FIXED-ORDER QCD PREDICTION OF DRELL-YAN OBSERVABLES

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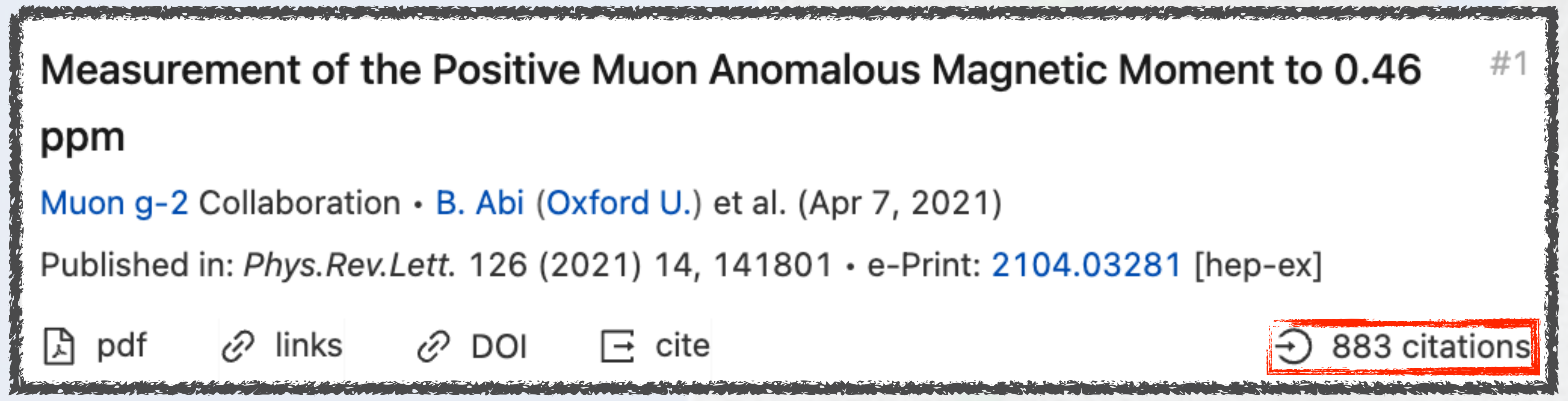
Swiss National Science Foundation

“THREE CLOUDS” IN PARTICLE PHYSICS SINCE 2021

Test of lepton universality in beauty-quark decays #1

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Mar 22, 2021)

Published in: *Nature Phys.* 18 (2022) 3, 277-282 • e-Print: 2103.11769 [hep-ex]



Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm #1

Muon g-2 Collaboration • B. Abi (Oxford U.) et al. (Apr 7, 2021)

Published in: *Phys.Rev.Lett.* 126 (2021) 14, 141801 • e-Print: 2104.03281 [hep-ex]

High-precision measurement of the W boson mass with the CDF II detector #1

CDF Collaboration • T. Aaltonen (Helsinki U. and Helsinki Inst. of Phys.) et al. (Apr 8, 2022)

Published in: *Science* 376 (2022) 6589, 170-176

Statistics from iNSPIRE-HEP by 22-07-2022

- Fitting the elephant with BSM free parameters
  - The “oblique corrections” S-T-U in vacuum polarisation:

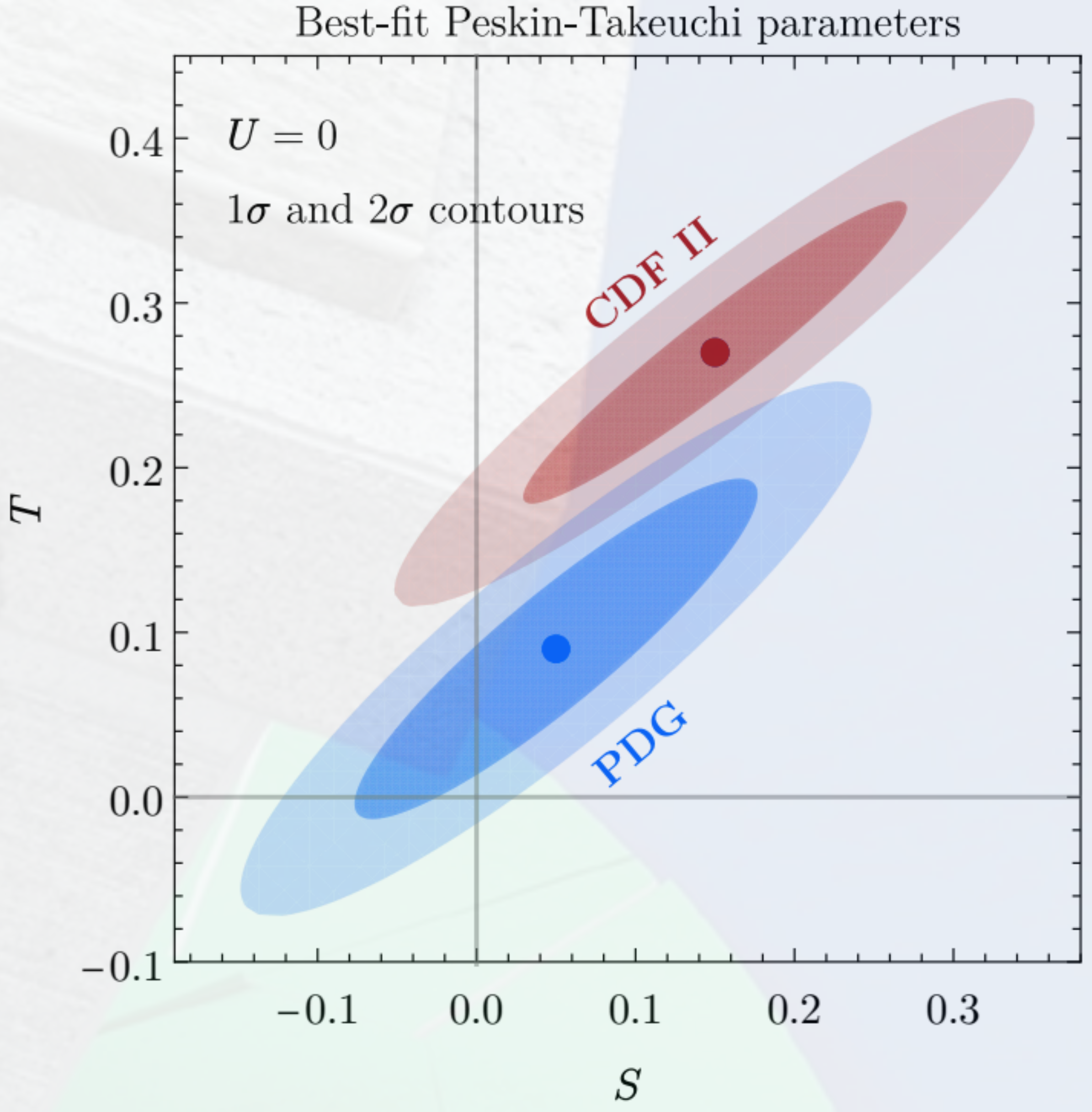
$$\alpha S = 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$

$$\alpha T = \frac{e^2[\Pi_{11}(0) - \Pi_{33}(0)]}{\sin^2(\theta_W)\cos^2(\theta_W)m_Z^2}$$

$$\alpha U = 4e^2[\Pi'_{11}(0) - \Pi'_{33}(0)]$$

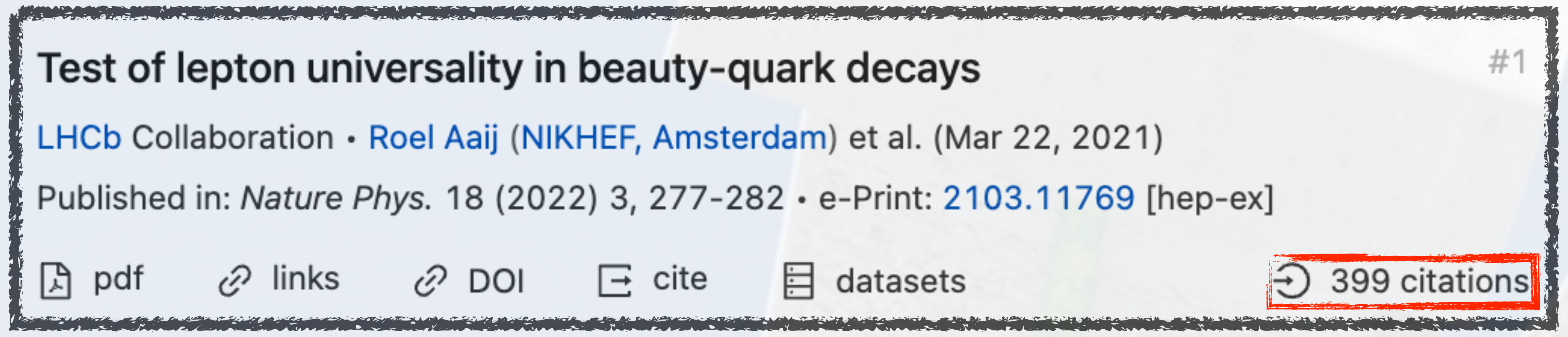
Peskin and Takeuchi `92

Carpenter, Murphy, Smylie `22



- Further experimental confirmation
  - Fermilab Run 2 ~ Run 5 analysis
  - LHCb Upgrade I (2025) and II (2030)
  - ATLAS, LHCb, CMS all have on-going analysis of W mass.

“THREE CLOUDS” IN PARTICLE PHYSICS SINCE 2021



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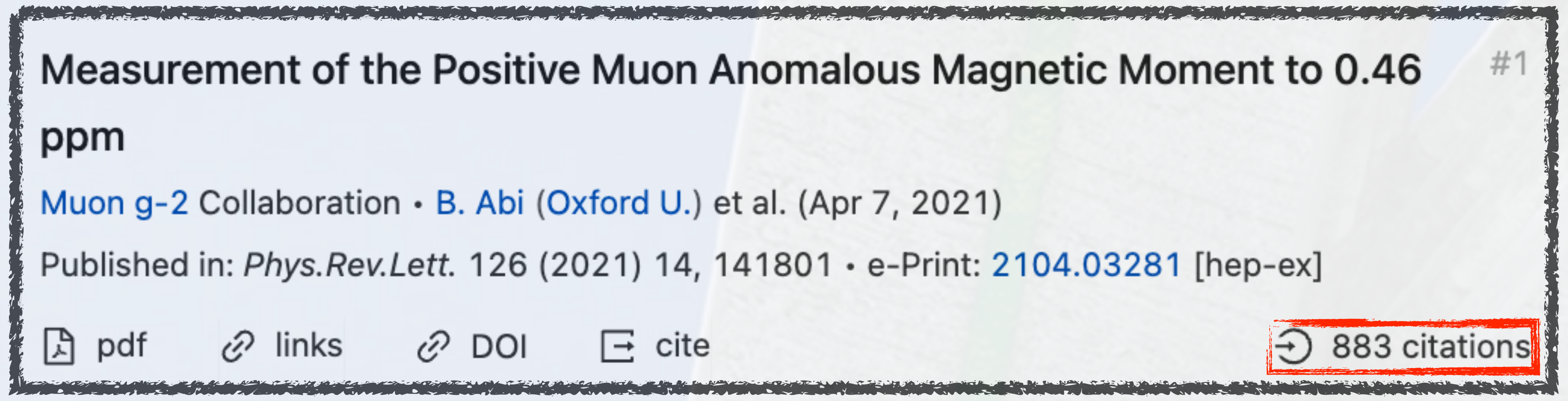

