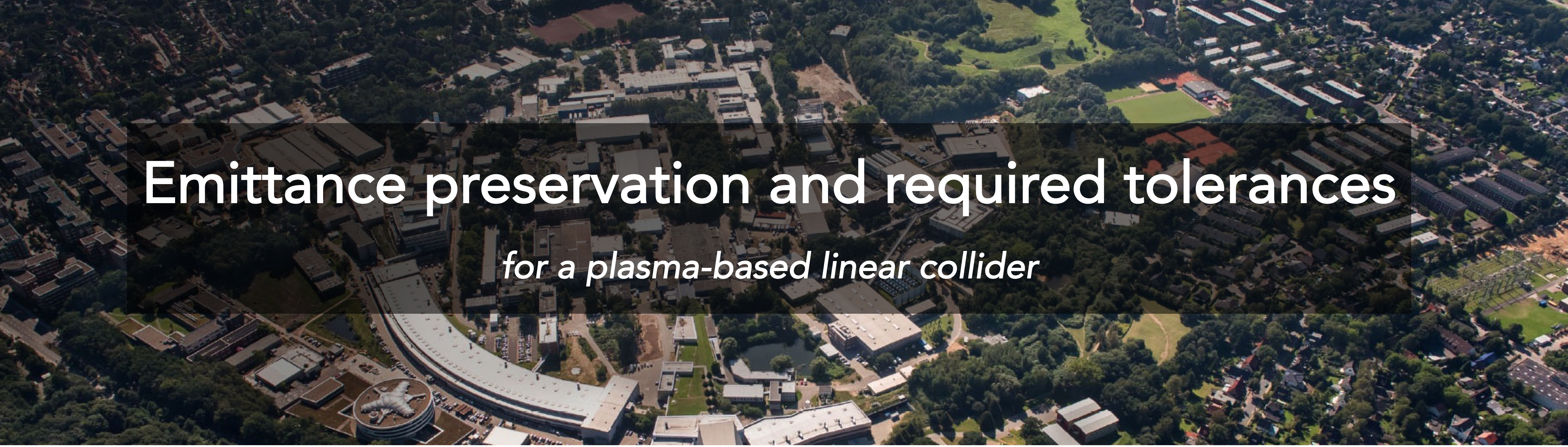


Emittance preservation and required tolerances

for a plasma-based linear collider

Maxence Thévenet – DESY

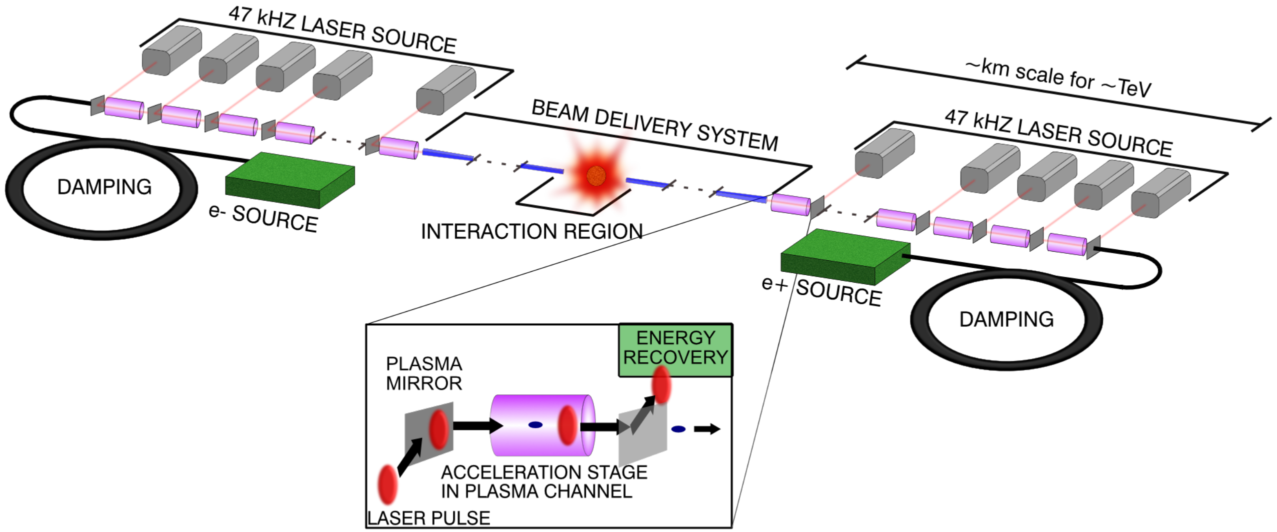
ALEGRO-2023, DESY, Hamburg, 23/03/2023

Based on
C. A. Lindstrøm & M. Thévenet
Emittance preservation in advanced accelerators
JINST **17** P05016 (2022)

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

A plasma-based linear collider needs low emittance



Source: Benedetti et al., White paper (Snowmass 2021)

$$\mathcal{L} = H_D \frac{N^2 f}{4\pi\sigma_x^*\sigma_y^*}$$

Energy $\mathcal{E} \simeq 0.1 - 15$ TeV

Luminosity $\mathcal{L} > 10^{34} cm^{-2} s^{-1}$

Emittance $\epsilon_{x/y} \sim 10 - 100\ nm$

➢ Short beams, flat or round beams

W. Leemans and E. Esarey *Phys. Today* 62.3: 44-49 (2009)
E. Adli et al. *arXiv:1308.1145* (2013)

Emittance is a measure of the beam quality

Normalized emittance $\epsilon \equiv \epsilon_n = \sqrt{\langle x^2\rangle\langle u_x^2\rangle - \langle xu_x\rangle^2}$

