Coupled maps for electron and ion clouds

Ubaldo Iriso and Steve Peggs

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R. Tomás, P. Thieberger, and S.Y. Zhang

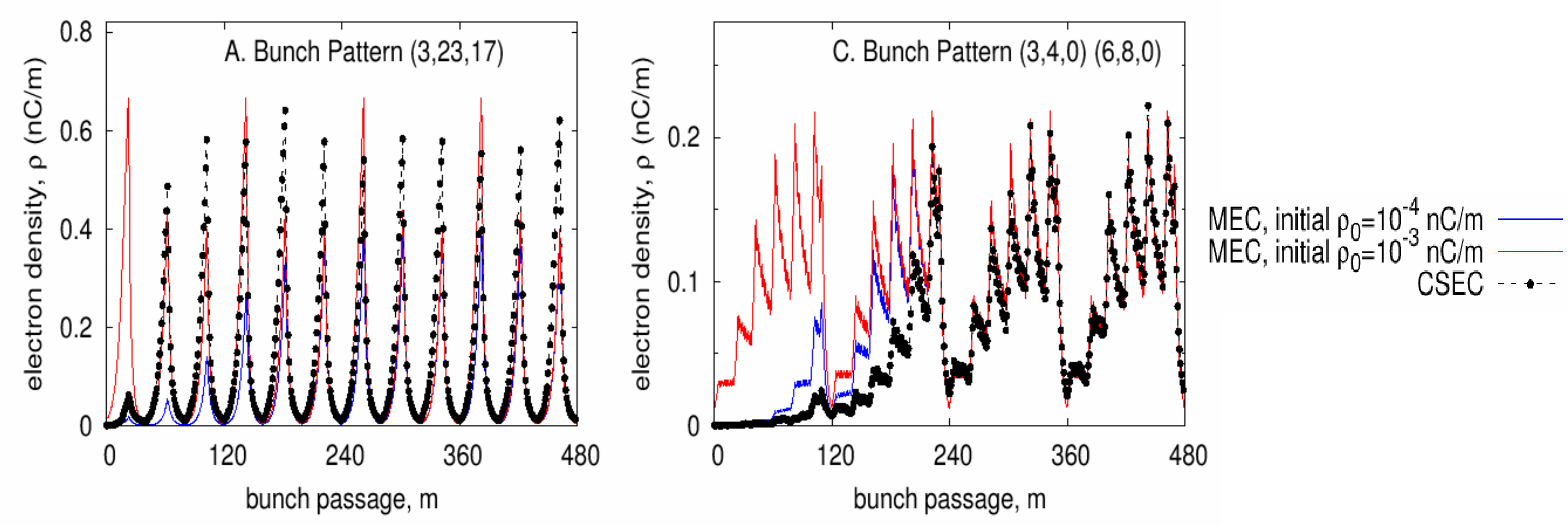
Contents:

1. Introduction: Maps for Electron Clouds

- A cubic iterative map model was presented to study e-clouds*, where the average e-density ρ at bunch passage ***m+1*** depends on the e-density at previous bunch passage, ***m*****:**

$$\rho_{m+1} = a\rho_m + b\rho_m^2 + c\rho_m^3$$

- Good agreement between maps (***MEC***) vs long computer sim codes (***CSEC***) for different bunch patterns, but map simulations run ~7 orders of magnitude faster:



- Maps also provide a level of abstraction to tackle electron clouds that can render fruitful conclusions. For instance, the way to minimize e-cloud density for different bunch patterns.

**U. Iriso and S. Peggs, PRST-AB, 8, 024403, 2005*

Holy grial: calculation of maps coefficients

But, the map coefficients were inferred after fitting results obtained from the detailed simulation codes (like *ECLOUD, CSEC*…)

➔ Ideally: calculate map coefficients analytically.

