“Efficiency vs. Stability” in novel and conventional accelerators

Sergey Antipov (DESY)

Many thanks Sergey Nagaitsev (JLAB), Alexey Burov (FNAL), Elias Metral (CERN), Alexander Novokhatski (SLAC), W. An (BNU), M. Vogt (DESY)

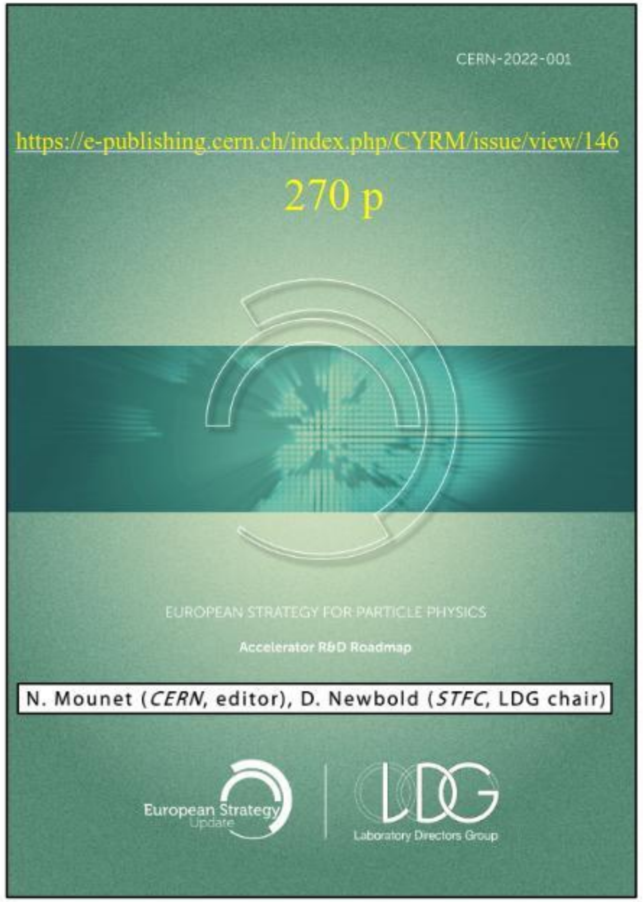
Alegro meeting, DESY, Mar 23, 2023

HELMHOLTZ

DESY.

Input from European Accelerator R&D Roadmap

Need a concrete and evidented statement of the basic feasibility to support future investment into larger-scale R&D



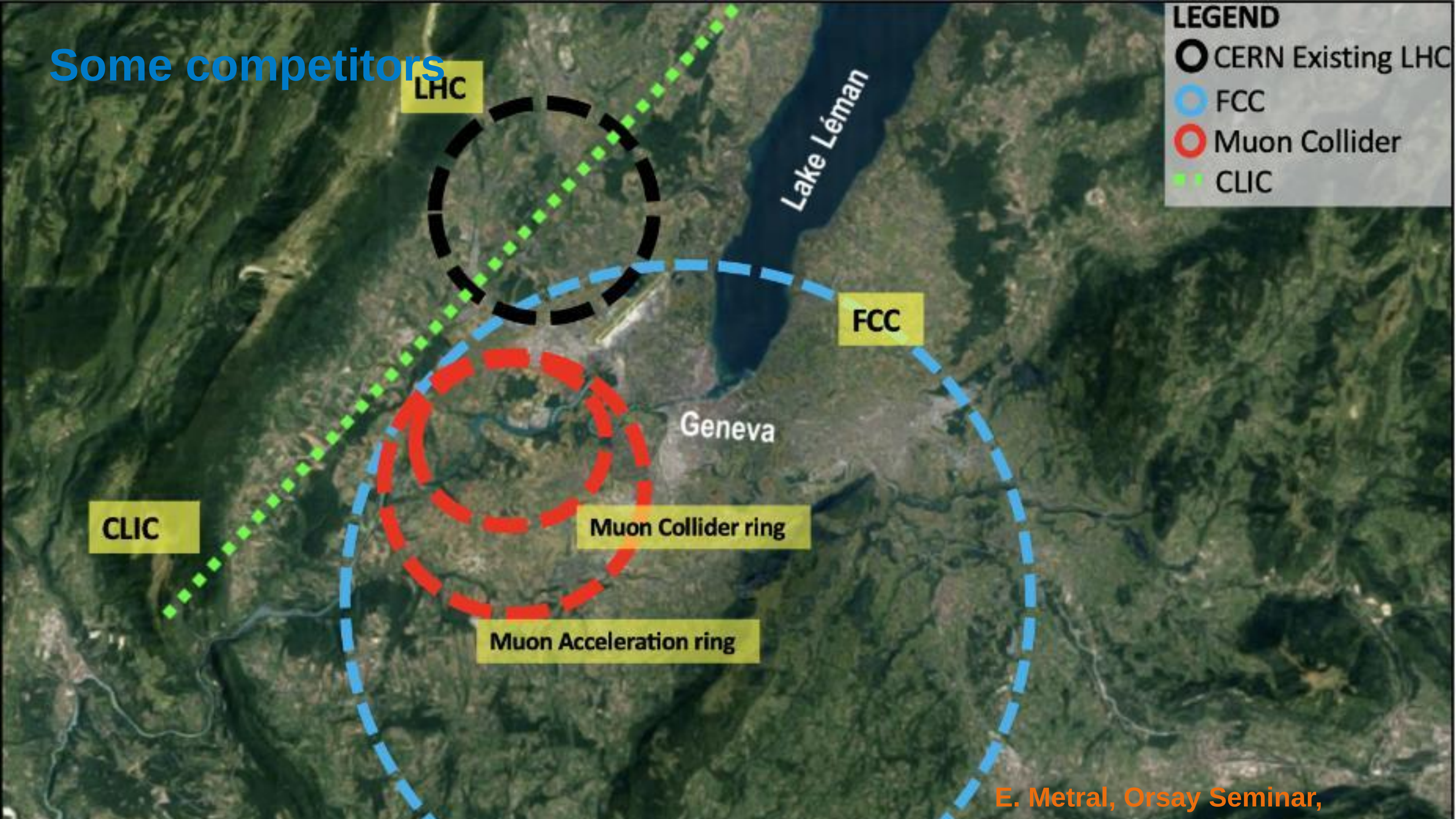
European Accelerator R&D Roadmap (17/03/2022)

- Conclusions on findings: **PLASMA**
  - The high-gradient plasma and laser accelerators panel has focused on the ambitious developments needed specifically for particle physics applications of the rapidly developing plasma-wakefield and dielectric-acceleration technologies
  - These include the further development of existing techniques for: **acceleration of high charge with low emittance and improved efficiency; acceleration of positrons; and combination of accelerating stages in a realistic future collider**
  - **The goal here will be to produce by 2026 a concrete and evidenced statement of the basic feasibility of such a machine to inform decisions on future investment into larger-scale R&D**

