Quenched jets beyond leading accuracy

Adam Takacs (University of Bergen)

Paul Caucal, Alba Soto-Ontoso

arXiv:2103.06566 vacuum substructure

arXiv:2111.14768 medium substructure

Johannes H. Isaksen, Konrad Tywoniuk

arXiv:2103.14676 energy-loss, q/g discrimination

arXiv:2206.02811 medium-induced emissions

Frederic Dreyer, Gregory Soyez

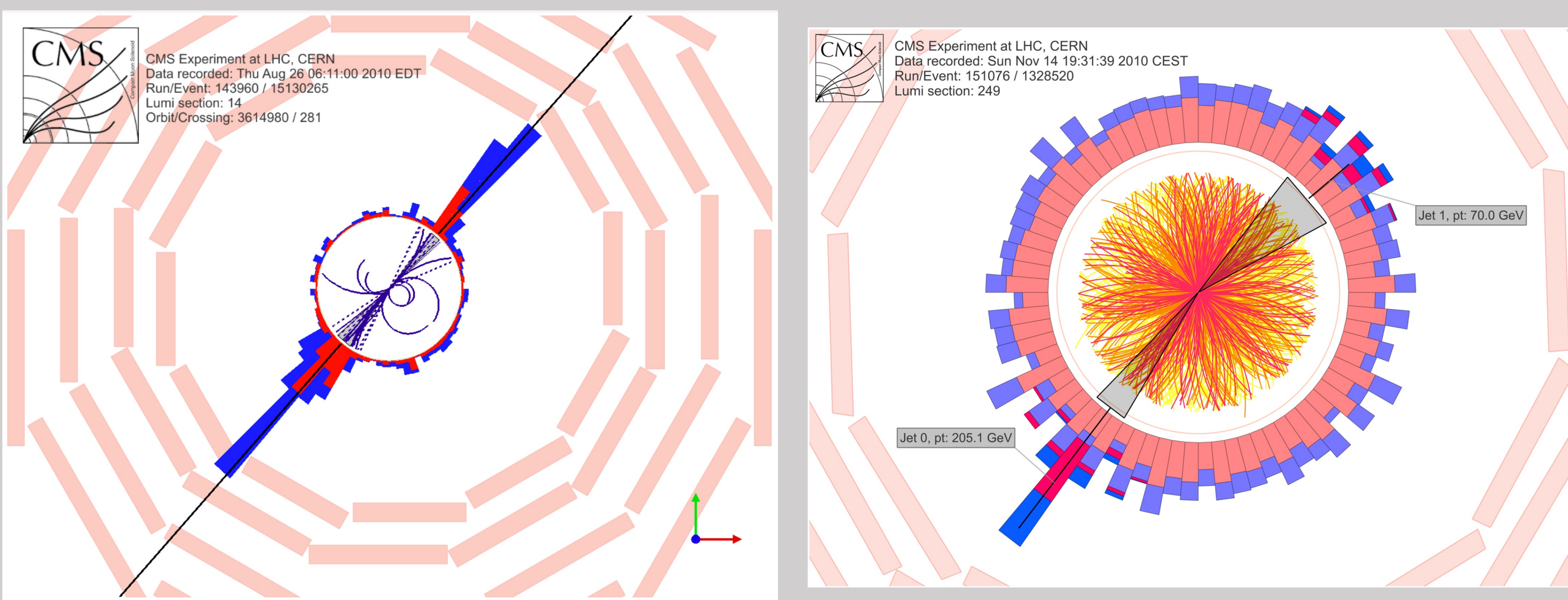
arXiv:2112.09140 q/g discrimination at NLL & ML

Introduction

Jets in QCD

pp

AA

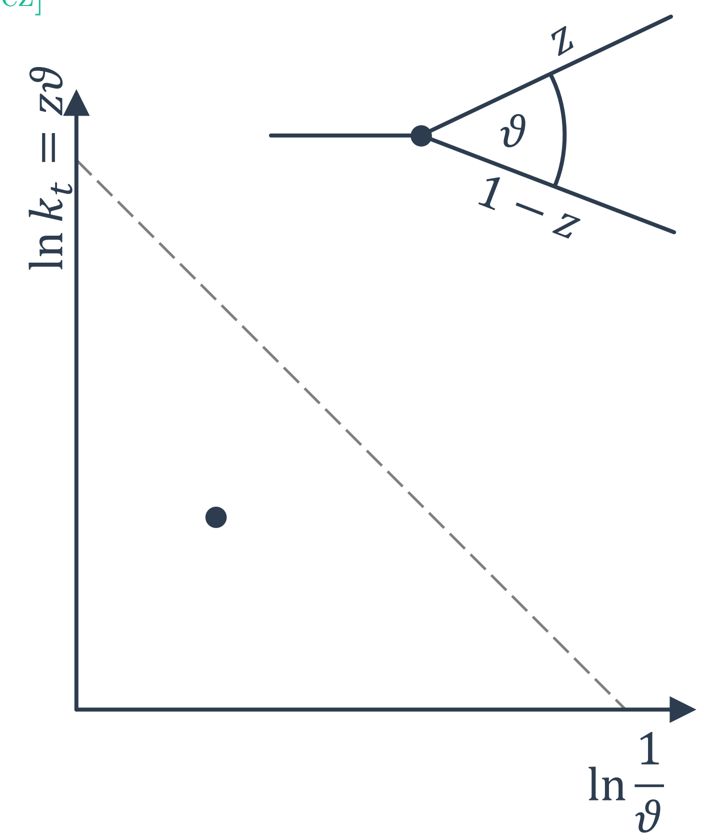


[O. Saarimäki's (2018)]

Grooming splittings in jets

The Lund plane: phase space of emissions [Dreyer,Salam,Soyez]

1. Find a jet
2. Recluster with C/A (widest angle first)
3. Follow the hardest branch ($z_i > {}^1/_2$)



Soft Drop grooming [Larkovski, Marzani, Soyez, Thaler]:

4. Stop if $z_i > z_{cut}\vartheta_i^\beta$ (with the widest angle)
   - Free parameters z_{cut} and β.

Dynamically grooming [Mehtar-Tani, Soto-Ontoso, Tywoniuk]:

4. Find the hardest $\max_i(z_i\vartheta_i^a)$
   - No cuts, autogenerated jet-by-jet
   - Clear physical meaning: hardest k_t ($a = 1$), or biggest m^2 ($a = 2$)

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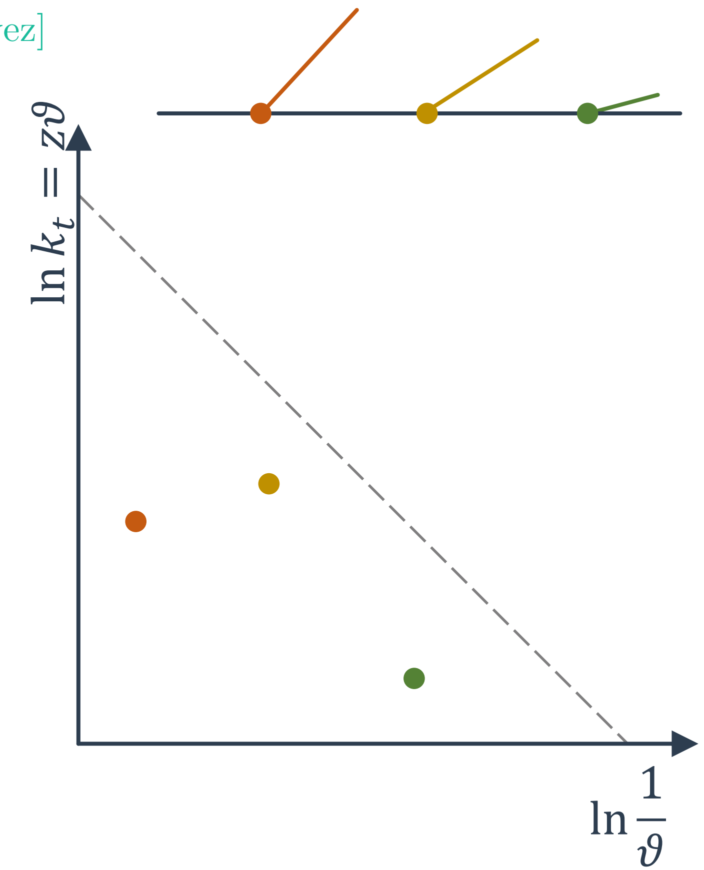

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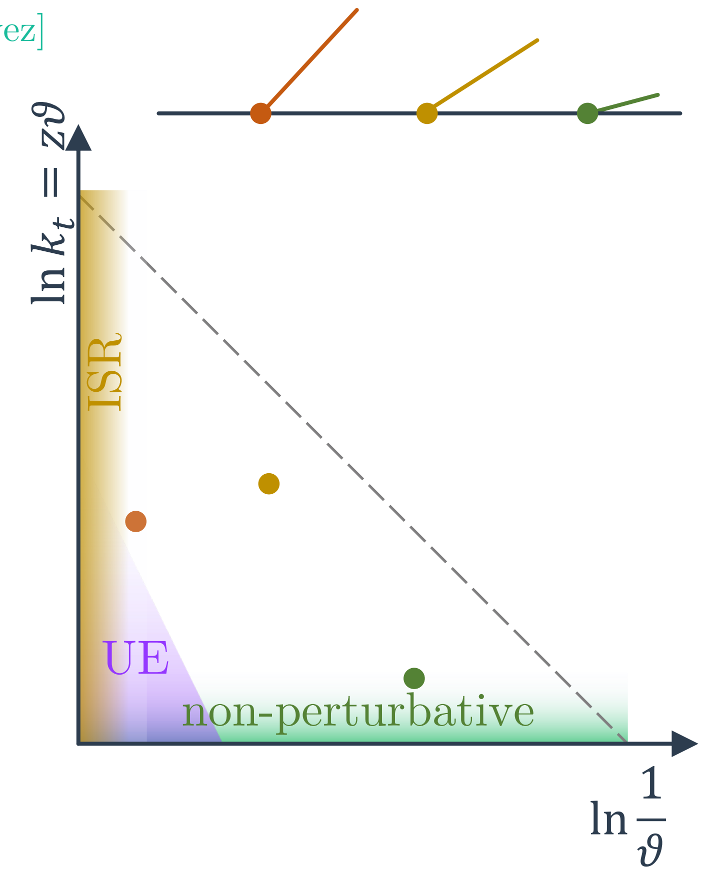

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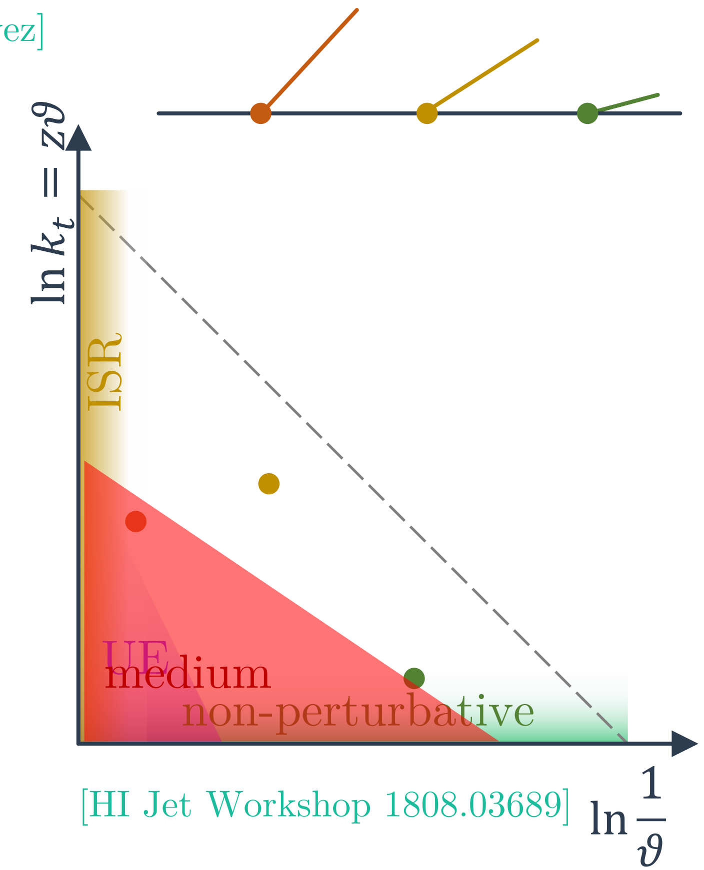

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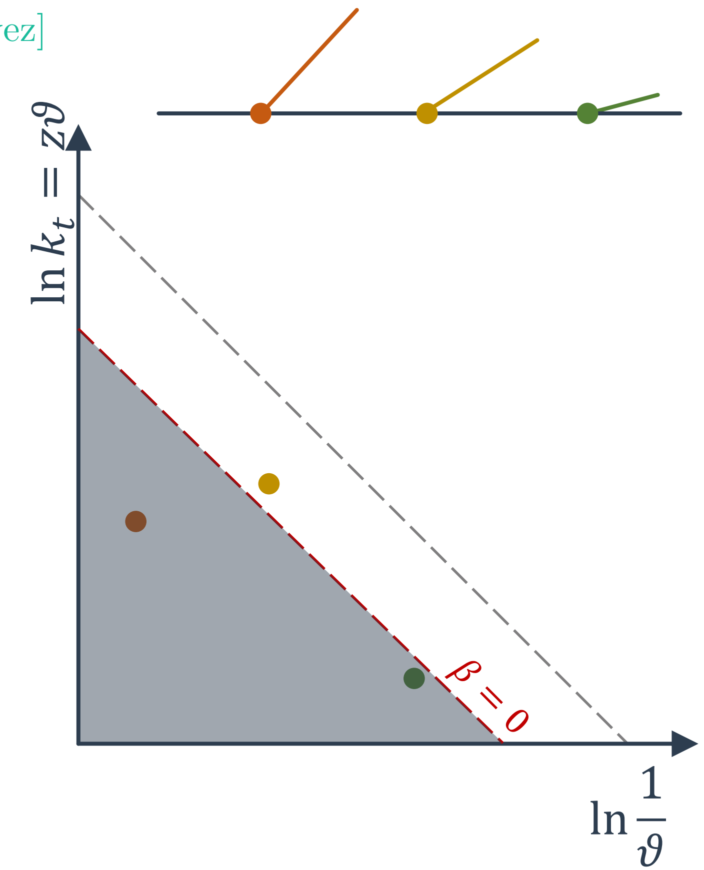

