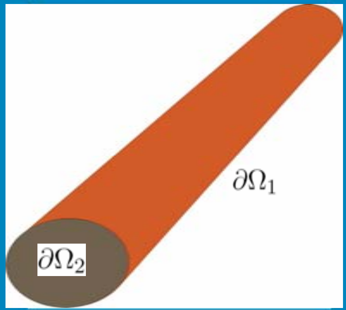
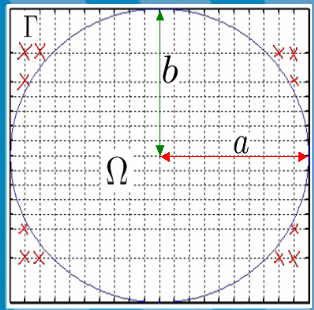
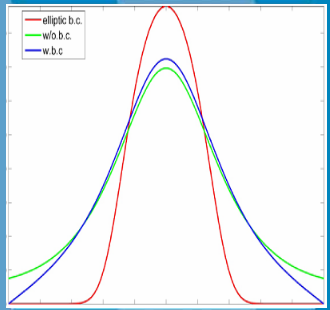
FAKULTÄT FÜR INFORMATIK
UND ELEKTROTECHNIK
UNIVERSITÄT ROSTOCK







SIMULATION PROGRESS AND PLANS AT ROSTOCK/DESY

Aleksandar Markovic
Gisela Pöplau

ECL2, CERN, March 1, 2007

Overview

Algorithms for 3D space charge calculations

- Multigrid based Poisson solver
- Software package MOEVE
- Simulations with GPT and ASTRA

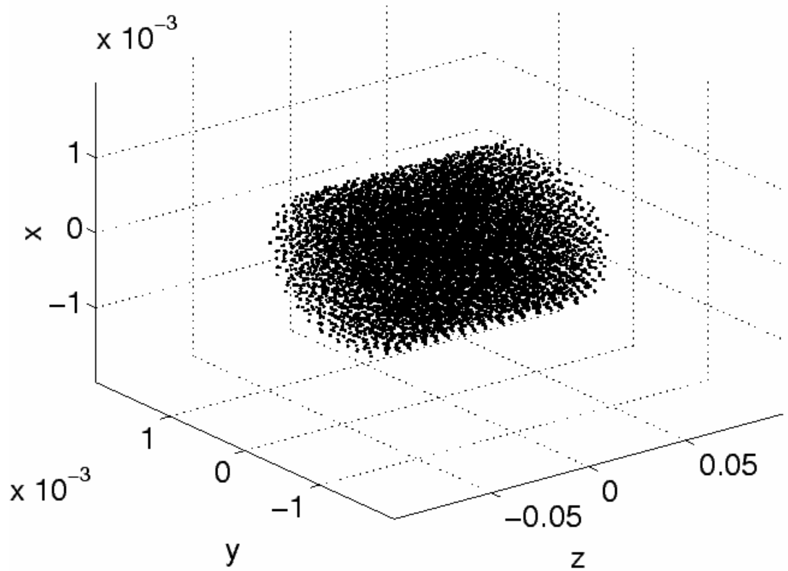
Particle tracking and e-cloud simulations

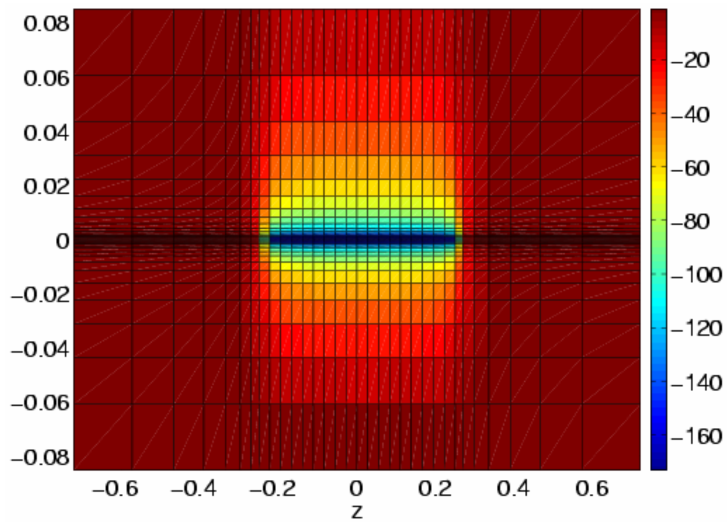
- Poisson solver for beam pipes with elliptical cross section
- Implementation of the tracking algorithm
- Plans for e-cloud simulations

Algorithms for 3D Space Charge Calculations

Development of a 3D Poisson solver
(Gisela, since 1999)

- Particle mesh method
- Poisson solver based on multigrid
- For non-equidistant tensor product meshes
- Software package MOEVE 2.0
- Part of the tracking code GPT 2.7 (General Particle Tracer, Pulsar Physics)
- Part of ASTRA (test phase)
- Plan: (self-) adaptive multigrid





Features of MOEVE

$$-\Delta\varphi = \frac{\rho}{\varepsilon_0} \quad \text{in } \Omega \subset \mathbb{R}^3 \quad \xrightarrow{\text{Finite difference discretization}} \quad Ax = b$$

iterative solvers:

- multigrid (MG)
- multigrid pre-conditioned conjugate gradients
- pre-conditioned conjugate gradients (Jacobi) (PCG)
- successive over relaxation (SOR)

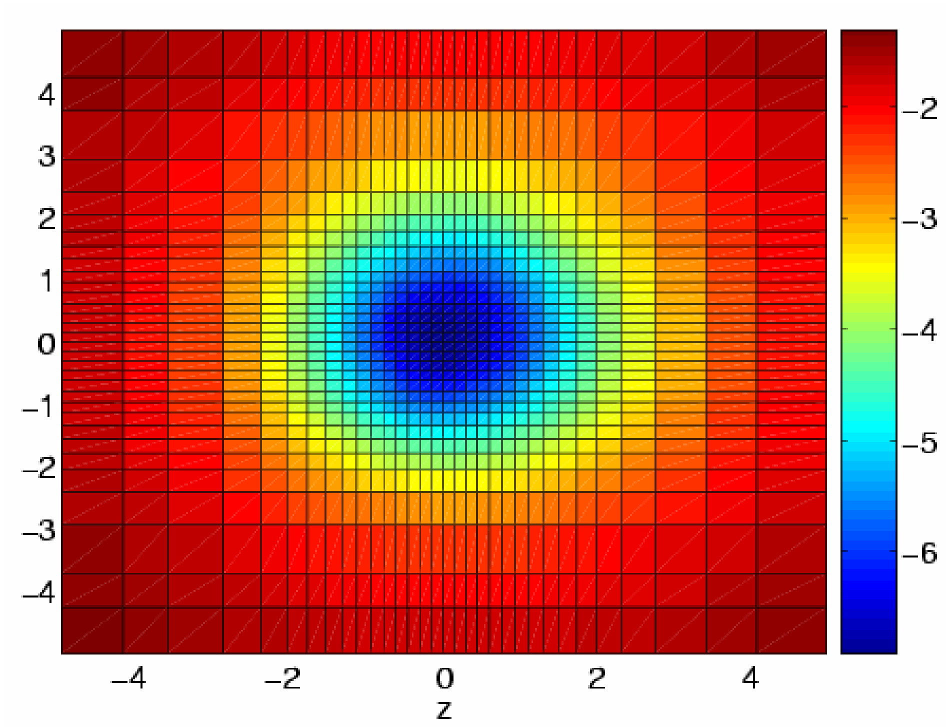
- step size: non-equidistant
- numerical effort: $O(M)$ — M: total number of grid points

boundary conditions

- free space boundary
- perfect conducting rectangular box
- perfect conducting beam pipe with elliptical cross section