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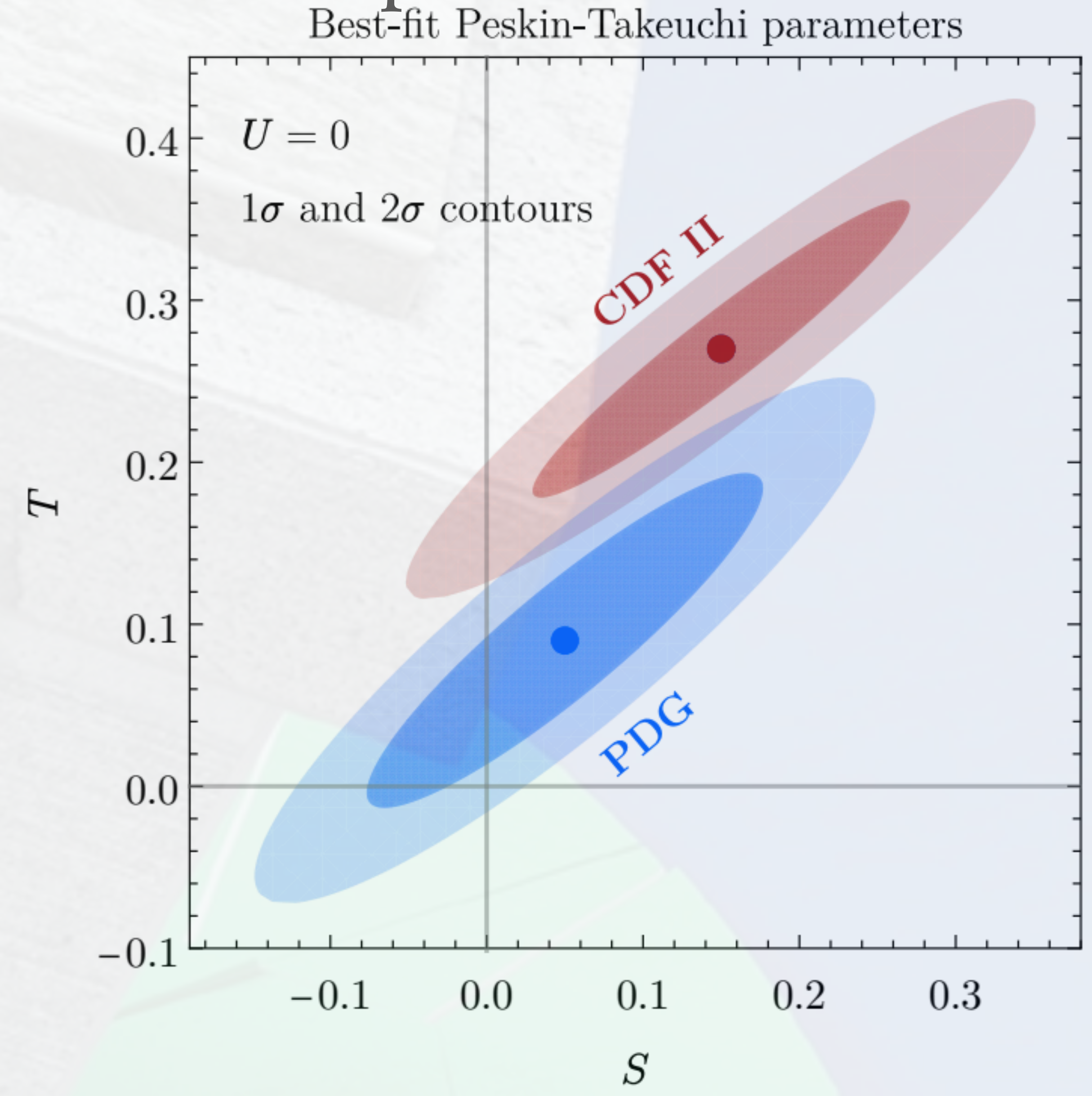
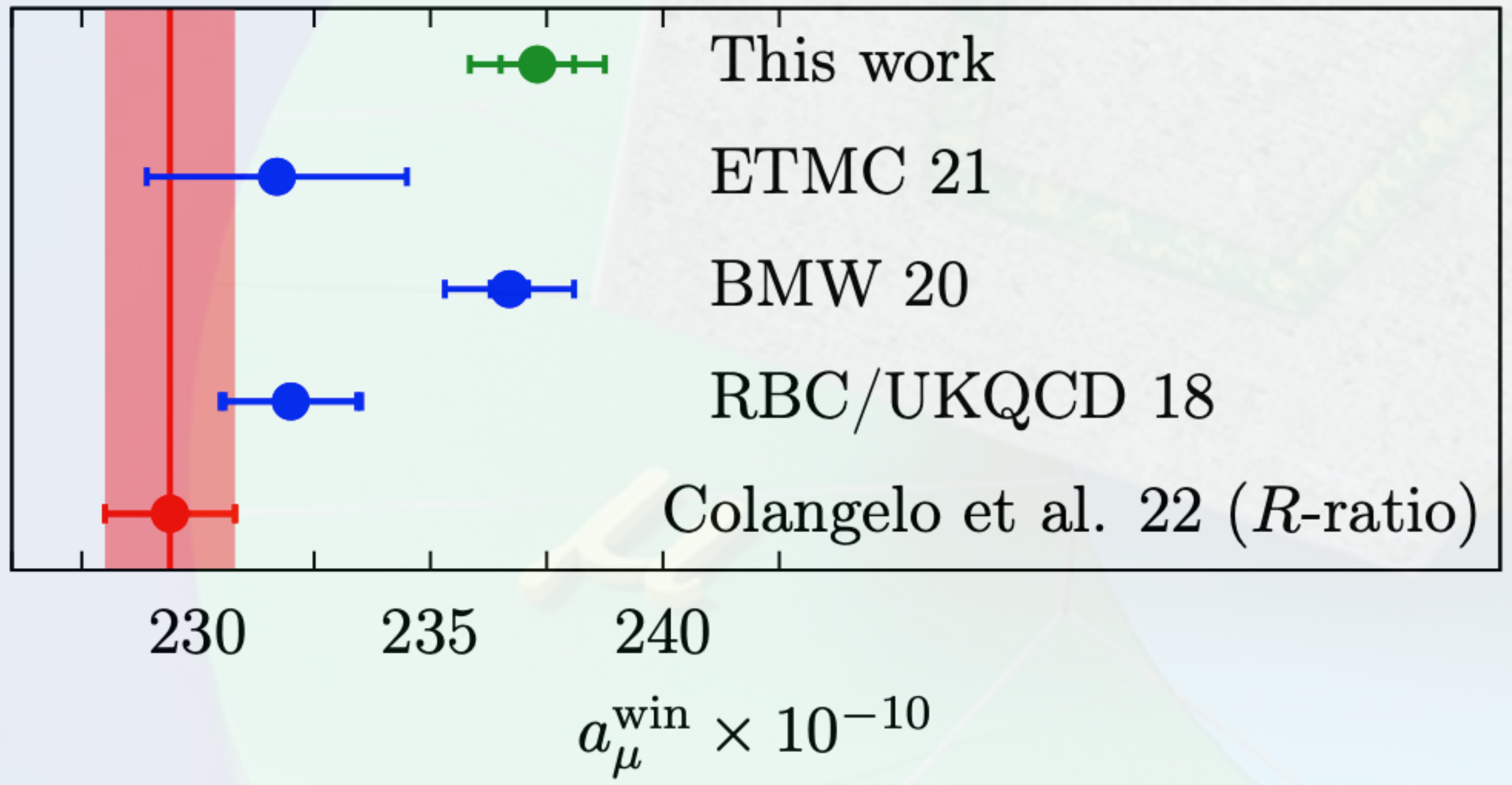
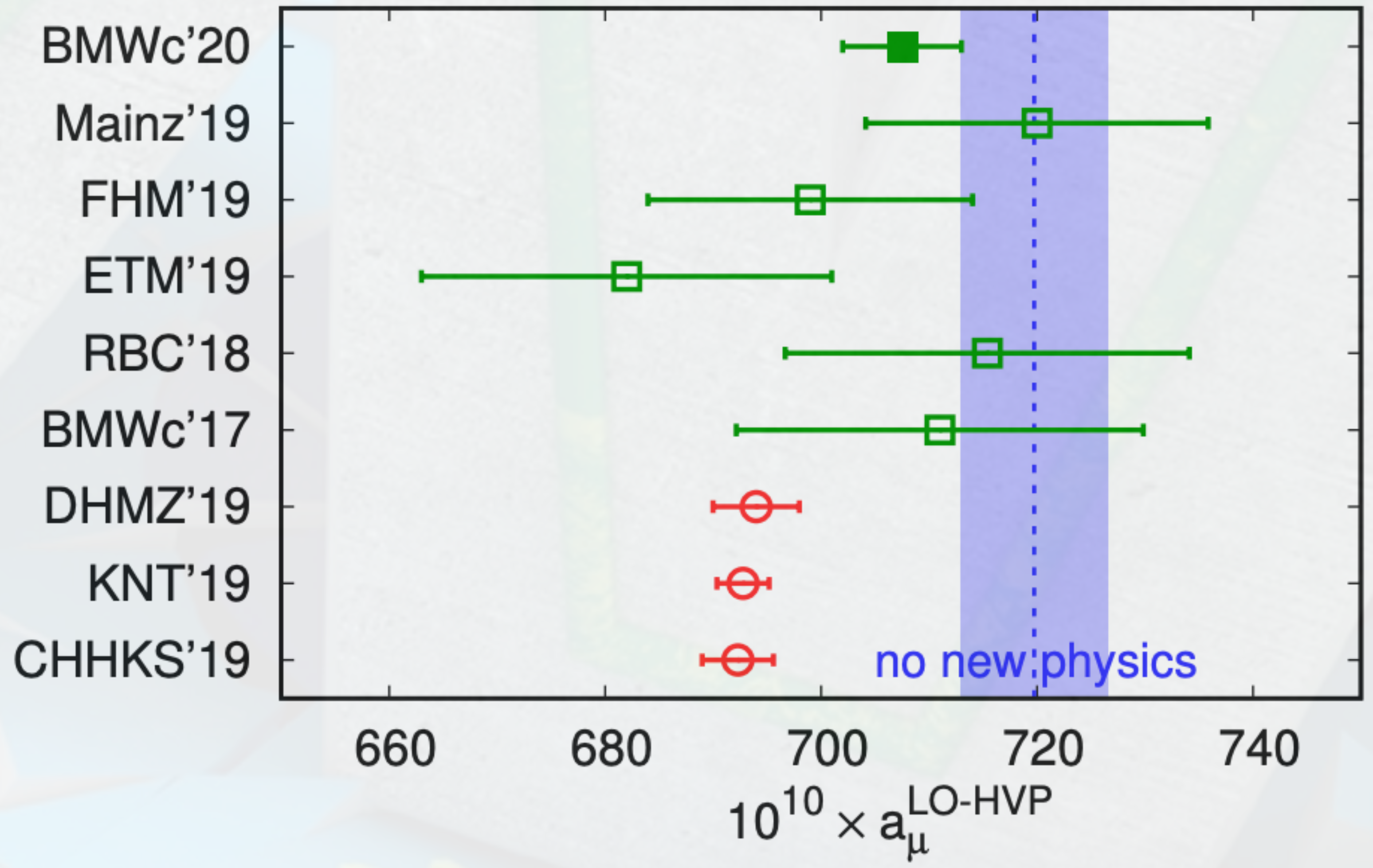


Ce, Gerardin, et. al. `22

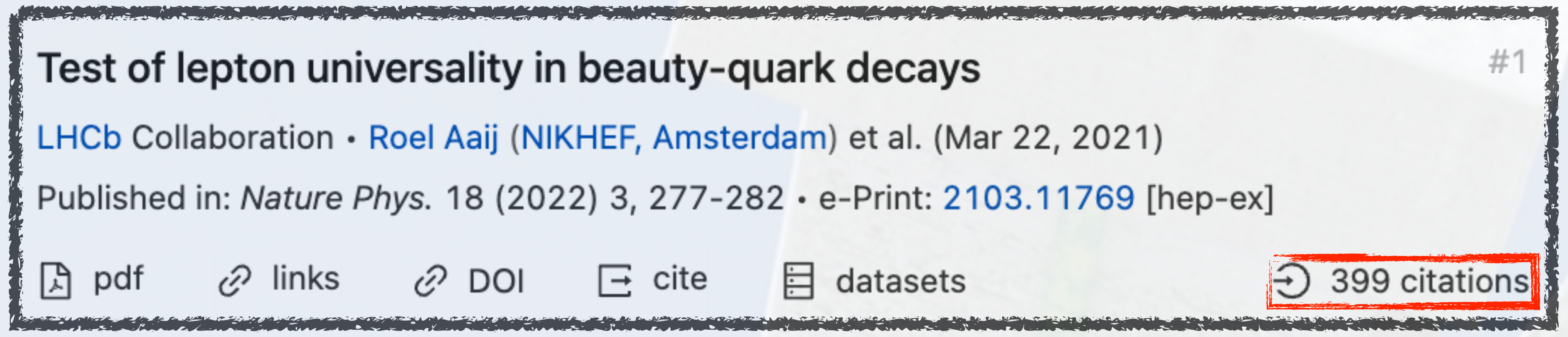


Borsanyi, Fodor et. al. `20



- Challenge experiment with better/alternative predictions
  - Lattice prediction of HVP in g-2
  - Improve template fit in CDFII (ResBos@NLO+NNLL)

“THREE CLOUDS” IN PARTICLE PHYSICS SINCE 2021



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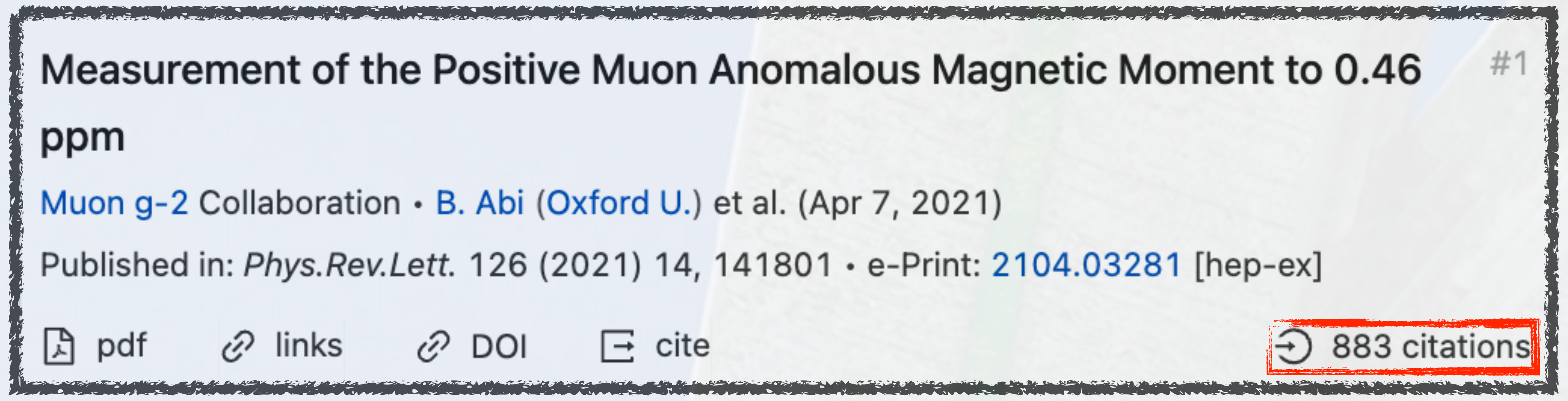