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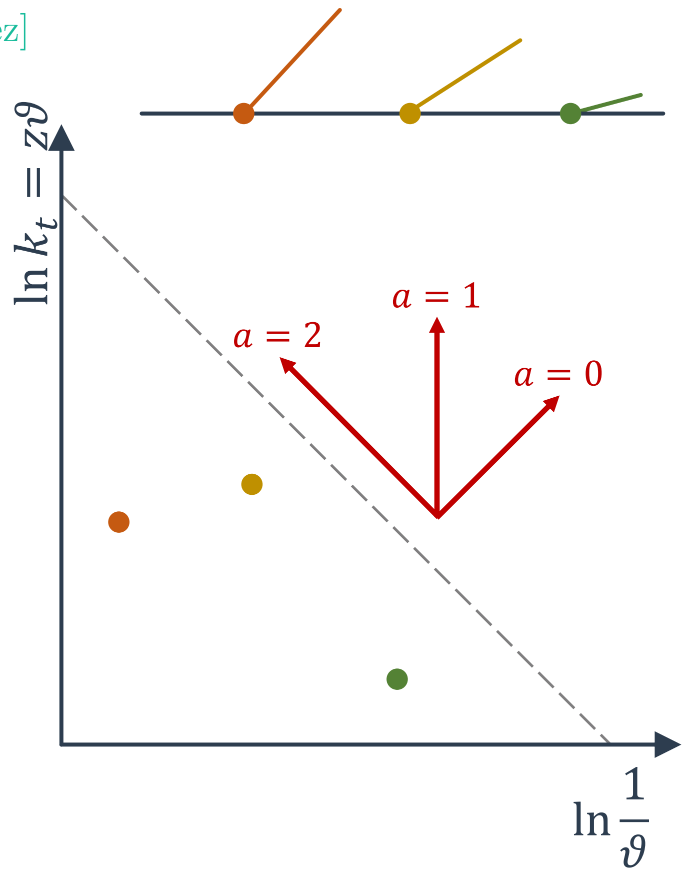

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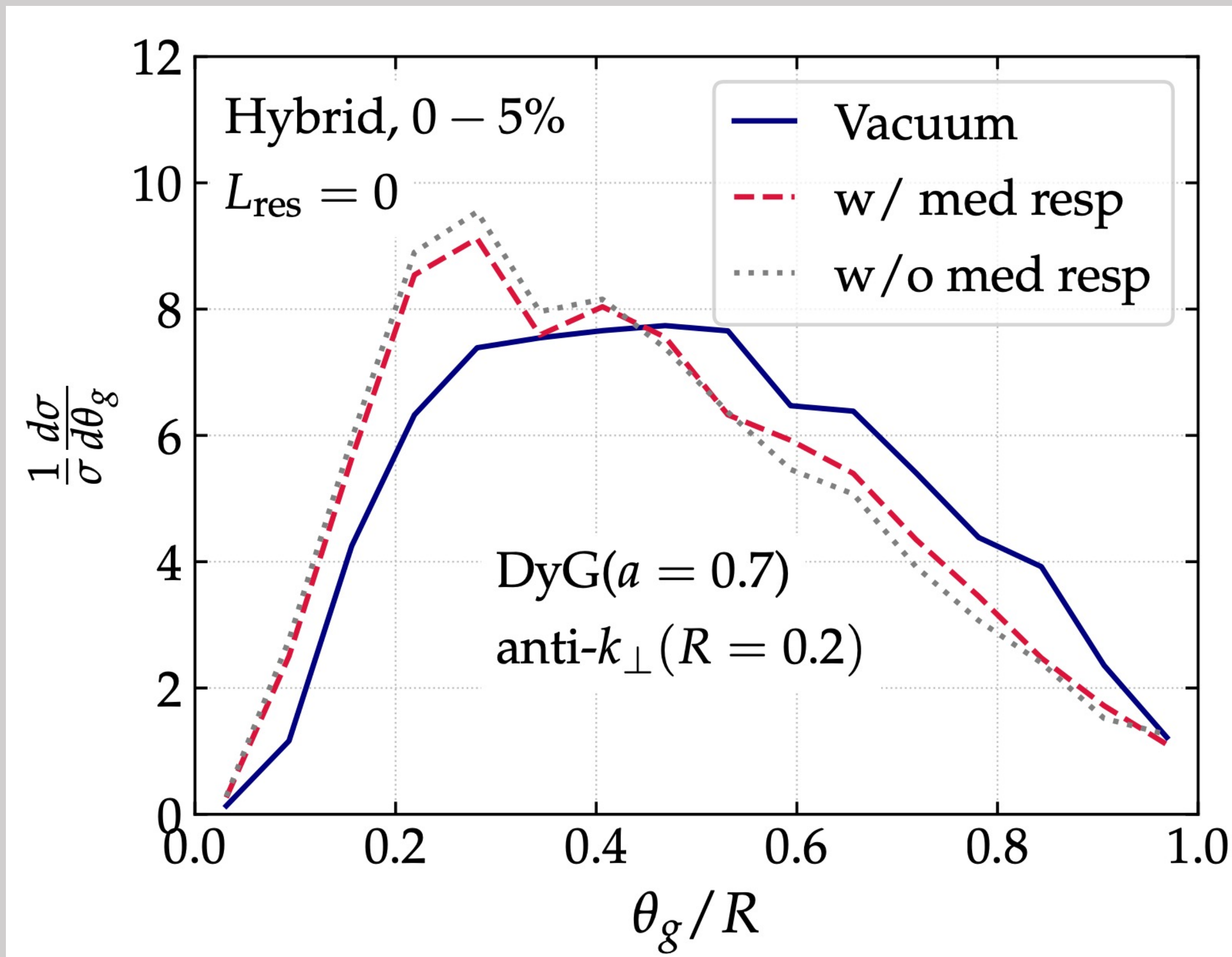
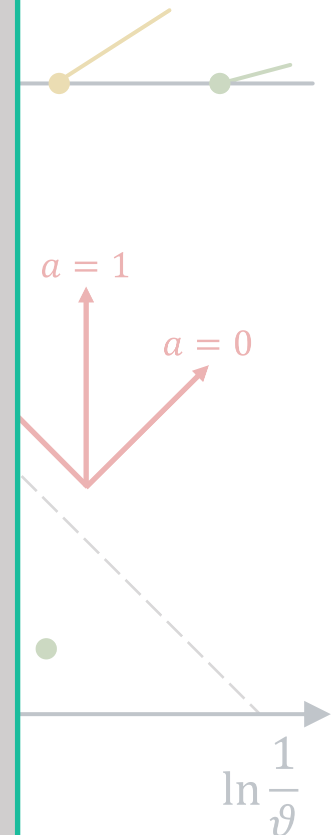


[Caucal,Soto-Ontoso,Takacs,2111.14768]



2)

Outline

1. Jets in pp beyond leading logarithmic accuracy
2. Medium-induced emissions
3. Medium cascade beyond leading accuracy
4. Quenched jets beyond leading accuracy?

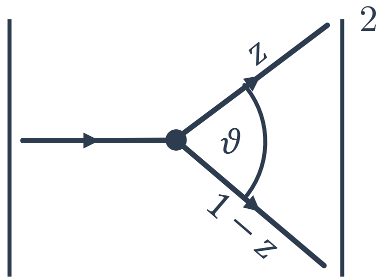
Proton-Proton Baseline for DyG

with Paul Caucal and Alba Soto-Ontoso

arXiv:2103.06566

Jets in QCD

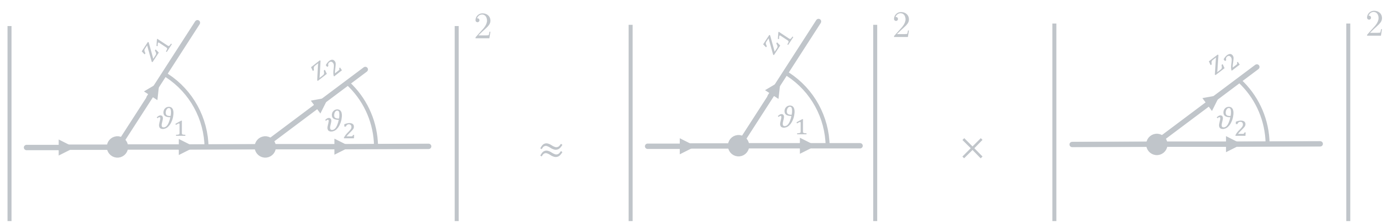
Soft and collinear divergence of QCD:



$$\sim \frac{\alpha_s}{\pi}\frac{C_i}{z}\frac{1}{\boldsymbol{k}^2}$$

soft & coll. divergences

Factorization of strongly ordered emissions (also virtual terms):



sequential algorithm: parton shower

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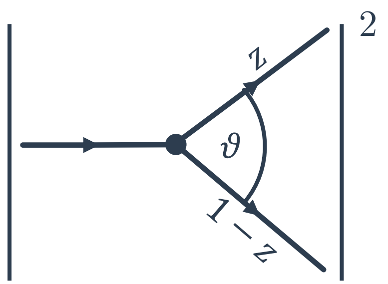

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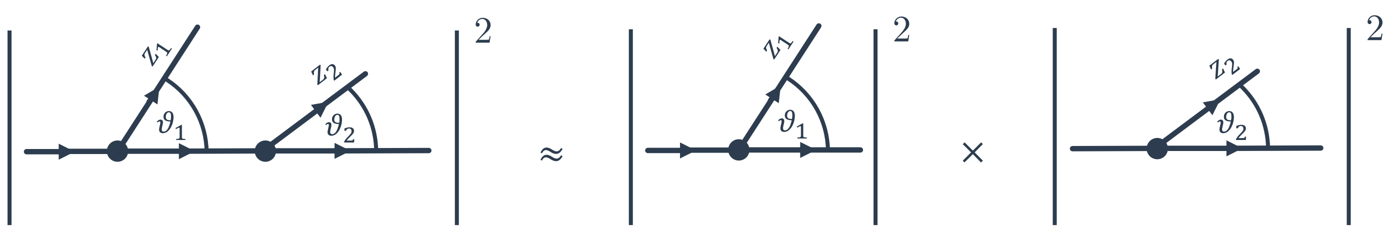

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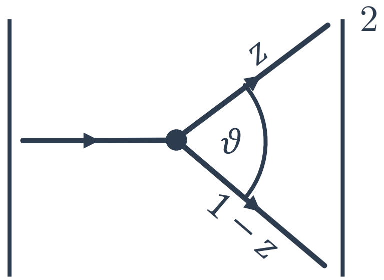

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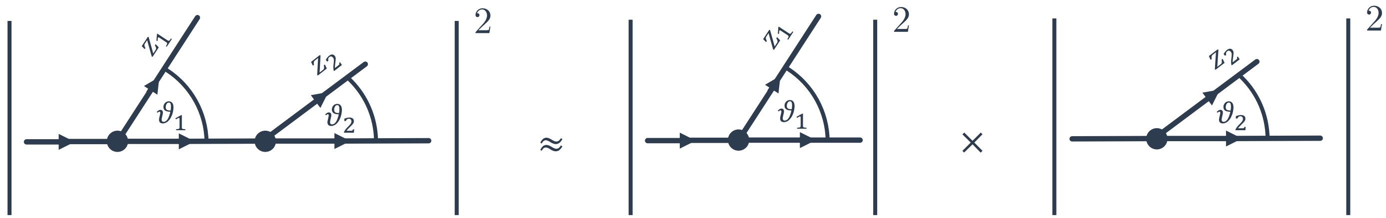

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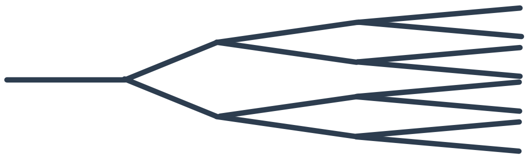


Basics of analytic calculation

Probability of (z,ϑ) is the hardest $(\kappa = z\vartheta^a)$:

$$P(z,\vartheta) = 2\bar{\alpha}\,\frac{1}{z}\frac{1}{\vartheta}$$

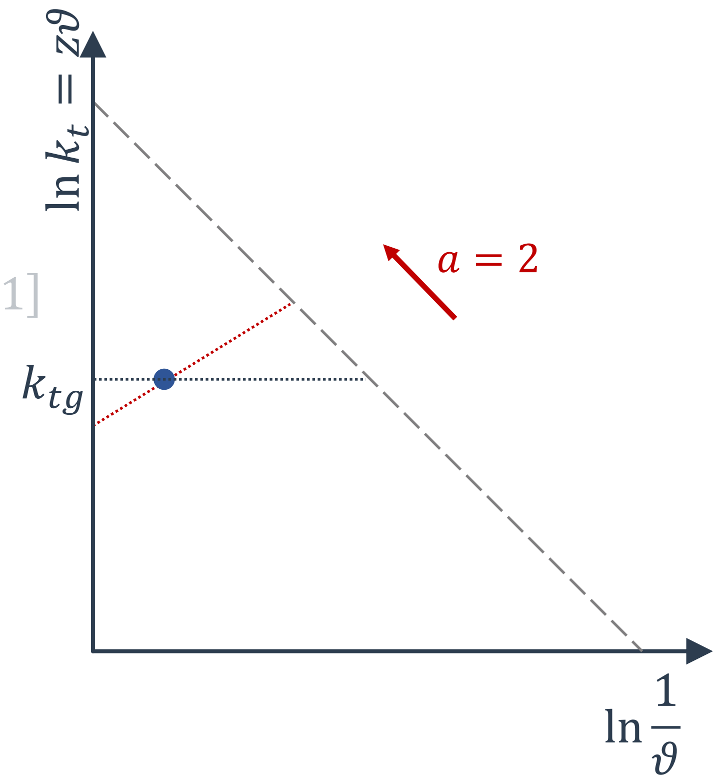
$$\Delta_i(\kappa|a) = \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{m=1}^{n}\int d\vartheta_m\, dz_m P_i(z_m,\vartheta_m)[\Theta(\kappa - z_m\vartheta_m^a) - 1]$$

$$= \exp\left[-\int d\vartheta \int dz\, P_i(z,\vartheta)\Theta(z\vartheta^a - \kappa)\right]$$

$$\frac{d^2\mathcal{P}_i(z,\vartheta|a)}{d\vartheta dz} = P_i(z,\vartheta)\Delta_i(z\vartheta^a|a)$$