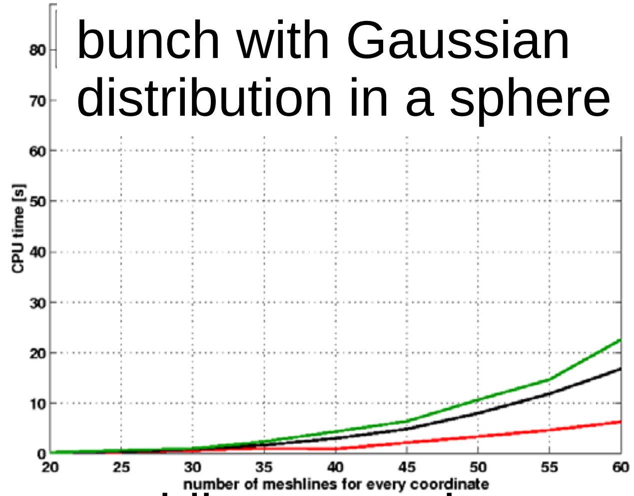
Why Multigrid?

- Other Poisson solvers are much easier to implement
- They slow down considerably on non-equidistant meshes

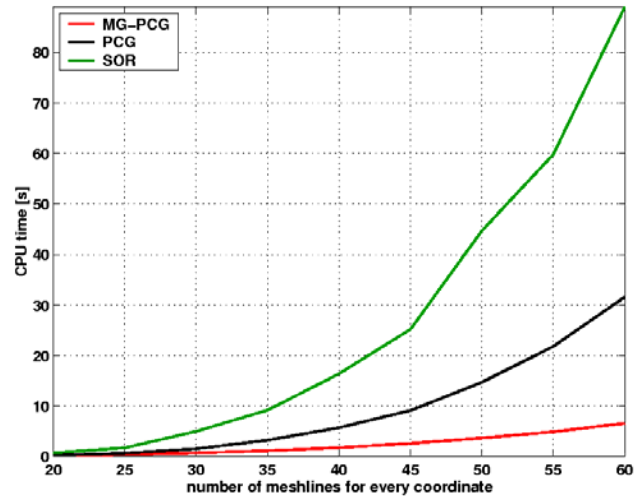


Discretization of a spherical bunch

bunch with Gaussian distribution in a sphere



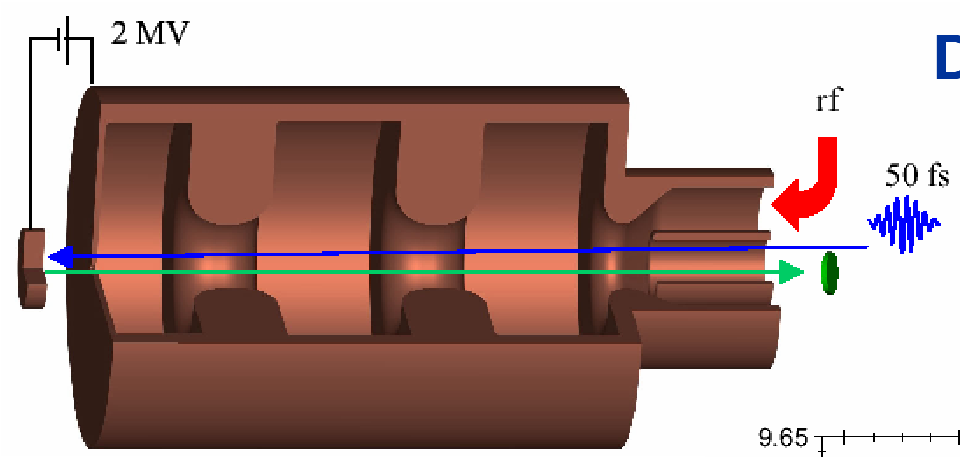
equidistant mesh



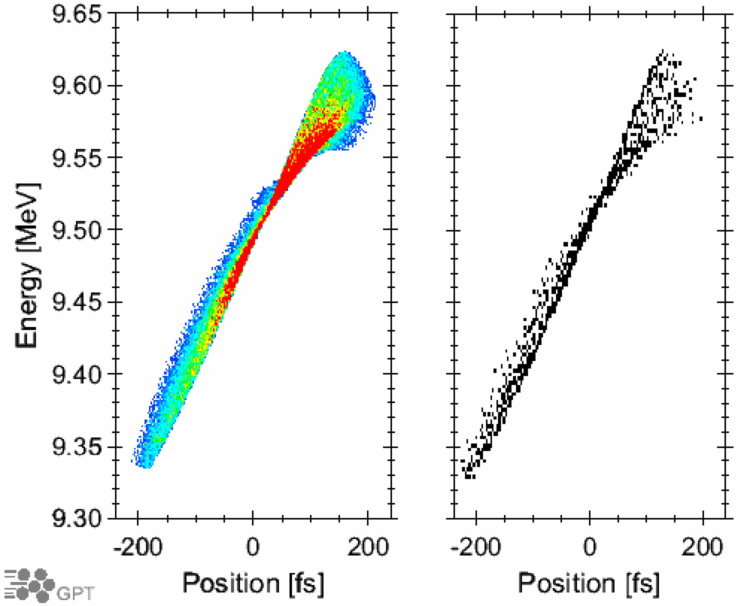
non-equidistant mesh

COMPUMAG 2003

DC/RF gun @ TU Eindhoven



Graphics: Pulsar Physics



3D:
100,000 particles

2D:
1,000 particles

Simulations with ASTRA

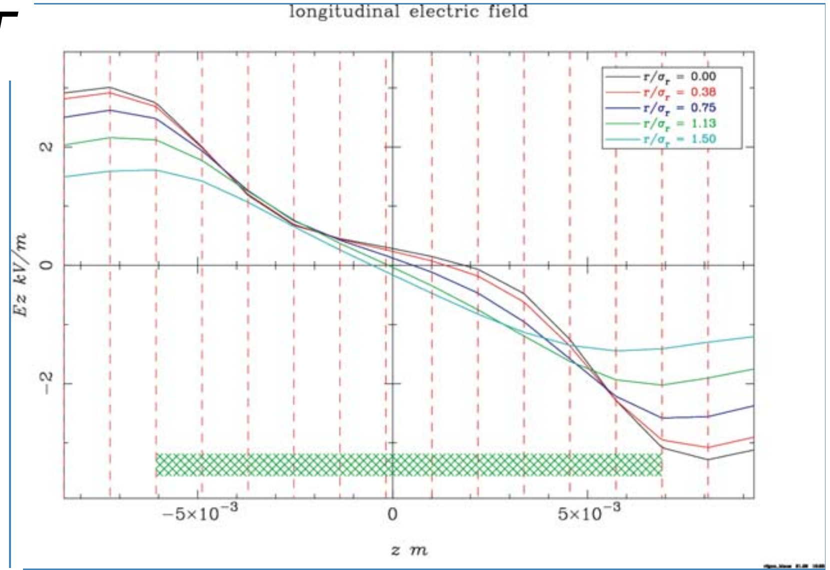
EPAC 2006

- Gaussian particle distribution:

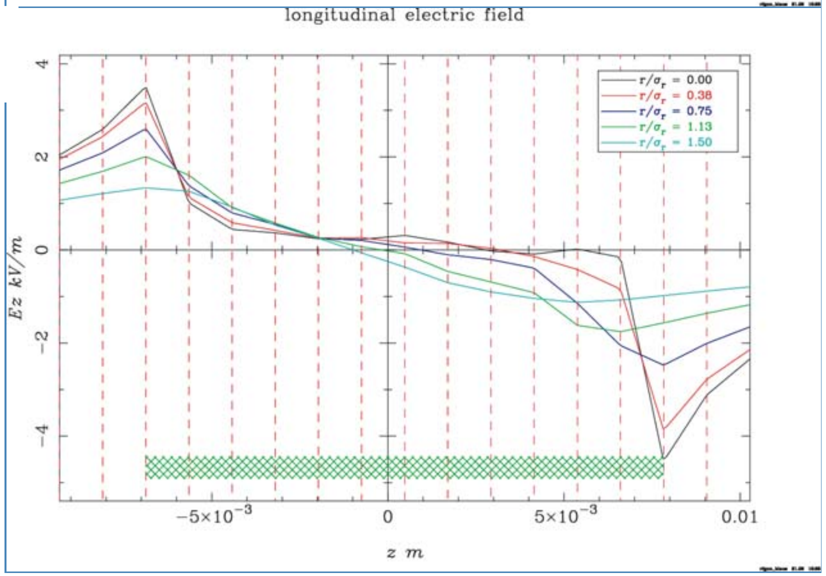
  $\sigma_x = \sigma_y = 0.75$ mm , $\sigma_z = 1.0$ mm

- 10,000 macro particles
- charge: -1 nC
- energy: 2 MeV
- tracking distance: 3 m
- quadrupol at z=1.2 m
- number of steps (Poisson solver): N=32

FFT



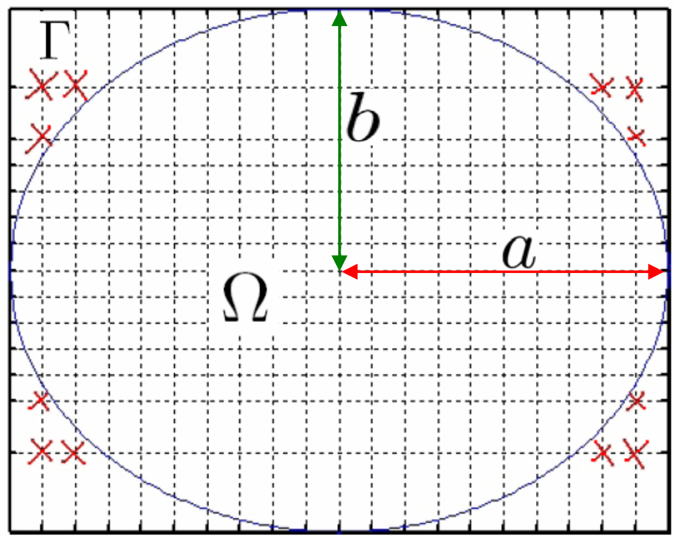
MG



Beam Pipes with Elliptical Cross Section

Development of a Poisson solver for beam pipes and development of a tracking procedure (Aleksandar, since 2005)

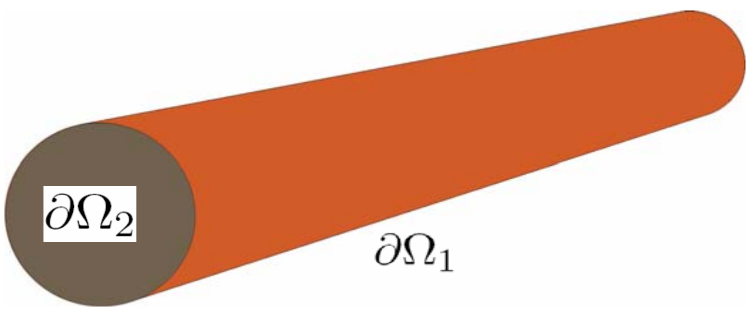
$$\Gamma = [-a, a] \times [-b, b] \times [-c, c]$$



- 3D Space charge fields of bunches in a beam pipe of elliptical cross section, part of MOEVE 2.0
  - iterative solvers: BiCG, BiCGSTAB
  - step size: non-equidistant

Poisson equation:

$$\begin{aligned} -\Delta\varphi &= \frac{\varrho}{\varepsilon_0} && \text{in } \Omega \subset \mathbb{R}^3, \\ \varphi &= 0 && \text{on } \partial\Omega_1, \\ \frac{\partial\varphi}{\partial n} + \frac{1}{r}\varphi &= 0 && \text{on } \partial\Omega_2, \end{aligned}$$

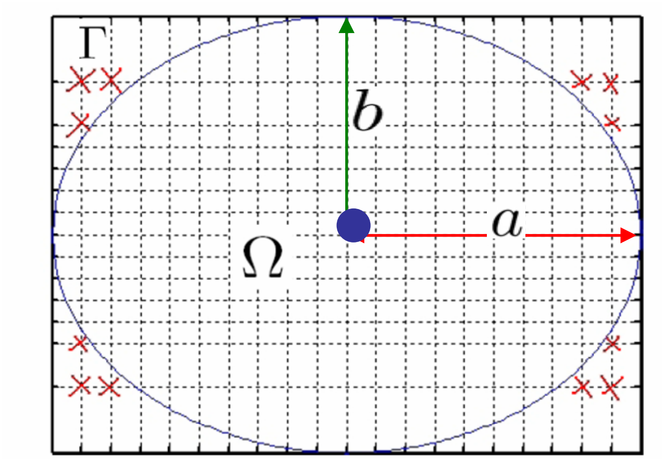


Space Charge Simulation Results

Motivated from the paper "*Simulation of transverse single bunch instabilities and emittance growth caused by electron cloud in LHC and SPS*".

E. *Benedetto, D. Schulte, F. Zimmermann, CERN, Switzerland, G. Rumolo, GSI, Germany*

- construction of a fast cosine transformation for the simulation of conducting boundaries



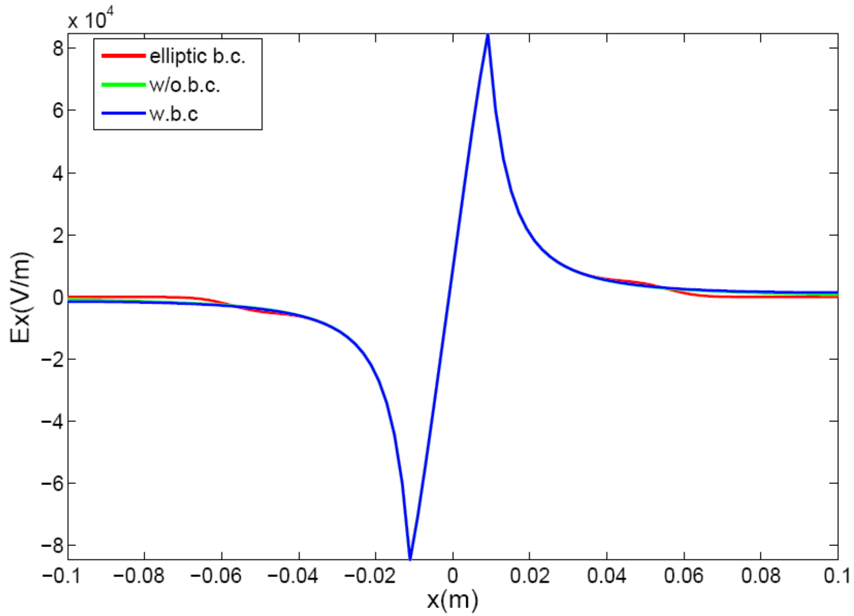
Numerical example:

- spherical bunch
- *r<<a,b*
  (*r*: radius of the bunch, *a*, *b:* the half axis of the elliptical beam pipe)
- uniformly distributed charge of **1nC**
- bunch is located the centre of the beam pipe

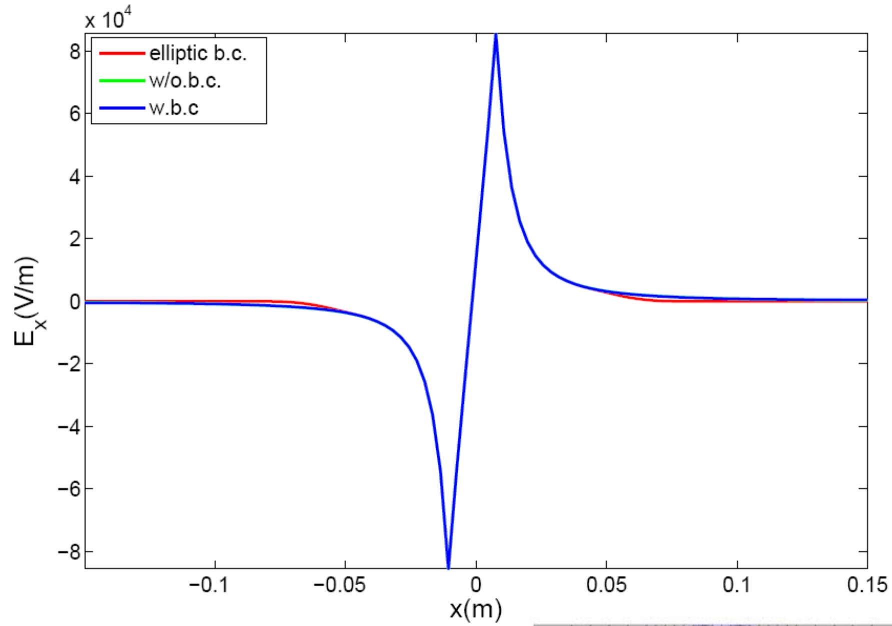
Space Charge Simulation Results

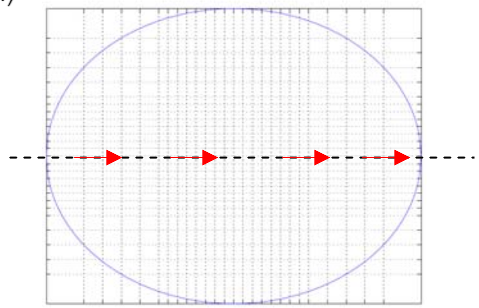
Electric field E_x along the x-axis of a

square
a=*b*



rectangular box
a = 1.5*b*



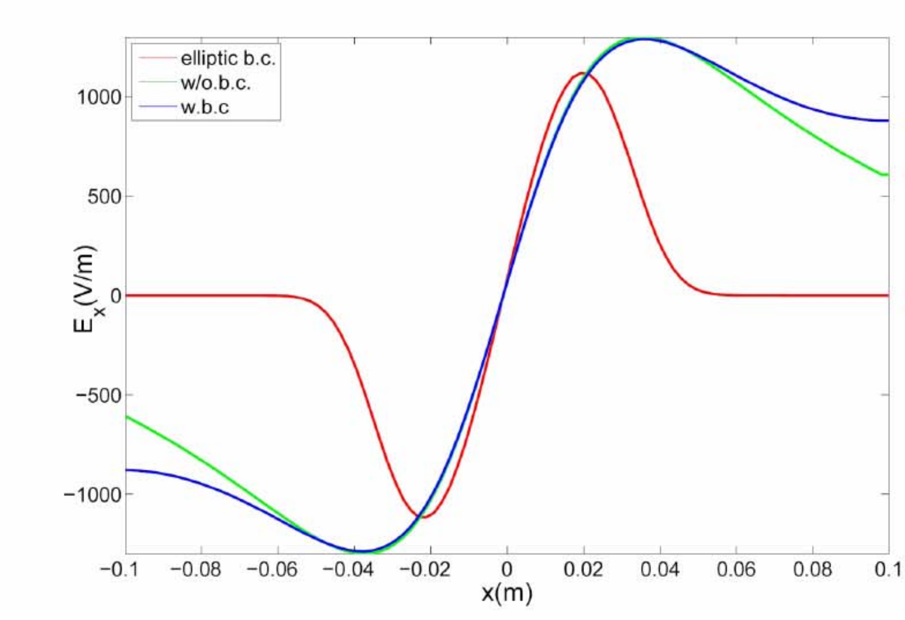


elliptic b.c.	beam pipe with elliptical cross section
w/o.b.c.	open b. c. on a rectangular
w.b.c.	conducting b.c. on a rectangular