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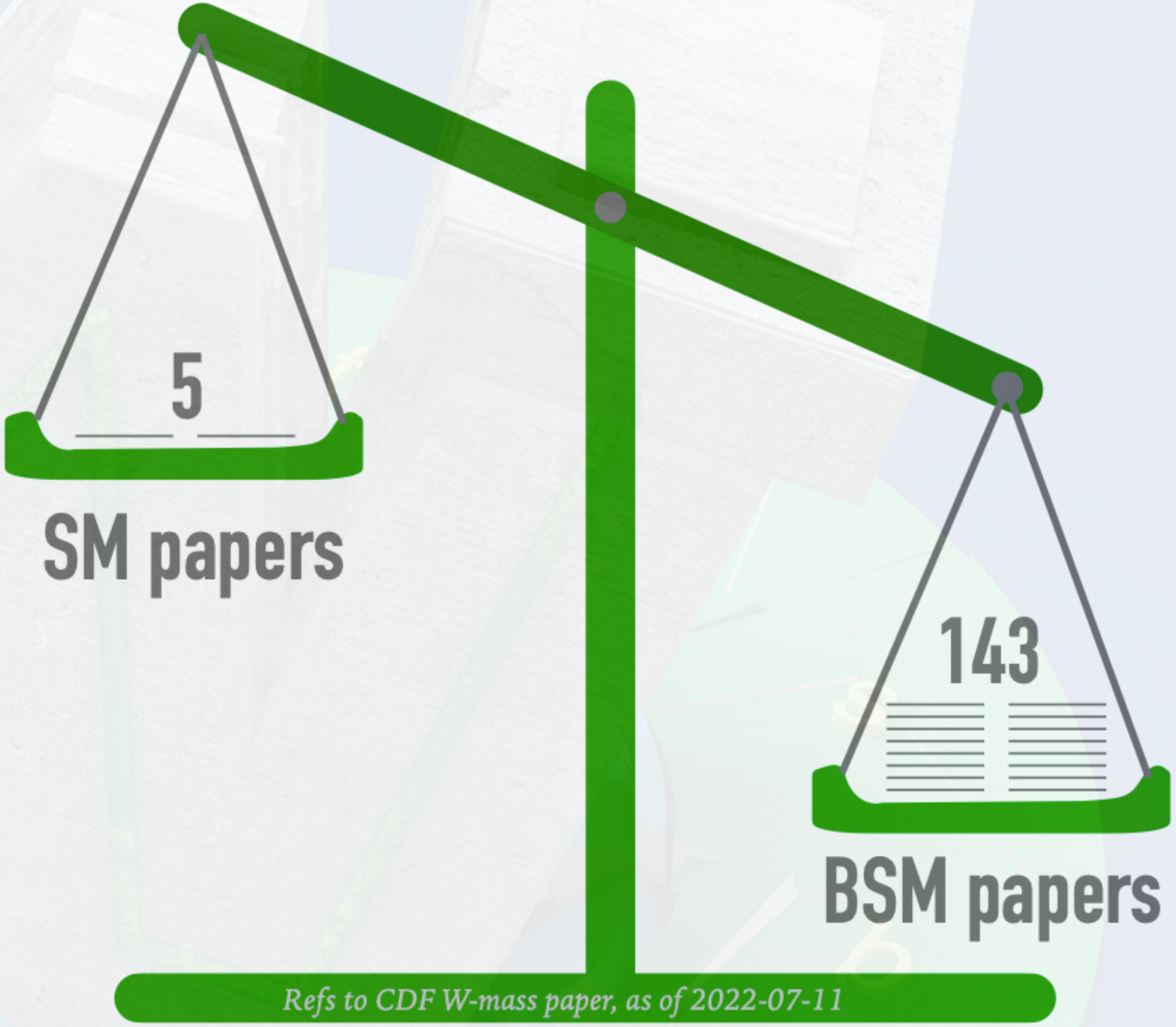
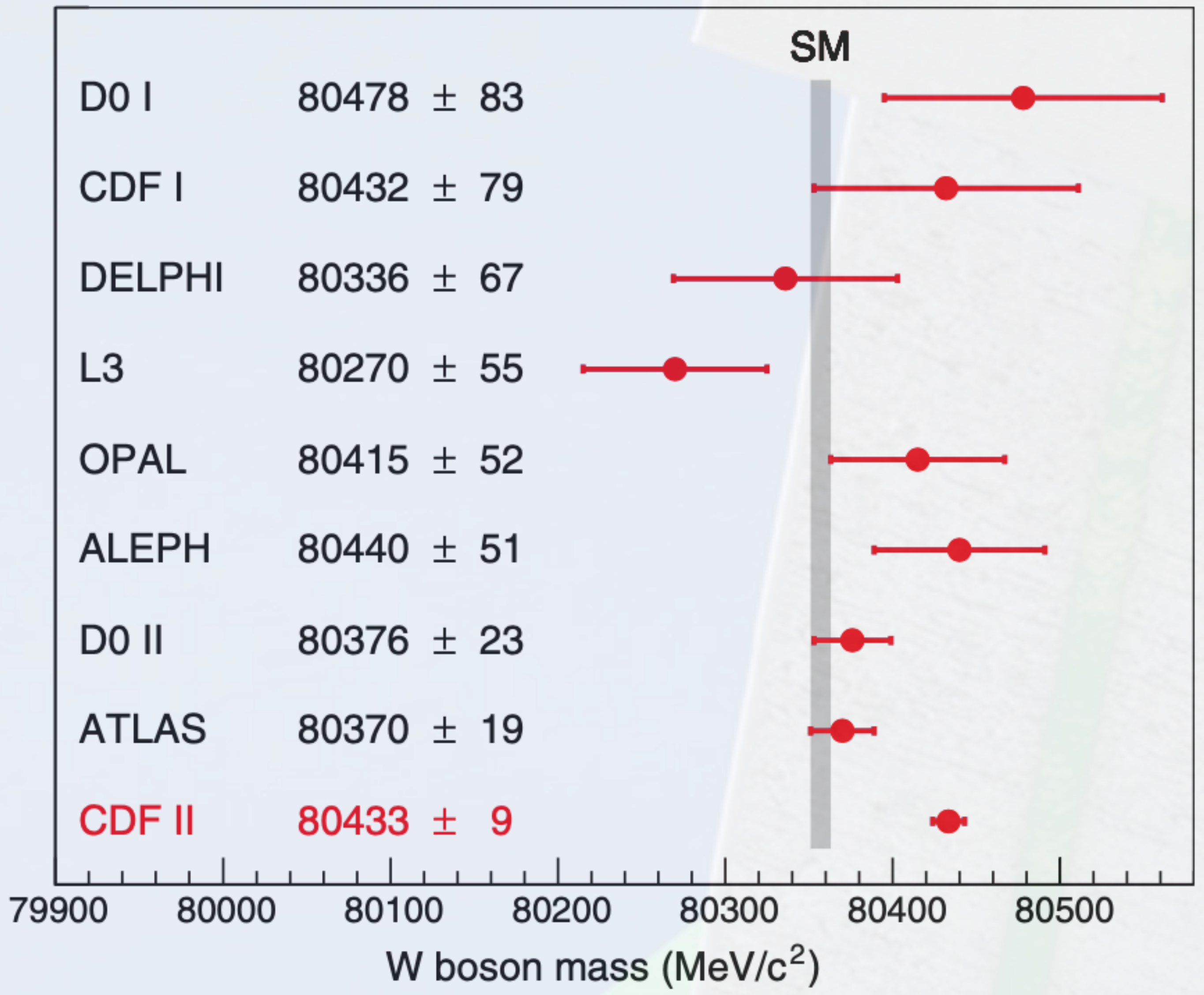
Slide by Gavin Salam ICHEP 2022 →



W MASS IN CDFII MEASUREMENT



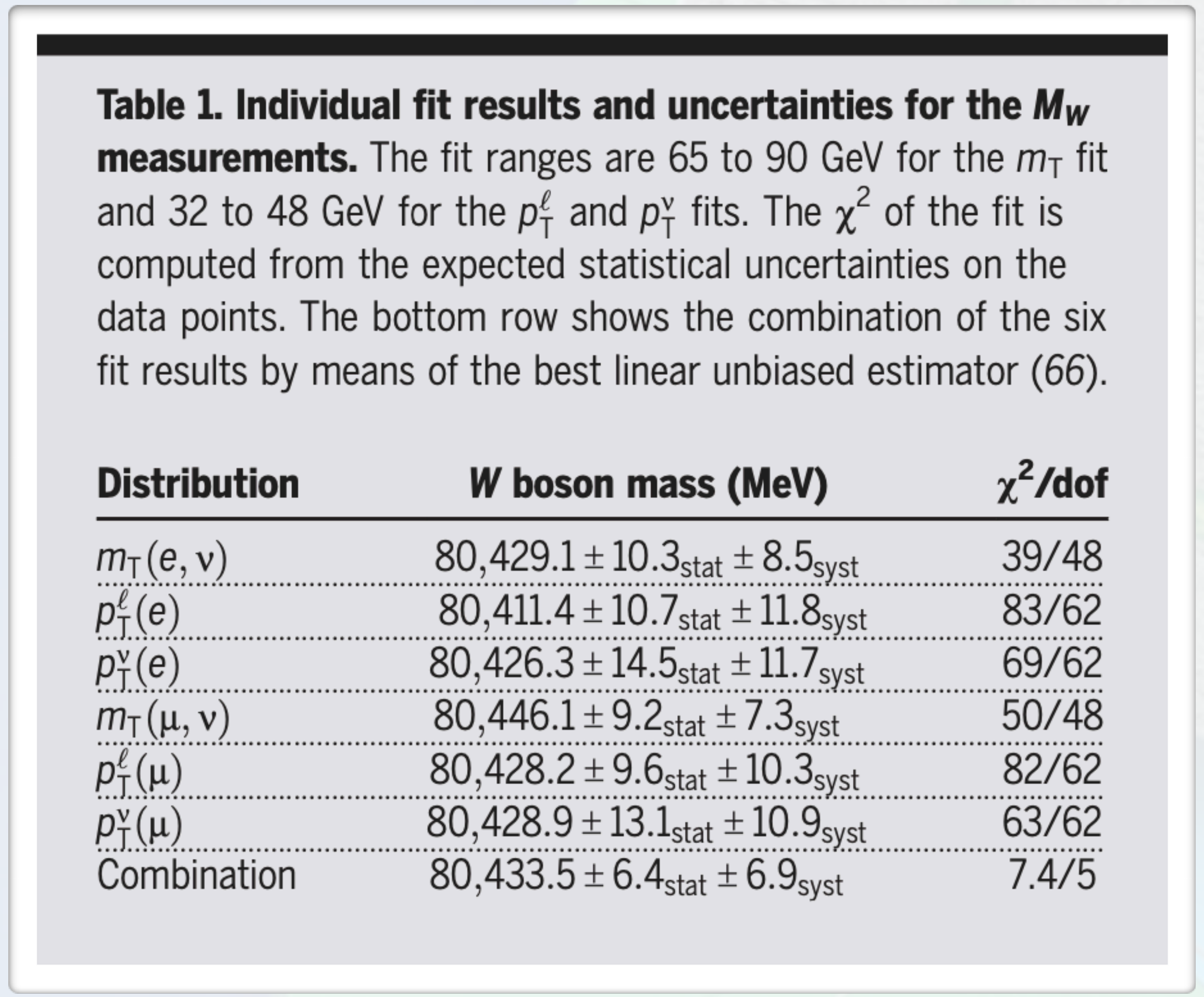


Table 1. Individual fit results and uncertainties for the M_W measurements. The fit ranges are 65 to 90 GeV for the m_T fit and 32 to 48 GeV for the p_T^ℓ and p_T^ν fits. The χ^2 of the fit is computed from the expected statistical uncertainties on the data points. The bottom row shows the combination of the six fit results by means of the best linear unbiased estimator (66).

Distribution	W boson mass (MeV)	χ^2/dof
$m_T(e, \nu)$	$80{,}429.1 \pm 10.3_{stat} \pm 8.5_{syst}$	39/48
$p_T^\ell(e)$	$80{,}411.4 \pm 10.7_{stat} \pm 11.8_{syst}$	83/62
$p_T^\nu(e)$	$80{,}426.3 \pm 14.5_{stat} \pm 11.7_{syst}$	69/62
$m_T(\mu, \nu)$	$80{,}446.1 \pm 9.2_{stat} \pm 7.3_{syst}$	50/48
$p_T^\ell(\mu)$	$80{,}428.2 \pm 9.6_{stat} \pm 10.3_{syst}$	82/62
$p_T^\nu(\mu)$	$80{,}428.9 \pm 13.1_{stat} \pm 10.9_{syst}$	63/62
Combination	$80{,}433.5 \pm 6.4_{stat} \pm 6.9_{syst}$	7.4/5

- PDG world average: $m_W = 80379 \pm 12\ MeV$ (PDG`20)
- CDFII latest result: $m_W = 80433 \pm 9\ MeV$ (CDF `22)
  - Indirect measurement of m_T^W, p_T^l, p_T^ν distributions

$$p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2}$$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)}$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu (1 - \cos\Delta\phi)}$$

  - Template fit to the best parameter values
  - Full error = Experiment + Theory model
  - Experiment statistics: $\pm 6.4\ MeV$
  - Experiment systematic: $\pm 5.3\ MeV$
  - Theory model: $\pm 5.2\ MeV$ $\pm ??\ MeV$