Courtesy E. Metral

E. Métral, US Accelerator Beam Physics (ABP) Roadmap Workshop, Washington DC, USA, 07/09/2022 12

Some competitors



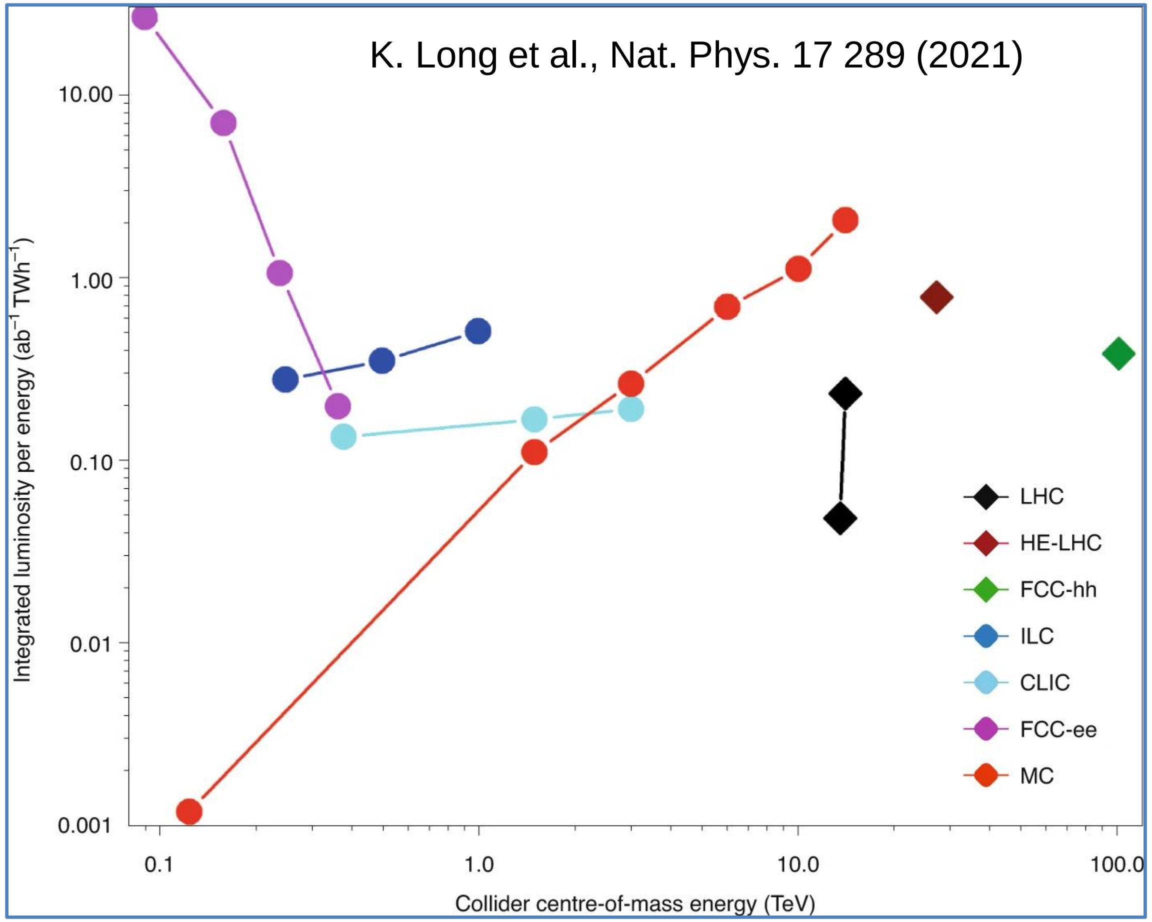
Focus on efficiency and sustainability

Need to achieve high efficiency while preserving beam quality

$$L \approx \frac{f_{rep} N^2}{4\pi\sigma_x^*\sigma_y^*} = \frac{P_b N}{4\pi E\sigma_x^*\sigma_y^*}$$

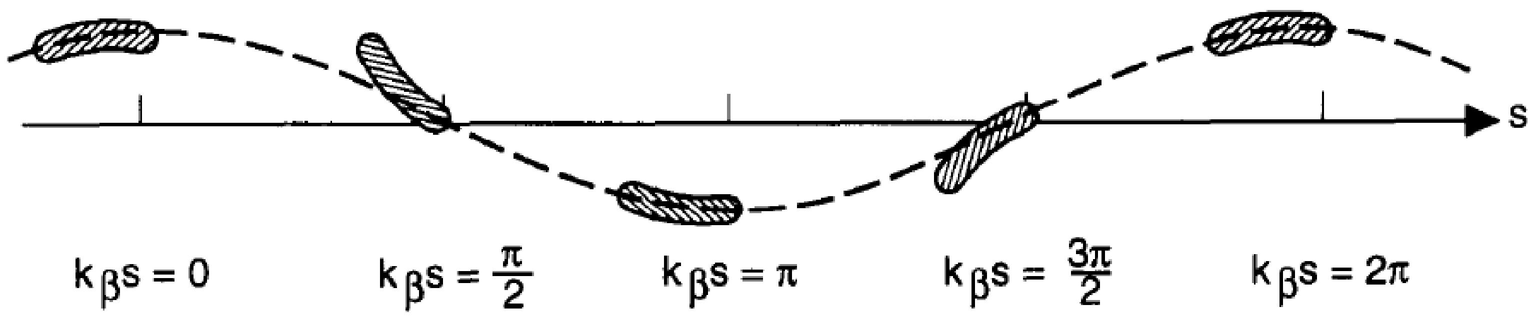
$$P_w = 2P_b/\eta_p$$

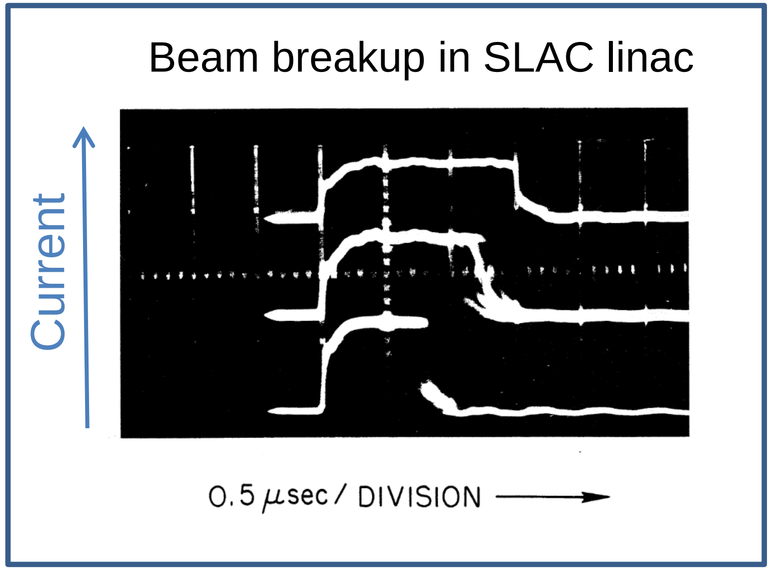
$$L/P_w \approx \frac{1}{E}\eta_p \frac{N}{8\pi\sigma_x^*\sigma_y^*}$$