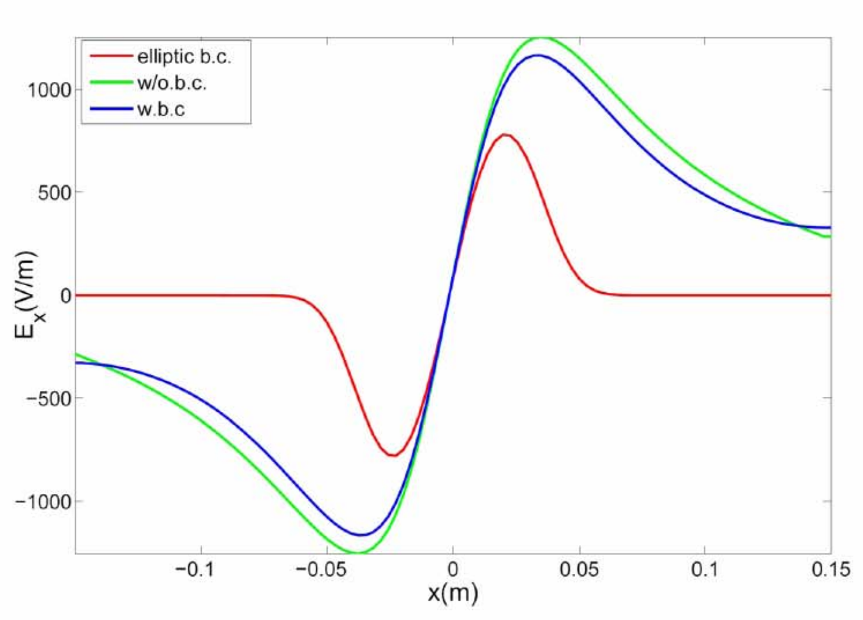
Space Charge Simulation Results

Electric field E_x along $y = \pm b/2$ of a

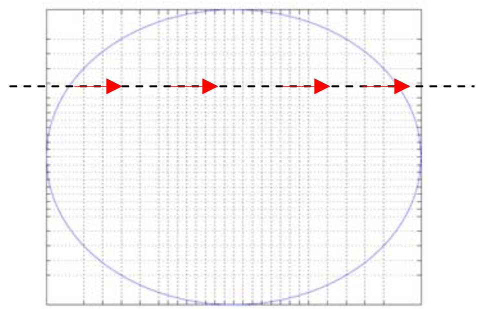
square *a*=*b*



rectangular box *a* = 1.5*b*



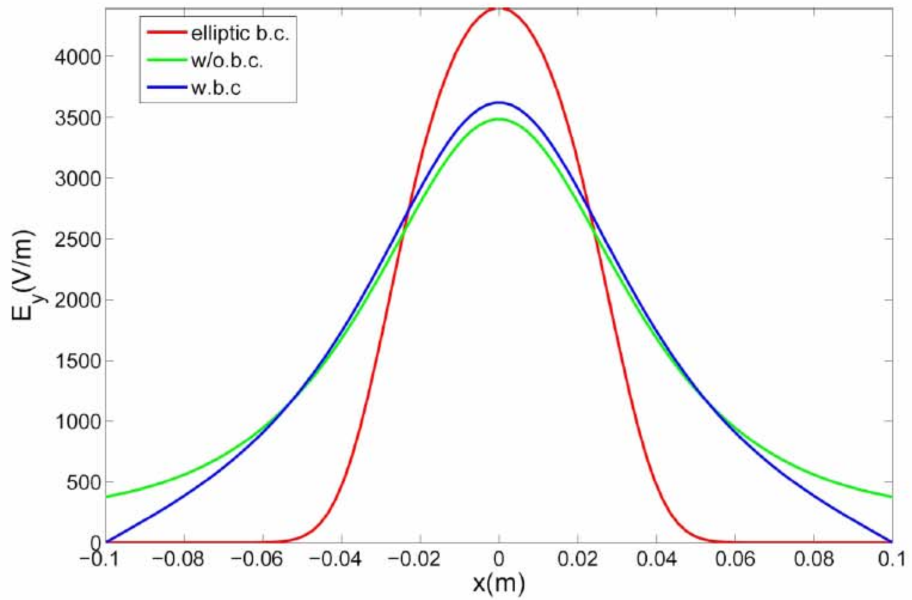
elliptic b.c.	beam pipe with elliptical cross section
w/o.b.c.	open b. c. on a rectangular
w.b.c.	conducting b.c. on a rectangular

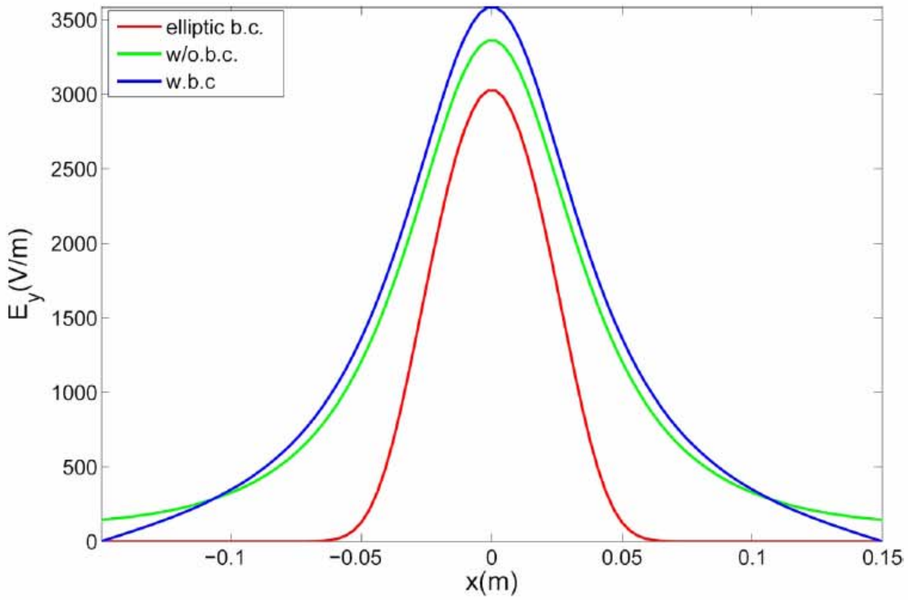


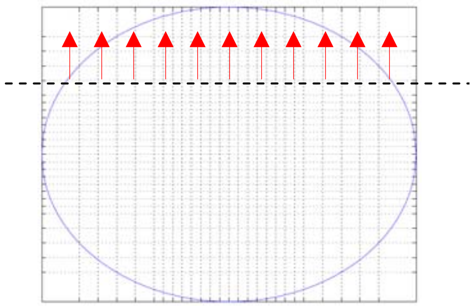
Space Charge Simulation Results

Electric field E_y along $y = \pm b/2$ of a

square	rectangular box
$a=b$	$a = 1.5b$







elliptic b.c.	beam pipe with elliptical cross section
w/o.b.c.	open b. c. on a rectangular
w.b.c.	conducting b.c. on a rectangular

Particle Tracking

Tracking Algorithm

- initial macro particle distribution
- deposit charges on the mesh nodes
- space charge computation in the center of mass system (3D Poisson solvers from MOEVE 2.0)
- interpolation of the fields for each macro particle in the laboratory frame
- time integration of the Newton-Lorentz equation for each macro particle (leap frog scheme)

Time Integration of Particle Equations *

$$p(t) = \gamma(t) v(t) \quad m = m_0 \gamma \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- relativistic generalisation of the Newton-Lorentz equation:

$$m_0 \frac{d\vec{p}}{dt} = q\vec{E} + q\left(\frac{\vec{p}}{\gamma} \times \vec{B}\right)$$

$$\frac{\vec{p}^{\,n+\frac{1}{2}} - \vec{p}^{\,n-\frac{1}{2}}}{\Delta t} = \frac{q}{m_0}\left[\vec{E}^{\,n} + \frac{\vec{p}^{\,n+\frac{1}{2}} - \vec{p}^{\,n-\frac{1}{2}}}{2\gamma^n} \times \vec{B}^{\,n}\right]$$

- electric impulse halves:

$$\vec{p}^{\,n+\frac{1}{2}} = \vec{p}^{\,+} + \frac{q\vec{E}^{\,n}\Delta t}{2m_0}$$

$$\vec{p}^{\,n-\frac{1}{2}} = \vec{p}^{\,-} - \frac{q\vec{E}^{\,n}\Delta t}{2m_0}$$

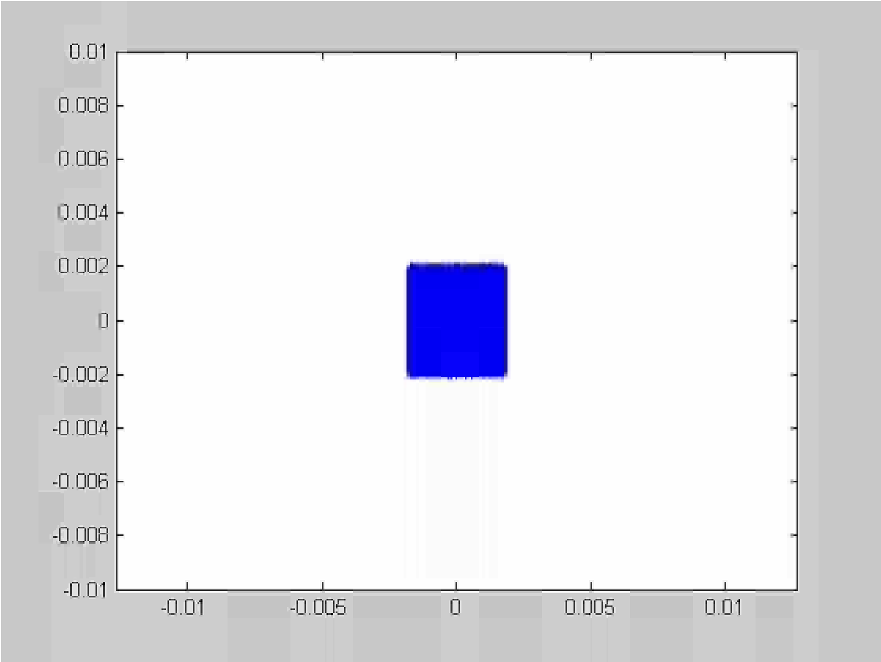
- Boris rotation for 3D fields:

$$\frac{\vec{p}^{\,+} - \vec{p}^{\,-}}{\Delta t} = \frac{q}{2\gamma^n m_0}\left(\vec{p}^{\,+} - \vec{p}^{\,-}\right) \times \vec{B}^{\,n}$$

*Birdsall, C.K. and A.B. Langdon, Plasma Physics via Computer Simulation, McGraw-Hill, New York, 1985.