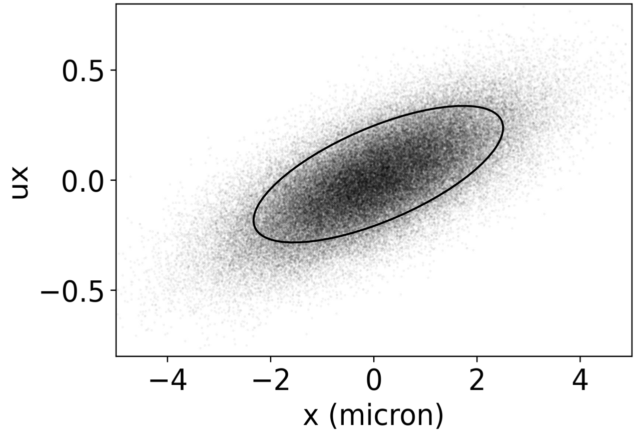
$$u_x = \frac{p_x}{m_e c}$$

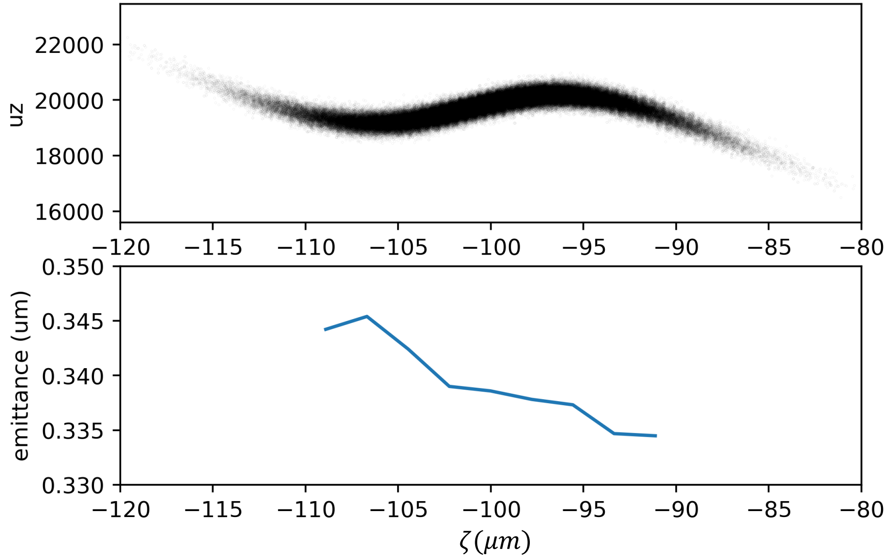
Trace-space emittance $\epsilon_{tr} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$

Low energy spread limit: $\epsilon_n \simeq \gamma\epsilon_{tr}$

$$\beta = \frac{\langle x^2\rangle}{\epsilon_{tr}} \simeq \gamma\frac{\langle x^2\rangle}{\epsilon_n}$$

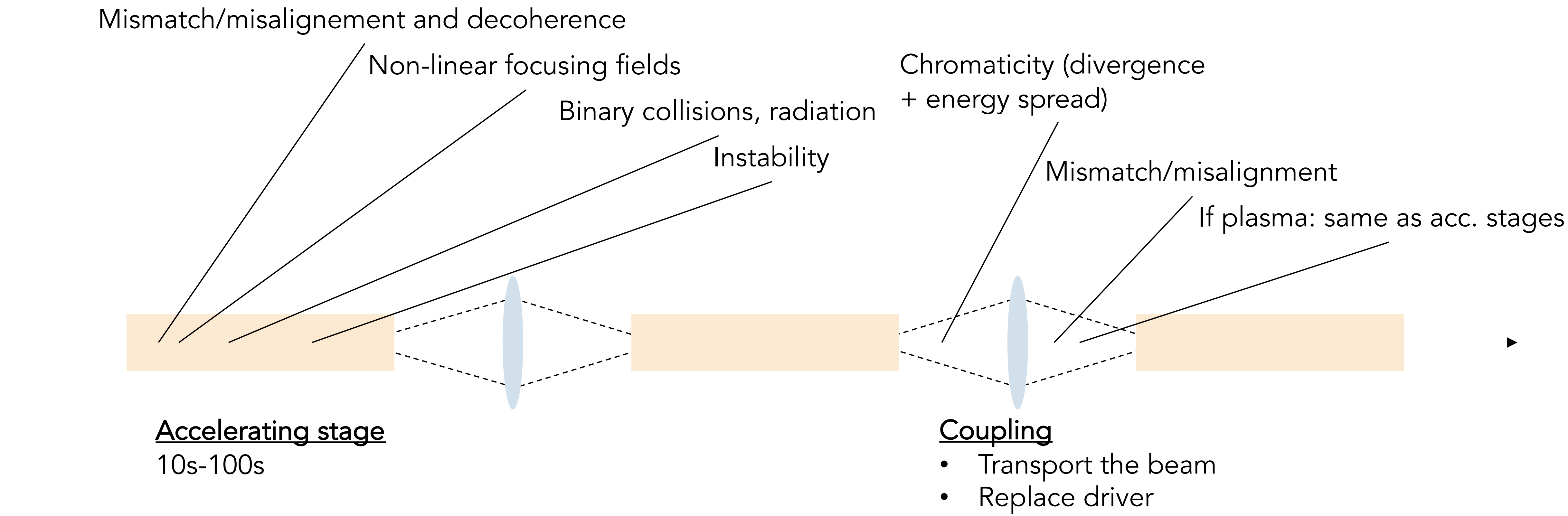
- Can be preserved in the presence of linear focusing fields
- Projected emittance & slice emittance





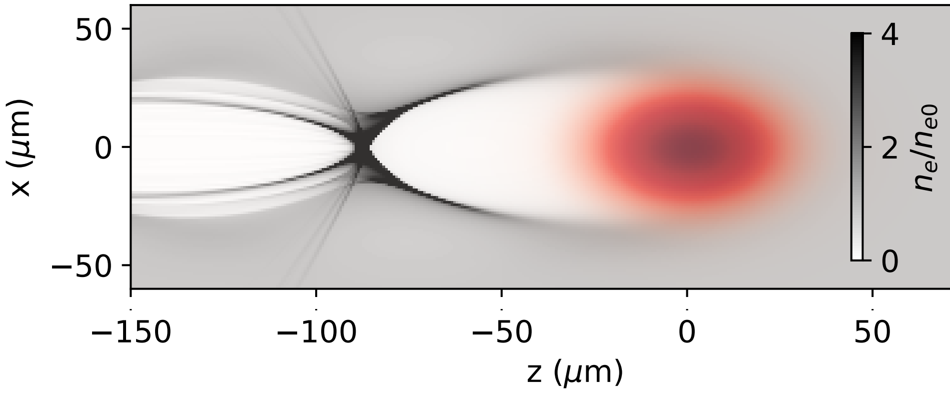
Floettmann PRSTAB 6, 034202 (2003)

Sources of emittance growth within a plasma-based collider



1. Energy spread and decoherence
2. Non-linear focusing fields
3. Hose instability
4. Additional sources

Beam dynamics within a stage: betatron oscillations



Blowout regime

Longitudinal: acceleration: $E_0 = \frac{m_e \omega_p c}{e}$; $\dot{\gamma} \simeq \omega_p$