With some simplifications, Ref* shows the linear map coefficient ***a*** is interpreted as the effective SEY – δ_{eff}:

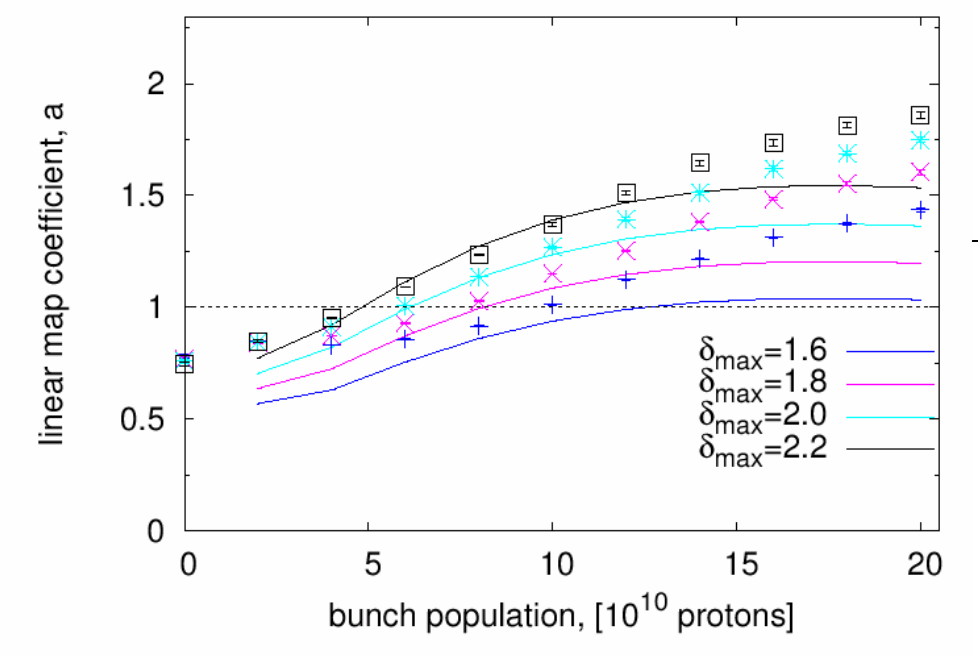
$$a = \int_0^\infty \left[\delta_r(E)^{n(E)} + \delta_t(E)\delta_{\text{sec}}^{\xi(E)} \frac{\delta_{\text{sec}}^{n(E)\xi(E)} - \delta_r^{n(E)}}{\delta_{\text{sec}}^{\xi(E)} - \delta_r(E)}\right] h(E)\ dE$$

h(E) = energy spectrum after bunch passage

n(E) = number of oscillations of an e- at energy E between two bunches

δ_r = secondary electrons coming from elastic reflections

δ_t = “true” secondary electrons



Acceptable agreement between fitting (points) and the analytical solution for ***a***

**U. Iriso, PhD Thesis, BNL/CAD-228*

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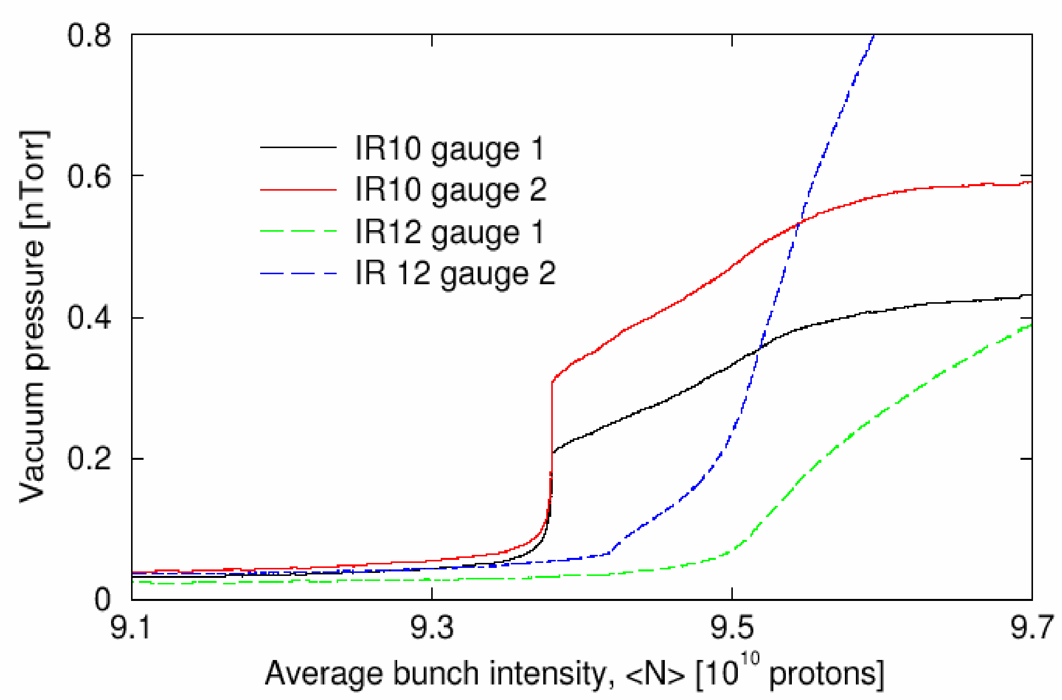
2. Experimental observations