ResBos, DYqT, PHOTOS, HORACE

Illustrator: Gaia Fontana

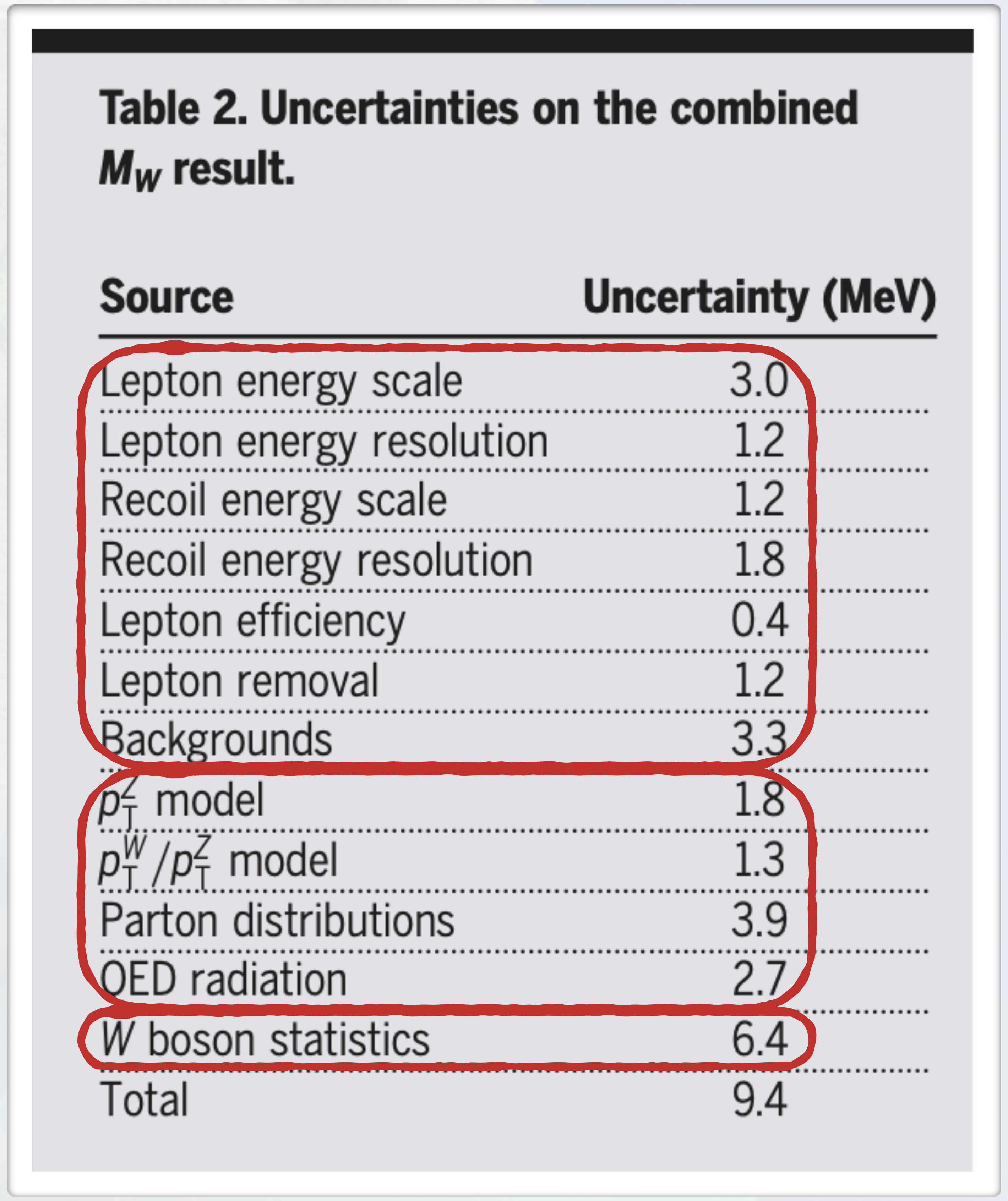
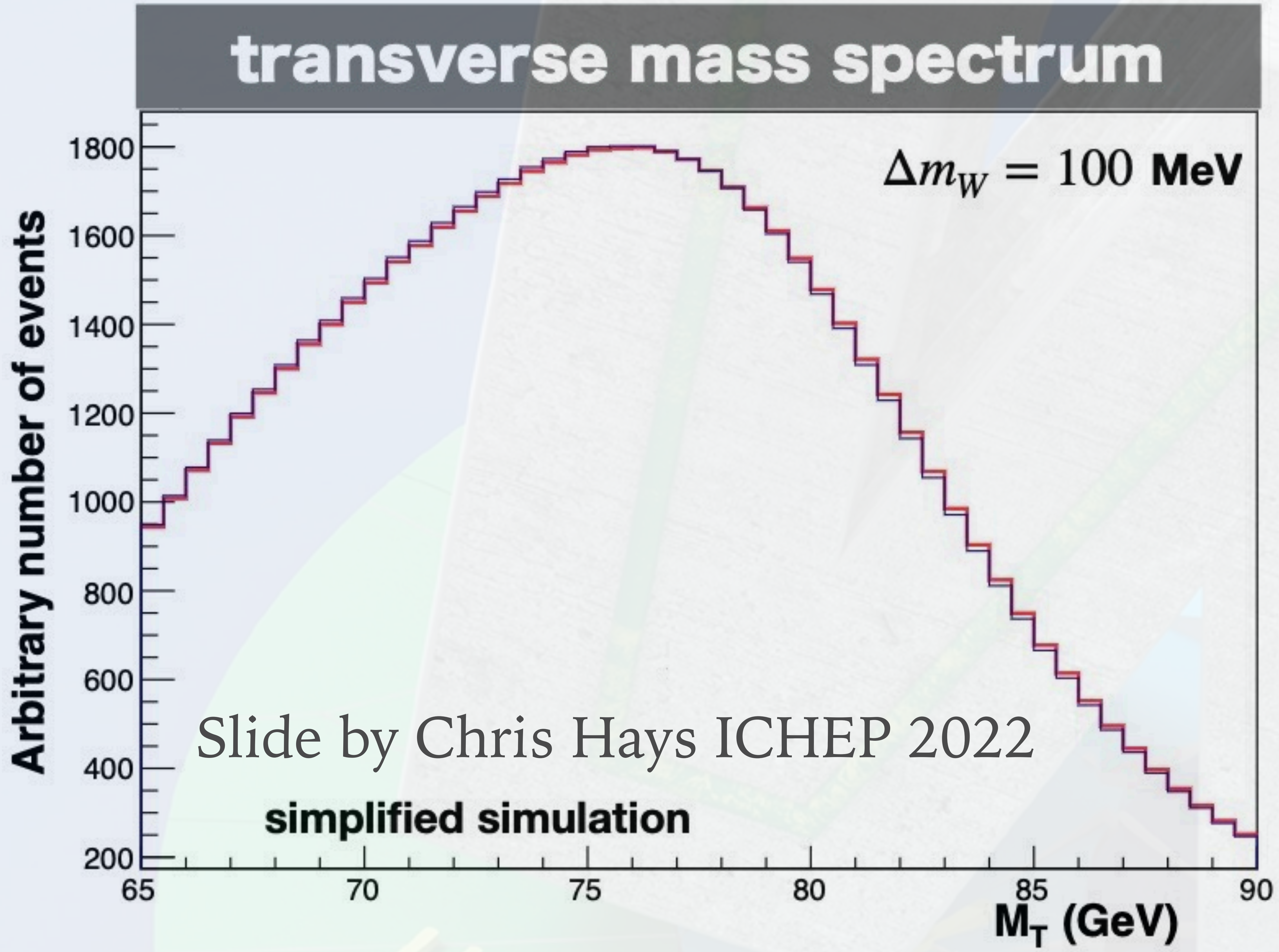


Table 2. Uncertainties on the combined M_W result.

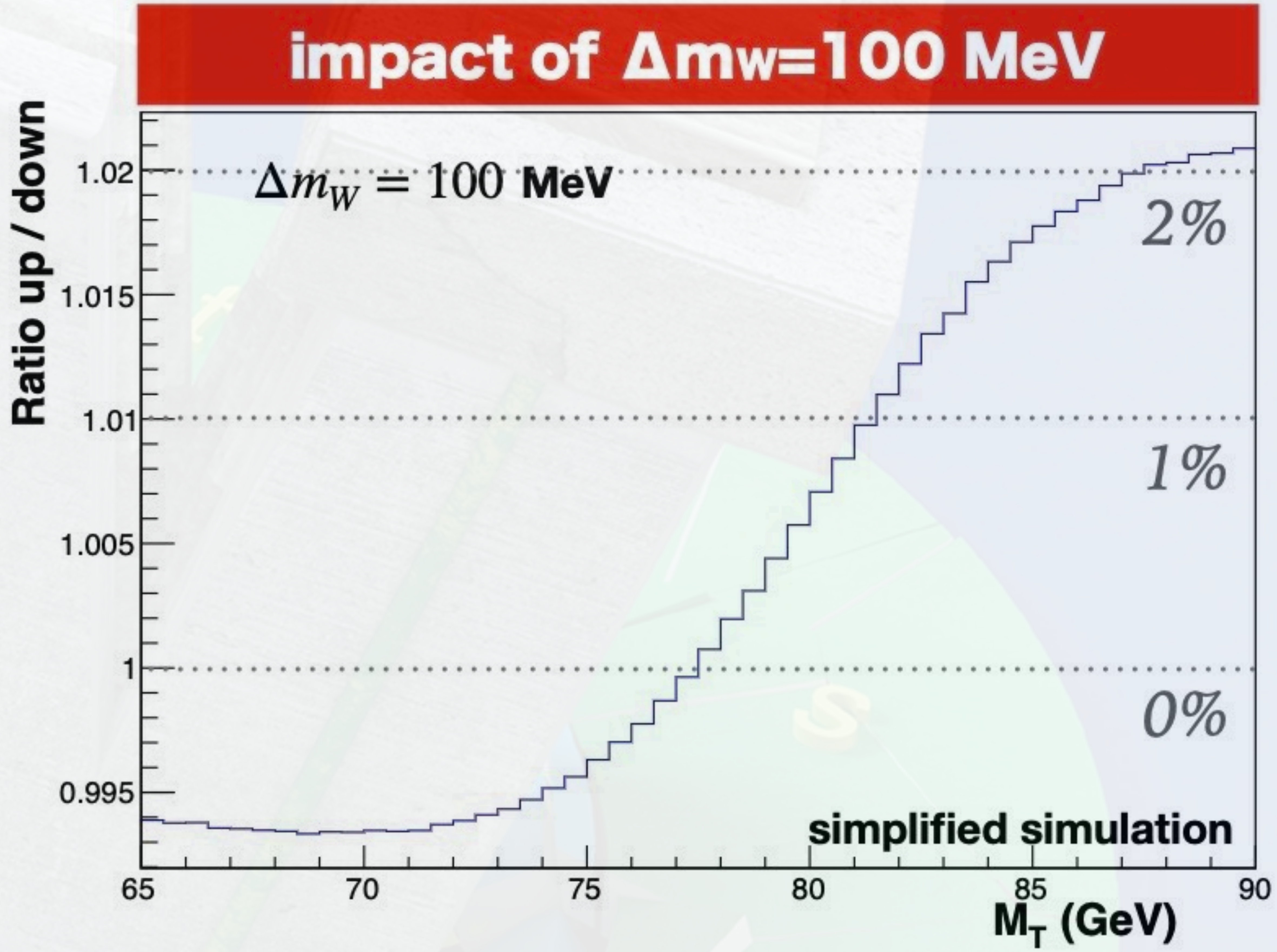
Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

W MASS IN CDFII MEASUREMENT

- ➤$d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV



Slide by Chris Hays ICHEP 2022

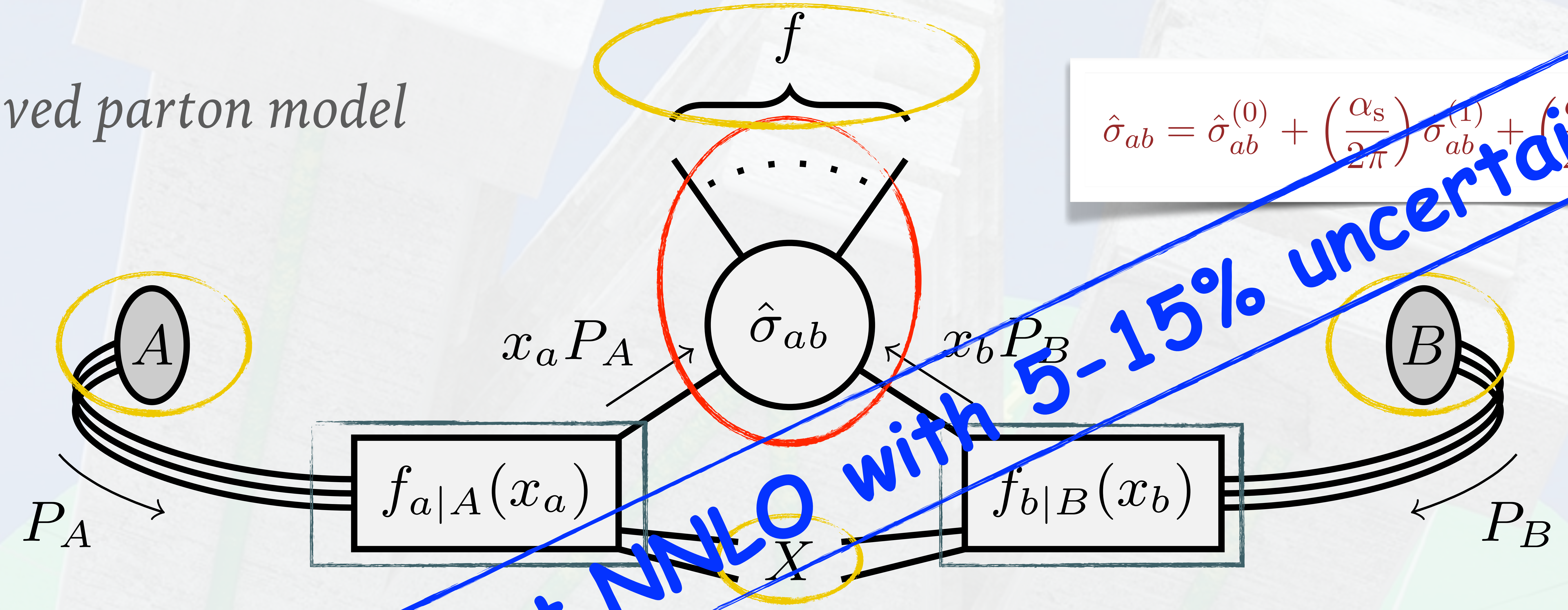


$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

PRECISION PREDICTIONS AT HADRON COLLIDER

QCD improved parton model

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}_{ab} + \ldots$$



Typical precision at NNLO with 5-15% uncertainties

$$\sigma_{AB} = \sum_{ab}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b)\, (1+\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q))$$

Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

Hard scattering
(Perturbative quantum field theory)
± 10 % level!

non-perturbative effects
(Fragmentation, hadronisation)
$\pm \Lambda/\sqrt{\hat{s}}$

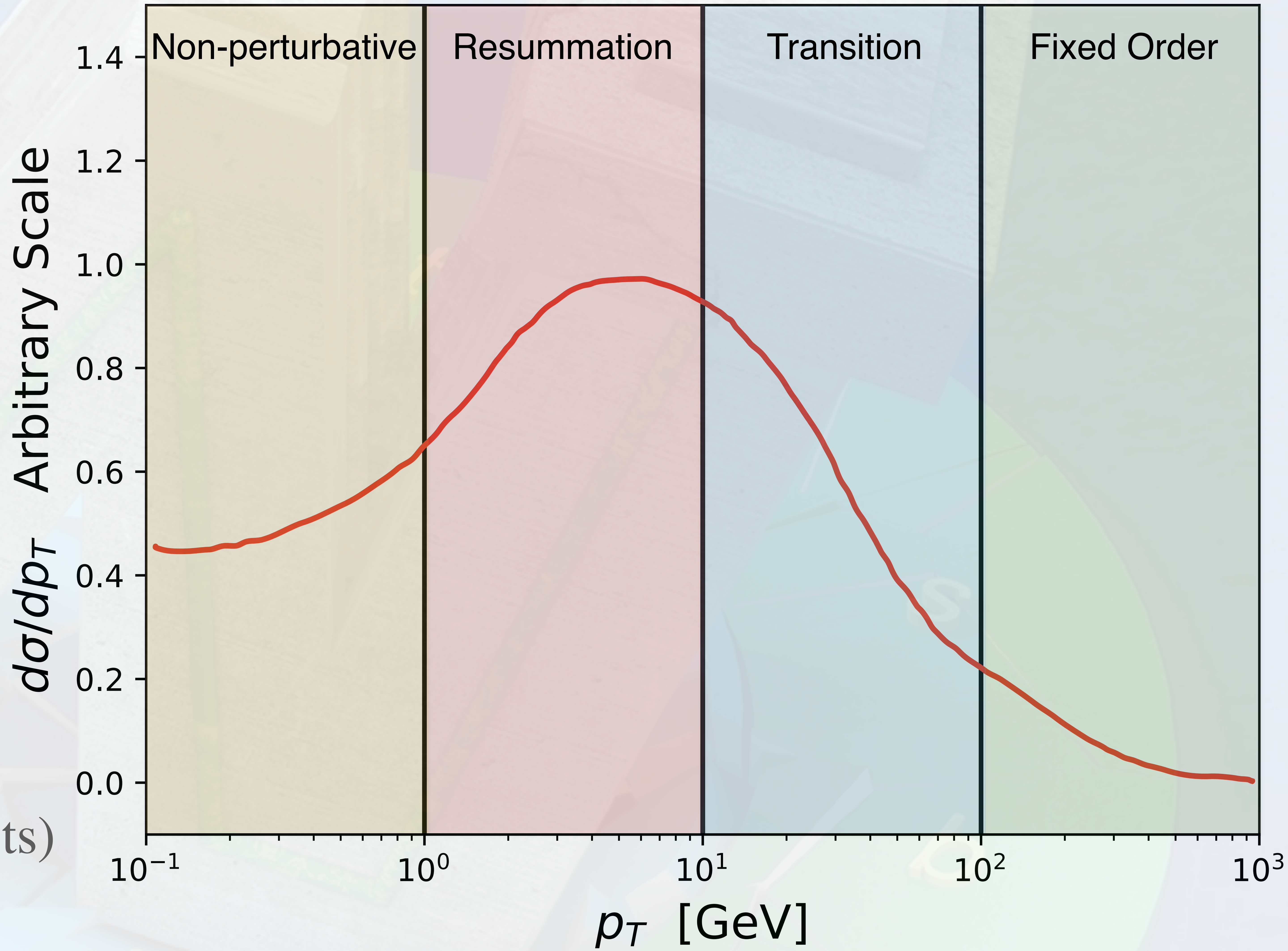
PRECISION PREDICTIONS AT HADRON COLLIDER

- ➤Beyond QCD improved parton model
  - ➤pQCD describes the tail of spectrum
  - ➤Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } \quad p_T \to 1 \text{ GeV}$$

  - ➤Various LP resummation schemes
  - ➤Multiple solutions in transition region
  - ➤Non-perturbative effects ~ 1 GeV (Short distance and long distance effects)

p_T Spectrum = multi-scale problem



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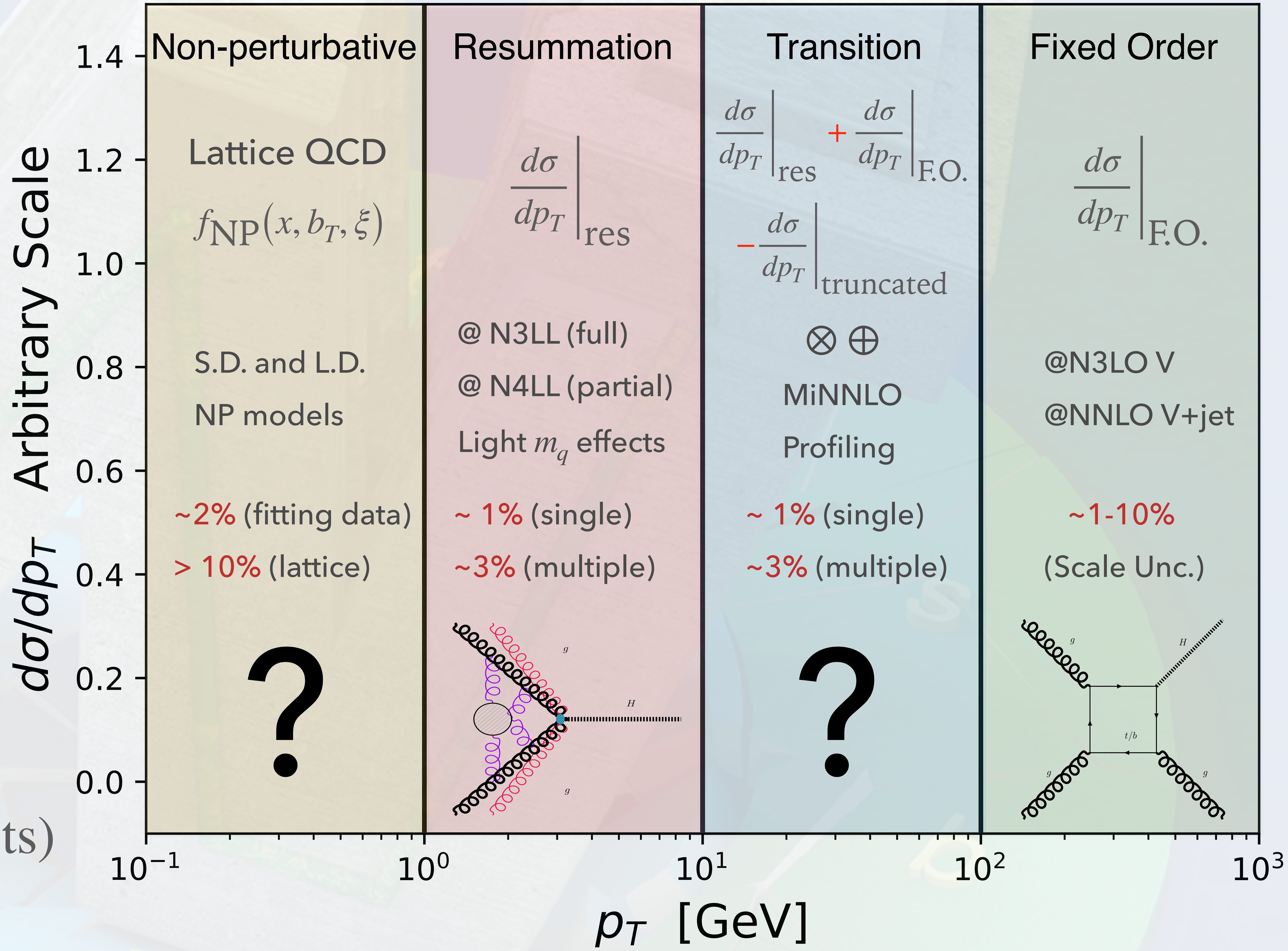

Non-perturbative

Lattice QCD

$f_{\mathrm{NP}}(x, b_T, \xi)$

S.D. and L.D.
NP models

~2% (fitting data)
> 10% (lattice)

Resummation

$$\left.\frac{d\sigma}{dp_T}\right|_{\mathrm{res}}$$

@ N3LL (full)
@ N4LL (partial)
Light m_q effects

~ 1% (single)
~3% (multiple)

Transition

$$\left.\frac{d\sigma}{dp_T}\right|_{\mathrm{res}} + \left.\frac{d\sigma}{dp_T}\right|_{\mathrm{F.O.}} - \left.\frac{d\sigma}{dp_T}\right|_{\mathrm{truncated}}$$

$\otimes$ $\oplus$

MiNNLO
Profiling

~ 1% (single)
~3% (multiple)

Fixed Order

$$\left.\frac{d\sigma}{dp_T}\right|_{\mathrm{F.O.}}$$

@N3LO V
@NNLO V+jet

~1-10%
(Scale Unc.)

PRECISION PREDICTIONS IN CDFII

- ➤CDF II use ResBos to generate theory templates
  - ➤NLO+NNLL accuracy for W/Z production

    Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03
  - ➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$
$$\times\, C \otimes f(x_1,\mu)\, C \otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

  - ➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

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  - Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S(b)=S_{\mathrm{NP}}S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b)=\int_{C_1^2/(b^*)^2}^{C_2^2Q^2}\frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right)A(\bar{\mu},C_1)+B(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}}=\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)-g_1g_3\ln(100x_1x_2)\right]b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

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- Use data driven method:

Fix	g1	g2	g3	α_s
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Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

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CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22

PRECISION PREDICTIONS IN CDFII

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson Ph.D. thesis`17
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

$$(1+\cos^2\theta)+\frac{1}{2}A_0\left(1-3\cos^2\theta\right)+A_1\sin 2\theta\cos\phi+A_2\sin^2\theta\cos 2\phi$$
$$+A_3\sin\theta\cos\phi+A_4\cos\theta+A_5\sin^2\theta\sin 2\phi+A_6\sin 2\theta\sin\phi+A_7\sin\theta\sin\phi$$

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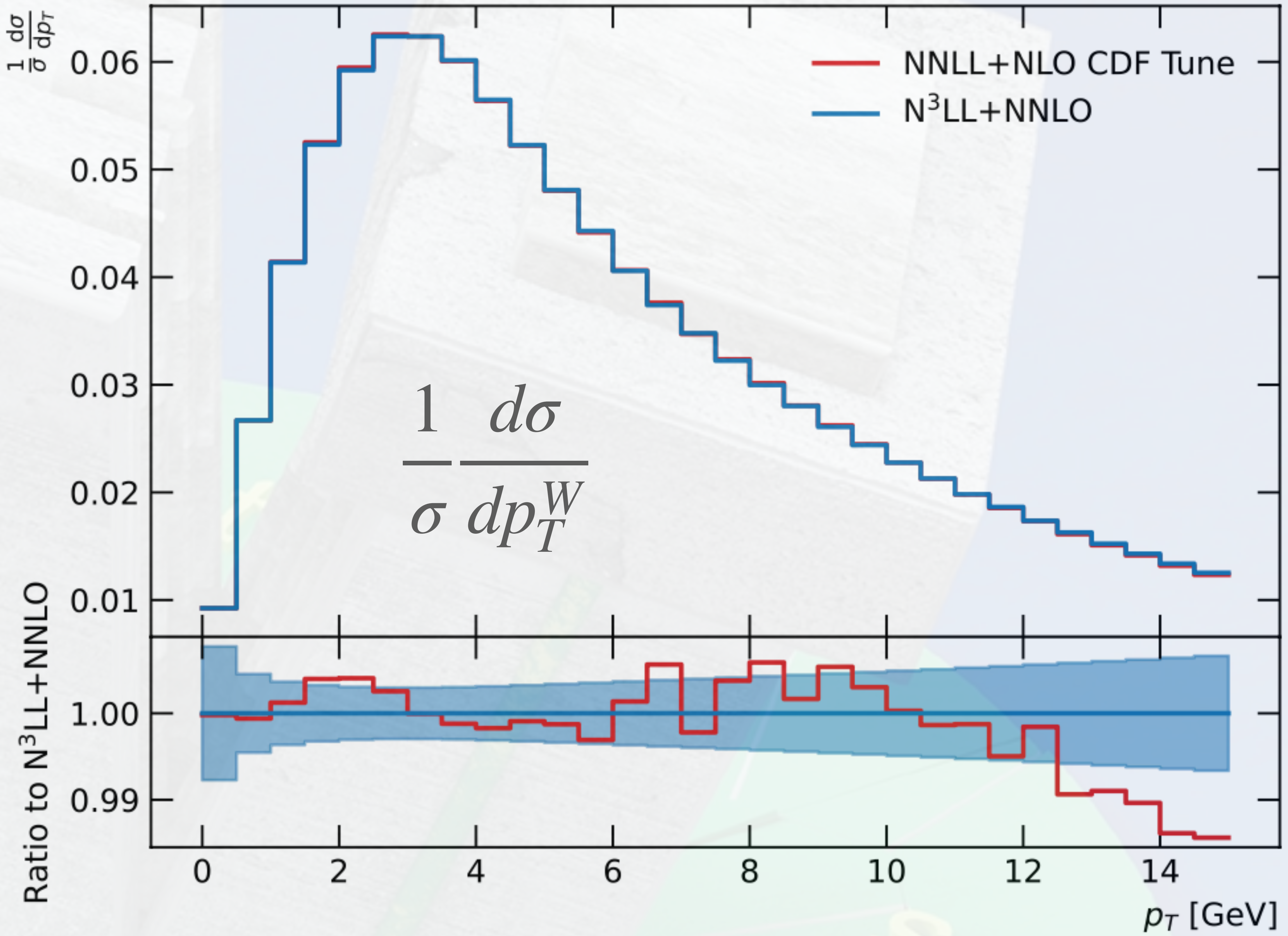
$$+A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22