Transverse: betatron oscillations $\frac{\mathrm{d}(\gamma m_e \dot{x})}{dt} = \mathrm{F_x}$. Full blowout, linear focusing force → $\omega_\beta = \frac{\omega_p}{\sqrt{2\gamma}}$

→ Betatron frequency depends on particle's energy

→ Betatron amplitude $\propto \gamma^{-\frac{1}{4}}$

→ Matching condition, beam envelope does not evolve: $\beta_m = \frac{\sqrt{2\gamma}}{k_p}$ or $\sigma_x = \left(\frac{2\epsilon^2}{\gamma k_p^2}\right)^{1/4}$ and $\sigma_{u_x} = \left(\frac{k_p^2 \epsilon^2 \gamma}{2}\right)^{1/4}$

J. B. Rosenzweig *et al.* *PRA* 44.10 R6189 (1991)
J. M. Dawson, John *Physical Review* 113.2: 383 (1959)

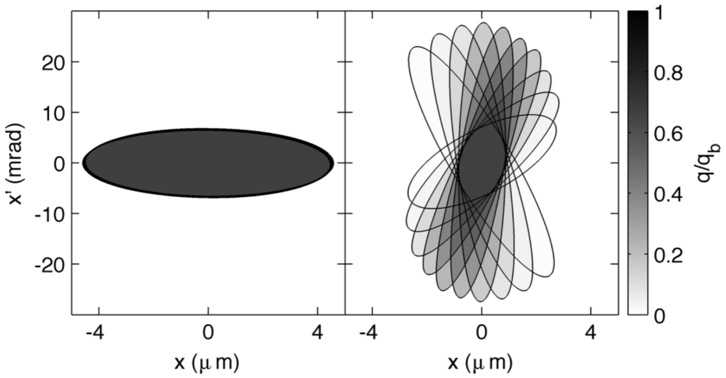
Mismatch and misalignment result in emittance growth

Mismatch results in envelope oscillations

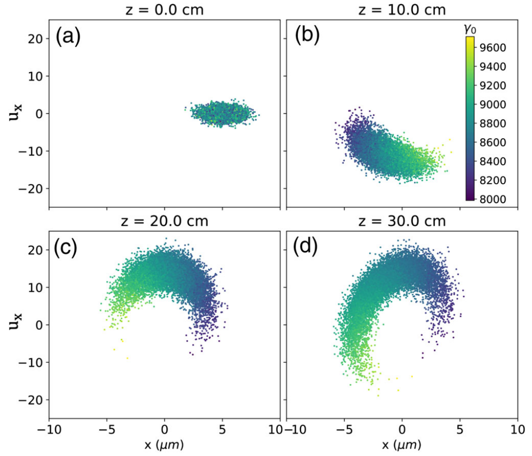
Different energy slices oscillate with different periods (decoherence)

This causes in (slice) emittance growth

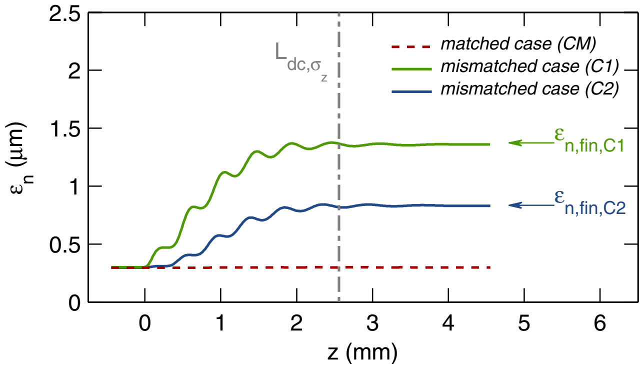
- Decoherence length $L_{decoherence} = \frac{\lambda_\beta}{\sigma_\gamma}$
- Saturated emittance $\frac{\epsilon_{\text{sat}}}{\epsilon_{\text{init}}} = \frac{1}{2}\left(\mathcal{M} + \frac{1}{\mathcal{M}}\right)$ $\qquad \mathcal{M} = \frac{1}{2}\left(\tilde{\beta} + \tilde{\gamma} + \sqrt{(\tilde{\beta}+\tilde{\gamma})^2 - 4}\right)$



Source: [1]



$$\Delta\epsilon_{\text{misalign}} \approx \frac{1}{2}\left(\frac{\Delta x^2}{\beta_m} + \beta_m \Delta x'^2\right)$$



[1] T. Mehrling et al. Phys. Rev. ST Accel. Beams 15 111303 (2012)
C. A.Lindstrøm et al., Proceedings of IPAC2016 (2016)
M. Thévenet et al., Phys. Rev. Accel. Beams 22: 051302 (2019)
S..Cheshkov, et al., PRSTAB 3: 071301 (2000)

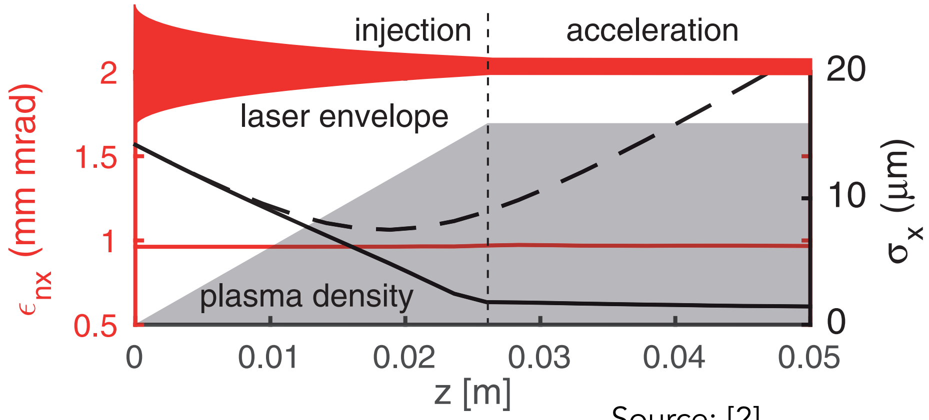
- ➢ First stages: full decoherence within 1 stage
- ➢ Later stages: (almost) no decoherence within 1 stage (but average emittance growth)
- ➢ 10 nm, μrad tolerances