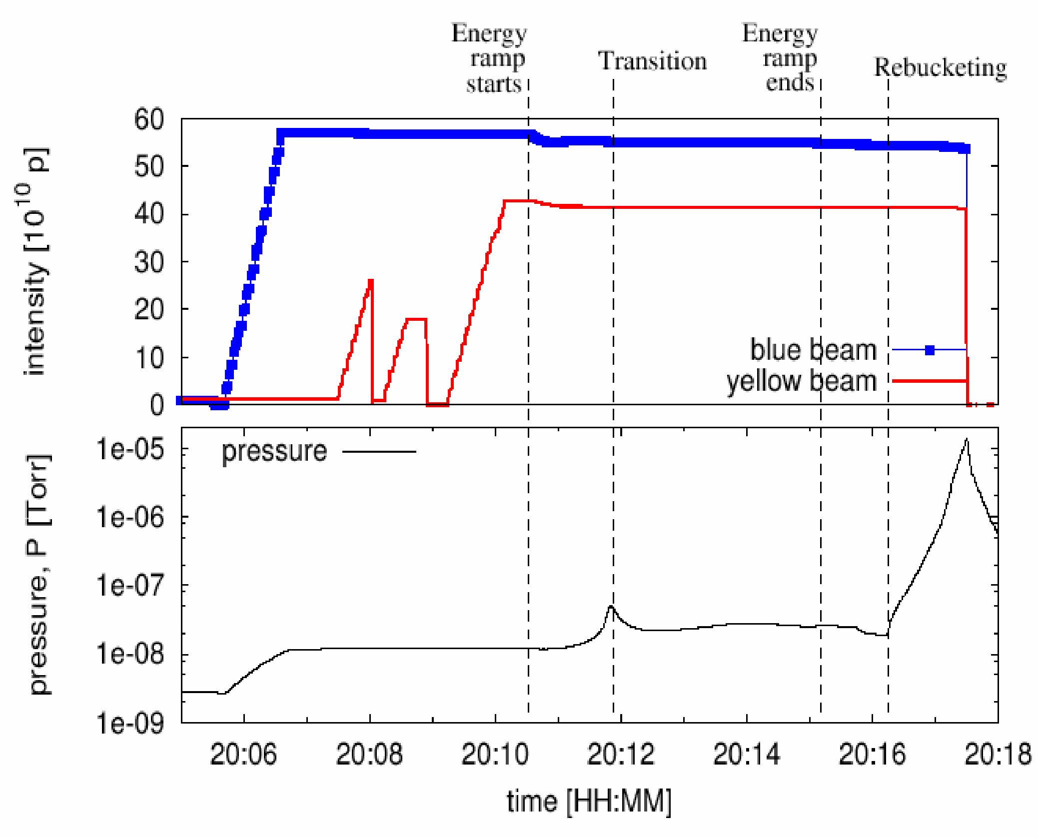
2.a. First and second order phase transitions



- Pressure (due to e-clouds) smoothly decays in IR12, it shows an abrupt decay in IR10, as the bunch intensity threshold for e-clouds is crossed.
- But contemporary simulation codes only reproduce a smooth transition from “cloud off” ←→ “cloud on” *
- How can both first and second order phase transition occur in e-clouds?

**S. Peggs and U. Iriso, Proceedings of ECLOUD’04, 2004*

2.b. A vacuum instability driven by e-clouds: ion clouds?



Proposed explanation*:
e-clouds and beam-gas collisions create ions, leading to a vacuum instability

- So, e-clouds can trigger an ion cloud (see Refs. * and **)

- Significant number of parameters to determine ions behaviour: different cross sections for different gases, backscattering probability, vacuum pumping…

**W. Fischer, U. Iriso, E. Mustafin, 33rd ICFA, 2004*

*** O. Grobner, CARE HHH, 2004*

→ But ion lifetimes are ~3-6 order of magnitude larger than e- → rather complex to introduce into the contemporary e-cloud codes (*CSEC, ECLOUD*) because of their prohibitely large CPU times

→ Can maps circumvent this prohibition?