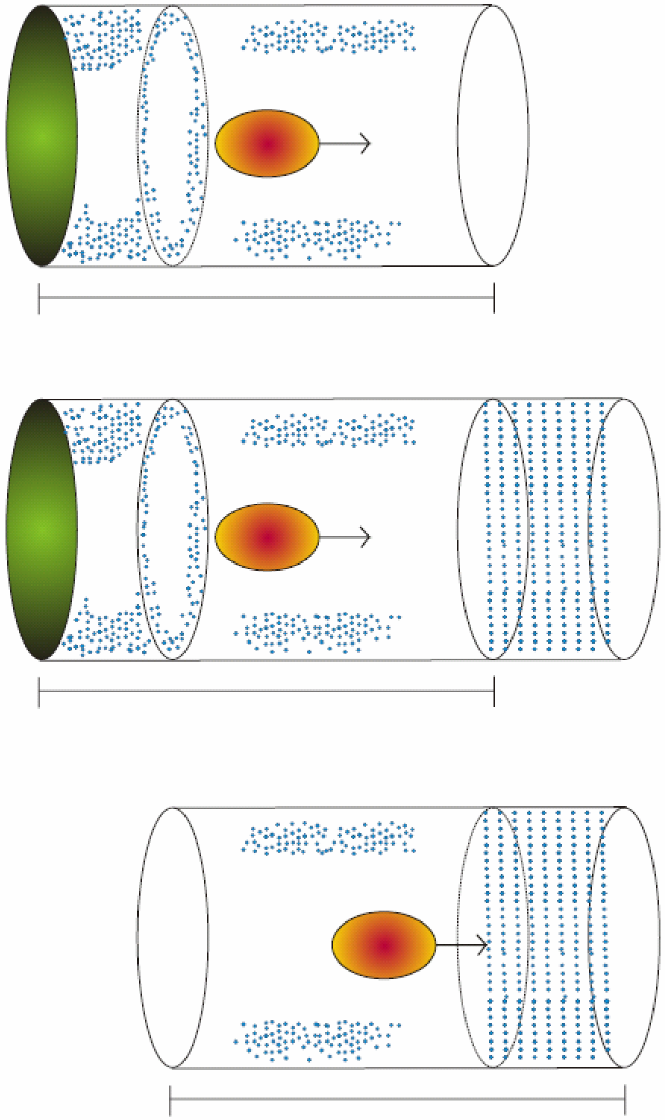
First Tracking Simulations

- cylindrical bunch
- 50,000 macro particles
- $\sigma_x=\sigma_y=\sigma_z=1.0$ mm
- uniform distribution
- charge –1 nC
- $E_{kin}=5$ MeV
- tracking time 110 ps
- time step 5 ps



Plans: Interaction Beam - E-Cloud

- starting with uniform particle distribution for the e-cloud
- separate space charge computation for the relativistic positron beam and the e-cloud
- each time step a new part of the computational domain gets uniform particle distribution for the e-cloud



The End - ECL2

Comparisson of the tracking with ASTRA*

Tracking with ASTRA (employing 3D space charge routine for elliptical domains and ASTRA‘s own FFT space charge solver)

- Gaussian bunch, 10,000 macro particles

$$\sigma_x = \sigma_y = 0.75 \text{ mm} \; \sigma_z = 1.0 \text{ mm}$$

- charge of -1nC, average energy of 2 MeV

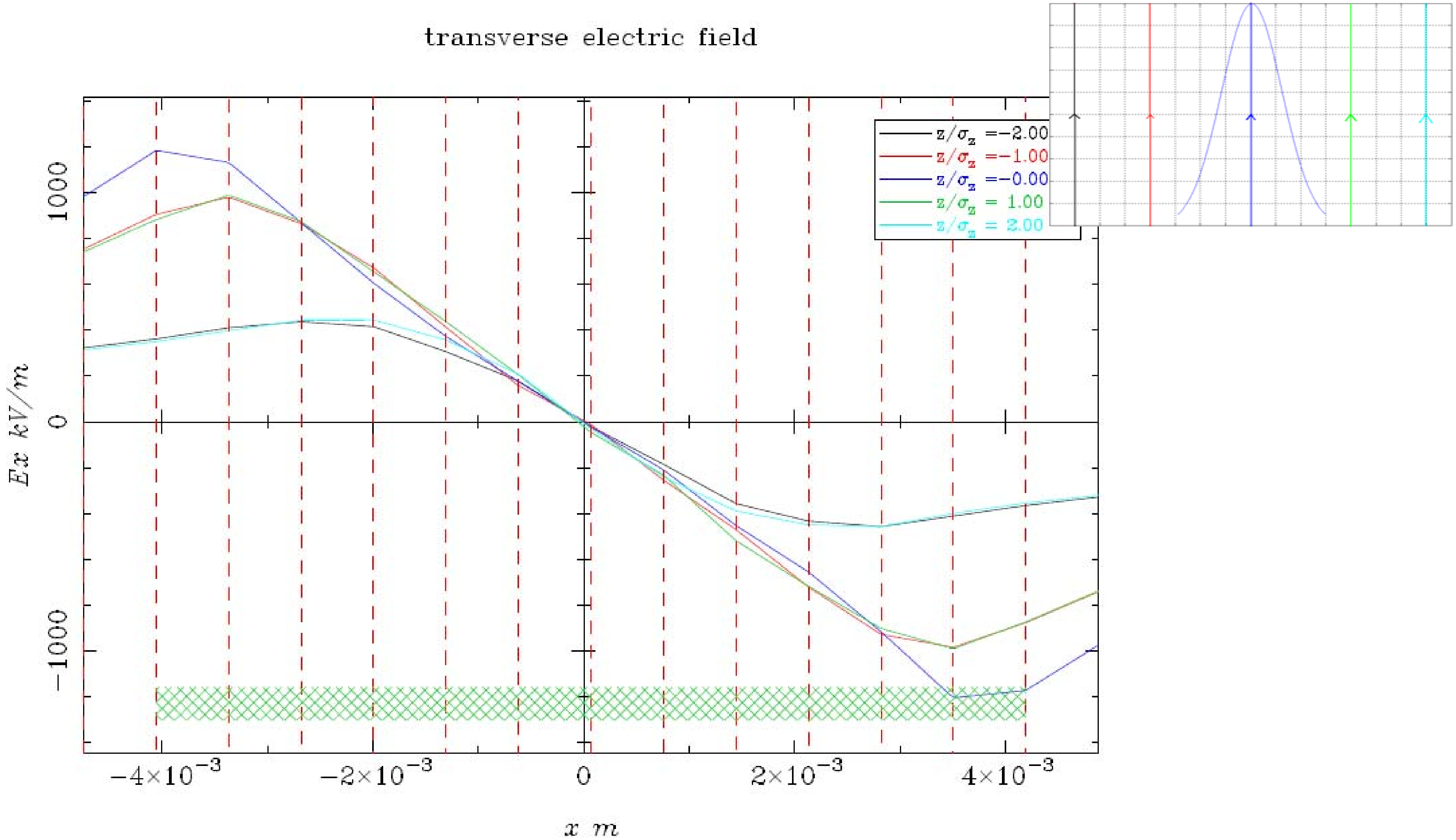
- beam pipe has a diameter of 24 mm

Investigate the transverse electric field after a drift of *z* = 0.3 m

* K. Flöttmann, “Astra”, DESY, Hamburg www.desy.de/ ˜mpyflo, 2000.