Adiabatic ramps mitigate emittance growth due to mismatch

- Adiabatic changes in focusing properties prevent emittance growth
- Ramp length $\gg \lambda_\beta$
- Useful for entrance (upramp) or exit (downramp) of plasma stage

$$K(z) = \frac{K_0}{(1+gz)^4}$$

- Ramp length unpractical for high-energy beams



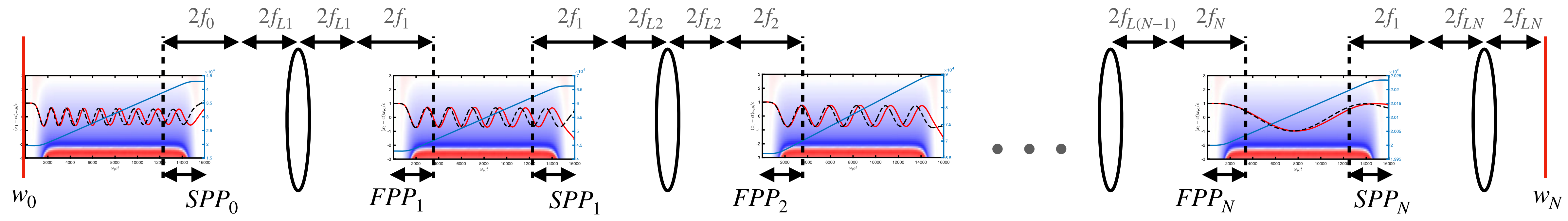
Source: [2]

[1] K. Floettmann PRSTAB 17: 054402 (2014)
[2] I. Dornmair et al., PRSTAB 18: 041302 (2015)

Energy spread is a key factor in this source of emittance growth

Chromatic focusing and mismatch:

"we find that for initial relative energy spreads below 10^{-3}, energy-spread growth below 10^{-5} of the energy gain per stage and normalized emittance below mm-mrad, the chromatic emittance growth can be minimal "



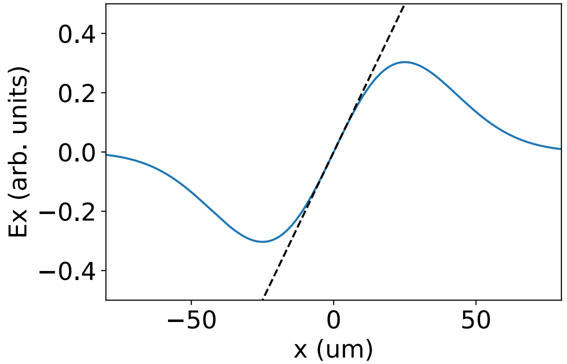
Source: A. G. R. Thomas & D. Seipt PRAB **24** 104602 (2021)

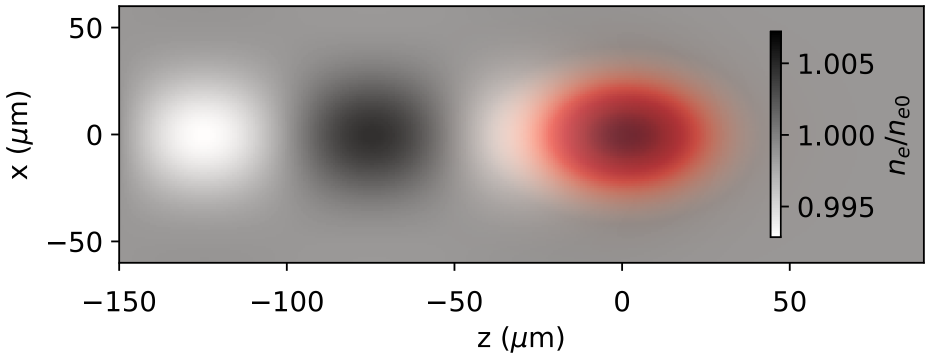
1. Energy spread and decoherence
2. Non-linear focusing fields
3. Hose instability
4. Additional sources

Non-linear focusing fields can cause emittance growth

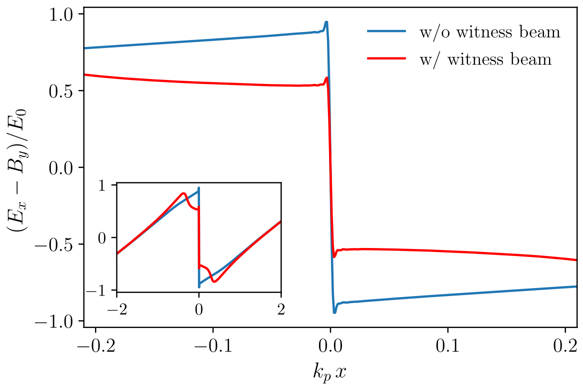
1/3: Non-linear plasma fields

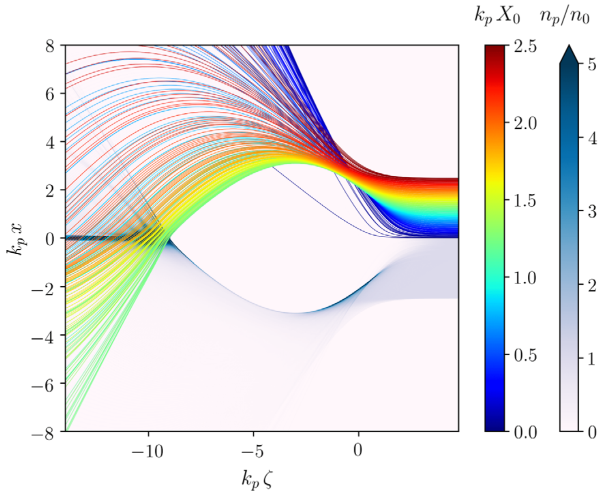
<u>Case 1: linear regime</u>





<u>Case 2: Positron acceleration</u>





Source: S. Diederichs PRAB 22, 081301 (2019)