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3. Coupled electron and ion clouds

Assume ion clouds can be formed and "couple" them to the electron cloud using maps:

$$\rho_{m+1} = f(\rho_m, R_m) \longrightarrow \text{for electron density}$$
$$R_{m+1} = g(\rho_m, R_m) \longrightarrow \text{for ion density}$$

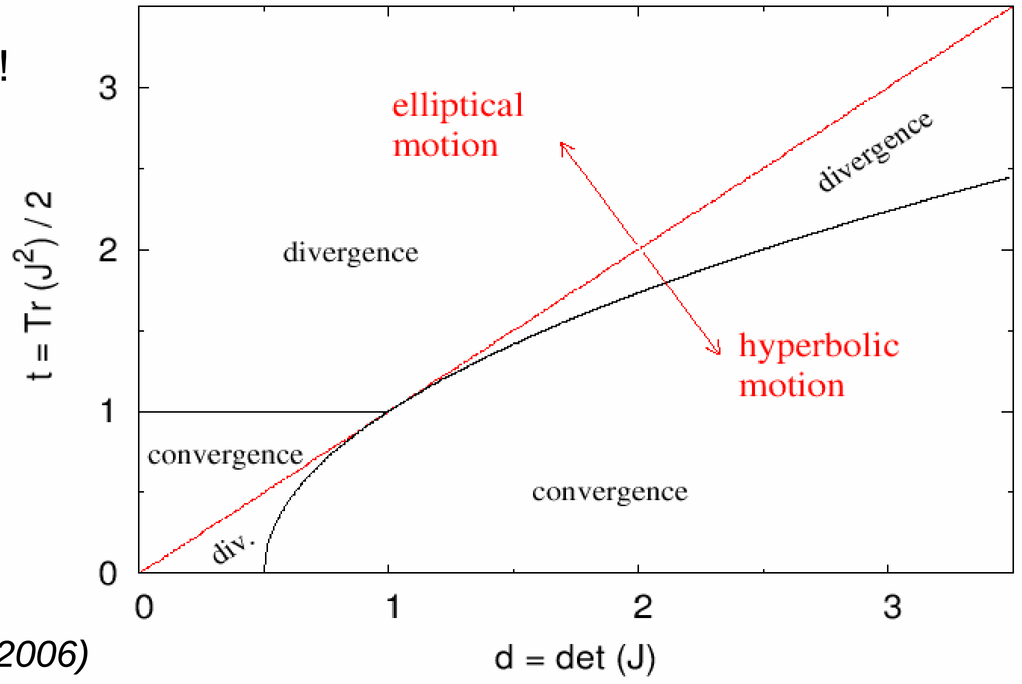
The 2-system is characterized by the vector $\vec{r}_m = \begin{pmatrix} \rho_m \\ R_m \end{pmatrix}$

Equilibrium is found if $\vec{r}_{m+1} = \vec{r}_m \equiv \vec{r*}$ at the so-called, "fixed points"

➔ But we need the fixed points to be ***stable***!!

- Stability condition of the fixed points depend on the Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial f}{\partial \rho_m} & \frac{\partial f}{\partial R_m} \\ \frac{\partial g}{\partial \rho_m} & \frac{\partial g}{\partial R_m} \end{pmatrix}_{\vec{r}*}$$