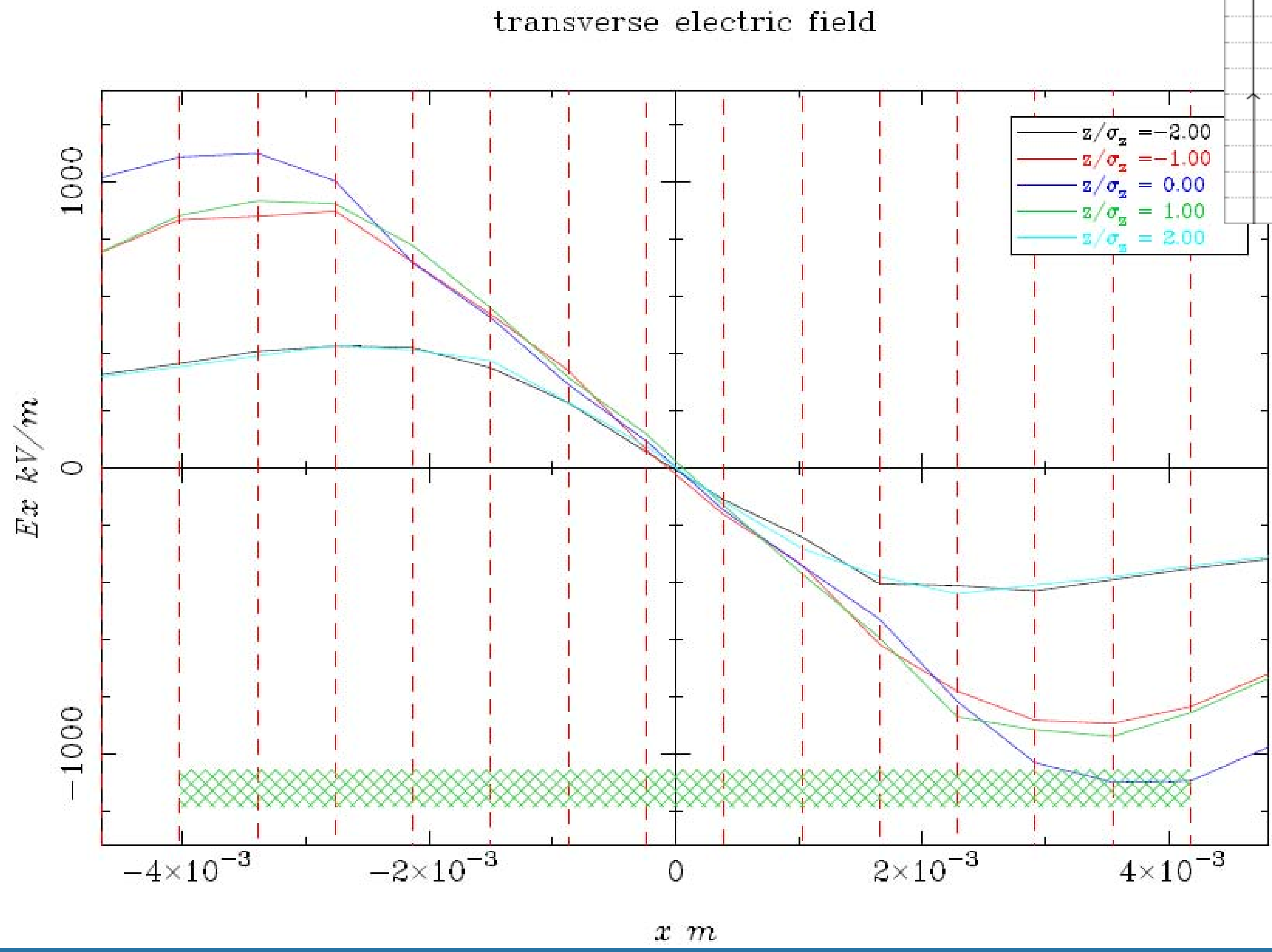
Comparisson of the tracking with ASTRA

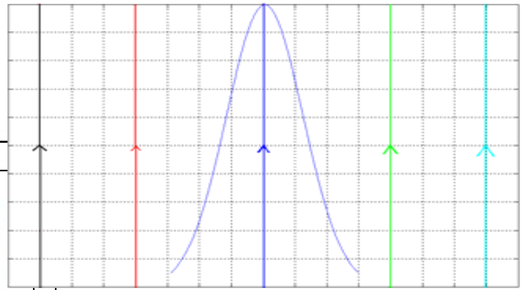
The transverse electric field after a drift z = 0.3 m in a circular beam pipe.



Comparisson of the tracking with ASTRA

The same drift was simulated with the 3D FFT space charge routine of Astra





Finite difference equation and the system of linear equations

Γ is generally discretized in N_x,N_y and N_z non-equidistant steps:

$$h_{x,0},\, h_{x,1},\, \ldots,\, h_{x,N_x-1}$$

$$\tilde{h}_{x,i} = \begin{cases} \dfrac{h_{x,i-1} + h_{x,i}}{2}, & i = 1, \ldots, N_x - 1 \\ \dfrac{h_{x,i}}{2}, & i = 0,\ N_x \end{cases}$$

$$\begin{aligned} & \tilde{h}_{y,j}\tilde{h}_{z,k}\left(-\frac{1}{h_{x,i-1}}\varphi_{i-1,j,k} + \left(\frac{1}{h_{x,i-1}} + \frac{1}{h_{x,i}}\right)\varphi_{i,j,k} - \frac{1}{h_{x,i}}\varphi_{i+1,j,k}\right) \\ + \; & \tilde{h}_{x,i}\tilde{h}_{z,k}\left(-\frac{1}{h_{y,j-1}}\varphi_{i,j-1,k} + \left(\frac{1}{h_{y,j-1}} + \frac{1}{h_{y,j}}\right)\varphi_{i,j,k} - \frac{1}{h_{y,i}}\varphi_{i,j+1,k}\right) \\ + \; & \tilde{h}_{x,i}\tilde{h}_{y,j}\left(-\frac{1}{h_{z,k-1}}\varphi_{i,j,k-1} + \left(\frac{1}{h_{z,k-1}} + \frac{1}{h_{z,k}}\right)\varphi_{i,j,k} - \frac{1}{h_{z,k}}\varphi_{i,j,k+1}\right) \\ = \; & \tilde{h}_{x,i}\tilde{h}_{y,j}\tilde{h}_{z,k}f_{i,j,k} \end{aligned}$$