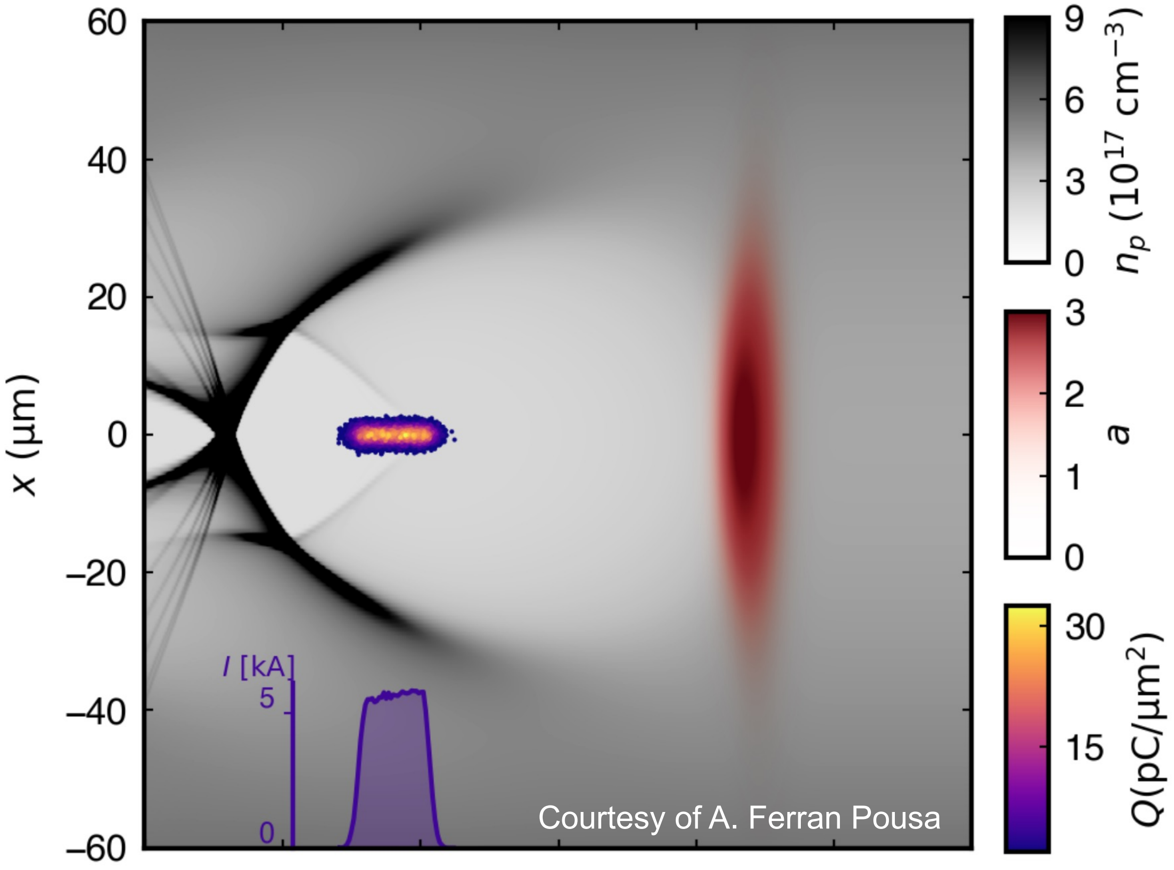
- (ζ-dependent) Equilibrium is reached, at the price of emittance growth
- Starting from a (non-trivial) matched distribution, emittance can be preserved

Non-linear focusing fields can cause emittance growth

2/3: Response of the plasma electrons: transverse beam loading

- Linear and non-linear (just not full blowout), plasma electrons can be present in the cavity
- A high-density beam perturbs the electron density
- ζ-dependent and non-linear focusing field

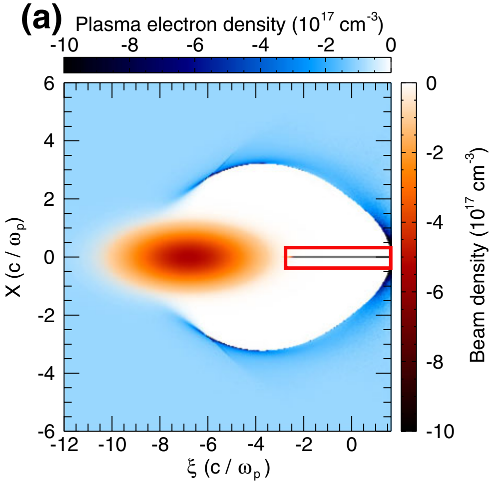
→ This picture evolves during propagation

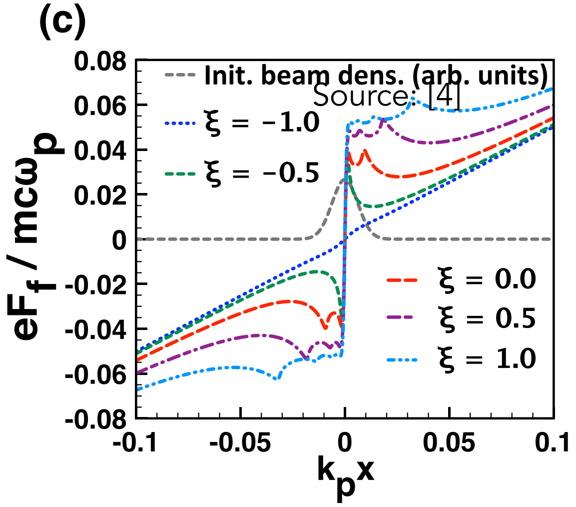


Non-linear focusing fields can cause emittance growth

3/3: Response of the plasma ion: ion motion

- Strong beams perturb ions, which create an ion density spike [1].
- Non-linear, leading to ζ-dependent emittance growth
- Mitigated with heavier ions
- Bound to happen as $\sigma_x \propto \gamma^{-1/4}$



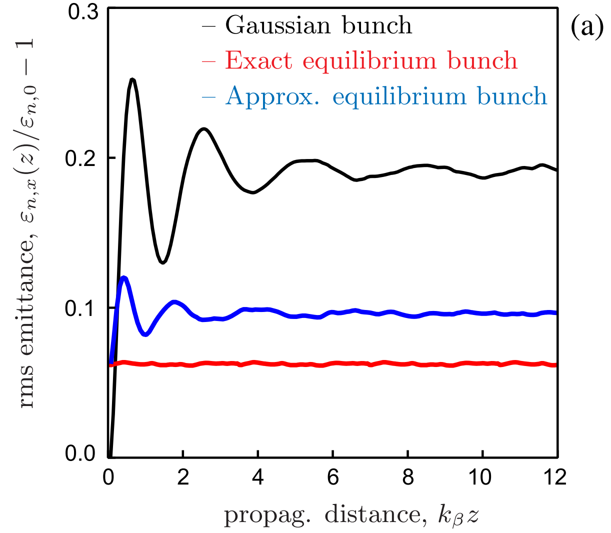


Source: [4]

Fortunately
- ➢ A matched beam will see no emittance growth [4]
- ➢ Such a beam can be created with adiabatic matching [5]

$$\Lambda = Z_i \frac{m}{M_i} \frac{I_b}{I_A} \frac{L_b^2}{\sigma_x^2} \gtrsim 1$$

Source: [3]



- Advanced beam profiles (transverse and longitudinal) can preserve emittance growth
- Such profiles can be achieved & preserved with adiabatic process (modest emittance cost)
- Can these beams be transported? Does this constraint stage design?