Beam breakup might limit performance

A known issue in conventional linacs

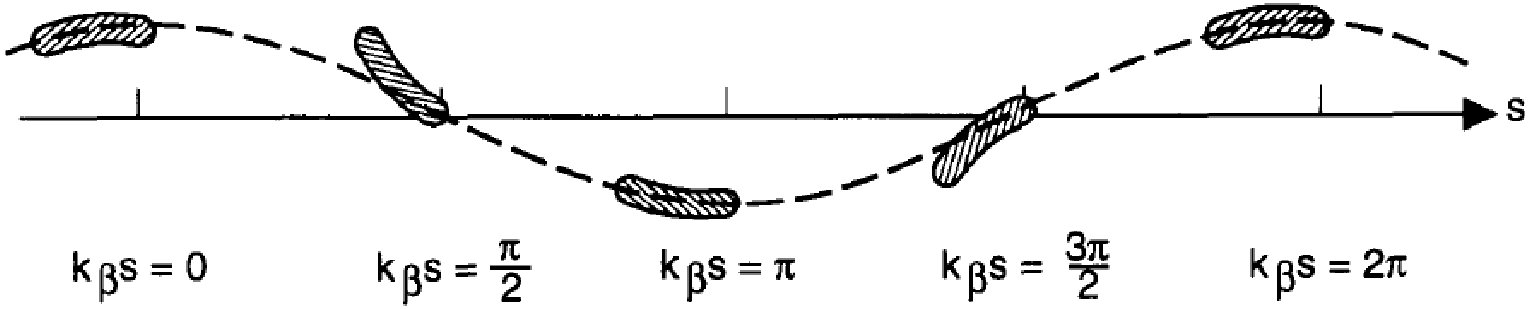




W. Panofsky and M. Bander, Rev.Sci.Instrum. 39 (1968)

Beam breakup might limit performance

A known issue in conventional linacs



PHYSICAL REVIEW ACCELERATORS AND BEAMS **20,** 121301 (2017)

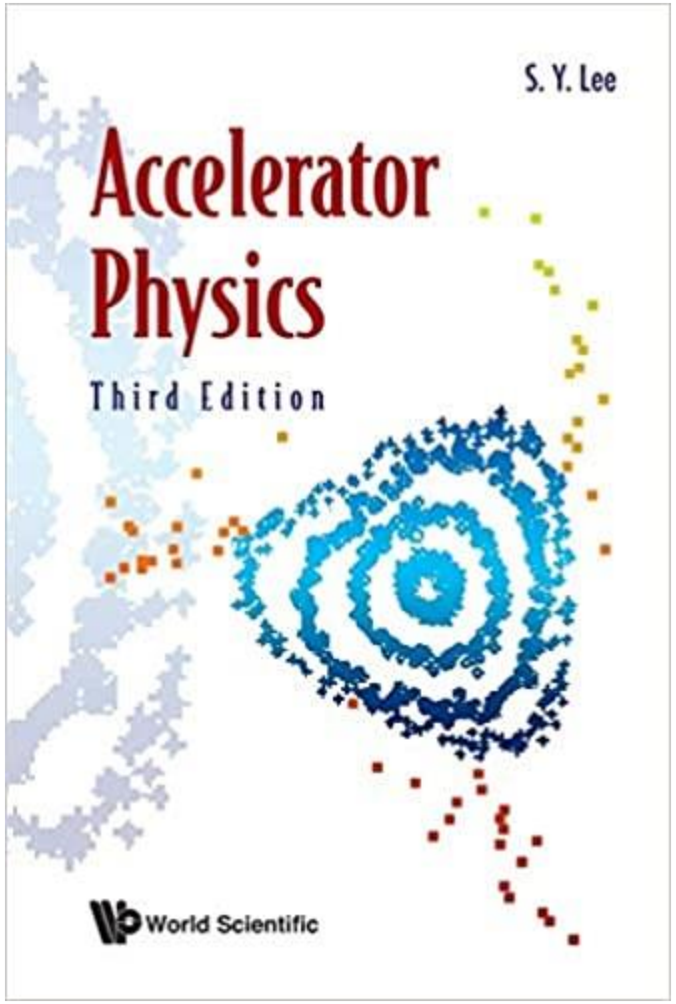
Efficiency versus instability in plasma accelerators

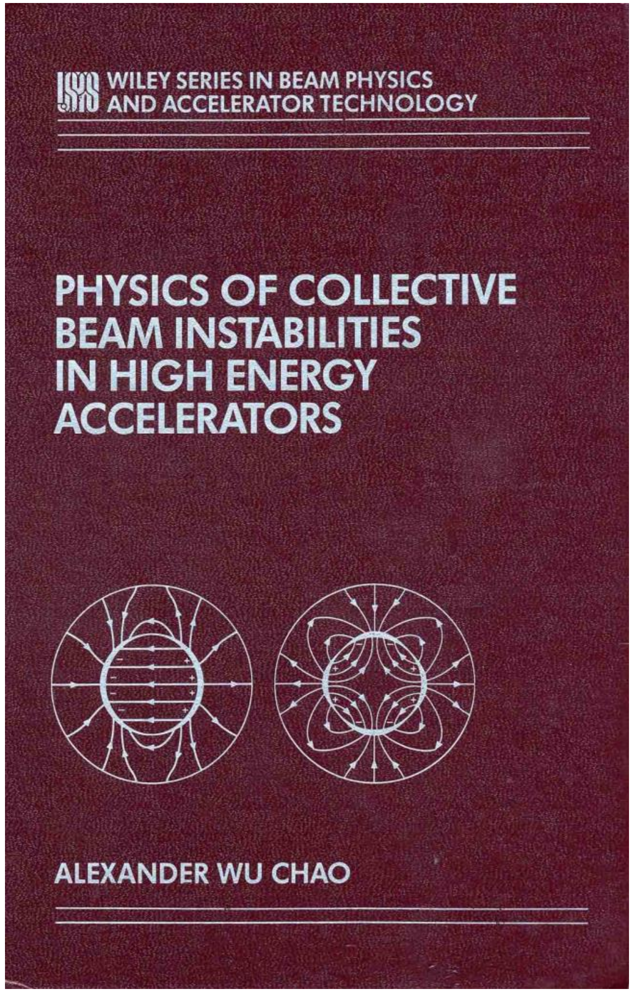
Valeri Lebedev,[1,*] Alexey Burov,[1] and Sergei Nagaitsev[1,2]
[1]*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510, USA*
[2]*Department of Physics, The University of Chicago, Chicago, Ilinois 60637, USA*
(Received 4 January 2017; published 20 December 2017)

$$\eta_t = \frac{{\eta_p}^2}{4(1-\eta_p)}$$

Choose one

Single partilce, nonlinear fields | **Many particles, linear fields**





Language warning

What do we call a wakefield?