We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV

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$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

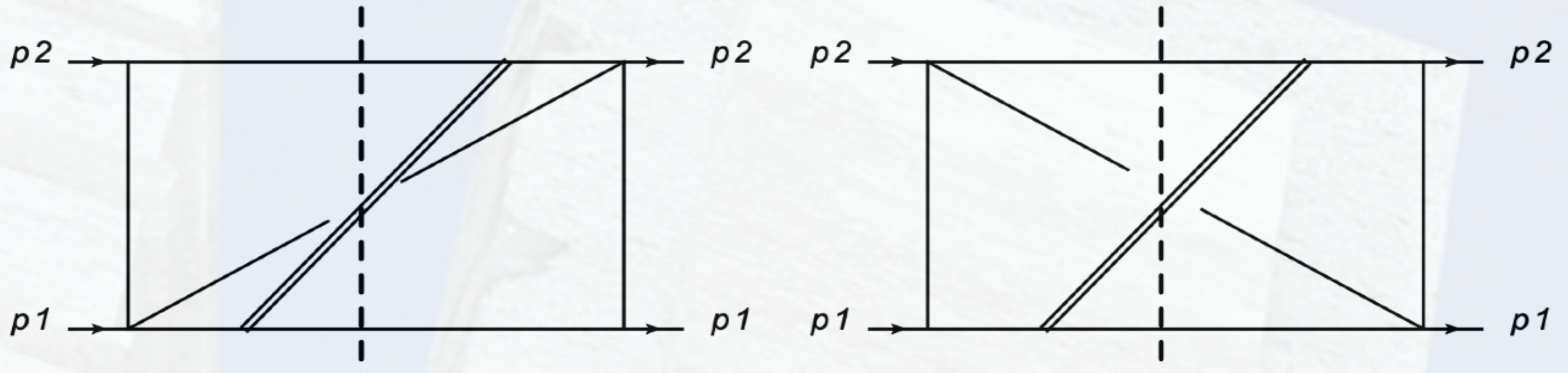
$$(1+\cos^2\theta) + \frac{1}{2}A_0\left(1-3\cos^2\theta\right) + A_1 \sin 2\theta \cos\phi + A_2 \sin^2\theta \cos 2\phi$$
$$+A_3 \sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22

- Alternative tools on the market
  - NNLO QCD
    - DYNNLO Catani, Cieri, Ferrera, Florian Grazzini `09
    - FEWZ Gavin, Li, Petriello, Quackenbush `13
    - MATRIX Grazzini, Kallweit, Wiesemann `17
    - MCFM Boughezal et. al.`16 Campbell, Neumann `19
    - Cross check Alekhin, Kardos, Moch, Trocsanyi `21
  - NNLO + PS
    - MiNNLO$_{PS}$ in POWHEG-BOX

      Monni, Nason, Re, Wiesemann, Zanderighi `20

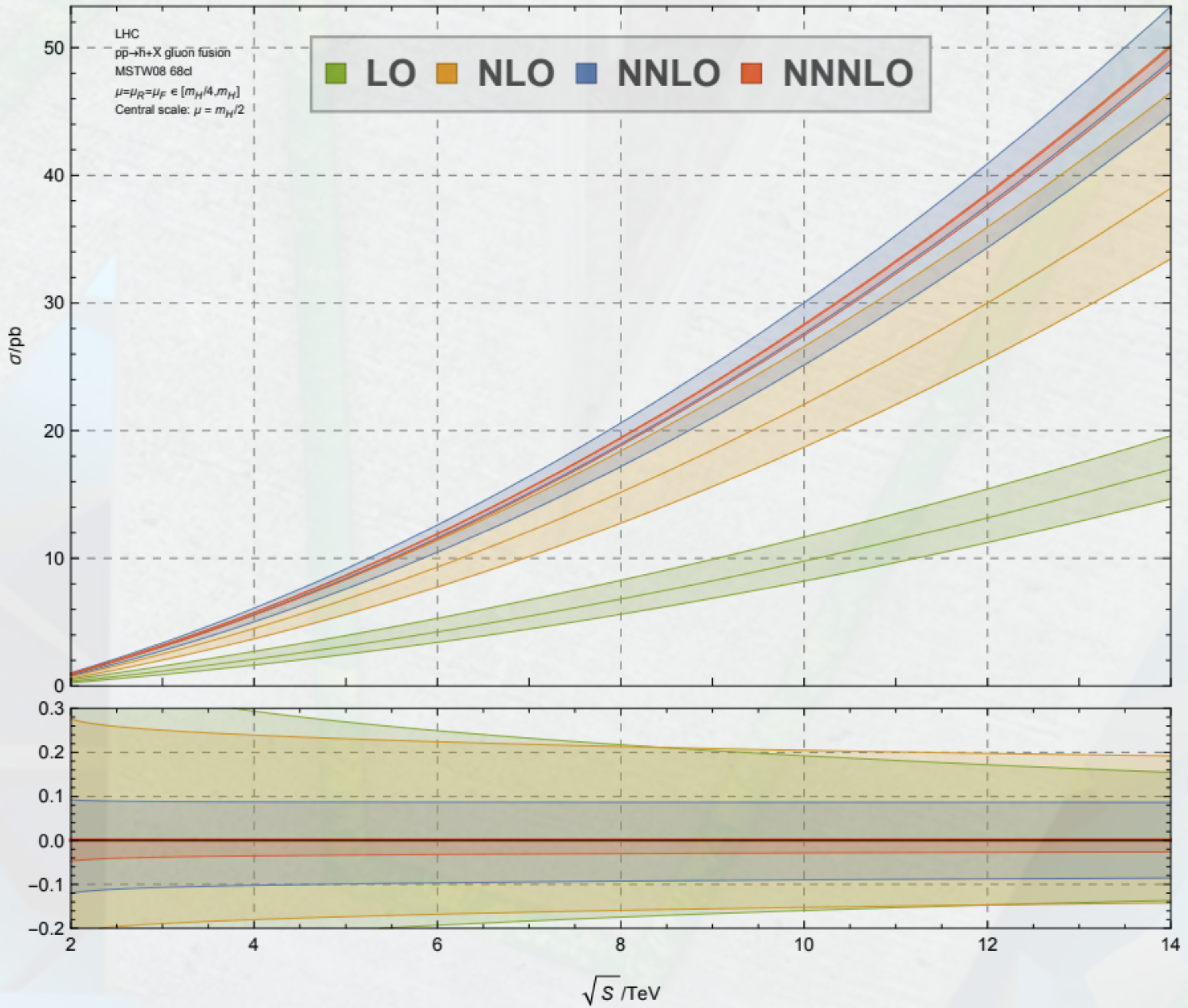
STATE-OF-THE-ART PREDICTIONS FOR σ_{N^3LO}



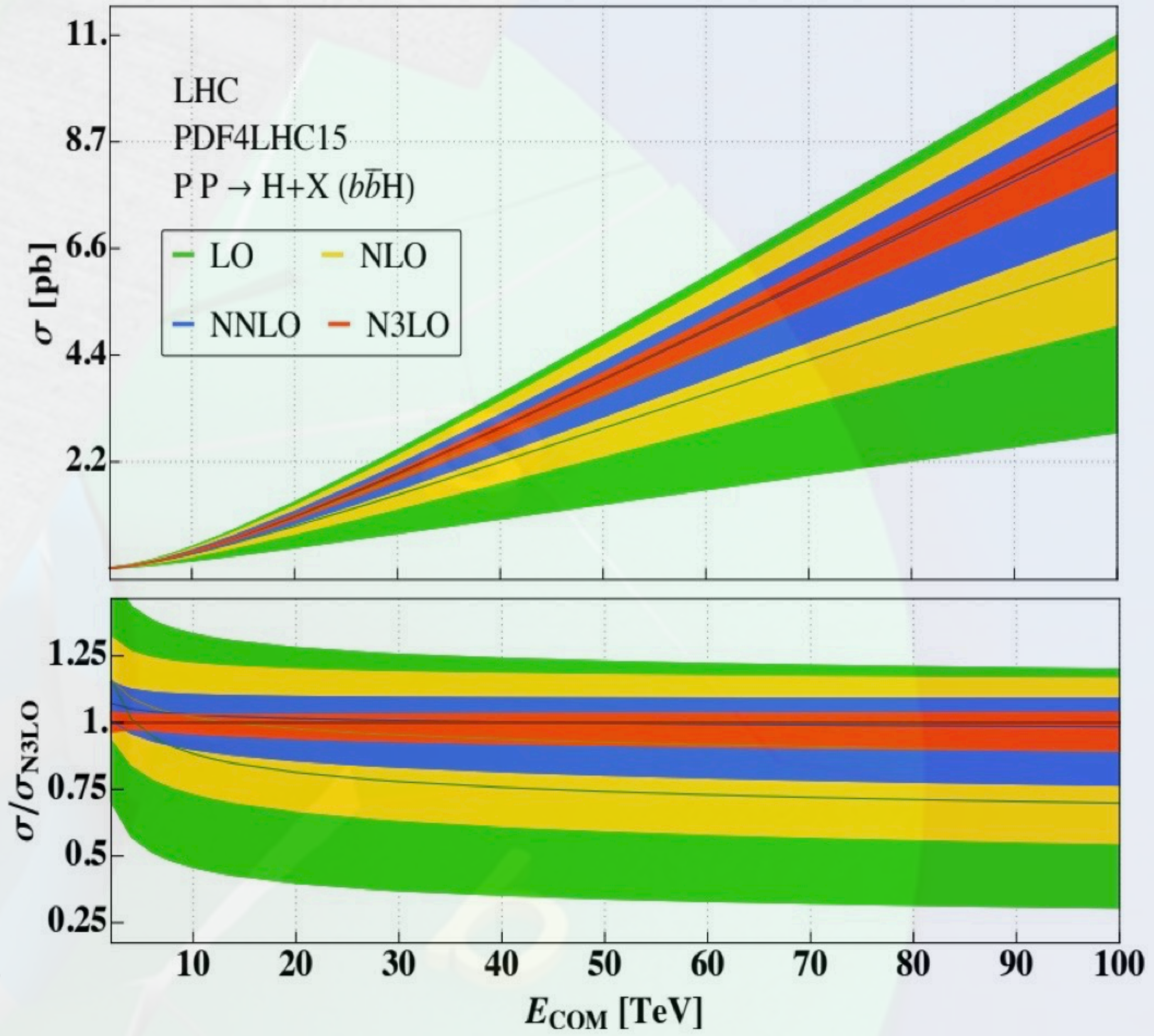
- ➤Assemble each $\hat{\sigma}_{ab}(x_a, x_b)$ at N3LO
  - ➤Integration of QCD radiation with unitarity cuts
  - ➤Standard treatment of multi-loop calculations except elliptic integrals with $\tau = m^2/\hat{s}$ where $\hat{s} = x_a x_b s$
  - ➤Use threshold expansion at different region of τ and truncate at sufficiently high orders ($\mathcal{O}(100)$). (Mistlberger `18)
  - ➤Use generalised power series ansatz to test the approximation and match coeff. of overlapping regions.