[1] J. B. Rosenzweig et al., PRL 95 195002 (2005)
[2] R. Golizadeh PRL 104, 155001 (2010)
[3] C. Benedetti et al., PRAB 20, 111301 (2017)
[4] W. An et al., PRL 118 244801 (2017)
[5] C. Benedetti et al., Phys. Plasmas 28, 053102 (2021)

1. Energy spread and decoherence
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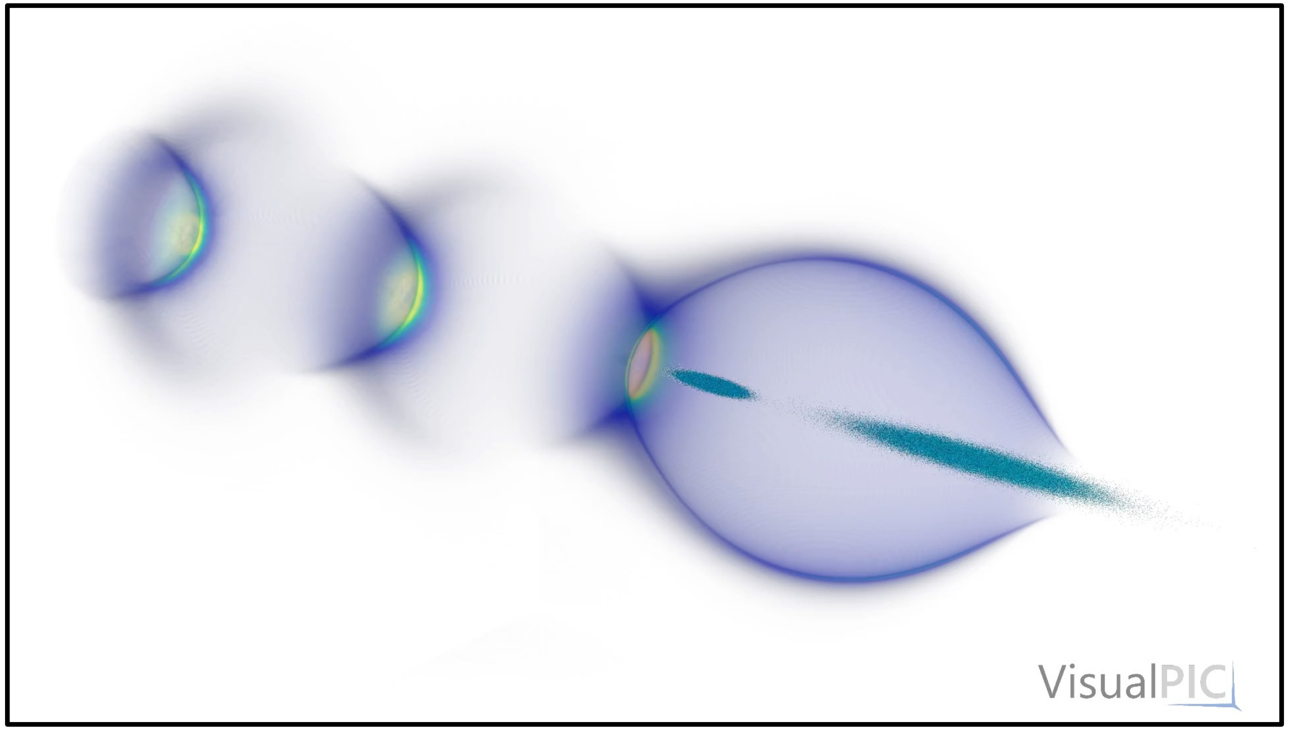
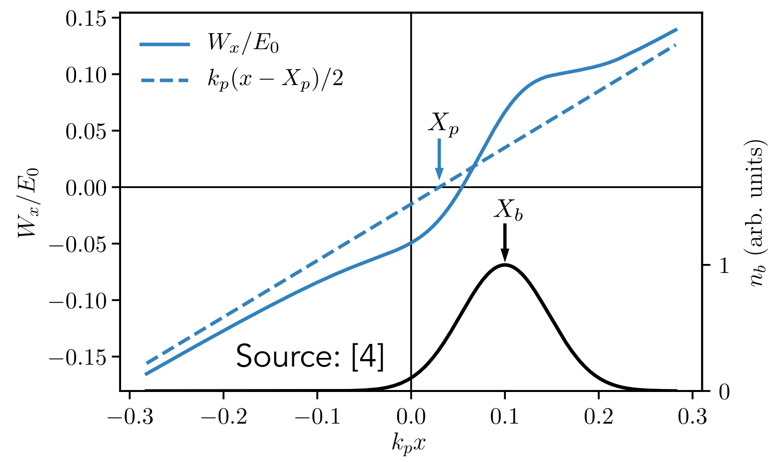
Hose instability can lead to catastrophic emittance growth

- Coupled transverse oscillations of beam centroid and wake centroid
- In particular for the driver
- Originally thought dramatic [1,2]
- Similar to beam breakup instability [3]



[1] D.H. Whittum, et al., PRL 67 991 (1991)
[2] C. Huang et al., PRL 99 255001 (2007)
[3] W. K. H. Panofsky and M. Bander RSI 39 206 (1968)