In plasma acceleration

Wake in plasma is the field generated by the driver that accelerates the witness beam.

In conventional acceleration

Wakefields are fields with which the witness bunch acts on itself. They are generated by the leading charges and act on the trailing charges.

Assumptions

Following a standard instability analysis procedure

- Flat optics approximation, $k_\beta = \ const$
- Nonlinear buble regime, $k_p r_b >> 1$
- Beam driven
  - Should, in principle, hold in general case

- Special longitudinal profile

- Linear plasma focusing

- Small perurabtion
  - Linear wakefield
  - Similar to a conventional RF cavity

Assumptions

Following a standard instability analysis procedure

- Flat optics approximation
- Blowout regime, $k_p r_b >> 1$
- Beam driven
  - Should in principle hold in general case
- Special longitudinal profile
- Linear focusing
- Small peturbation
  - Linear wakefield
  - Similar to a conventional RF cavity

A fair assumption in conventional accelerators but might not hold in general for plasma acceleratio

Instability growth rate depends only on local bubble radius

(Under some fairly strong assumptions)

Instability

- Nonlinear buble regime, $k_p r_b >> 1$
- Special longitudinal profile $\rho(\zeta) \; = \; const$
- Linear focusing $F_r(r) = -2\pi e^2 n_0 r$
- Linear wakefield $W_\perp(\zeta) = 8\zeta / r_b^4$

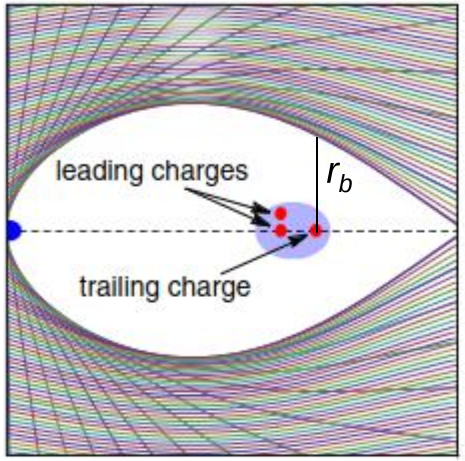


Illustration from
G. Stupakov, PRAB 21 041301 (2018)

- Growth rate depends only on one parameter:
  the local bubble radius at the location of the witness

$$\eta_t = -\frac{F_t}{F_r} = \frac{e^2}{F_r(r)} \int_0^L \rho(\zeta) r W_\perp(\zeta) d\zeta$$

Courtesy S. Nagaitsev

Instability growth rate depends only on local bubble radius

(Under some fairly strong assumptions)

Instability

- Nonlinear buble regime, $k_p r_b >> 1$
- Special longitudinal profile $\rho(\zeta) = const$
- Linear focusing $F_r(r) = -2\pi e^2 n_0 r$
- Linear wakefield $W_\perp(\zeta) = 8\zeta / r_b^4$

- Growth rate depends only on one parameter: the local bubble radius at the location of the witness

$$\eta_t = -\frac{F_t}{F_r} = \frac{e^2}{F_r(r)} \int_0^L \rho(\zeta) r W_\perp(\zeta) d\zeta$$

Efficiency

- Bubble shape, W. Lu, Phys. Plasmas 12 063101 (2005)

$$r_b \frac{d^2 r_b}{d\zeta^2} + 2\frac{dr_b^2}{d\zeta} + 1 = \frac{2}{\pi n_e r_b^2} \rho(\zeta)$$

- Power to the plasma $P = \frac{\pi^2 e^2 n_0^2 c}{4} R_b^4$
- Power to witness $P_a = \frac{\pi^2 e^2 n_0^2 c}{4} (r_{b2}^2 - r_{b1}^2)(\frac{R_b^4}{r_{b2}^2} + r_{b1}^2)$
- Efficiency depends only on the local bubble radius at the location of the witness

$$\eta_p = \frac{P_a}{P} = f(r)$$

$$\eta_t = \frac{\eta_p^2}{4(1-\eta_p)}$$

Courtesy S. Nagaitsev

Tail amplitude increases rapidly

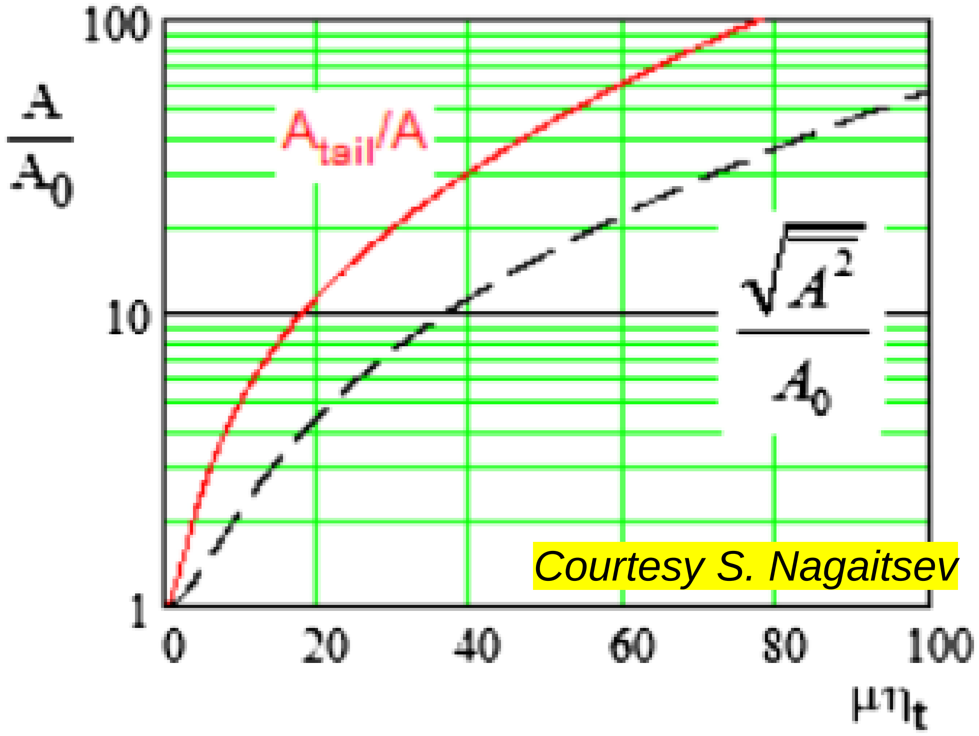
Normalized coordinates

- $X = \frac{x}{\sqrt{\beta}}\sqrt{\frac{p}{p_0}},\ \beta = k_p^{-1}\sqrt{2\gamma}$
- $\mathrm{d}\mu = \mathrm{d}s\, k_p/\sqrt{2\gamma}$

