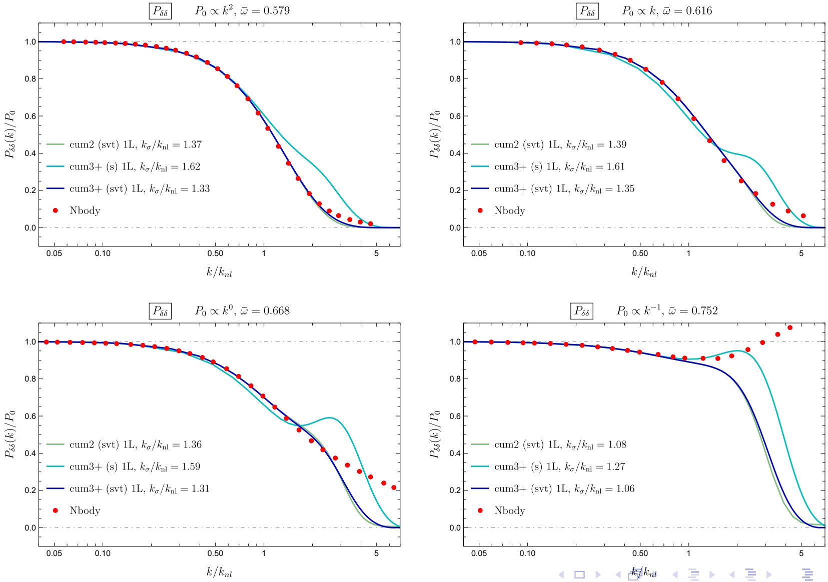
- The perturbation variables are coupled to the background values and vice versa, so it is necessary to determine all the expectation values, which then introduce a new scale, e. g. the dispersion scale k_σ:

1. measure directly from simulations, as they are physical quantities,
2. calculate self-consistently from their equations of motion,
3. calculate from halo models,
4. matching them to the density power spectrum only.

Non-Linear Kernels: $F_{2,\delta/\theta}$ and $F_{3,\delta/\theta}$ vs q/k_σ (here: $k/k_\sigma = 0.2$)



Comparison with simulations: Impact of hierarchy assumptions



- **VPT vs EFT** (Pajer & Zaldarriaga, 2013)

$$P_{\text{EFT}}/P_0 = 1 + \left(\frac{k}{k_{\text{nl}}}\right)^{n_s+3}\left[\alpha(n_s) + \bar{\alpha}(n_s)\ln\frac{k}{k_{\text{nl}}}\right] + \beta\left(\frac{k}{k_{\text{nl}}}\right)^2 + \gamma\left(\frac{k}{k_{\text{nl}}}\right)^{4-n_s} \quad \text{(2-parameter fit)}$$