Hose instability mitigation through BNS damping



$$\omega_\beta = f(\zeta)$$

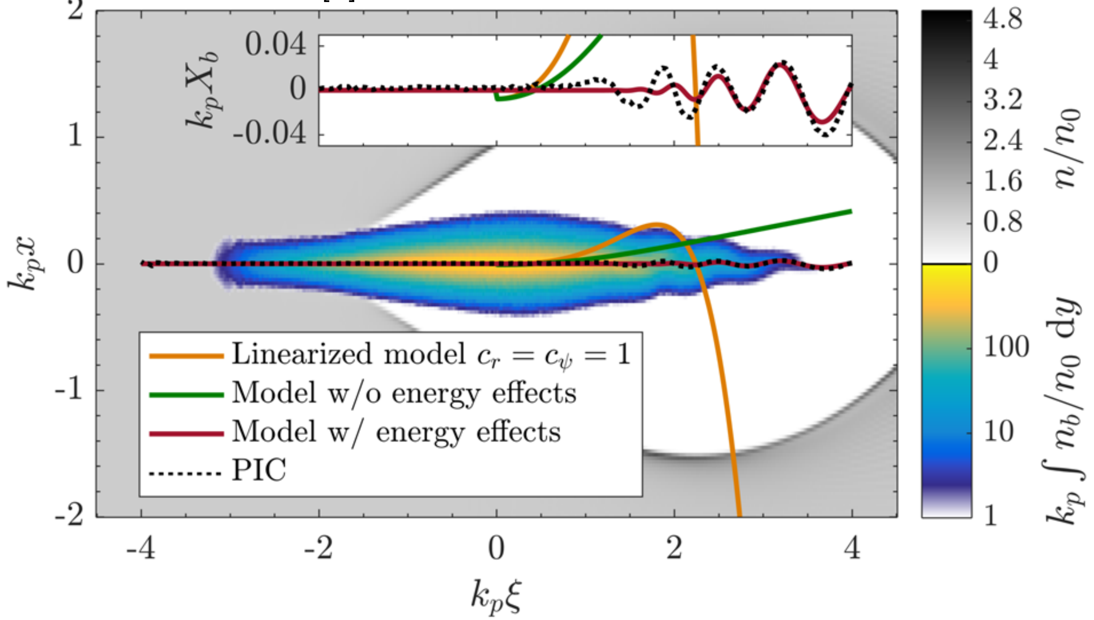
[1]: linear regime: saturates as $F_x = f(\zeta)$ due to transverse beam loading

[2]: blowout regime: saturates as $\gamma = f(\zeta)$ due to strong chirp (driver)

[3]: Large beam driver for $F_x = f(\zeta)$

[4]: ion motion suppresses it

Source: [2]



V. E. Balakin et al., Proceedings of HEACC1983, 119–120 (1983)
[1] R. Lehe et al., PRL 119 (2017)
[2] T. J. Mehrling et al., PRL 118 174801 (2017)
[3] A. Martinez de la Ossa et al. PRL 121 064803 (2018)
[4] T. J. Mehrling PRL 121: 264802 (2018)

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Other sources/sinks of emittance within accelerator stage

Coulomb scattering

- Increases beam divergence as

$$\frac{d\langle\theta^2\rangle}{ds} = \frac{k_p^2 r_e}{\gamma^2}\left[Z_i^2 \ln\left(\frac{\lambda}{R_a}\right) + 1.78Z(Z+1)\ln\left(\frac{287}{\sqrt{Z}}\right)\right],$$

- And related emittance growth (matched beam)

$$\frac{d\epsilon_n}{ds} = \frac{\beta_x}{2}\frac{d\langle\theta^2\rangle}{ds}\gamma,$$

- Does not depend on matched density
- In practice, prevents the use of high-Z species
- Mitigation: hollow core plasma channels

Radiation cooling

- Particles with larger betatron amplitude loose more energy to synchrotron radiation
- Classical radiation reaction

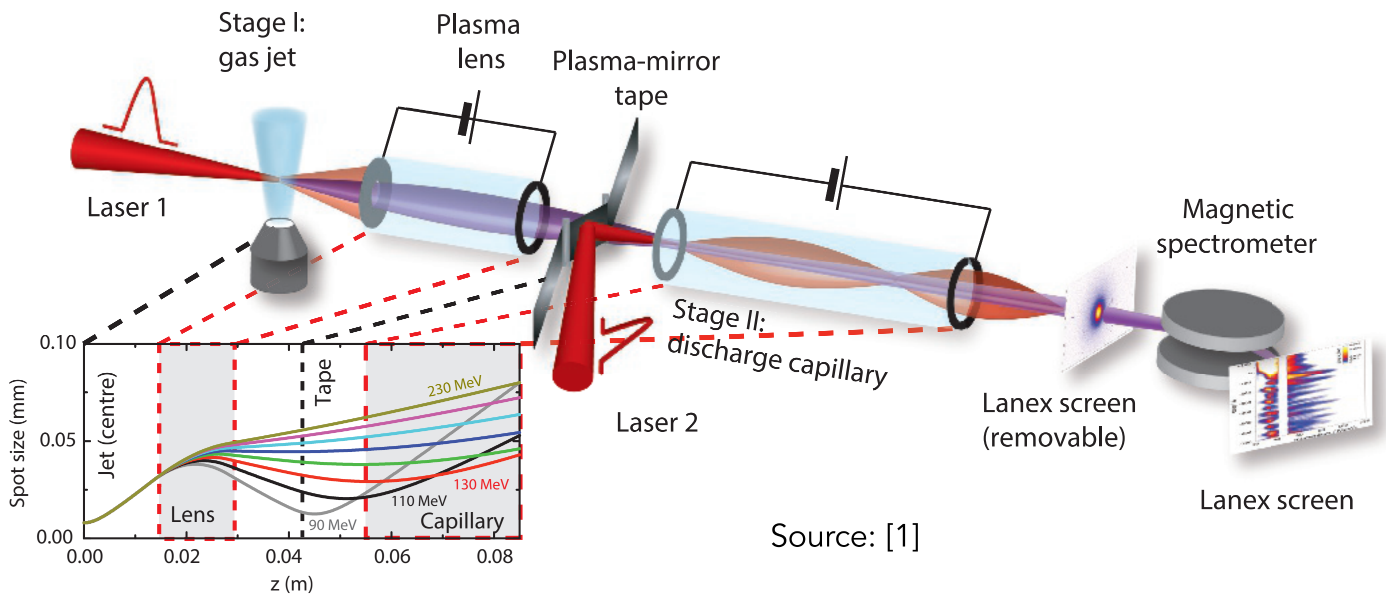
$$\ddot{x} + \tau_R c^2 K^2 \dot{x} + K^2 c^2 x/\gamma = 0$$

→ Growth in energy spread

$$\frac{\sigma_\gamma}{\gamma} \propto n_e \gamma^{\frac{3}{2}} \epsilon$$

B.W. Montague Proceedings of CAS-ECFA-INFN pp. 208–218 (1984)
C.B. Schroeder et al., JINST 17 P05011 (2022)
C. B. Schroeder et al., PRSTAB 13, 101301 (2010)
P. Michel et al., PRE 74, 026501 (2006)