Equation of motion

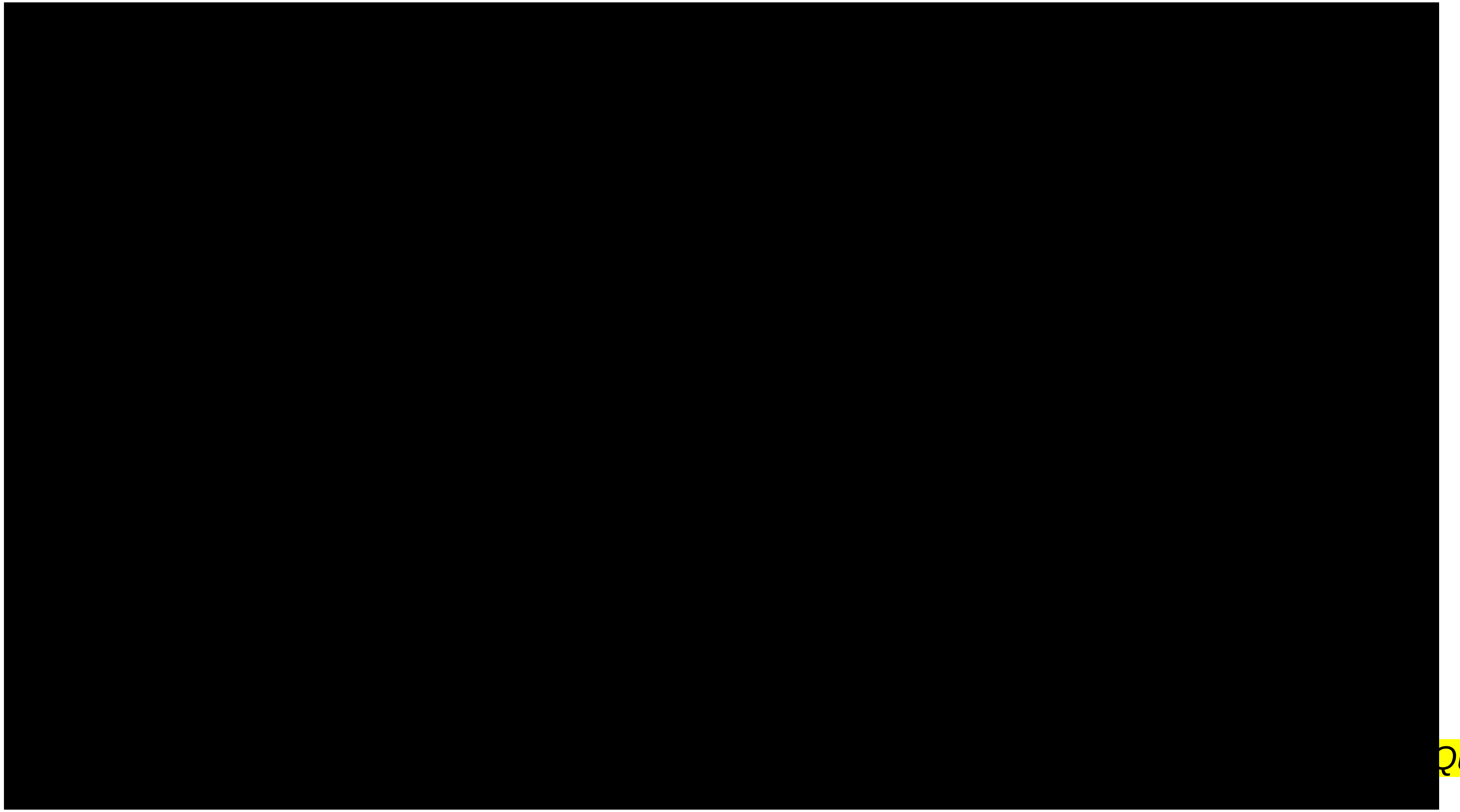
- $\frac{d^2X(\mu,\zeta)}{d\mu^2} + X(\mu,\zeta) = \frac{2\eta_t}{L_t^2}\int_0^{\zeta} X(\mu,\zeta')\,(\zeta-\zeta')d\zeta'$
- Solved for $\eta_t << 1$ and $\Delta p/p = 0$

C. Schroeder et al., PRL 82 6 (1999)



Numerical example

Power efficiency $\eta_p = 0.5$, instability parameter $\eta_t = 0.13$



QuickPIC simulation by Weiming An

What is the chance that the beam is perfectly on axis?

Mitigation strategies

Multiple mechanisms exist

Good review: Mehrling et al., PRAB 22 031302 (2019)

Reducing initial amplidues

- Alignment
- Tapering

Landau damping

- Utilize nonlinearity of transverse focusing
- Costs an increase in beam emittance

BNS damping

- Destroy the resonance condition
- Most effective method in linacs

Not in this talk

J. Rosenzweig, PRL 95 195002 (2005)

W. An, PRL 118 244801 (2007)

Mitigation strategies

Just good alignment is not good enough

Good review: Mehrling, PRAB 22 031302 (2019)