Measuring an observable:

$$\left.\frac{1}{\sigma}\frac{d\sigma}{dk_g}\right|_a = \int_0^1 d\vartheta \int_0^1 dz\ \mathcal{P}_i(z,\vartheta|a)\delta(k_g - z\vartheta) = \frac{1}{k_g}\frac{\sqrt{\pi a\bar{\alpha}}}{a-1}\left[\operatorname{erf}\left(\sqrt{\frac{\bar{\alpha}}{a}}\ln k_g\right) - \operatorname{erf}\left(\sqrt{a\bar{\alpha}}\ln k_g\right)\right]$$



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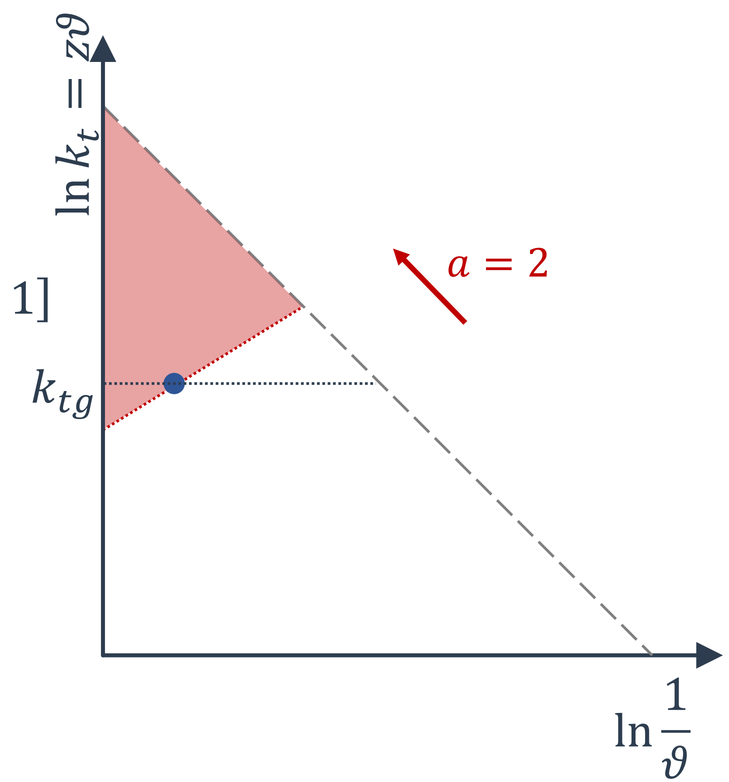

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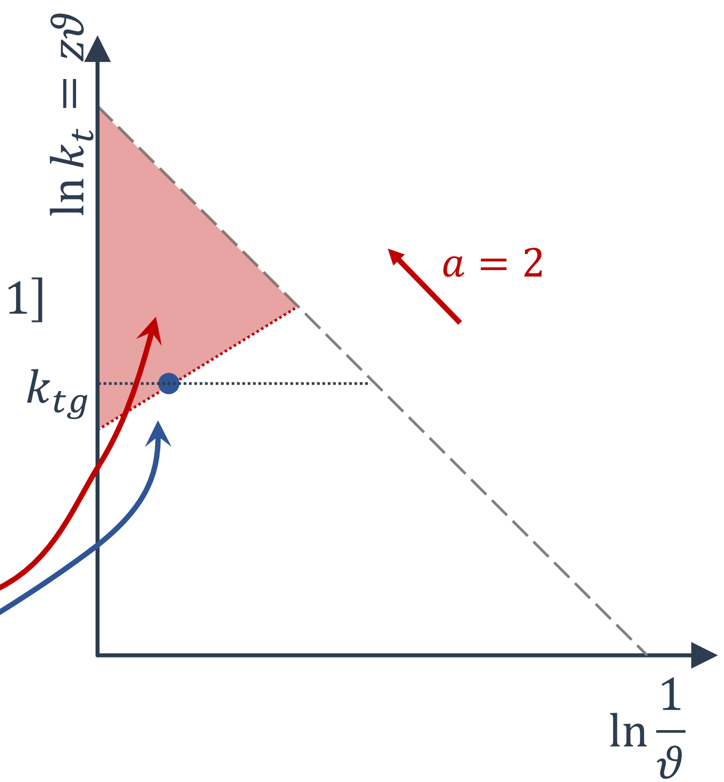

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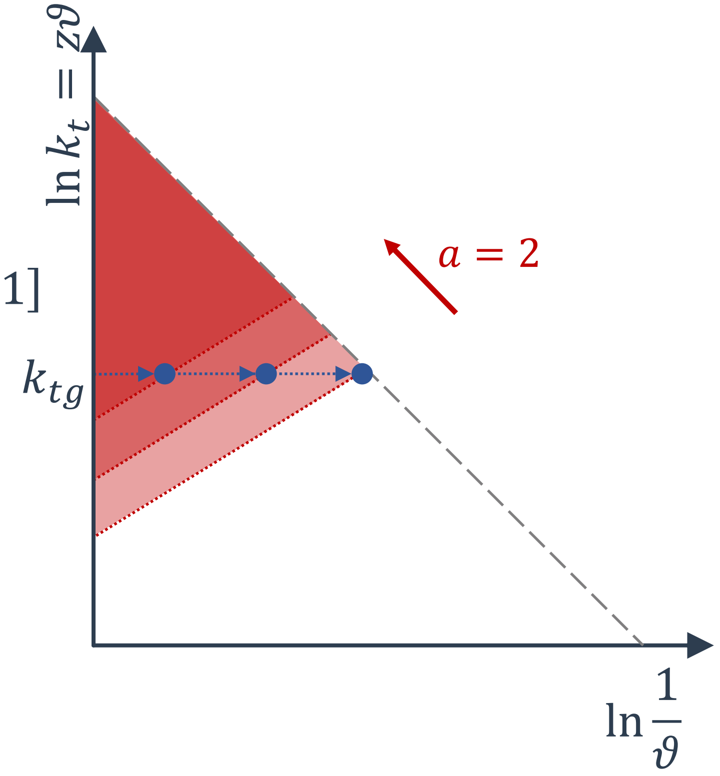

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What did the resummation do?

Probability:

$$\Sigma(k_g|a) = \int_0^{k_g} dk_g' \frac{1}{\sigma}\frac{d\sigma}{dk_g'}\bigg|_a = 1 - \bar{\alpha}\ln^2\frac{1}{k_g} + \frac{1+a+a^2}{6a}\bar{\alpha}^2\ln^4\frac{1}{k_g} + \mathcal{O}(\bar{\alpha}^3)$$

important when $\bar{\alpha}\ln^2\frac{1}{k_g} \sim 1$!

Hard-collinear correction:

$$\delta P_{hc} = \int_0^1 dz\left[P_i(z) - \frac{2C_i}{z}\right] = 2C_iB_i$$

$$\Delta_i(\kappa|a) = e^{-\frac{\bar{\alpha}}{a}\ln^2\kappa} \longrightarrow \delta\Delta_{hc}(\kappa|a) = e^{-\frac{\bar{\alpha}}{a}(2B_i\ln\kappa + B_i^2)}$$

Running-coupling:

$$\alpha_s^{1l}(k_t) = \frac{\alpha_s(p_tR)}{1+2\beta_0\alpha_s(p_tR)\ln(k_t/p_tR)} \approx \alpha_s(p_tR)[1 - 2\beta_0\alpha_s(p_tR)\ln(k_t/p_tR)]$$

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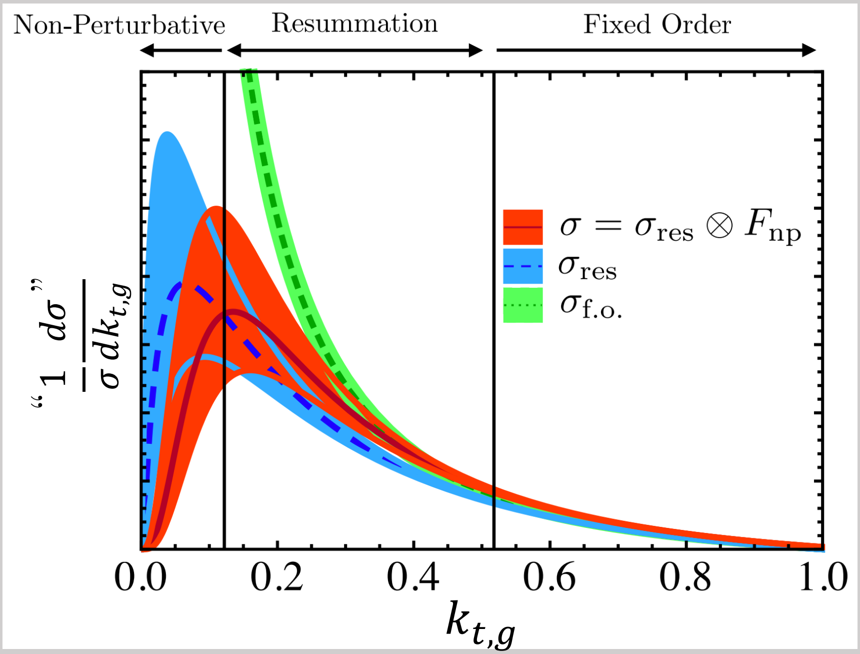

[Larkoski,Moult,Nachman,1709.04464]

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Defining accuracy for jets

[Banfi,Salam,Zanderighi,0407286]

Accuracy based on Σ: N^{p}DL where $2n - p \leq m \leq 2n$

$$\Sigma(k_g|a) = \sum_{n=0}^{\infty} \sum_{m=0}^{2n} C_{mn} \alpha_s^n \ln^m(k_g)$$

Accuracy based on $\ln \Sigma$: N^{n}LL, LL is g_1, then g_{n+1}

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Systematically well organized expansion to achieve higher accuracy!

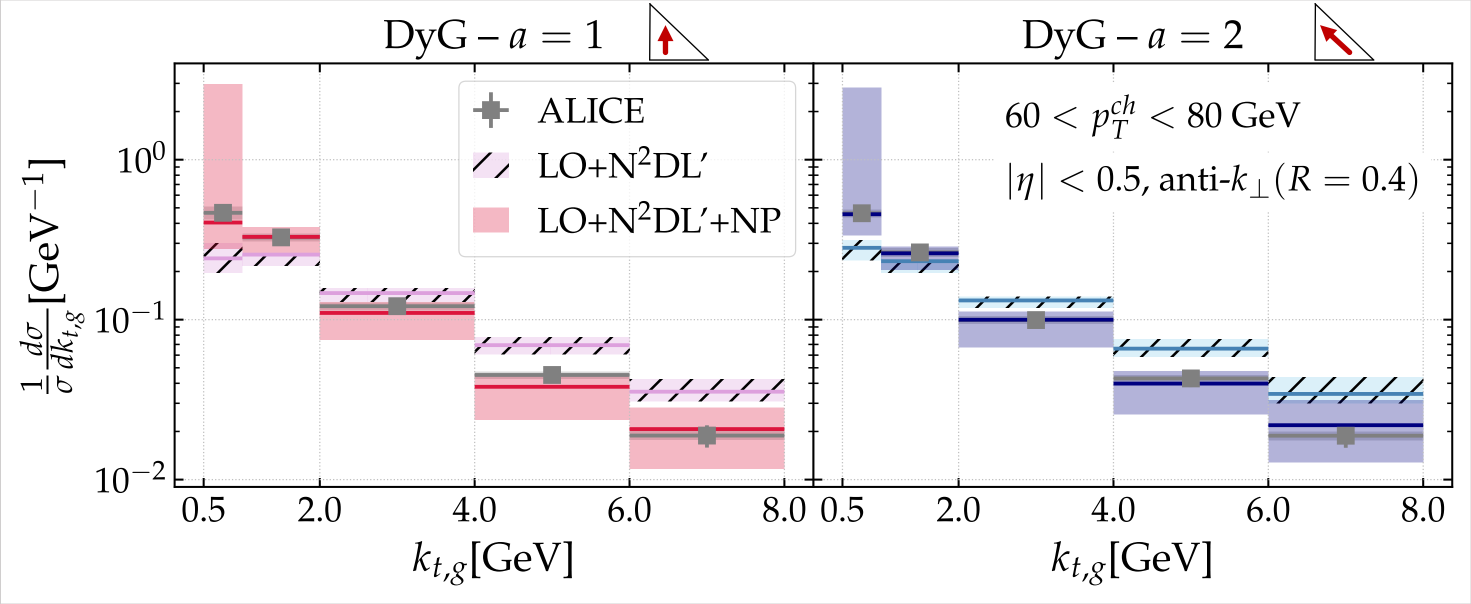
Predictions for DyG

Targeted accuracy is LO+N^{2}DL:

- Splitting function at 2-loop
- Running coupling at 2-loop
- Non-global contributions (large-N_c, small-R)
  - There is no clustering log
  - Boundary logs present
- No multiple emission contribution
- Matching to MadGraph5
- Non-perturbative corrections

Results - Comparison to ALICE preliminary

Hardest emission inside the jet, $k_{t,g}$ distribution



[ALICE 2009.07172, 2009.12247]

- Sensitive to NP effects.
- Good agreement with data.
- Baseline for AA calculation.