Inter-stage coupling is also a source of emittance growth



Source: [1]

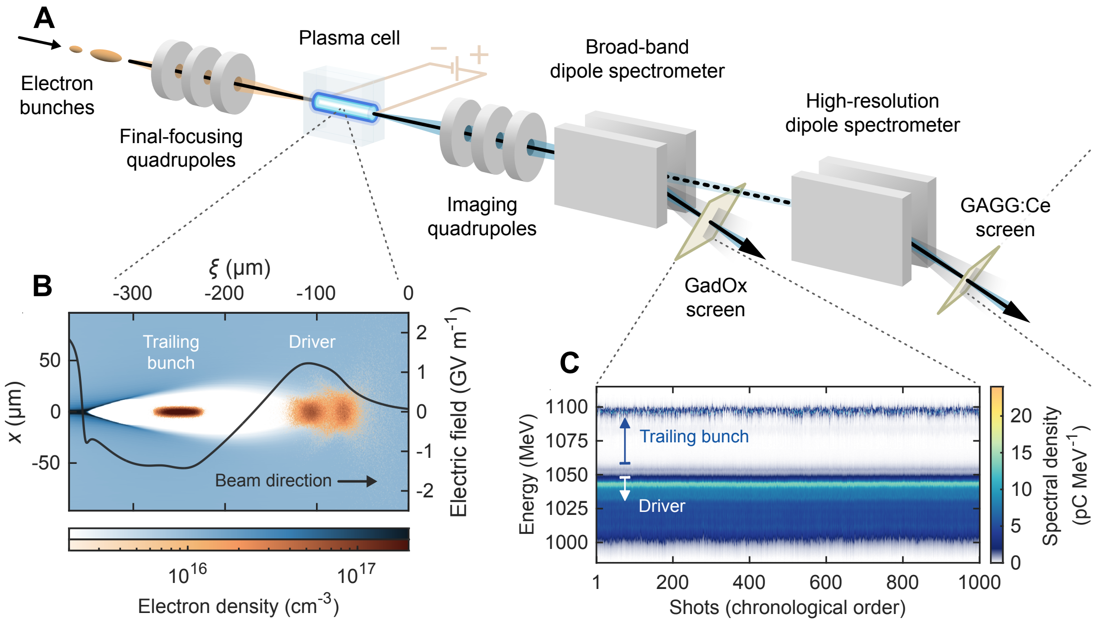
- Relatively large divergence and large energy spread beams
- Sources of emittance growth during coupling:
  - Drift space + energy spread
  - Chromatic focusing $\frac{\Delta(\epsilon^2)}{\epsilon_{\text{init}}^2} = \frac{4L^2}{\beta_m^2}\sigma_\delta^2$
  - For plasma-based focusing optics: all of the above
  - Mismatch in the following stage
- Mitigated with advanced beam optics

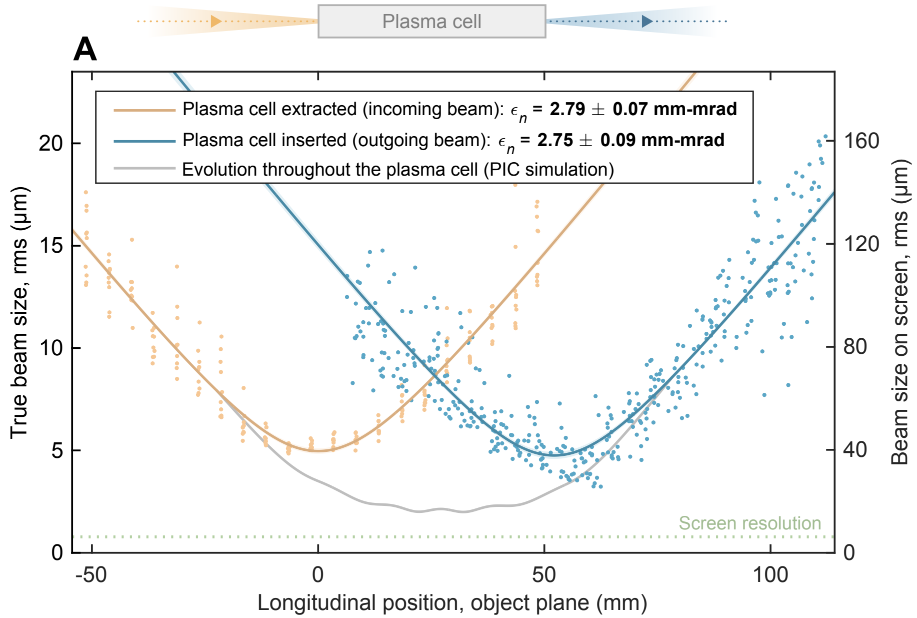
[1] S. Steinke et al., Nature 530 190 (2016)
[2] M. Migliorati et al. PRSTAB 16 011302 (2013)

Encouraging: recent demonstration of emittance preservation

Material provided by Carl A. Lindstrøm, Univ. Oslo

Experiment at FLASHForward





Experimental setup: 1 GeV beam driver with 400 pC charge
50 mm plasma cell: peak density ~ $1.2 \times 10^{16} cm^{-3}$ (with ramps)

Stable operating point: 40 MeV energy gain, 22% transfer efficiency
(1.4 GV/m estimated peak field)

Preservation of: **Charge** (40 pC), in 41% of shots
Energy spread (0.12% FWHM or lower), in 62% of shots
Emittance in x dfirection

Lindstrøm, Carl Andreas, et al. "Preservation of beam quality in a plasma-wakefield accelerator." (2022).

Conclusion

- Various effects cause emittance growth, but mitigation methods exist
  - Misalignment/mismatch, non-linear focusing fields, Coulomb collisions, coupling
  - Preliminary studies suggest tolerances on the order of 10 nm and 1 μrad (can be relaxed, e.g. plasma shaping)
- Energy spread can also lead to emittance growth via phase mixing
  - Sources of energy spread must also be monitored
  - Preliminary studies suggest tolerances on the order of 10^{-3} (initial) and 10^{-5} (per stage)
- Recent results show encouraging results
  - Hose instability can be mitigated, with a potential cost on emittance/shaping
  - Emittance preservation in a full plasma stage was demonstrated
- Interplay between different effects would need to be scrutinized
  - Understanding and mitigation methods for separate sources of emittance growth
  - Coupling & transversely tailored profiled; coupling and longitudinally-tailored profile; …

Thank you for your attention