Reducing initial amplidues

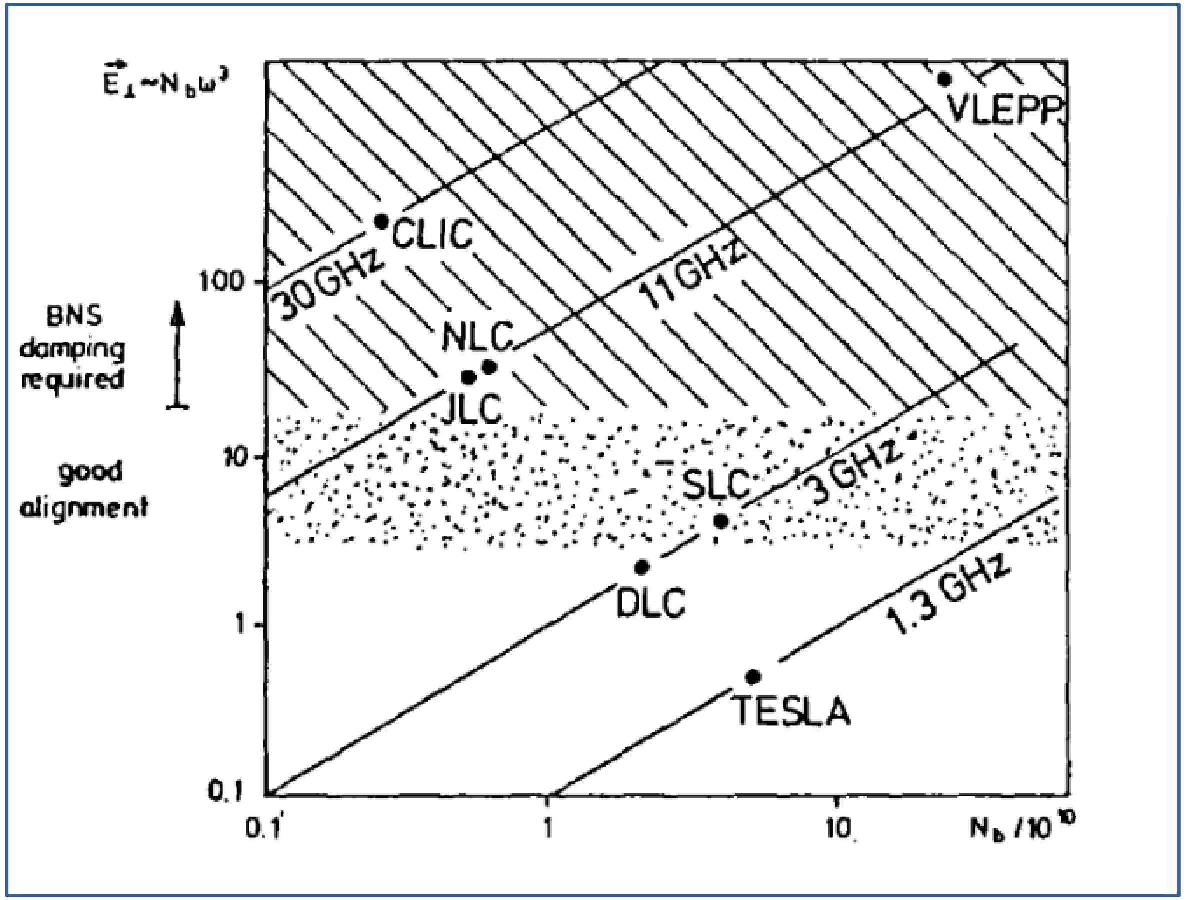
- Alignment
- Tapering

Landau damping

- Utilize nonlinearity of transverse focusing
- Costs an increase in beam emittance

BNS damping

- Destroy the resonance condition
- Most effective method in linacs



J. Rossbach, An Overview of Linear Collider Plans, PAC'92

Physics of the BNS damping

- The particles of the head of a bunch do not experienced action of the wake field and freely oscillate in the focusing lattice at the betatron frequencies.
- However this oscillations produce a periodical force for the particles of the tail of the bunch, which experience the action of the wake field.
- As the frequency of the force and the frequency of free oscillations are the same then amplitude of oscillations of the tail's particles will grow in time because of the resonance.
- An immediate solution for this situation is to destroy the resonance, than means to give different betatron frequencies to the particle of the bunch head and particles of the bunch tail.
- It can be done in many different way, but a simple solution is to utilize the fact that the betatron oscillation frequency depends by virtue of the chromaticity on the energy of the beam particles.

Courtesy A. Novokhatski

Two particle model

- Two particles (head and tail) have different betatron frequencies and
  $\gamma = const, \; g_{\perp}(s) = 2$

$$\frac{\partial^2}{\partial \tau^2} X_H(\tau) + \mu_H^2 X_H(\tau) = 0 \qquad X_H(\tau) = cos(\mu_H \tau)$$

$$\frac{\partial^2}{\partial \tau^2} X_T(\tau) + \mu_T^2 X_T(\tau) = X_H(\tau) \qquad X_T(\tau) = 1 + \frac{cos(\mu_H \tau) - cos(\mu_T \tau)}{\mu_H^2 - \mu_T^2}$$

- To keep the amplitude of $X_T(\tau)$ around 1 we need $\Delta\mu = \mu_T - \mu_h \geq \frac{1}{2\mu}$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\mu}{\mu} \geq \frac{1}{2\,\mu^2} = \frac{1}{2\,\nu^2\,(L^*)^2} = \frac{eQ}{2\,\nu^2 4\pi\varepsilon_0 a_W^3\,\gamma_0 mc^2}$$

BNS damping condition

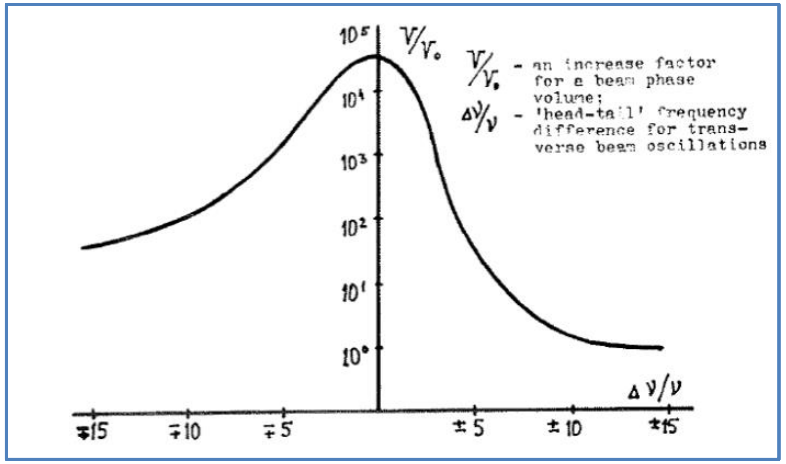
Courtesy A. Novokhatski

Larger energy spread is beneficial for stability

Damping instabilities in intense beams in colliders and beoynd

Use cases in colliders:

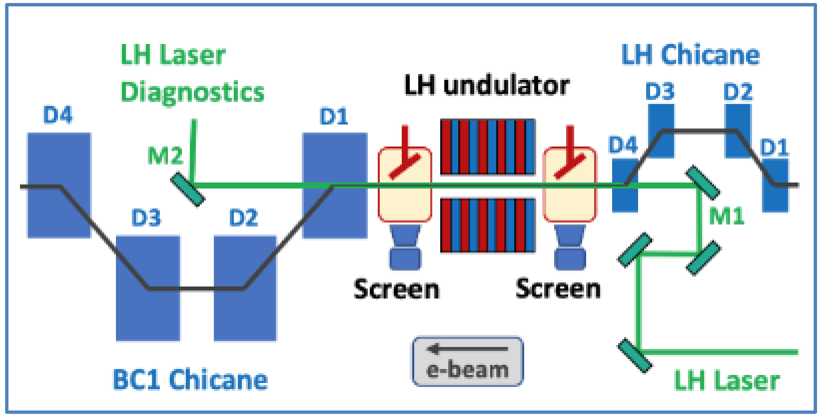
- VLEPP (on paper)
- SLC
- SLAC PEP-II B-factory



VLEPP. A. Novokahski, MCBI'19

FEL:

- FLASH uses a laser heater to improve beam stability in the linac

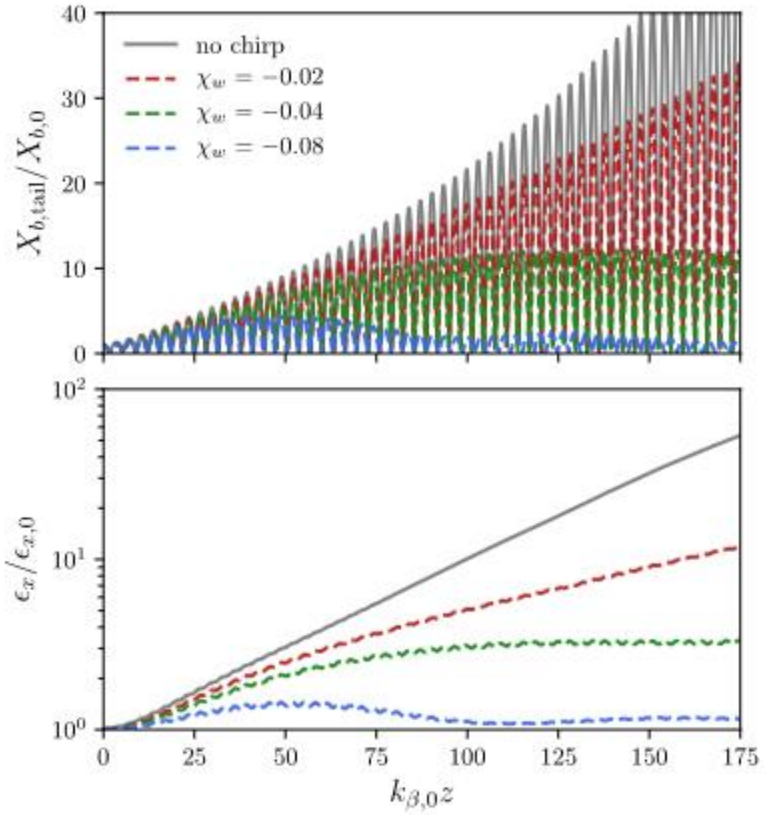


Ch. Gerthet al., IPAC'21

Larger energy spread is beneficial for stability

Significant energy chirps might be required

- Might need relative energy chirps ~10% level
- Complicates transport between stages
- Spread has to be significantly reduced for final focusing



T. Mehrling et al., PRAB 22 031302 (2019)

Ion motion also produces BNS damping

Theoretical description (S. Nagaitsev, AAC'18)

- Ions being dragged by the field of the bunch
- Generates a **head-tail chirp** of betatron frequencies

$$\frac{d^2 X(\mu,\zeta)}{d\mu^2} + (1 + 2\frac{\Delta\omega_\perp}{\omega_\perp})X(\mu,\zeta) = \frac{2\eta_t}{L_t^2}\int_0^\zeta X(\mu,\zeta')\,(\zeta - \zeta')d\zeta'$$

- Need a small enough beam size

$$\frac{N r_i L_t}{2\sigma^2} \geq \eta_t$$

- Projected emittance degrades due to accumulation of betatron phase advance between the head and the tail

$$\mu_i = \sqrt{2 r_p N L_t / M_i \sigma^2}$$

  - Having heavier ions helps

Ion motion also produces BNS damping

Theoretical description (S. Nagaitsev, AAC'18)

- Ions being dragged by the field of the bunch
- Generates a **head-tail chirp** of betatron frequencies

$$\frac{d^2X(\mu,\zeta)}{d\mu^2} + \left(1 + 2\frac{\Delta\omega_\perp}{\omega_\perp}\right)X(\mu,\zeta) = \frac{2\eta_t}{L_t^2}\int_0^\zeta X(\mu,\zeta')\,(\zeta - \zeta')d\zeta'$$

- Need a small enough beam size

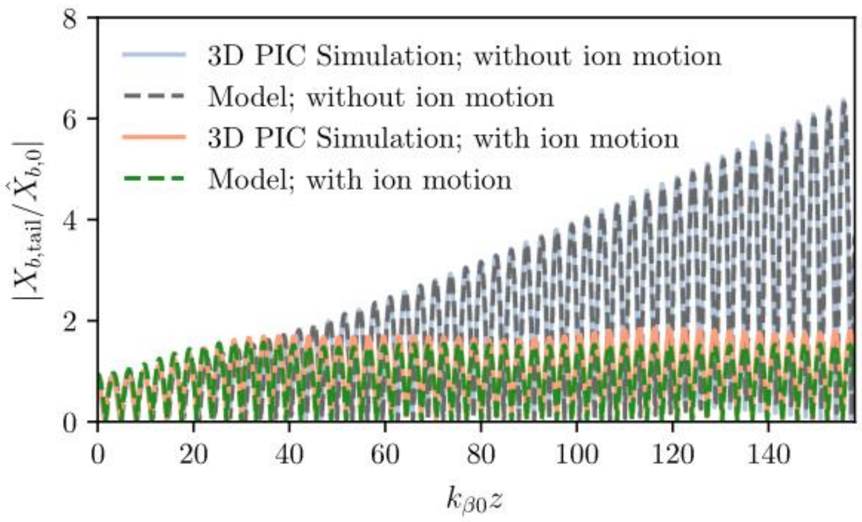
$$\frac{N r_i L_t}{2\sigma^2} \geq \eta_t$$

- Projected emittance degrades due to accumulation of betatron phase advance between the head and the tail

$$\mu_i = \sqrt{2 r_p N L_t / M_i \sigma^2}$$

  - Having heavier ions helps

Works in simulation!



T. Mehrling et al., PRL 121 264802 (2018)

How applicable is the standard instability analysis?

More accurate wake treatment might be needed

PHYSICAL REVIEW ACCELERATORS AND BEAMS **21,** 041301 (2018)

Short-range wakefields generated in the blowout regime of plasma-wakefield acceleration

G. Stupakov
SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

(Received 10 November 2017; published 2 April 2018)

Also presented
G. Stupakov, DESY/ U. Hamburg sem. 21.11.17

Standard instability analysis assumes linear wakes:

- Only then one can obtain the wake of the bunch by integrating wake Green's function with the charge distribution
- Not true in general: the plasma wake is generated by a **nonlinear** flow of plama e.

$$F_t(\zeta) = e^2 \int_0^{\zeta} \underline{\rho(\zeta')rW_\perp(\zeta - \zeta')d\zeta'}$$

?

How applicable is the standard instability analysis?

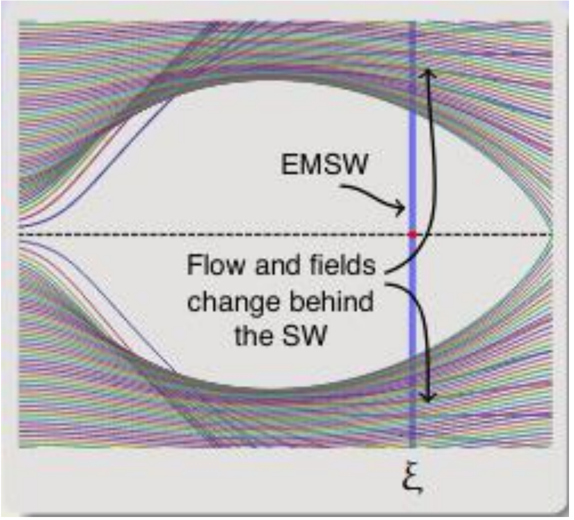
More accurate wake treatment might be needed

Standard instability analysis assumes linear wakes:

- Only then one can obtain the wake of the bunch by integrating wake Green's function with the charge distribution
- Not true in general: the plasma wake is generated by a **nonlinear** flow of plama e.