Medium-induced emissions

with Johannes Isaksen and Konrad Tywoniuk

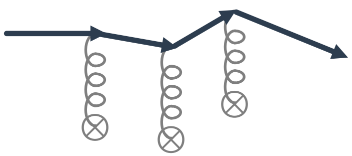
arXiv:2206.02811

QCD in a background medium

[Zakharov, BDMPS, GLV, Wiedemann (1996-2000)
Blaizot, Iancu, Salgado, CGC formalism (2012-)]

QCD with medium bkg:

- Colored background $\mathcal{A}_0(t, x)$
- Energy is conserved (p^+), transverse kick ($\boldsymbol{p}$)
- Multiple scatterings

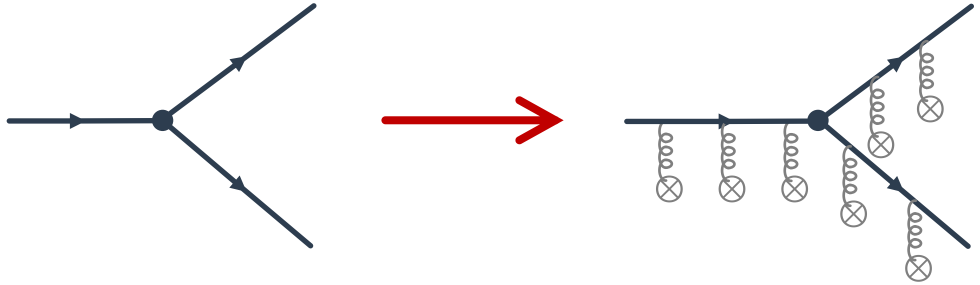


Keeping space-time: mixed Fourier space (p^+, $\boldsymbol{p}$, p^-) $\rightarrow$ (p^+, $\boldsymbol{x}$, t)

- Effective propagator:

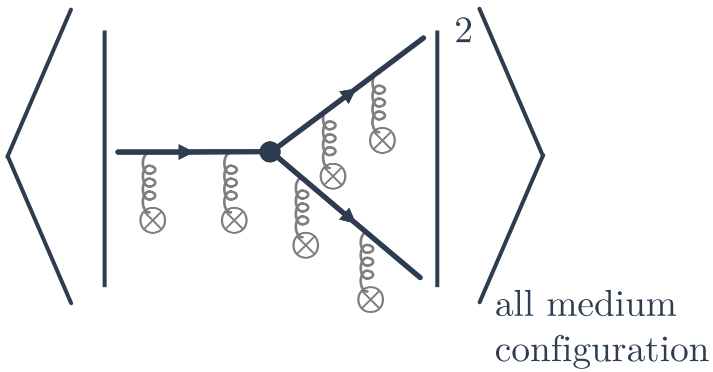
- In/out-coming legs

- Effective vertices:



Medium-induced radiation

LO radiation in the soft and collinear limit:



Elastic broadening

[AMY,HTL]
[Casalderrey-Solana,Teaney]
[EQCD,Caron-Huot]
[Moore,Schlichting,Schlusser,Soudi]

Medium averages:

$$\left\langle A_0^a(t,\boldsymbol{r}) A_0^b(t',\boldsymbol{r}')\right\rangle = \delta^{ab} n(t)\delta(t-t')\gamma(\boldsymbol{r}-\boldsymbol{r}',t)$$

Transverse momentum broadening:

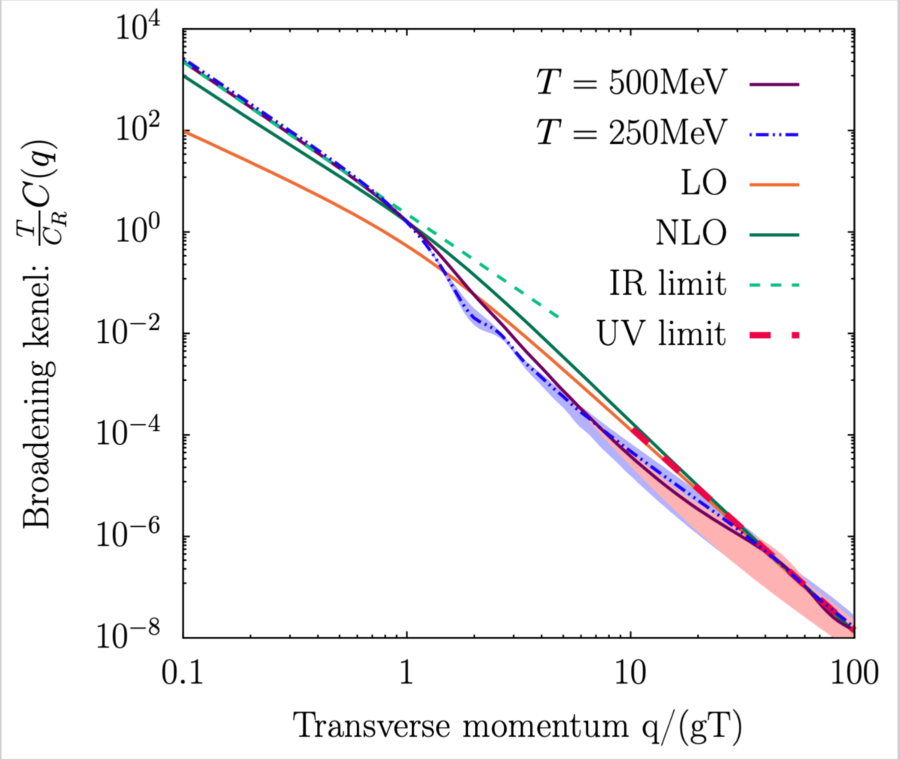
$$\gamma(\boldsymbol{r},t) = \int_{\boldsymbol{q}} e^{i\boldsymbol{q}\cdot\boldsymbol{r}}\, C_{el}(\boldsymbol{q},t)$$

In practice:

$$C_{el}(\boldsymbol{q},t) = \frac{4\pi\hat{q}_0(t)}{(\boldsymbol{q}^2+\mu^2)^2}$$

Important nPT parameters:

- mean free path: $\lambda = \frac{\mu^2}{\hat{q}_0}$
- screening mass: μ



[Schlichting,Soudi,2111.13731]

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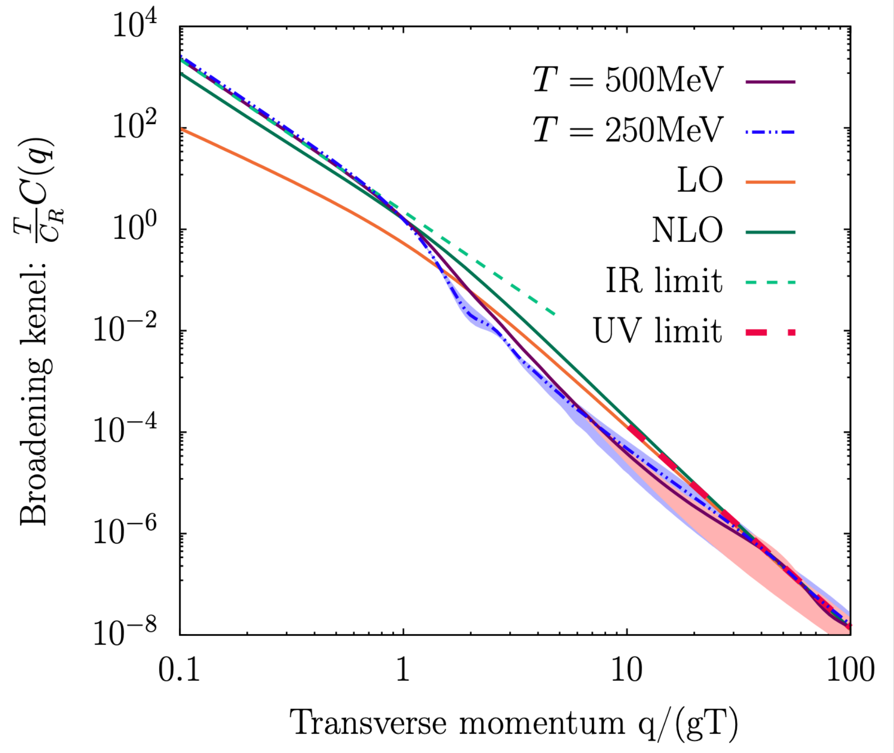

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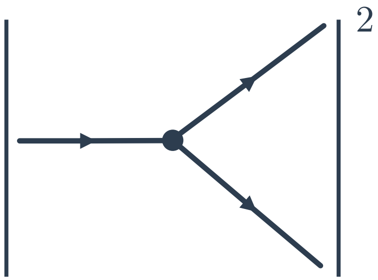

$$\sim \frac{\alpha_s}{\pi}\frac{C_i}{z}\frac{1}{\boldsymbol{k}^2}$$

soft and collinear divergences!

Medium-induced radiation

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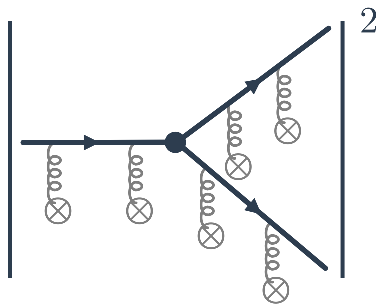

$$\omega \frac{dI^{mie}}{d\omega} = \omega \frac{dI^{med}}{d\omega} - \omega \frac{\text{``}dI^{vac}\text{''}}{d\omega}$$

$\neq 0$ medium induced emissions!
soft but no collinear divergence!

$$\mathcal{K}(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1) = \delta(\boldsymbol{p}_2 - \boldsymbol{p}_1)\mathcal{K}_0(\boldsymbol{p}_1, \mathrm{t}_2 - \mathrm{t}_1)$$

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analytically

[BDMPS-Z(1997)]
[GLV, Wiedemann(2000)]
[AMY(2000)]

numerically

[Feal,Vazquez(2018)]
[Andres,Dominigues,Martinez(2020)]
[Schlichting,Soudi(2021)]

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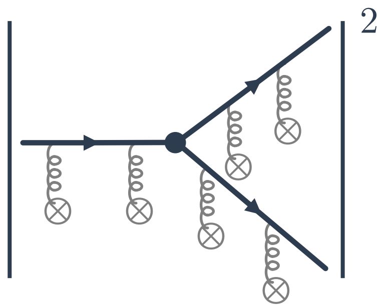

$$\omega \frac{dI^{mie}}{d\omega} = \frac{2\alpha_s C_i}{\omega^2} \,\mathrm{Re} \int dt_2 \int dt_1 \int_{\boldsymbol{p}_1,\boldsymbol{p}_2} \boldsymbol{p}_2 \cdot \boldsymbol{p}_1 \, \mathcal{K}(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1) - \text{vacuum}$$

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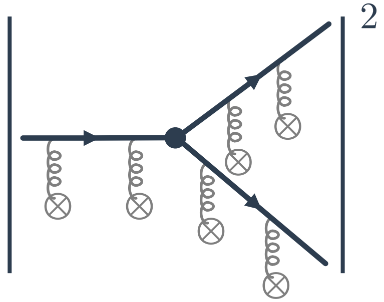

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[BDMPS-Z(1997)]
[GLV, Wiedemann(2000)]
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?

numerically

[Feal,Vazquez(2018)]
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Opacity expansion

[GLV, Wiedemann(2000)]

Exclusive number of scatterings: iterate the kernel

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$$\omega\frac{dI^{N=1}}{d\omega}=2\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}\int_0^\infty dx\,\frac{1}{1+\bar{\omega}_c/\omega}\frac{x-\sin x}{x^2}$$

$$=\begin{cases}2\bar{\alpha}\frac{L}{\lambda}\left(\gamma_E-1-\ln\frac{\bar{\omega}_c}{\omega}\right), & \omega\ll\bar{\omega}_c=\frac{\mu^2L}{2}\\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \bar{\omega}_c\ll\omega\end{cases}$$

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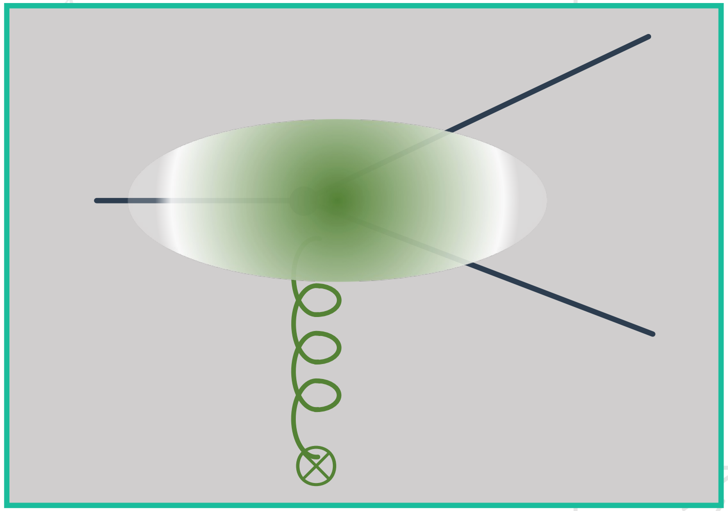

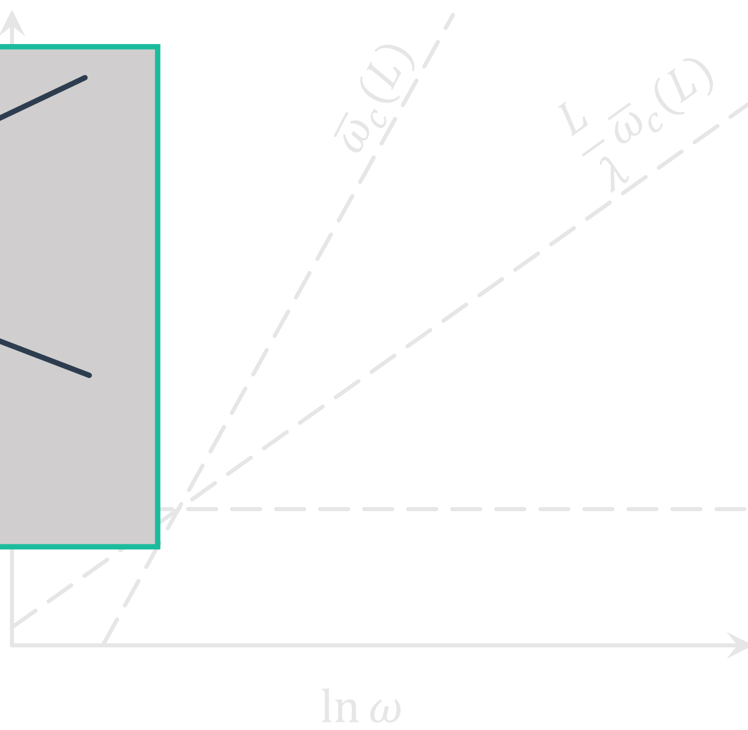

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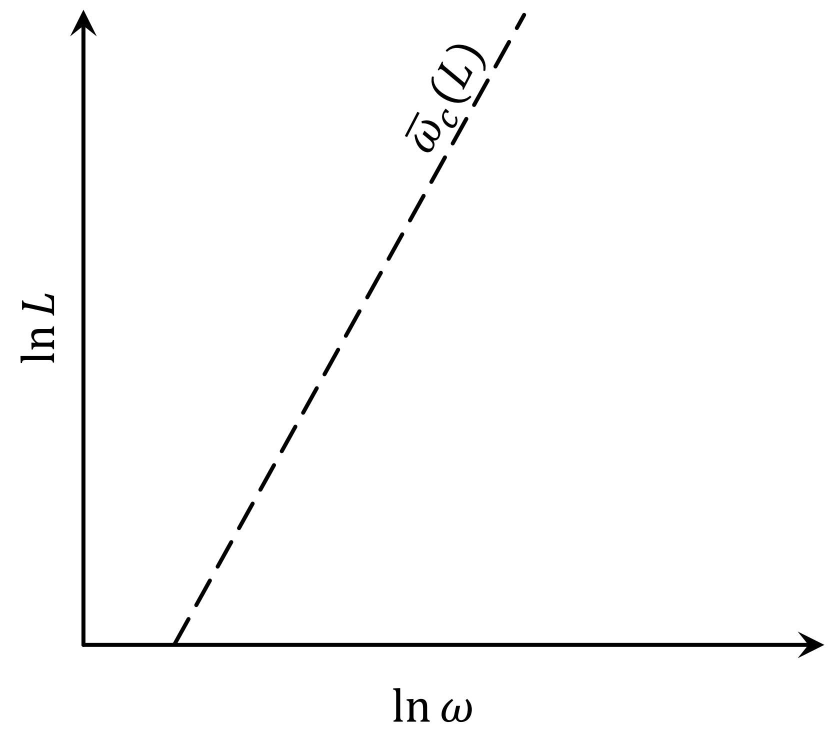