Finite difference equation and the system of linear equations

Boundary adapted 7-point - star of grid points inside the elliptical domain Ω

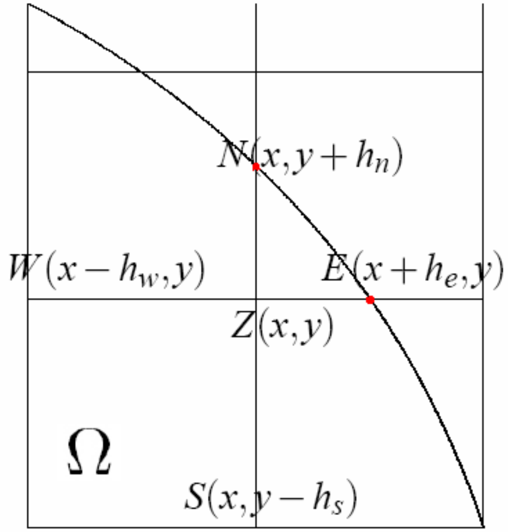
Linear system :

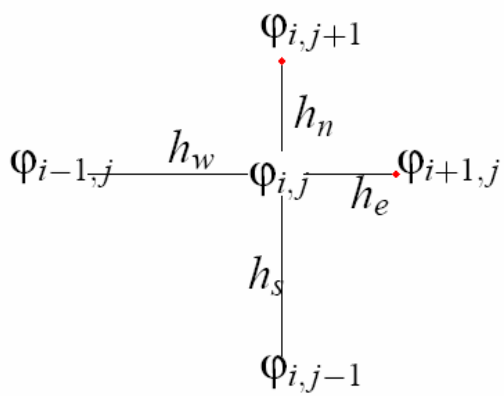
$$Au = b$$

System matrix A is:

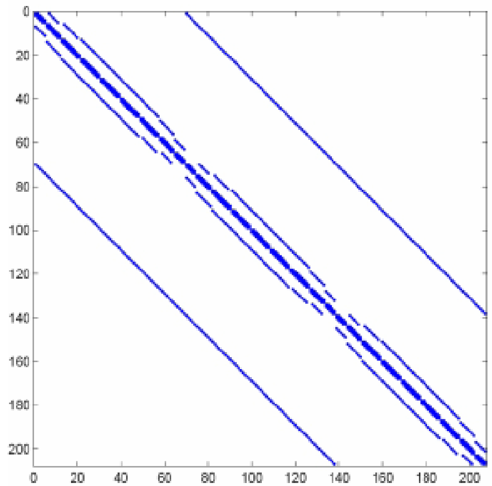
- block structured
- nonsymmetric
- positve definite

Use of BiCGSTAB to solve the system

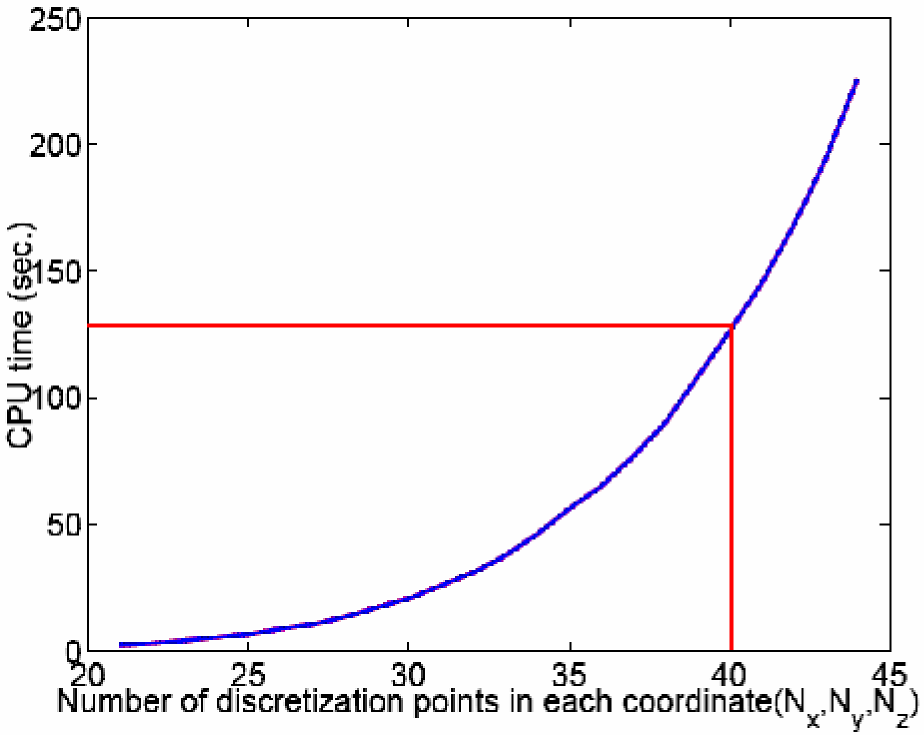


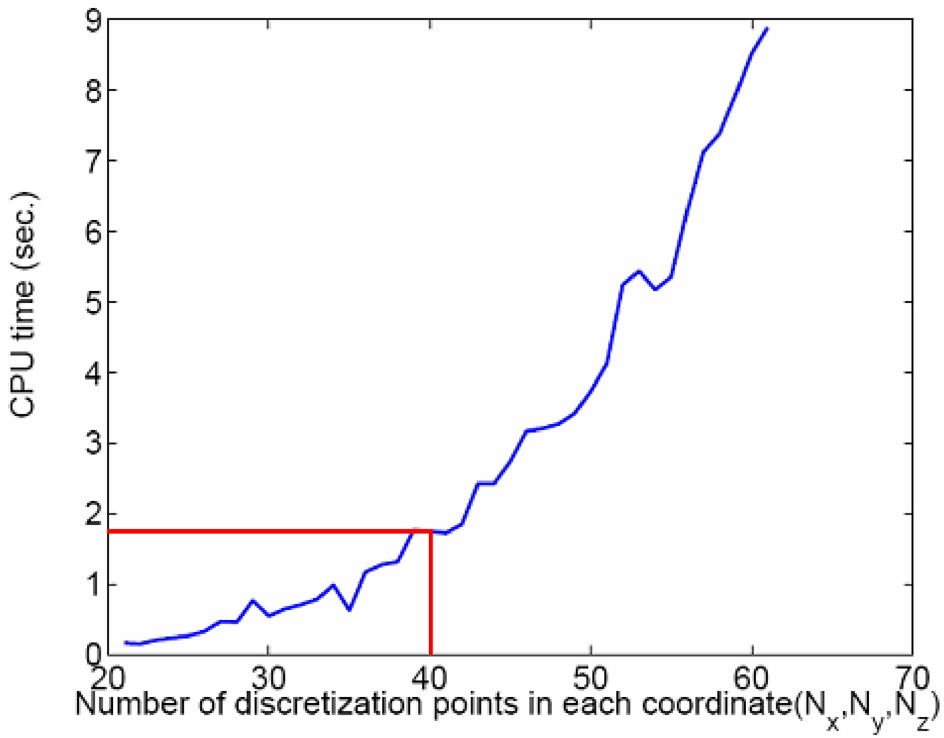


Non-symmetric 2-D Shortley-Weller Star.



Solver performance





Computation time of BiCG (left) and BiCGSTAB (right) for gradually increasing number of discretization points in each coordinate (N_x,N_y,N_z).