Not exact analytical solution of elliptic integrals but numerically precise enough for phenomenology

- ➤Application of ggF Higgs production
  - ➤Remarkable precision of the first N3LO XS (Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger `15 to `18)
  - ➤Available in public code iHixs 2 (Dulat, Lazopoulos, Mistlberger `18)
  - ➤Further application to bbF Higgs (Dulat, Lazopoulos, Mistlberger `19)
- ➤ VBF to Higgs and HH using DIS structure function (Dreyer, Karlberg `17 `19)



Phys.Rev.Lett. 114 (2015) 212001



Phys.Rev.Lett. 125 (2020) 5, 051804

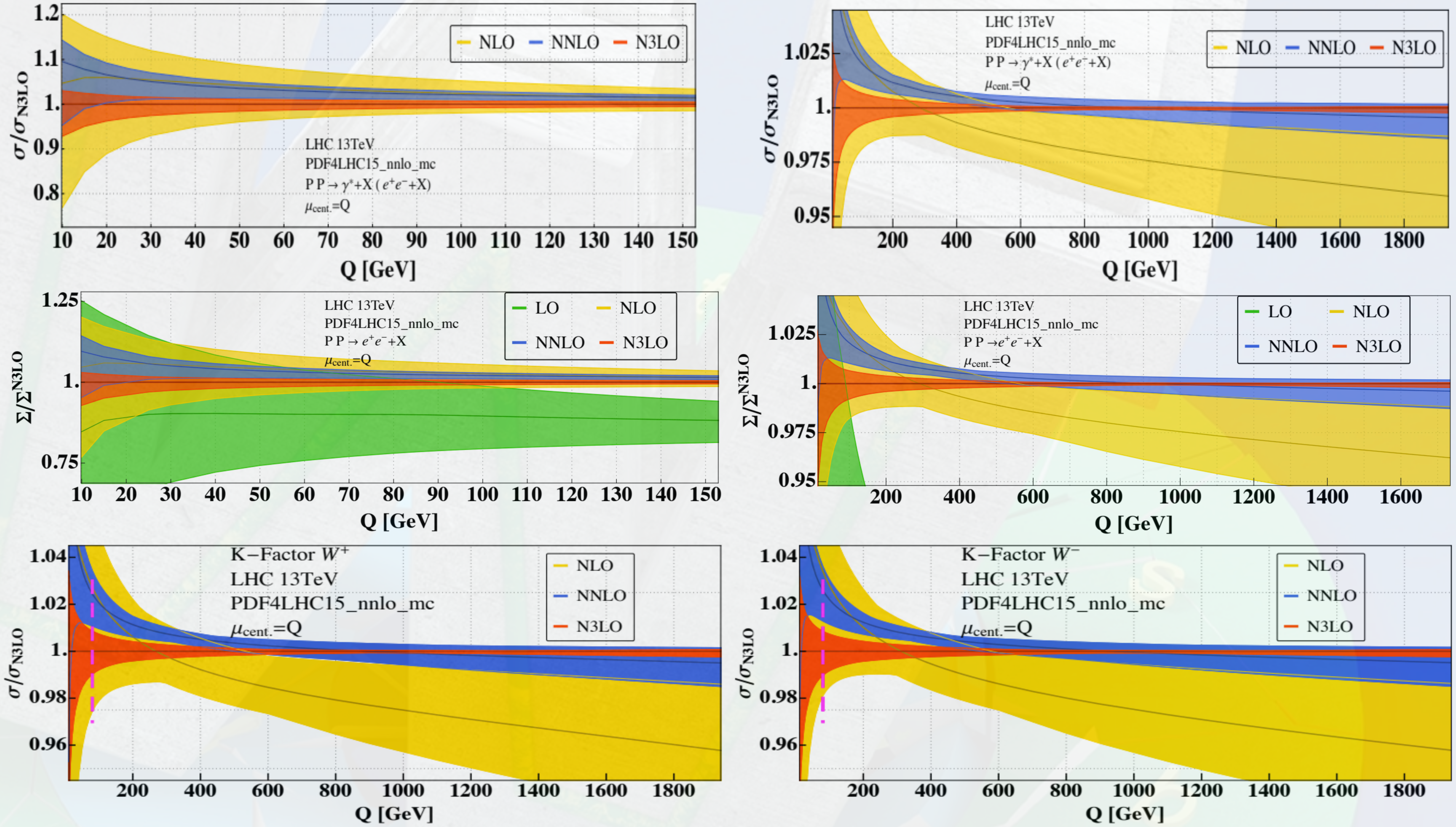
STATE-OF-THE-ART PREDICTIONS FOR σ_{N^3LO}

➤Application to 2 → 1 colour singlet production at the LHC (Duhr, Dulat, Mistlberger `20 `21)

$pp \to \gamma^*$

$pp \to Z/\gamma^* \to e^+e^-$

$pp \to W^\pm$



➤Application to 2 → 2 colour singlet production at the LHC (Baglio, Duhr, Mistlberger, Szafron `22)

GOING DIFFERENTIAL

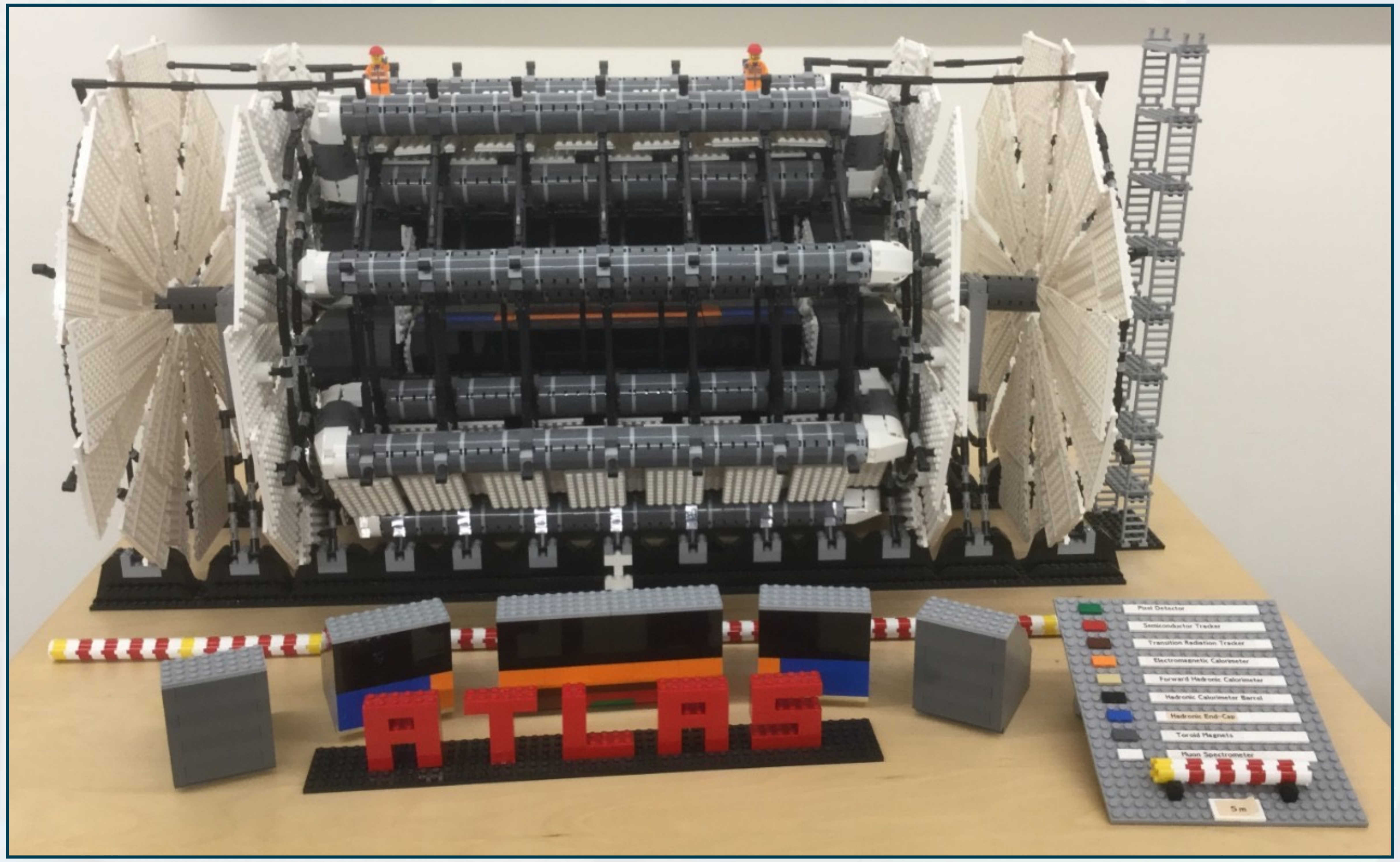
$\sigma^{\mathrm{pp}\to\mathrm{H}}_{tot}$

- What is the probability of producing a Higgs boson?

… where the Higgs decays into a pair of photons, $\mathrm{H}\to\gamma\gamma$, and the leading and sub-leading photon have a transverse momentum that is larger than 35% and 25% of the Higgs boson mass, respectively, and are produced within the rapidity interval $|y_\gamma| < 2.37$, where the barrel-endcap region $1.37 < |y_\gamma| < 1.52$ is excluded. Photons are further required to be isolated from additional QCD activity by requiring that the scalar sum of the transverse momenta of hadrons in a cone of $\Delta R = 0.2$ around the photons is less than $5\,\%$ of the photon transverse energy E_{T}.

$\mathrm{d}\sigma^{\mathrm{pp}\to\mathrm{H}}$

Measurements are done within a fiducial volume:



want direct comparison

(extrapolation $\leftrightsquigarrow$ source of uncertainties)

Slide by Alexander Huss MIAPbP 2022

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO accuracy
  - Projection to Born

$$\frac{d\sigma^{F}_{N^kLO}}{d\mathcal{O}} = \left(\frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\mathcal{O}} - \frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\tilde{\mathcal{O}}}\right) + \frac{d\sigma^{F}_{N^kLO}}{d\tilde{\mathcal{O}}}$$

    - Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18
    - Higgs decay to $b\bar{b}$ (MCFM) Mondini, Schiavi, Williams `19
    - Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21
  - qT slicing

$$d\sigma^{F}_{N^kLO} = \mathcal{H}^{F}_{N^kLO} \otimes d\sigma^{F}_{LO}\Big|_{\delta(\tau)} + \left[d\sigma^{F+jet}_{N^{k-1}LO} - d\sigma^{F\;CT}_{N^kLO}\right]_{\tau>\tau_{cut}} + \mathcal{O}(\tau^2_{cut}/Q^2)$$

    - Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18
    - Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19
    - Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21 `22
- Combined with resummation (N3LL at small qT)
  - Drell-Yan production (DYTurbo) Camarda, Cieri, Ferrera `21 (RadISH+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, et. al. `18 `19 `22 (CuTe-MCFM with partial N4LL) Neumann and Campbell `22
  - Higgs production via ggF (SCET+NNLOJET) XC, Gehrmann et. al.`18 (SCETlib) Billis, Dehnadi, et. al.`21

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO predictions for neutral current production
  - Fully differential N3LO Drell-Yan production (XC, Gehrmann, Glover, Huss, Yang, Zhu `21)
  - Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp\cdot b_\perp} \sum_q \sigma_{\rm LO}^{\gamma^*} H_{q\bar{q}}\Bigg[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1}\mathcal{I}_{qk}\left(z_1,b_T^2,\mu\right) f_{k/h_1}(x_1/z_1,\mu)$$

$$\times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2}\mathcal{I}_{\bar{q}j}\left(z_2,b_T^2,\mu\right) f_{j/h_2}(x_2/z_2,\mu)\mathcal{S}\left(b_\perp,\mu\right) + \left(q\leftrightarrow\bar{q}\right)\Bigg] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

  - All factorised functions are recently known up to N3LO:
    1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (Gehrmann, Glover, Huber, Ikizlerli, Studerus `10)
    2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp,\mu)$ at α_s^3 (Li, Zhu `16)
    3) Matching kernel of TMD beam function I_{qk} at α_s^3 (Luo, Yang, Zhu, Zhu `19, Ebert, Mistlberger, Vita `20)
  - Apply qt cut to factorise N3LO contribution into two parts:

$$d\sigma^{\gamma^*}_{N^3LO} = [\mathscr{H}^{\gamma^*}\otimes d\sigma^{\gamma^*}]_{N^3LO}\Big|_{\delta(p_{T,\gamma^*})} + \left[d\sigma^{\gamma^*+jet}_{NNLO} - d\sigma^{\gamma^*\ CT}_{N^3LO}\right]_{p_{T,\gamma^*}>q_T^{cut}} + \mathcal{O}((q_T^{cut}/Q)^2)$$

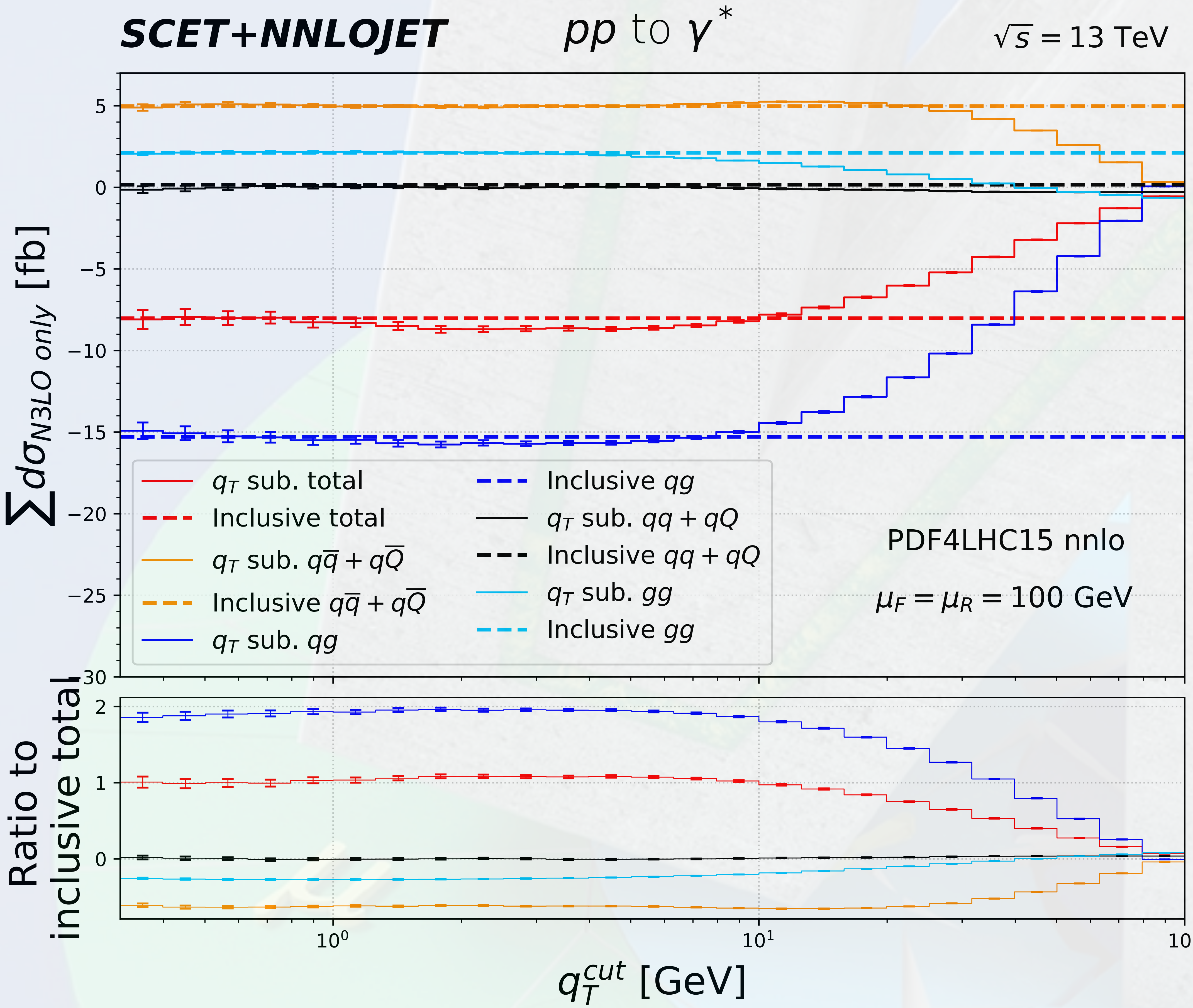
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO predictions for neutral and charged current production $\mathcal{O}(\alpha\alpha_s^3)$
  - Computational setup for $pp \to \gamma^* \to l^+l^-$
    (identical setup in the inclusive calculation by Durh, Dulat and Mistlberger in *Phys.Rev.Lett.* 125 (2020) 17, 172001)
    - Fix Q value for γ^* at 100 GeV (NNLO and N3LO scale variations deviate)
    - Use central value of PDF4LHC15_nnlo_mc as benchmark input
    - $\mu_R = \mu_F = 100$ GeV for central QCD scale and use 7-point variations for uncertainty estimation
    - Apply $p_{T,\gamma^*} > 0.25$ GeV constrain for NNLO $\gamma^* + Jet$ without jet definition
  - Computational setup for $pp \to W^\pm \to l^\pm\nu$
    - Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations and $m_{l\nu} \in [0, +\infty]$
    - Use NNPDF31_nnlo PDFs and $p_{T,l\nu} > 0.5$ GeV
    - Unit CKM matrix for LHC process
  - Common setup
    - Consider LO decay with $m_e = m_\mu = 0$, $\alpha_s(m_Z) = 0.118$, G_μ EW-scheme with fixed α value