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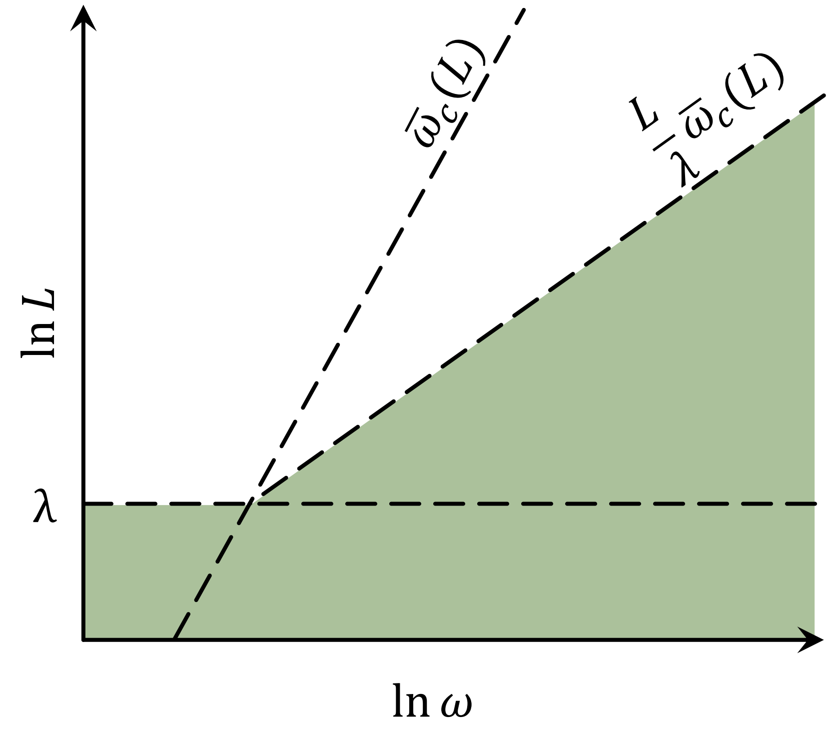

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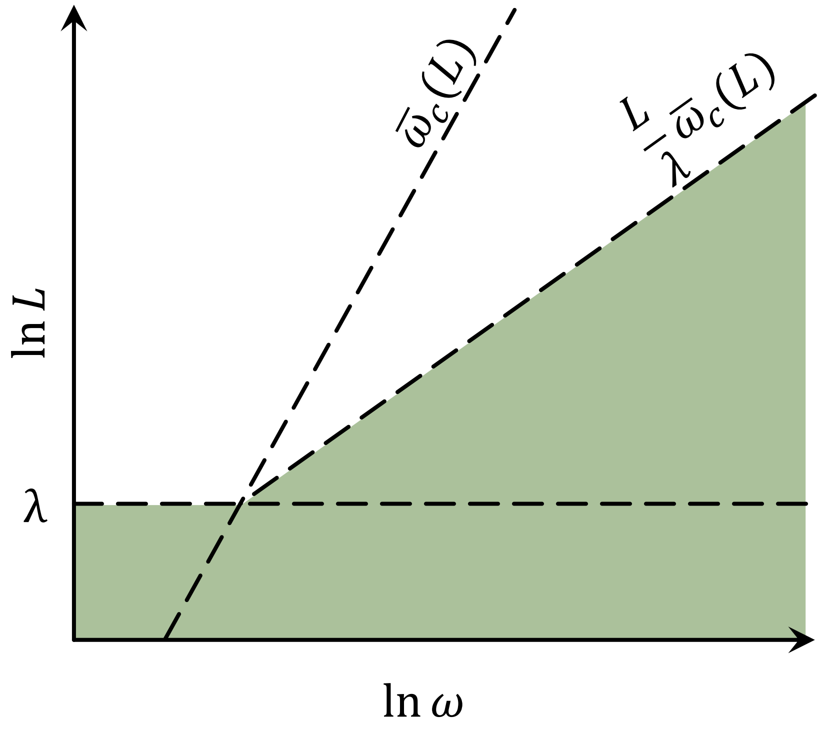

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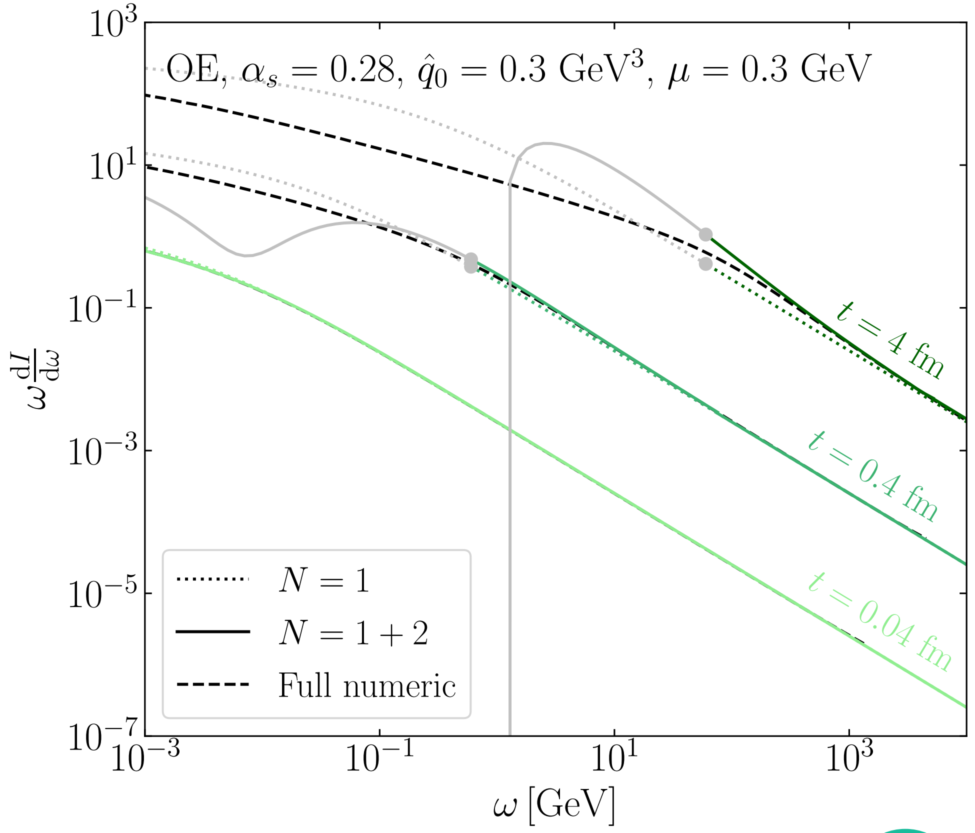

Resummed opacity expansion

[Wiedemann(2000)]
[Andres,Dominguez,Gonzales Martinez(2020)]
[Isaksen,Takacs,Tywoniuk(2022)]

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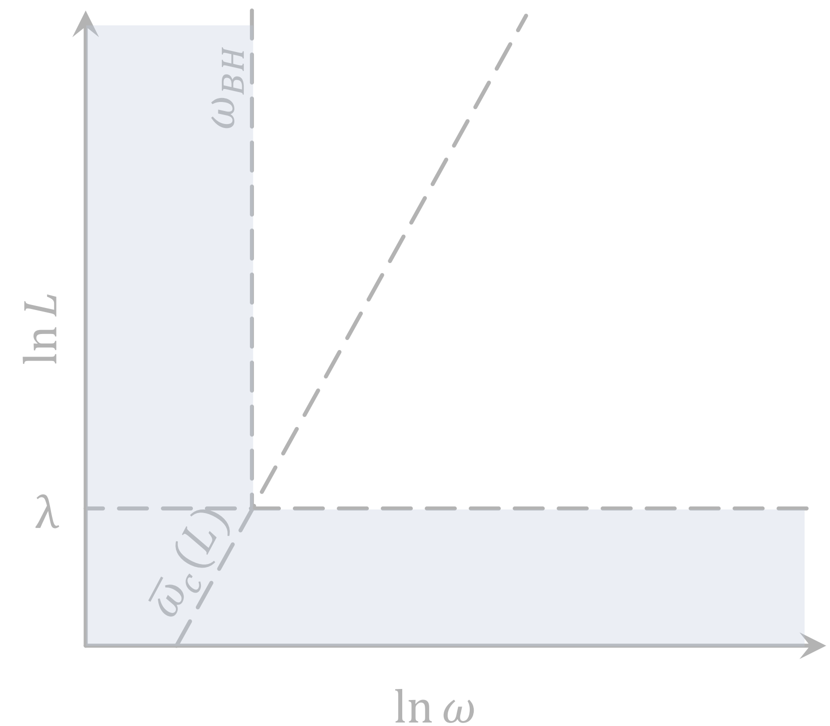

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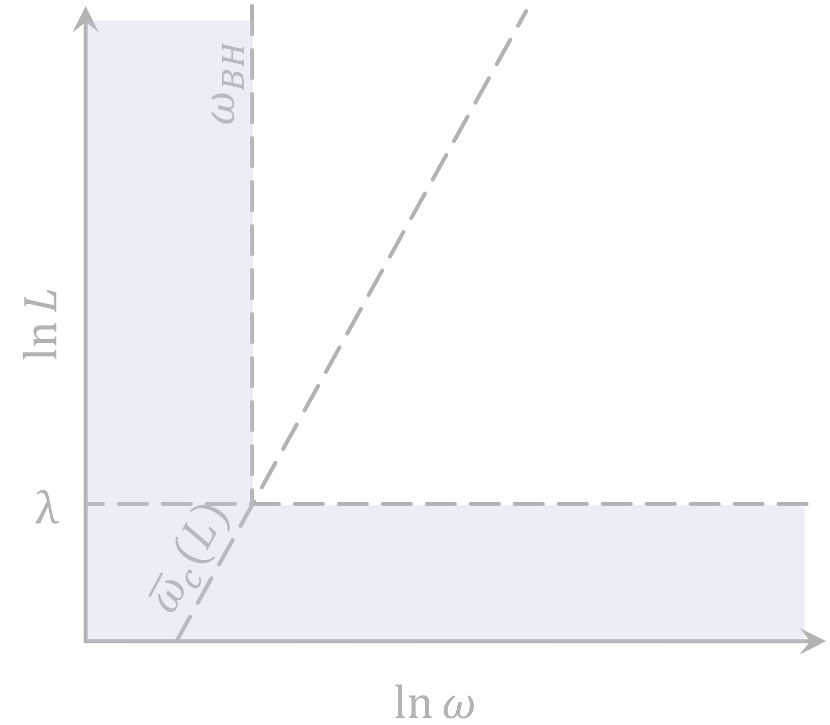

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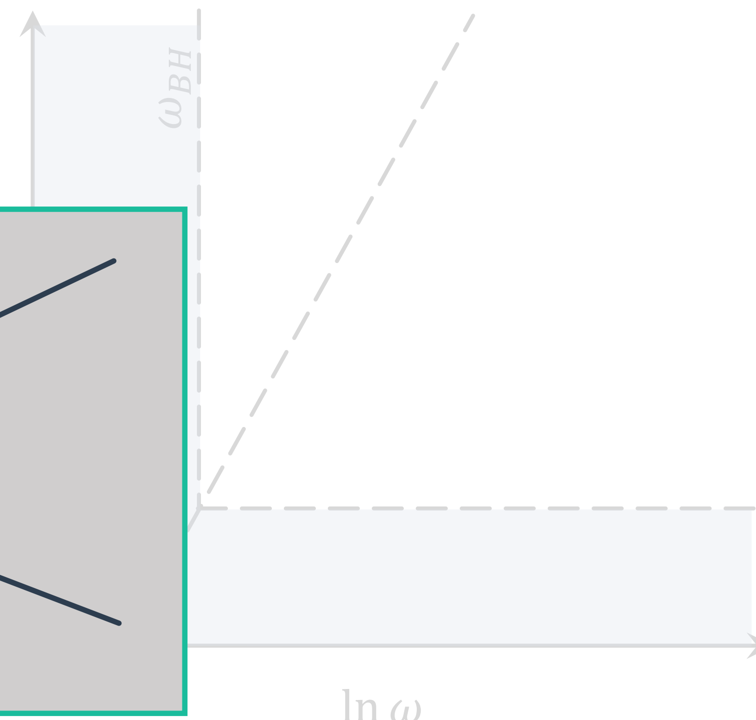

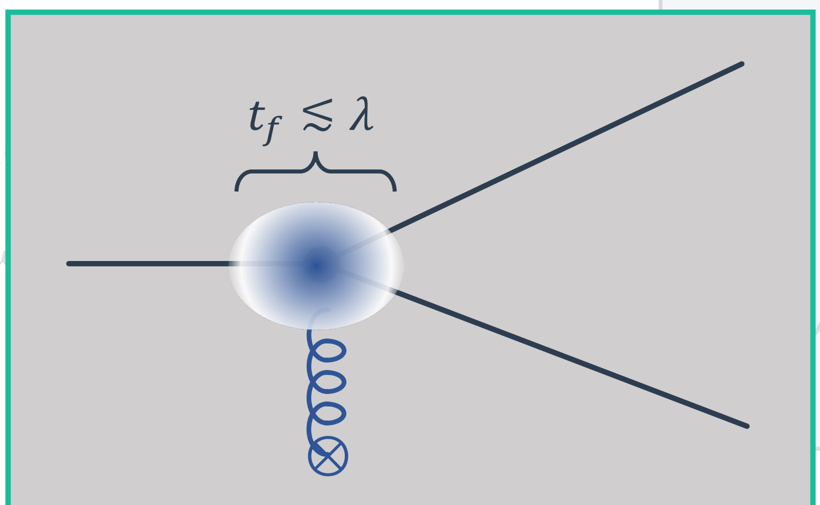

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Opacity expansion, $L \ll \lambda$