Can move forward only in special cases

- Special case 1: **charge** is **small**
- Special case 2: **length** of the witness is **small** $k_p \sigma_z << 1$



G. Stupakov, DESY 11 Nov. 2017

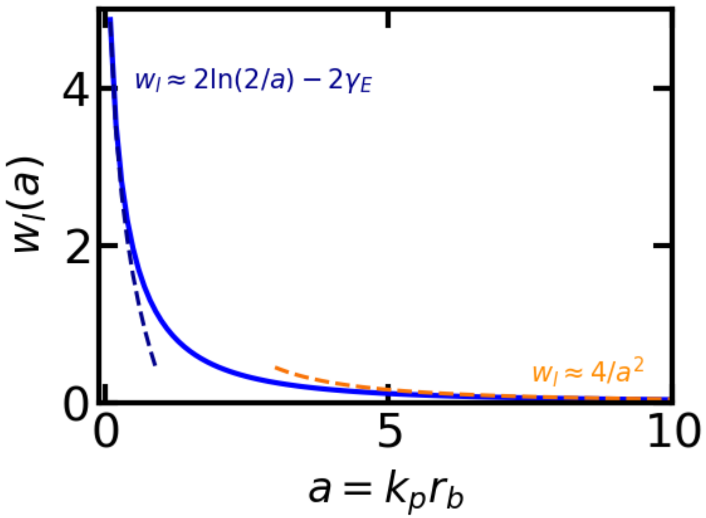
Fields in EM shock wave are **linear functions of charge**

Transverse wake can be significantly smaller

Assumption length of the witness is small $k_p\ \sigma_z << 1$

Hollow cillindrical channel

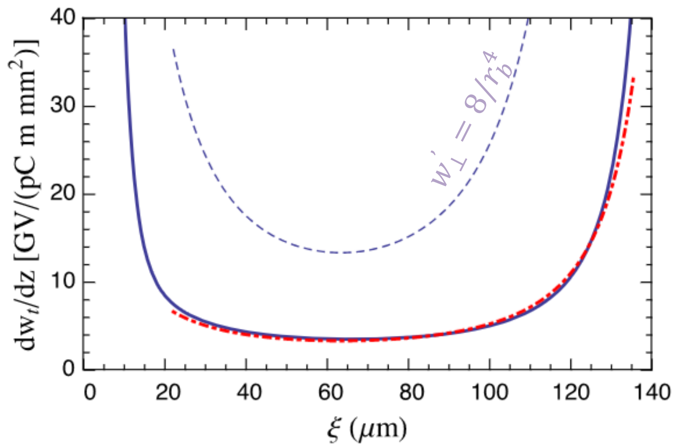
$$w_{\parallel}(\zeta) \approx \frac{4K_0(a)}{a^2 K_2(a)}, \qquad a = k_p r_b$$



- At a >> 1: a **universal expression** for short-distance wake in plasma, resitive, dilectric, or corrugated structure

General case

- Numerical simulation required



G. Stupakov, PRAB 21 041301 (2018)

$n_0 = 4\mathrm{x}10^{16}$ cm^{-3}
k_p = 26 um
σ_z = 13 um
σ_r = 5 um
Q = 1 nC

- EM field penetrates into plasma $\Delta r \sim k_p^{-1}$
  - Possible **correction**:

$$w_\perp'(\zeta) = 8/(r_b(\zeta) + 0.75\ k_p^{-1})^4$$

“Careless limit[1]”

An exact upper limit on the strength of defocusing wakes

On “Efficiency versus Instability in plasma accelerators”

S. S. Baturin*
Department of Electrical Engineering and Department of Physics, Northern Illinois University, DeKalb, IL 60115, USA
(Dated: October 5, 2020)

$$\eta_t \leq \frac{\eta_p{}^2}{4(1-\eta_p)}$$

The ‘geometric’ wake is an exact upper bound for the longitudinal wake potential:

- Converges to an exact solution in the limit $k_p r_b >> 1$
- The defocusing force can only be smaller

Efficiency-instability relation becomes an ineqality

- Lower bound (what happens if not designed carefully)
- The upper limit is yet to be established

At large bubble radius

$$W_{\parallel}(\zeta) \approx \frac{4L}{r_b{}^2}(1-\frac{2\Delta E_z(r_b)}{r_b}) \leq \frac{4L}{r_b{}^2}$$

$$W_r(\zeta) \approx \frac{r_0}{r_b{}^2}\int_0^{\zeta} W_{\parallel}(s)\, ds$$

[1] In conventional accelerators the broadband impedance of a ring does not exceed |Z/n| < 1/2 Z_0 aka “careless limit”

There seems to be no fundamental limit

Dipole beam breakup is a significant threat to performance

- Both for conventional and advanced accelerators
- Have to make sure damping mechanisms are in place

Multiple mitigation strategies exist

- BNS damping through ion motion seems promising

Efficiency-instability relation is a lower bound

- Provides insight for tolerance and instabilty analysis
- Shows how bad the system can perform if not designed carefully

Takeaway for a conventional accelerator physicist

- We are laking a satisfactory theory for collective beam instabilities with nonlinear wakefields

Thank you

Let’s keep things stable

Parameter comparison

Note: the numbers for plasma designs might be somewhat outdated

$$L \approx \frac{f_{rep} N^2}{4\pi \sigma_x^* \sigma_y^*} = \frac{P_b N}{4\pi E \sigma_x^* \sigma_y^*}$$

$$P_w = 2P_b / \eta_p$$

$$L/P_w \approx \frac{1}{E} \eta_p \frac{N}{8\pi \sigma_x^* \sigma_y^*}$$

Table 1. RMS emittances and momentum spreads for various linear collider proposals.

	ILC	CLIC	LPA	PWFA
Beam energy, TeV	0.25	1.5	0.5	1.5
Luminosity, 10^{34} cm^{-2}s^{-1}	1.8	6	2	6.3
Particles per bunch, 10^{10}	2	0.37	0.4	1
Bunch rep. rate, kHz	6.5	15.6	15	10
σ_x, nm	474	40	10	194
σ_y, nm	6	1	10	1.1
RMS norm.h.emit., ε_x, μm	10	0.66	0.1	10
RMS norm.v.emit., ε_y, μm	0.035	0.020	0.1	0.035
RMS mom. spread, %	0.1*	0.35	N/C	N/C
RMS bunch length, μm	300	44	1	20
IP size ratio, $R = \sigma_x/\sigma_y$	79	40	1	176
$R_\varepsilon = (1 + \varepsilon_x \beta_y^* / \varepsilon_y \beta_x^*)/2$	7	0.84	1	1.8
Emit. margin, $\varepsilon_{ny}\|_{\max}/\varepsilon_y$	12	4	16	2

V. Lebedev et al., Rev. Acc. Sci. Tech. 9 (2016)