**see U. Iriso and S. Peggs, PRST-AB, 9, 071002 (2006)*

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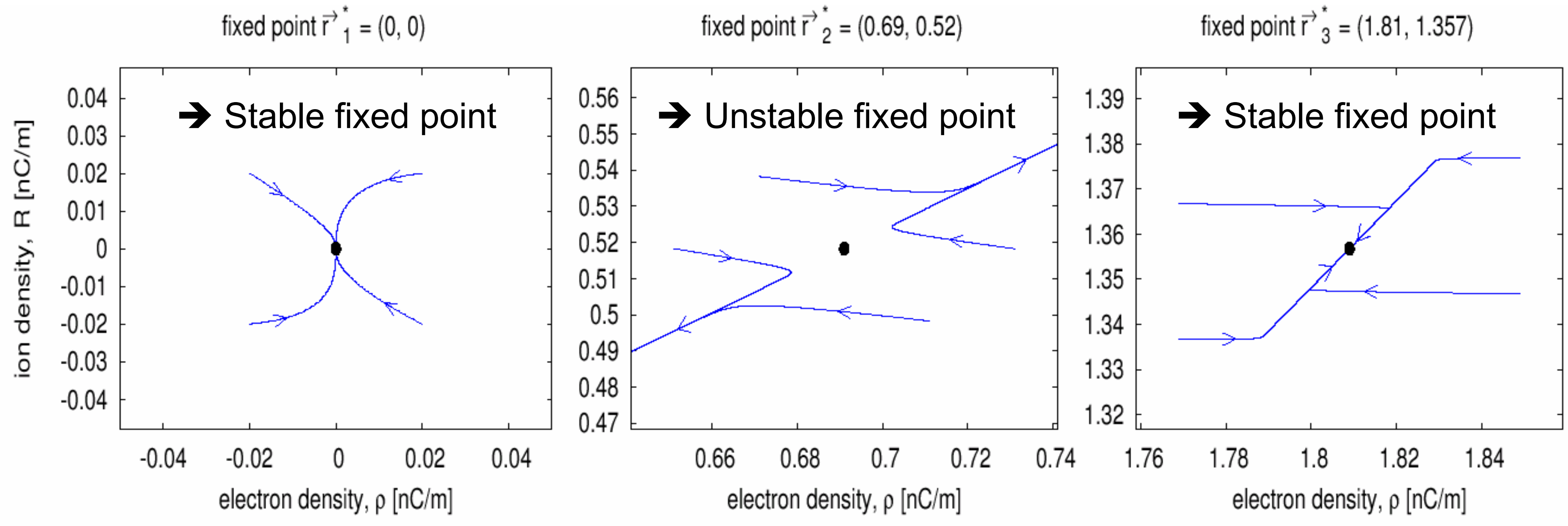
4. “Proof-of-principle” coupled maps

$$\begin{aligned}\rho_{m+1} &= (a + b\rho_m + yR_m)\rho_m + c\rho_m^3 \\ R_{m+1} &= AR_m + Y\rho_m\ .\end{aligned}$$

- Electrons collide with rest gas and create ions: $Y\rho_m$
- Ions enlarge ***a*** (enlarge e- survival between bunches): $b\rho_m R_m$

→ Assume now ***a(N)*** changes linearly and keep rest of parameters constants
→ For a given bunch population ***N***, more than one solution can be found;

→ Example: the 3 fixed points are found for N=5·10^{10} protons/bunch*. Their stability depends on the jacobian matrix and can be checked in the ***(ρ, R)*** space:


fixed point $\vec{r}^{\,*}_1 = (0, 0)$
➔ Stable fixed point

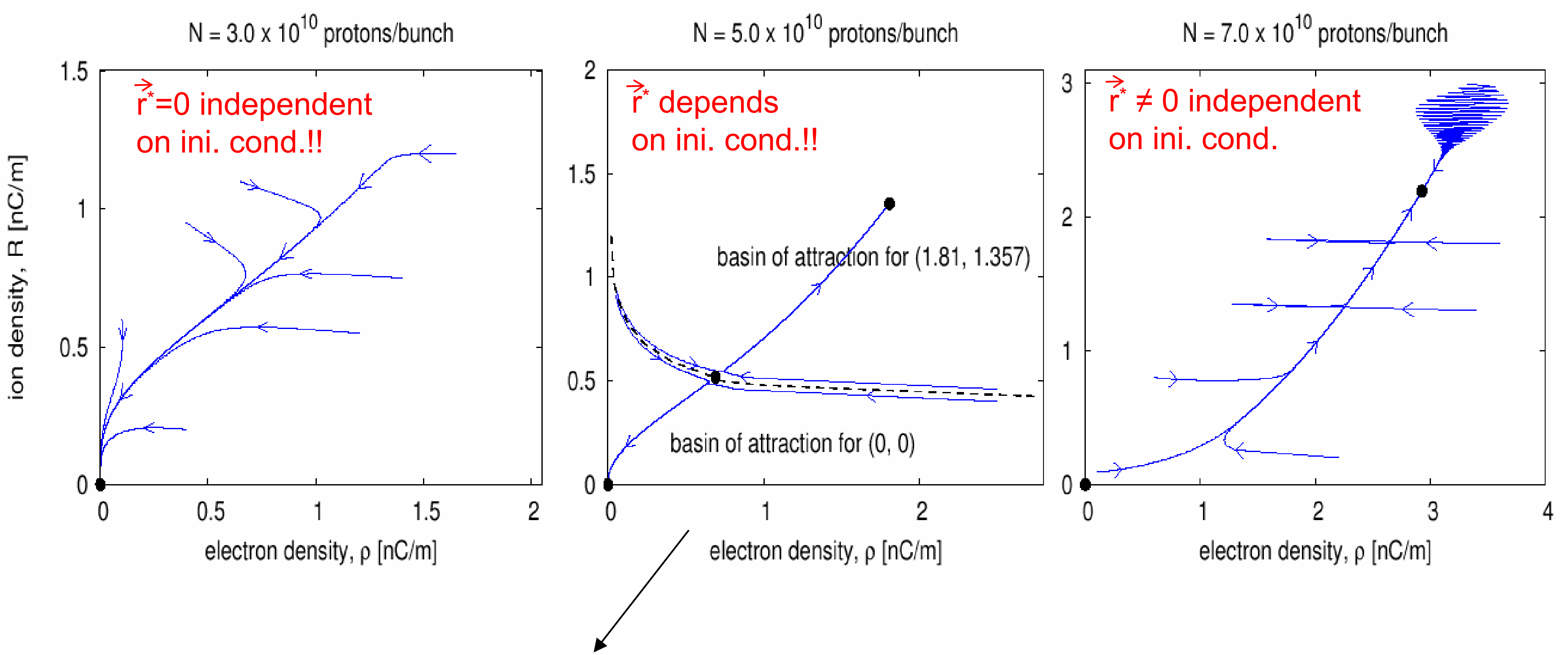
fixed point $\vec{r}^{\,*}_2 = (0.69, 0.52)$
➔ Unstable fixed point

fixed point $\vec{r}^{\,*}_3 = (1.81, 1.357)$
➔ Stable fixed point

**see values of the coefficients at U. Iriso and S. Peggs, PRST-AB, 9, 071002 (2006)*

4.a. Stability and importance of the initial conditions

- $\vec{r}^{\,*}$ depends on the initial conditions (memory effects) for different N
- ***(ρ, R)*** space behaviour differs with different bunch populations:

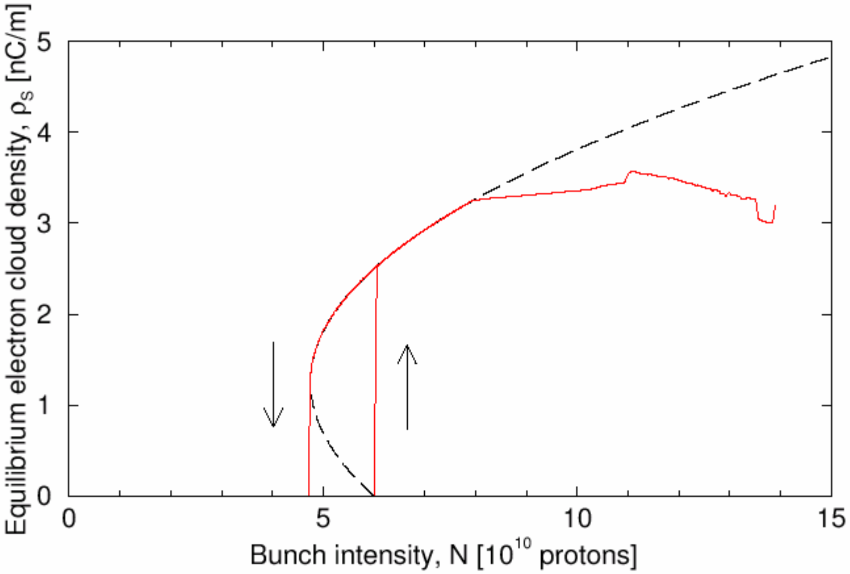


- The two basins of attraction in the ***(ρ, R)*** space produce the hysteresis

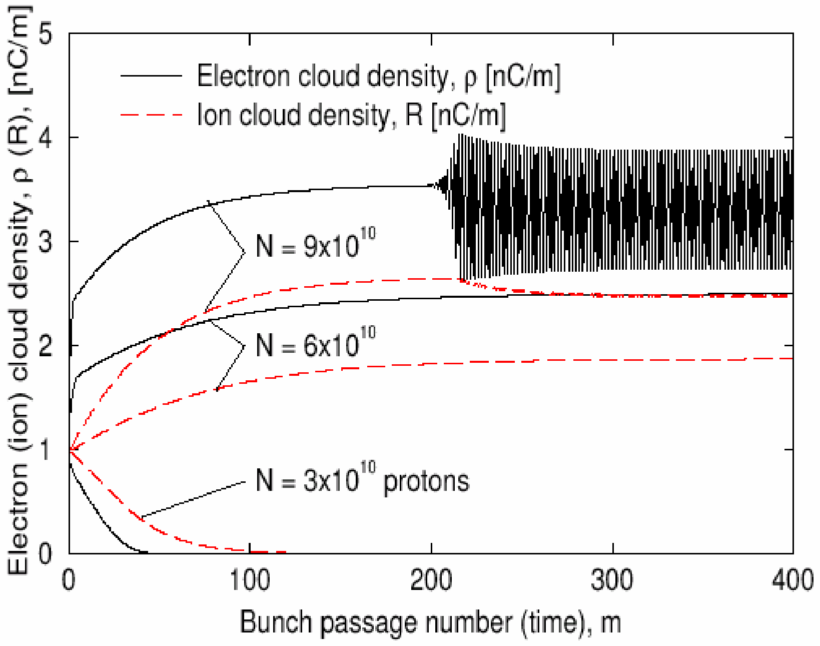
4.b. Hysteresis and additional dynamical phases

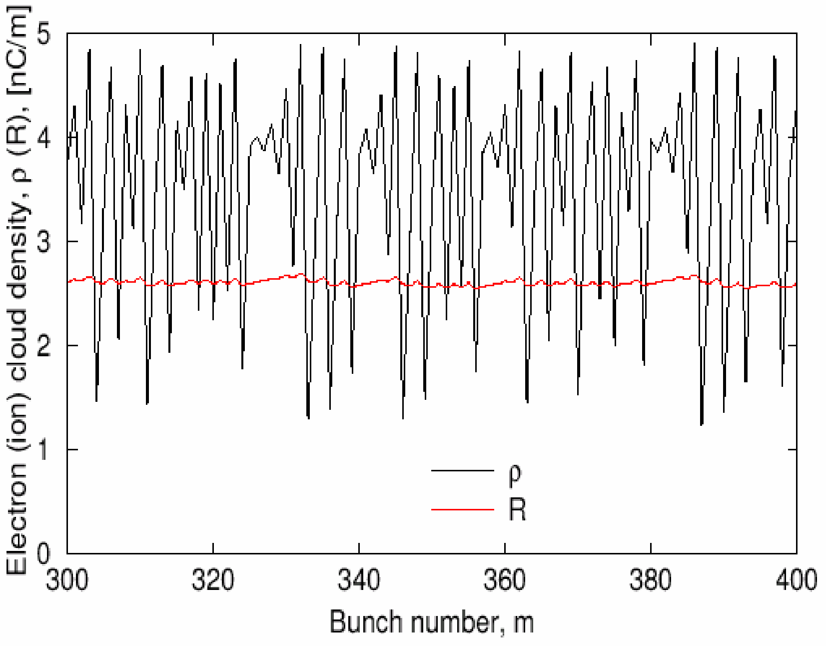
Results from a dynamical simulation based on the coupled maps first as ***N*** is slowly increased, then as ***N*** is slowly decreased

Hysteresis is observed because the final state depend on the initial conditions for some bunch intensities.



…chaotic regimes can be found…





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5. Conclusions

- Maps are a suitable tool to overcome CPU limitations presented by possible electron and ion clouds coupling.

- The development of stability conditions for a broad spectrum of potential coupling mechanisms is presented.

- Final solution $\vec{r}^{\,*}$ for e- and ion densities can depend on the initial conditions → hysteresis effects.

- The model reproduce the first order phase transitions seen in practice for the pressure.

- They also predict that chaotic regimes may appear near machine operating conditions.