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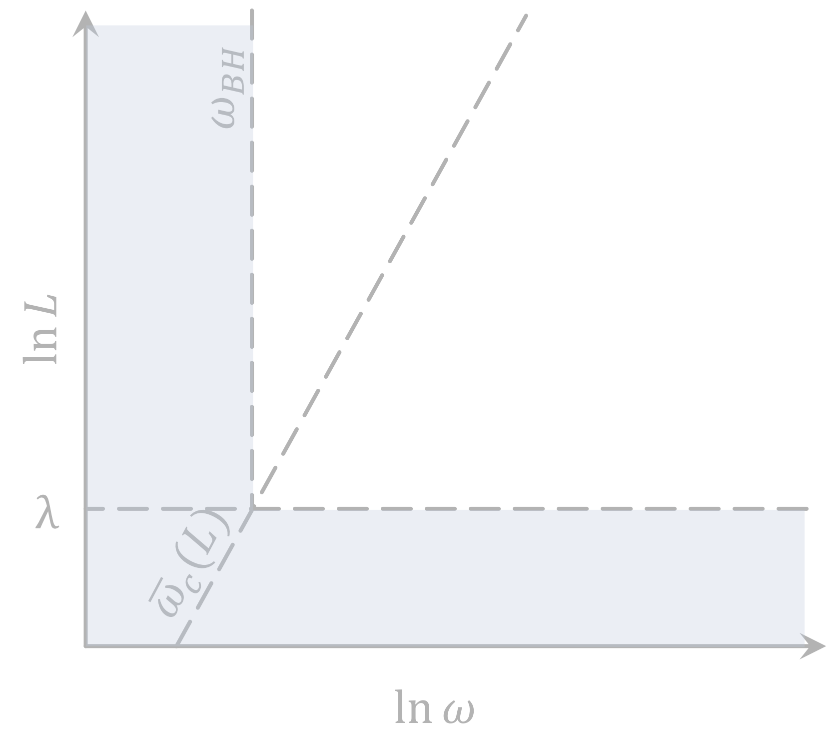

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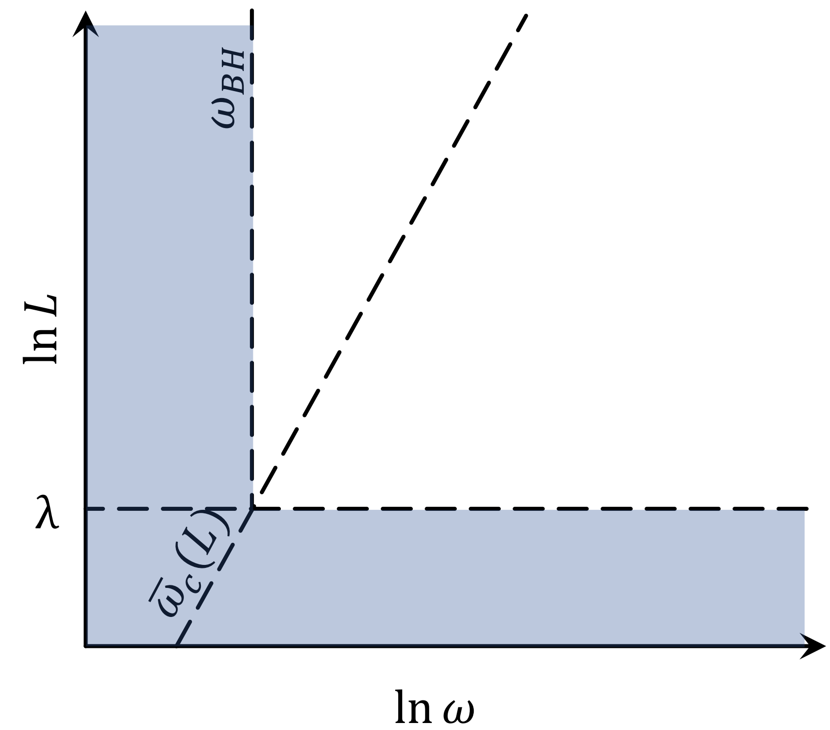

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[Isaksen,Takacs,Tywoniuk(2022)]

Exclusive number of **real** scatterings:

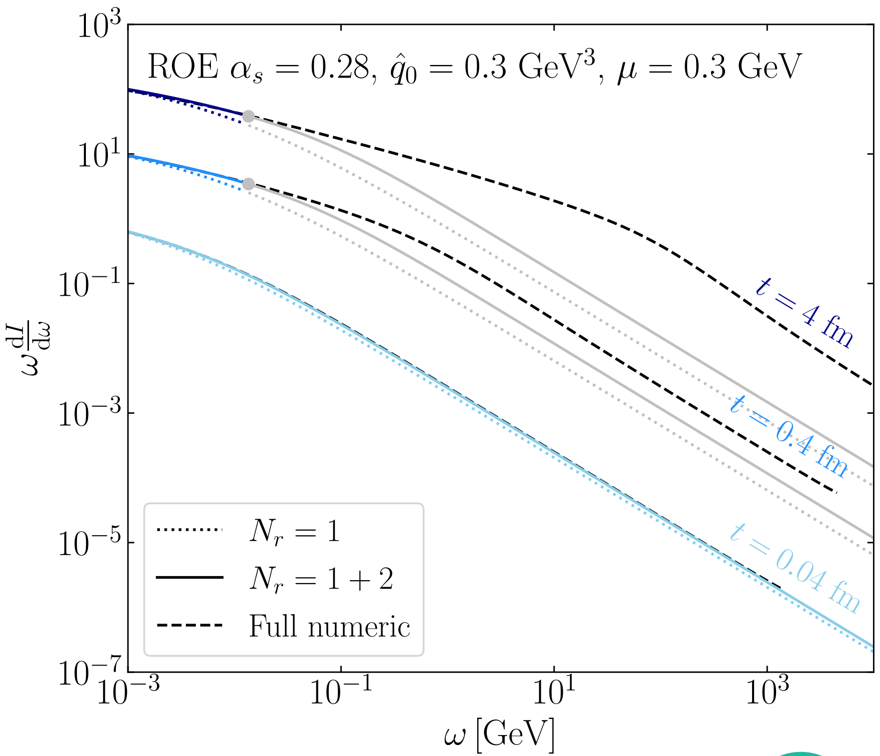
$$\mathcal{K}(\boldsymbol{p}_2, t_2; \boldsymbol{p}_1, t_1) = \delta(\boldsymbol{p}_2 - \boldsymbol{p}_1)\Delta(t, t_0)\, \mathcal{K}_0(\boldsymbol{p}_1, \mathrm{t}_2 - \mathrm{t}_1)$$

$$- \int_{t_1}^{t_2} ds \frac{\Delta(t, t_0)}{\Delta(s, t_0)} \int_{\boldsymbol{q}} \mathcal{K}_0(\boldsymbol{p}_2, \mathrm{t}_2 - s)\, C_{el}(\boldsymbol{q}, s) \mathcal{K}(\boldsymbol{p}_2 - \boldsymbol{q}, s; \boldsymbol{p}_1, t_1)$$

$$\Delta(t, t_0) = \exp\left[- \int_{t_0}^{t} ds \int_{\boldsymbol{q}} C_{el}(\boldsymbol{q})\right]$$

Opacity expansion, $L \ll \lambda$

$$\omega \frac{dI}{d\omega} = \begin{cases} \bar{\alpha} \frac{L}{\lambda} \sum_{n=1}^{\infty} g_n\left(\frac{\omega}{\omega_{BH}}\right), \omega \ll \omega_{BH} = \frac{\lambda}{L} \bar{\omega}_c = \frac{\mu^2 \lambda}{2} \end{cases}$$



Improved opacity expansion

[Mehtar-Tani,Tywoniuk,Barata,Soto-Ontoso]

Expansion around soft scatterings: $v(\boldsymbol{x},t) = v^{HO}(\boldsymbol{x},t) + \delta v(\boldsymbol{x},t)$

$$\mathcal{K}_{HO}(\boldsymbol{x}_2,t_2;\boldsymbol{x}_1,t_1) = \mathcal{K}_0(\boldsymbol{y},s;\boldsymbol{x}_1 t_1) - \int_{t_1}^{t_2} ds \int_{\boldsymbol{y}} \mathcal{K}_0(\boldsymbol{x}_2,t_2;\boldsymbol{y},s)\, v^{HO}(\boldsymbol{y},s)\mathcal{K}_{HO}(\boldsymbol{y},s;\boldsymbol{x}_1 t_1)$$

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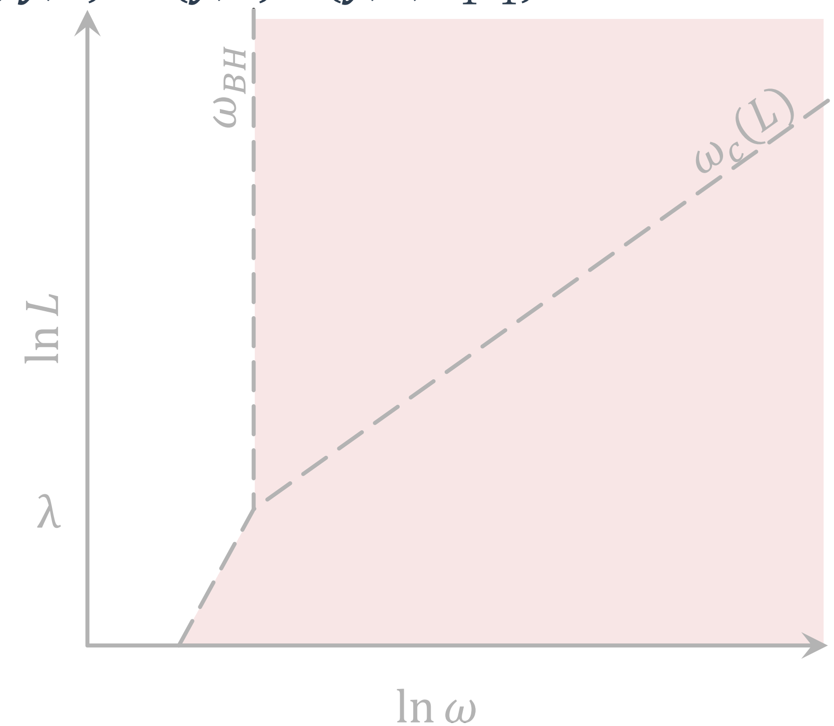

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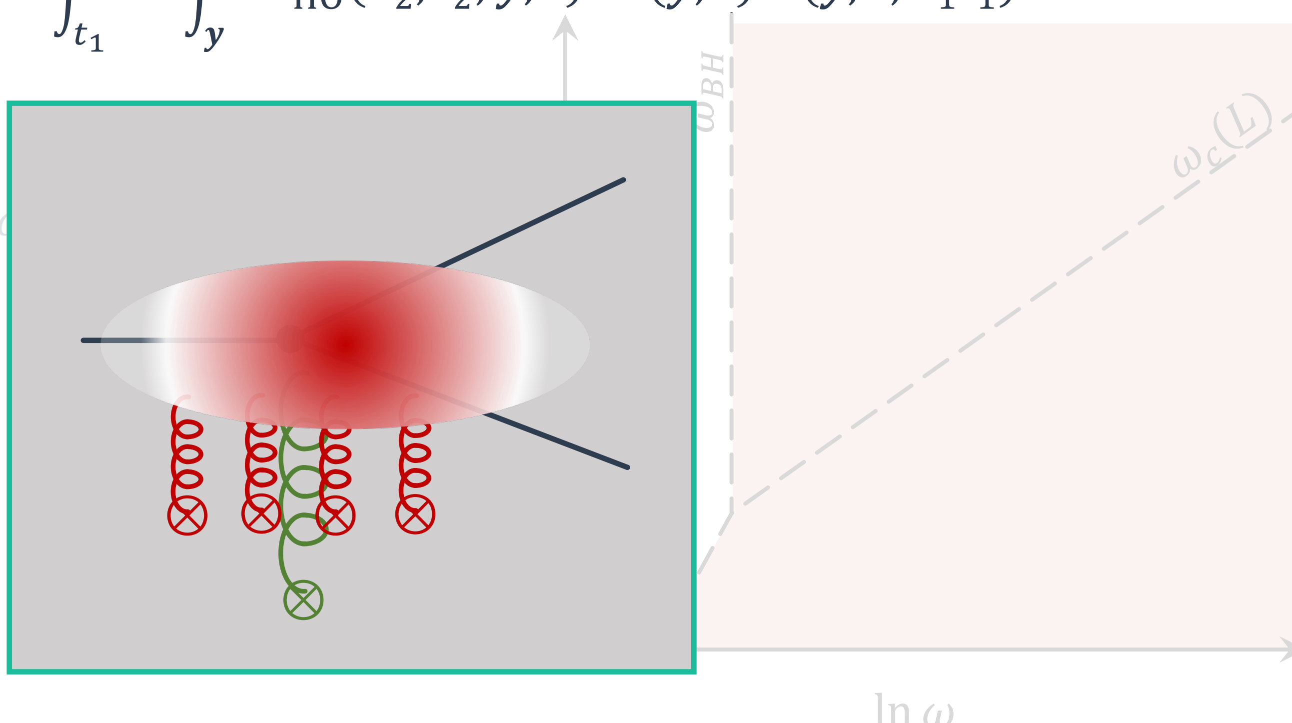

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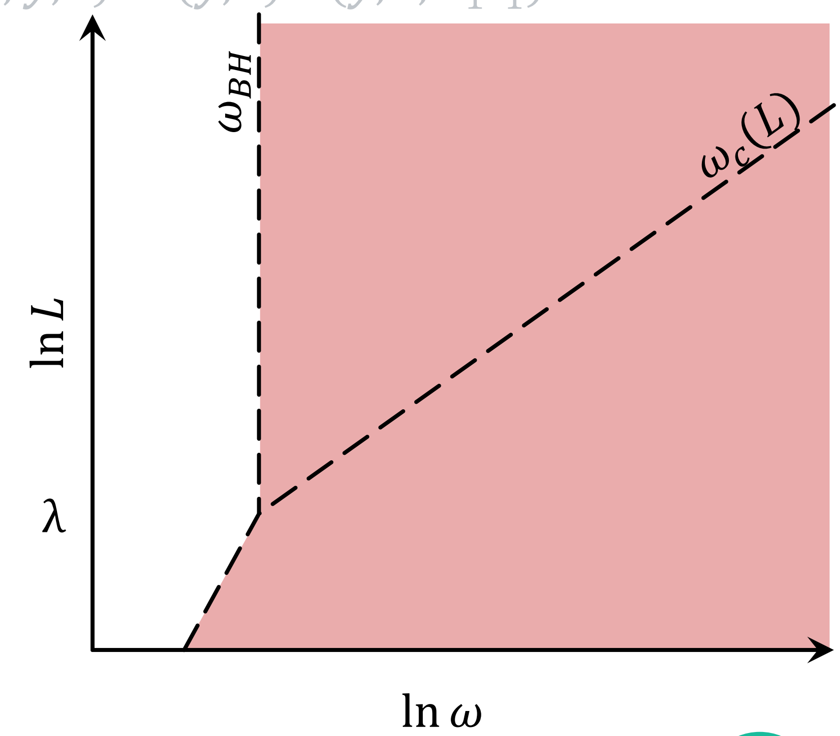

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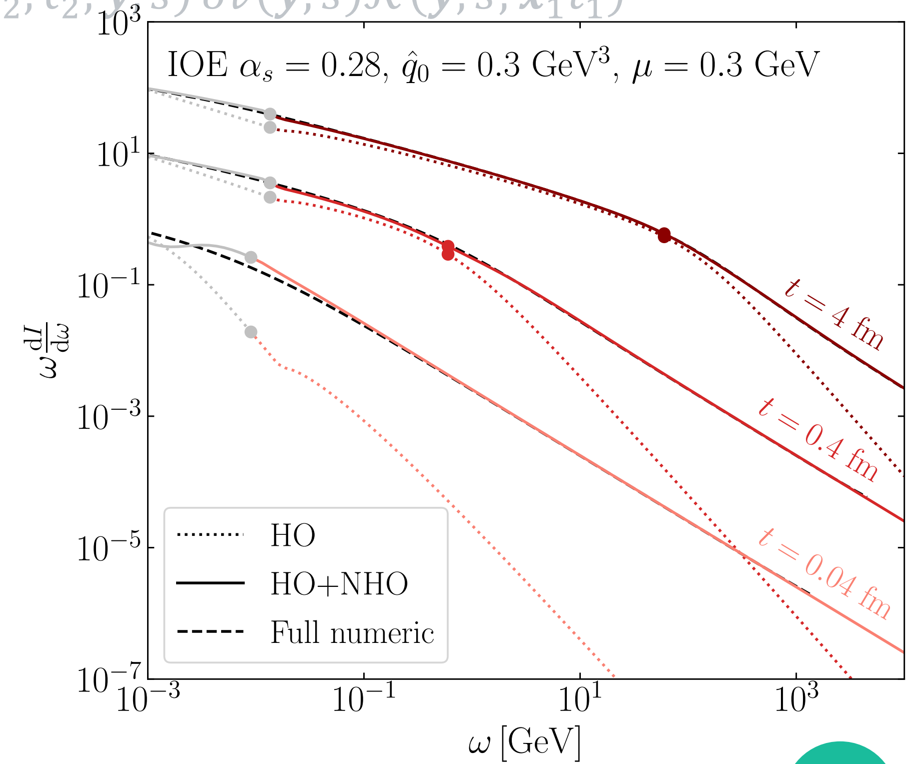
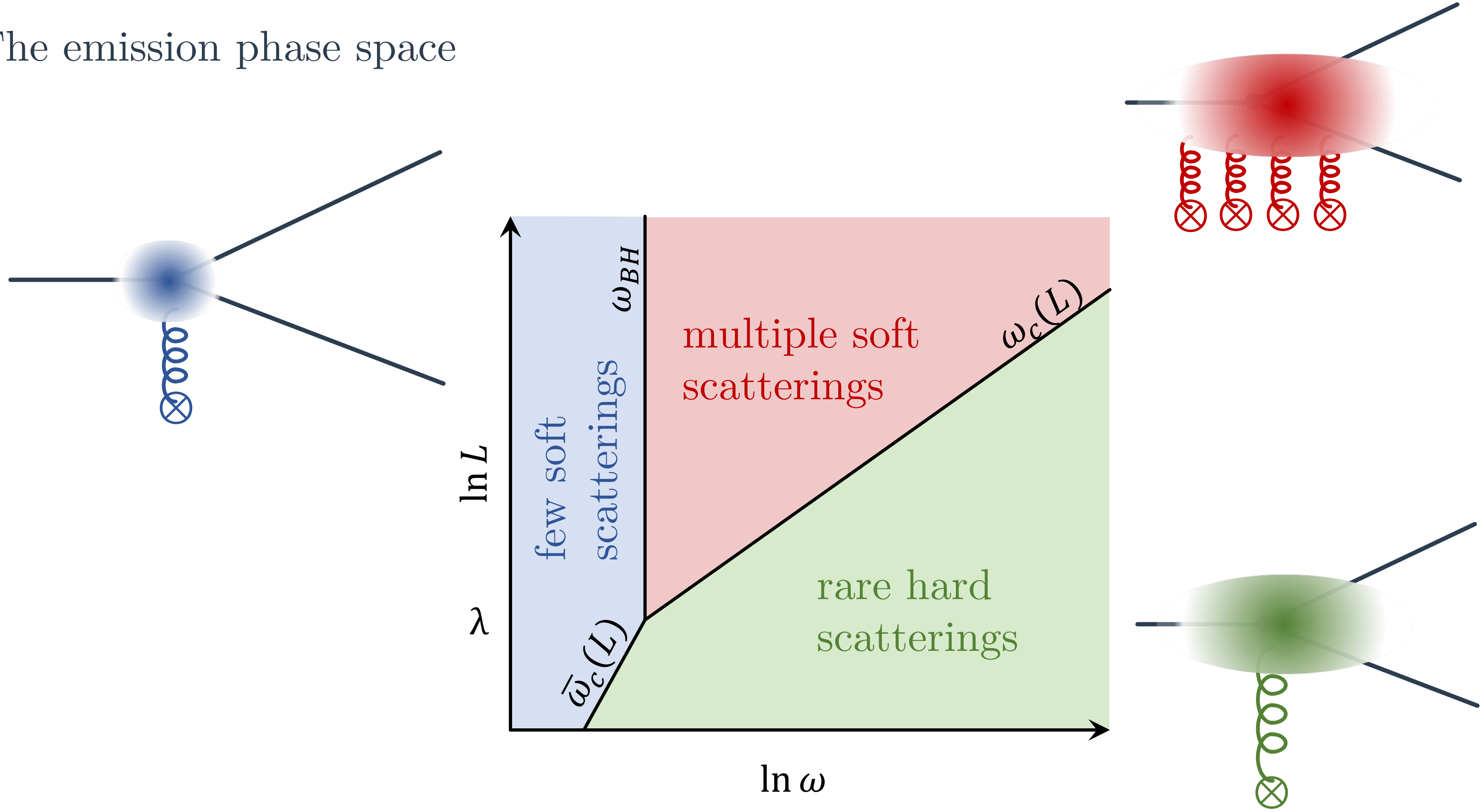


Summary of medium-induced emissions

The emission phase space



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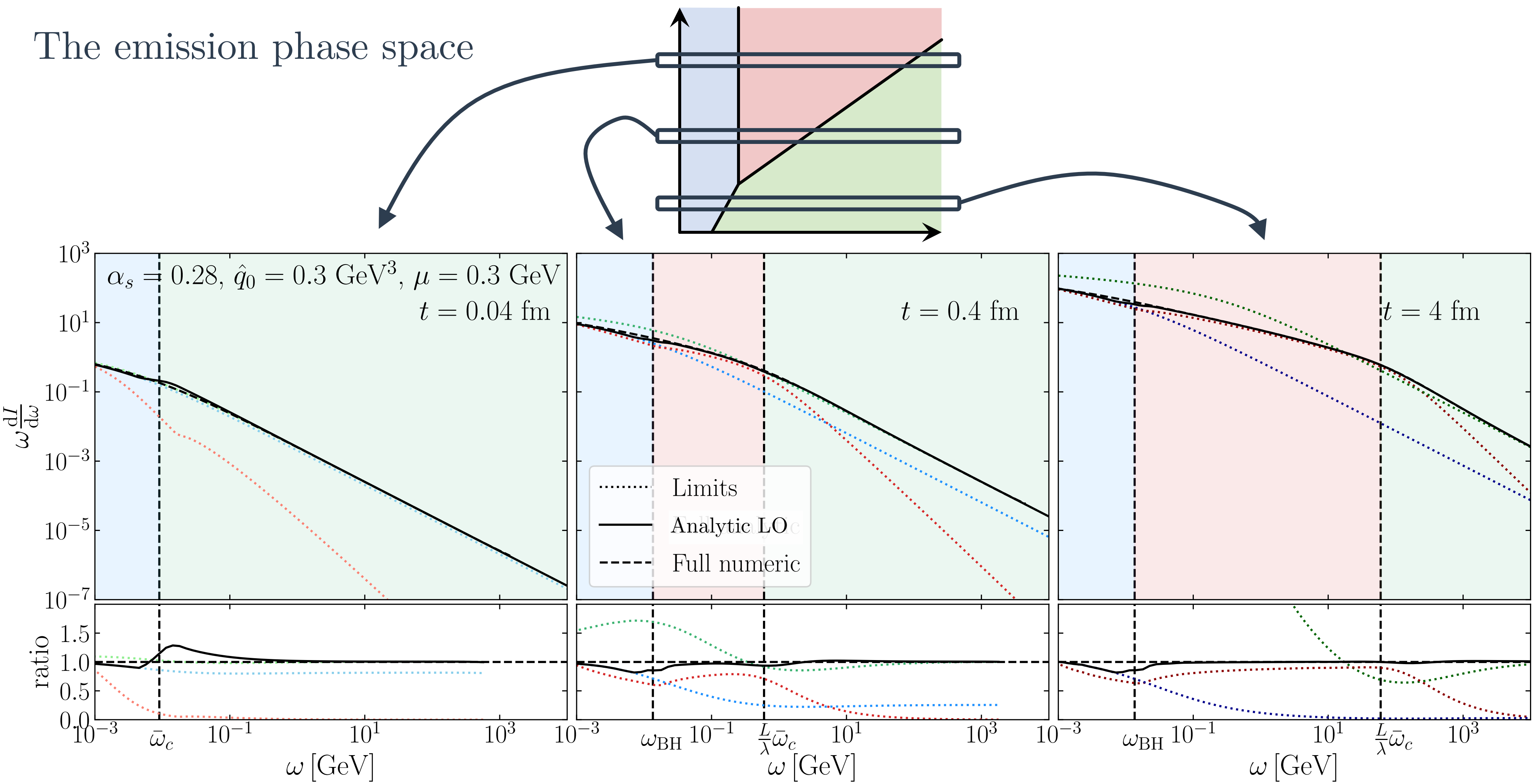
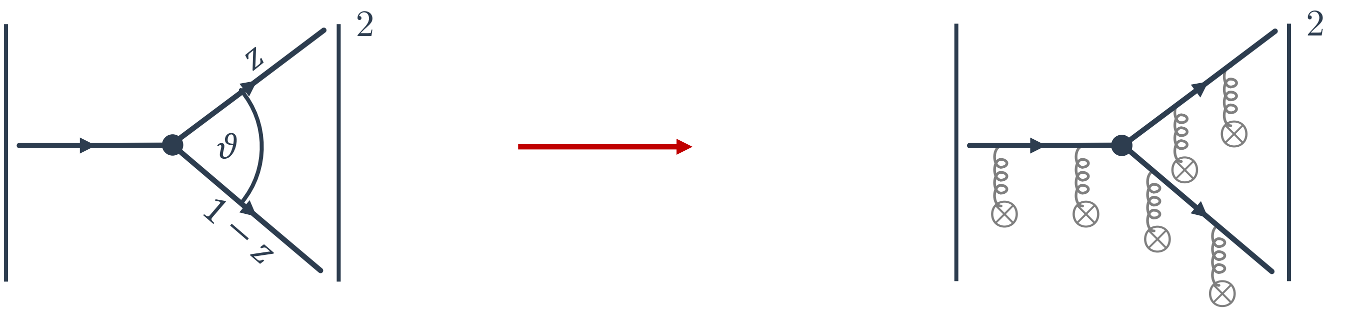


Defining accuracy for the medium-induced cascade

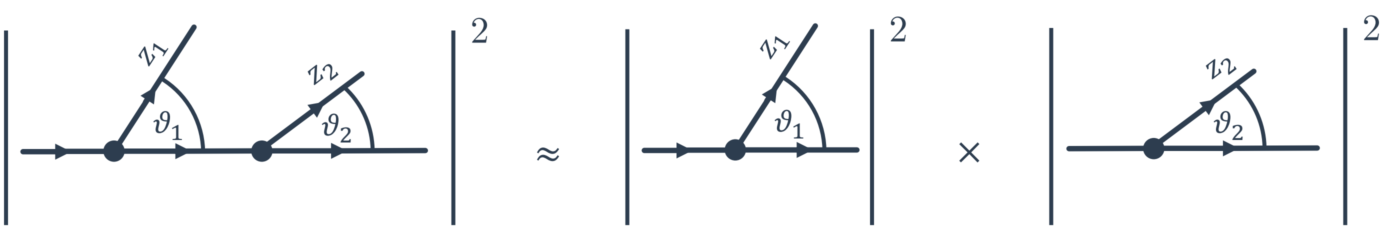
Multiple induced-emissions in the plasma

[BDMPS]
[Blaizot,Dominiguez,Iancu,Mehtar-Tani]

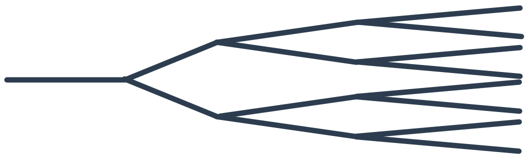
Soft and collinear divergence of QCD:



Factorization of strongly ordered emissions (also virtual terms):



sequential algorithm: parton shower



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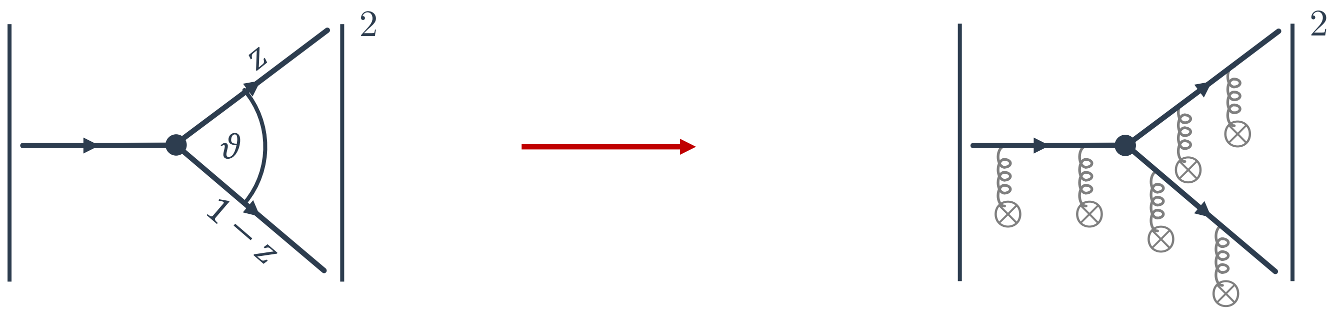

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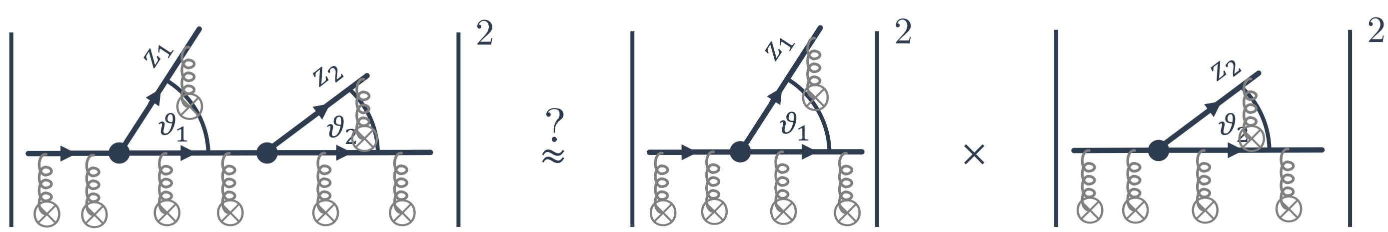

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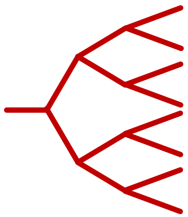
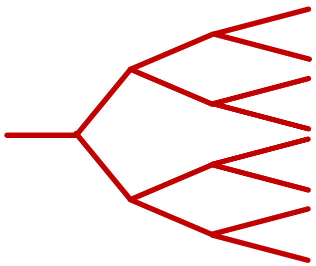


Application: medium-induced cascade

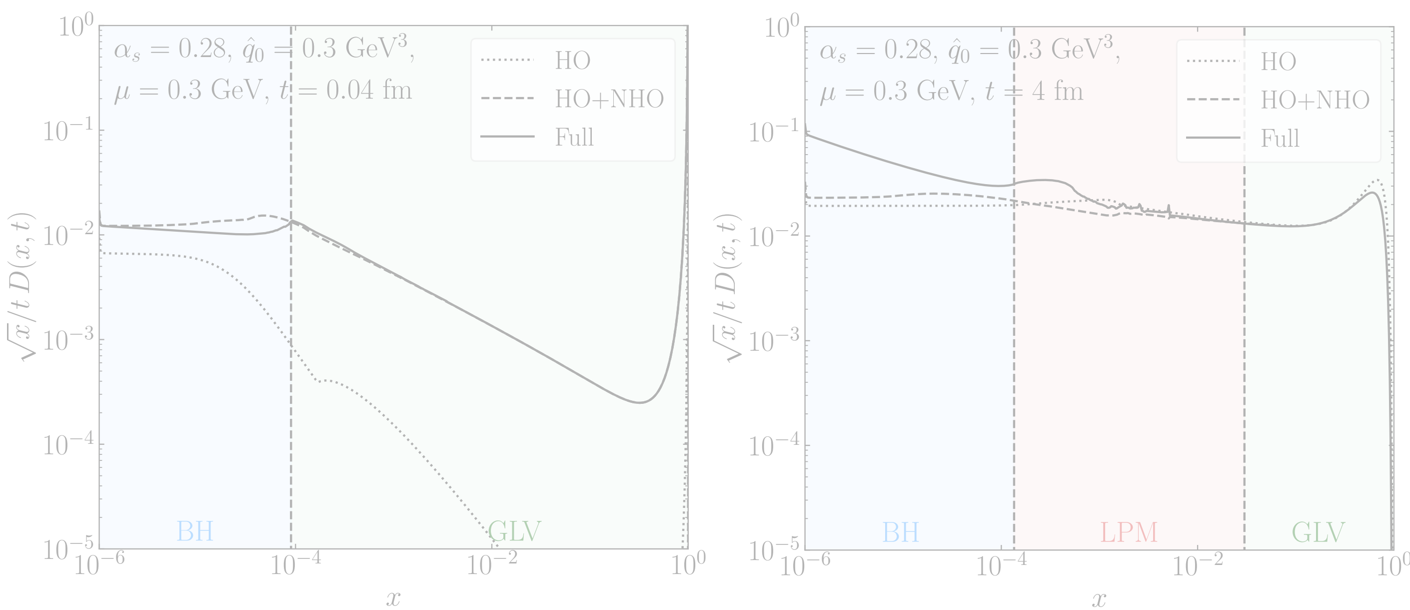
Medium-induced fragmentation function:



$$D(x,t) = x\frac{dN}{dx}$$

$$\partial_t D(x,t) = \int_x^1 dz \left.\frac{d^2 I}{dz dt}\right|_{\frac{x}{z}E} D\left(\frac{x}{z},t\right) - \int_0^1 dz\, z \left.\frac{d^2 I}{dz dt}\right|_{xE} D(x,t)$$

similar to DGLAP!



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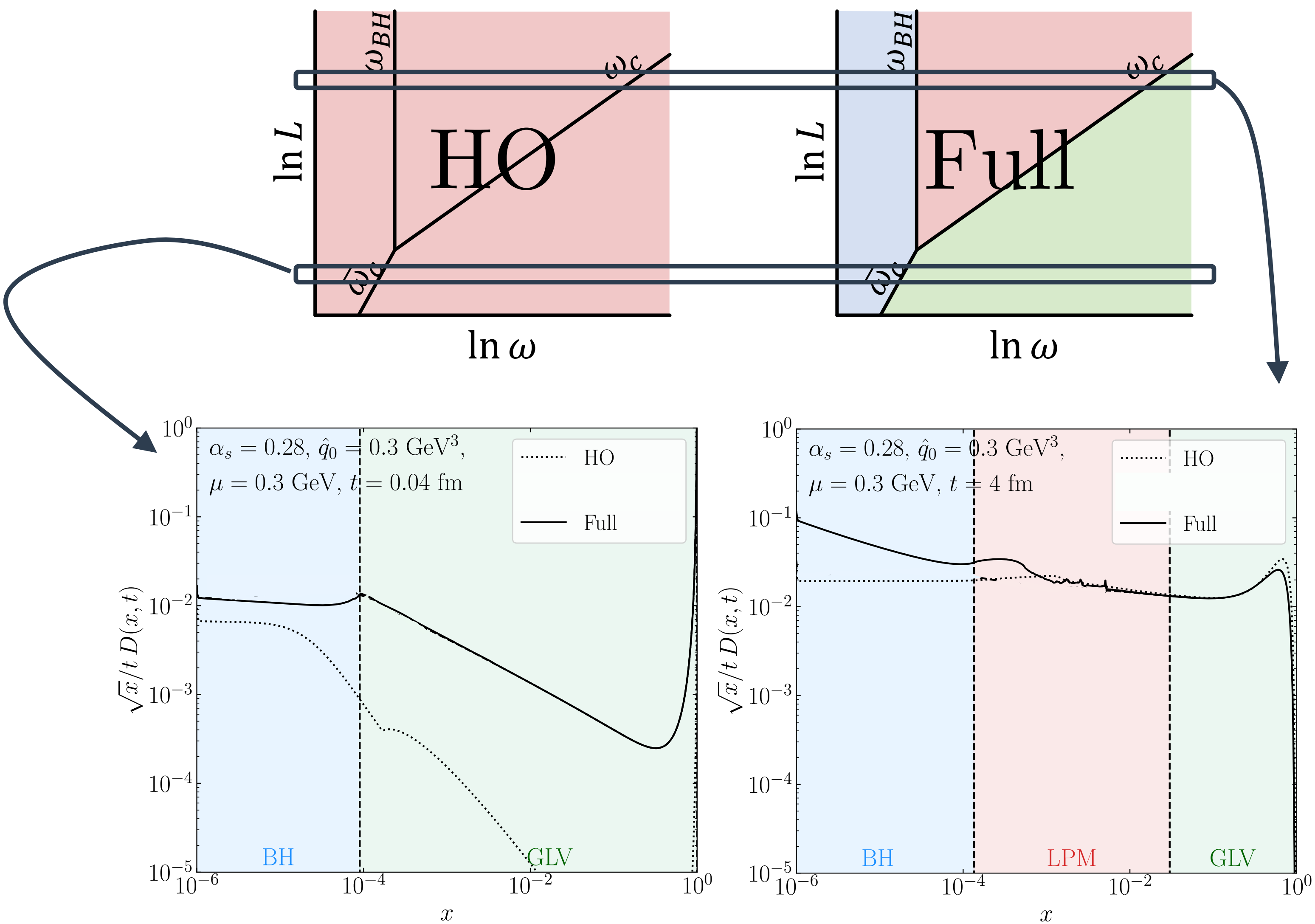

there are differences, but how to quantify?

Examples for the accuracy

Energy-loss probability:

$$\mathcal{P}(\varepsilon) = \sum_{k=0}^{\infty} \frac{1}{k!} \prod_{m=1}^{k} \int_0^L dt_m \int_0^{p_t} d\omega_m \frac{d^2 I}{d\omega_m dt_m} \left[\delta\left(\varepsilon - \sum_m \omega_m\right) - 1\right]$$

Quenching weight:

$$\mathcal{Q}(\nu) = \int_0^{\infty} d\varepsilon\, \mathcal{P}(\varepsilon) e^{-\nu\varepsilon} = \exp\left[\int_0^L dt \int_0^{p_t} d\omega \frac{d^2 I}{d\omega dt} \left(e^{-\nu\omega} - 1\right)\right]$$

Medium-induced fragmentation function:

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No emission probability (Sudakov):

$$\Delta(t, t_0) = \exp\left[-\int_{t_0}^{t} dt \int d\omega \frac{d^2 I}{d\omega dt}\right]$$

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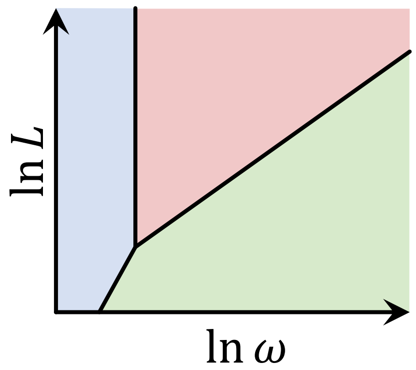
The accuracy of the medium-induced cascade

Hint of accuracy $\lambda \ll L$:

$$\omega L \frac{dI}{d\omega dL} = \begin{cases} \bar{\alpha}\frac{L}{\lambda}\sum_{n=0}^{\infty} f_n\left(\frac{\omega}{\omega_{BH}}\right), & \omega \ll \omega_{BH}, \\ \bar{\alpha}\sqrt{\frac{\omega_c}{\omega}}\sum_{n=0}^{\infty}\frac{1}{\ln Q_r^2(\omega)/\mu^2} g_n, & \omega_{BH} \ll \omega \ll \omega_c, \\ \bar{\alpha}\sum_{n=0}^{\infty}\left(\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}\right)^n h_n\left(\frac{\omega}{\bar{\omega}_c}\right), & \bar{\omega}_c \ll \omega \end{cases}$$



Accuracy using the Sudakov:

$$-\ln\Delta = \int_0^L dt \int d\omega \frac{dI}{d\omega dt}$$

$$\sim \bar{\alpha}\left(f_1\cdot\ln\frac{L}{\lambda} + f_2\right) + \bar{\alpha}\left(g_1\cdot\frac{L}{\lambda} + g_2\cdot\ln\frac{L}{\lambda} + g_3\right) + \bar{\alpha}\frac{L}{\lambda}\left(h_1\cdot\ln^2\frac{\omega_{BH}}{\omega} + h_2\cdot\ln\frac{\omega_{BH}}{\omega} + h_3\right) + \cdots$$

- g_1, h_1: soft limit, fixed coupling, infinite medium size
- f_1, g_2, h_2: finite medium, hard-collinear correction, running coupling, etc.
- See also: small medium, finite mass, etc.

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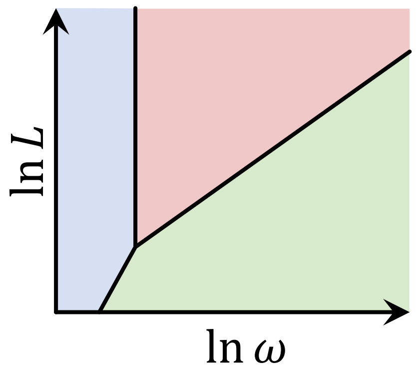

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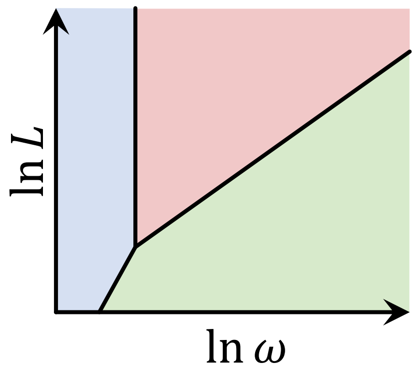

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Summary

- Understanding jet modification in medium

- Jets in “vacuum”:
  - all order α_s expansion
  - resummation and accuracy: large logarithms

- Medium induced cascades:
  - all “medium” order expansion
  - resummation and accuracy: large medium power correction and logarithms

- Interplay between vacuum and medium accuracy
  - Factorization of vacuum and medium emissions
  - Low-pT jets vs. small medium, heavy-quark jets

Thank you for the attention!