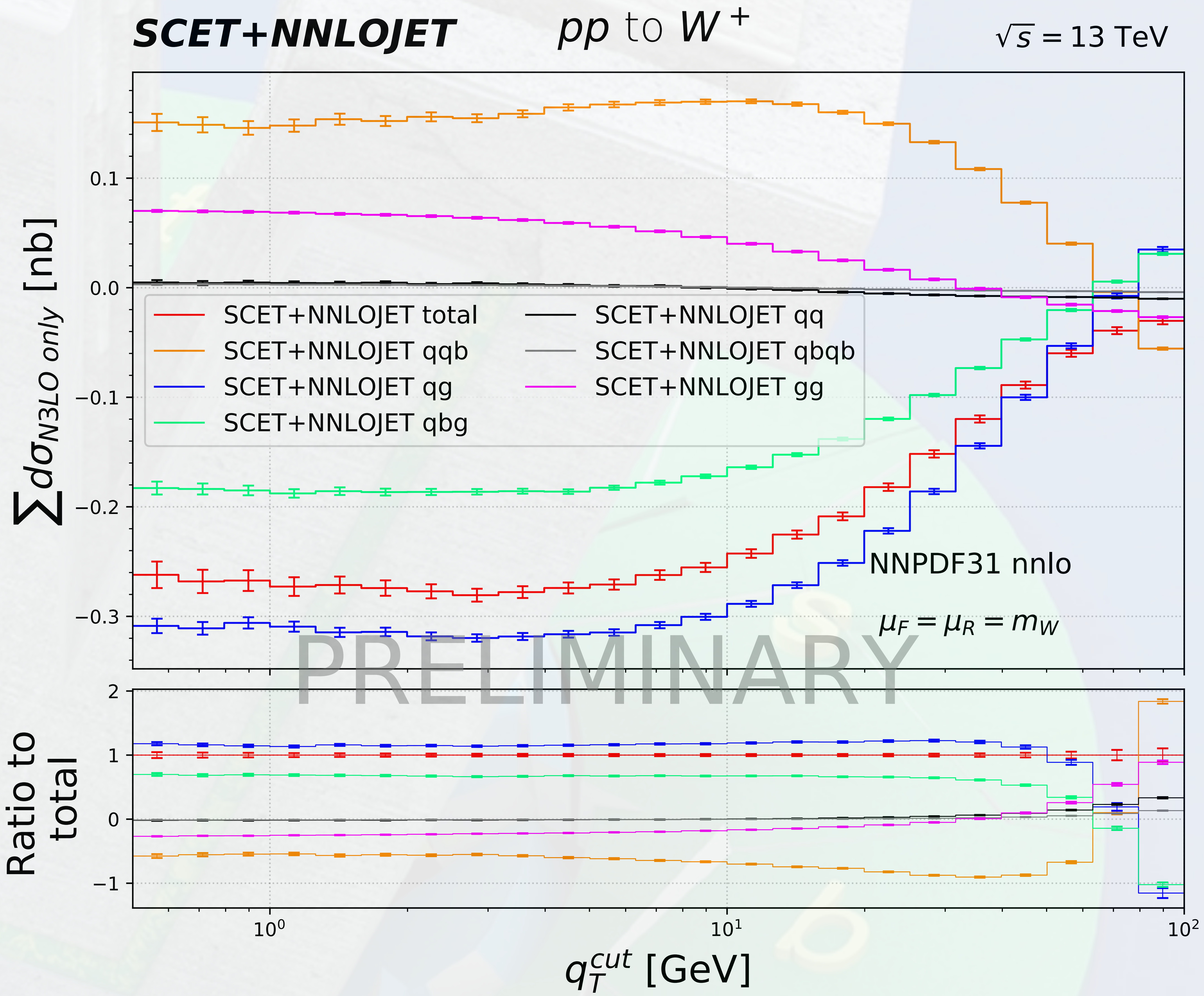
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- ➤Differential N3LO predictions for neutral and charged current production

$$\sum d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}\big|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}\big|_{p_{T,V}\in[0,q_T^{cut}]}$$



XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett*. 128 (2022) 5

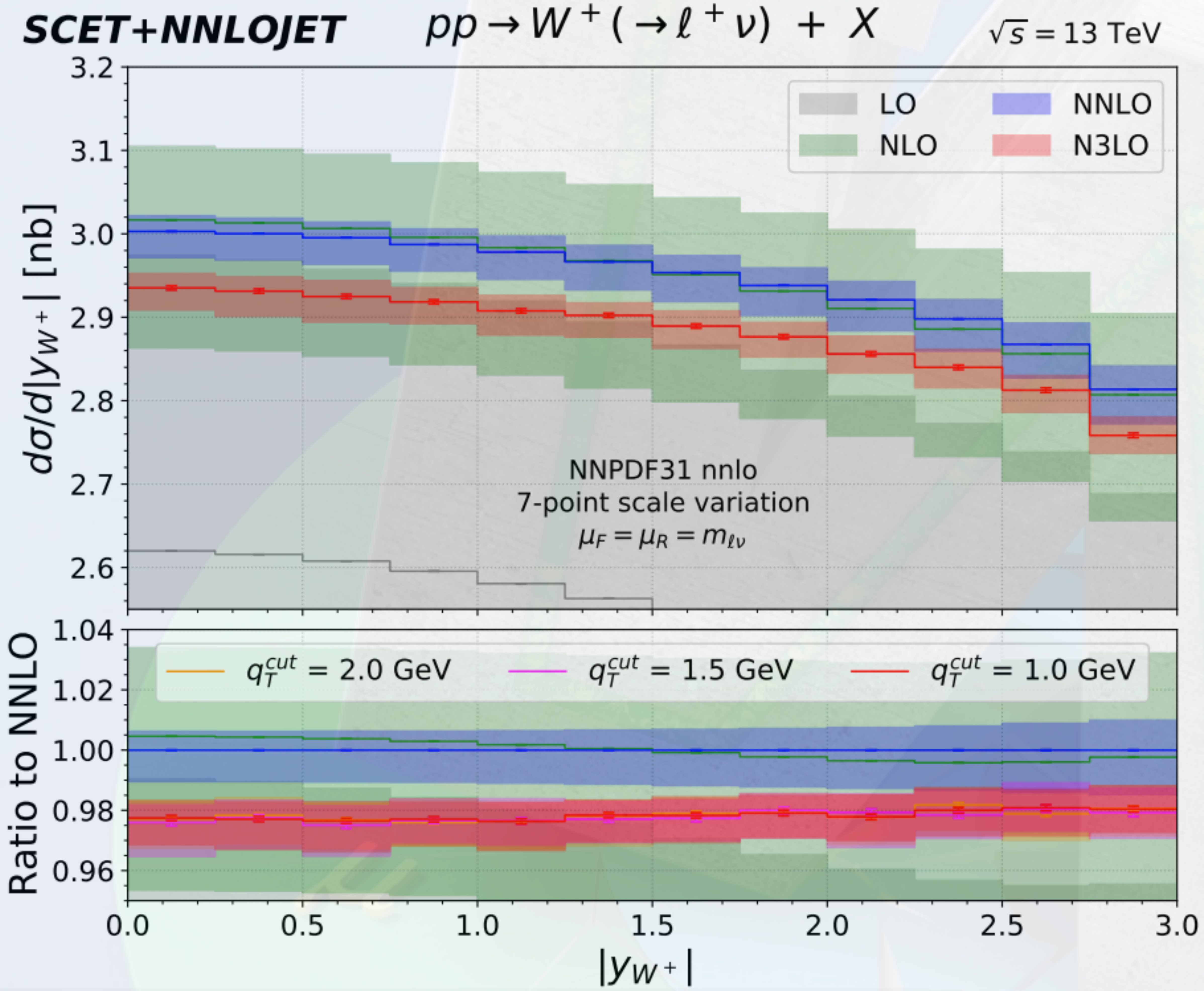


XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

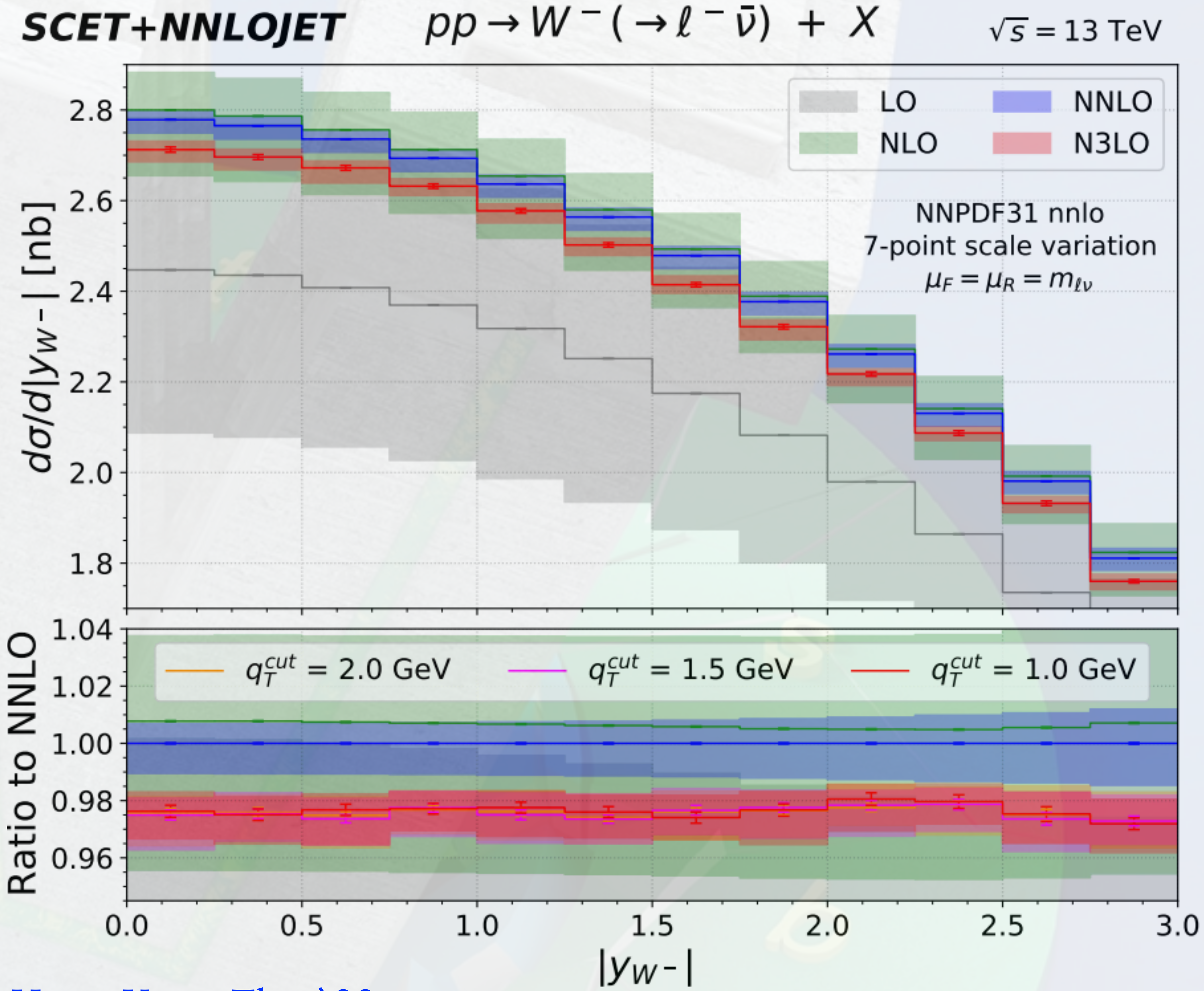
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- ➤Differential N3LO predictions for charged current production

$d\sigma^{W^+}_{FO}/d|y_{W^+}|$



$d\sigma^{W^-}_{FO}/d|y_{W^-}|$

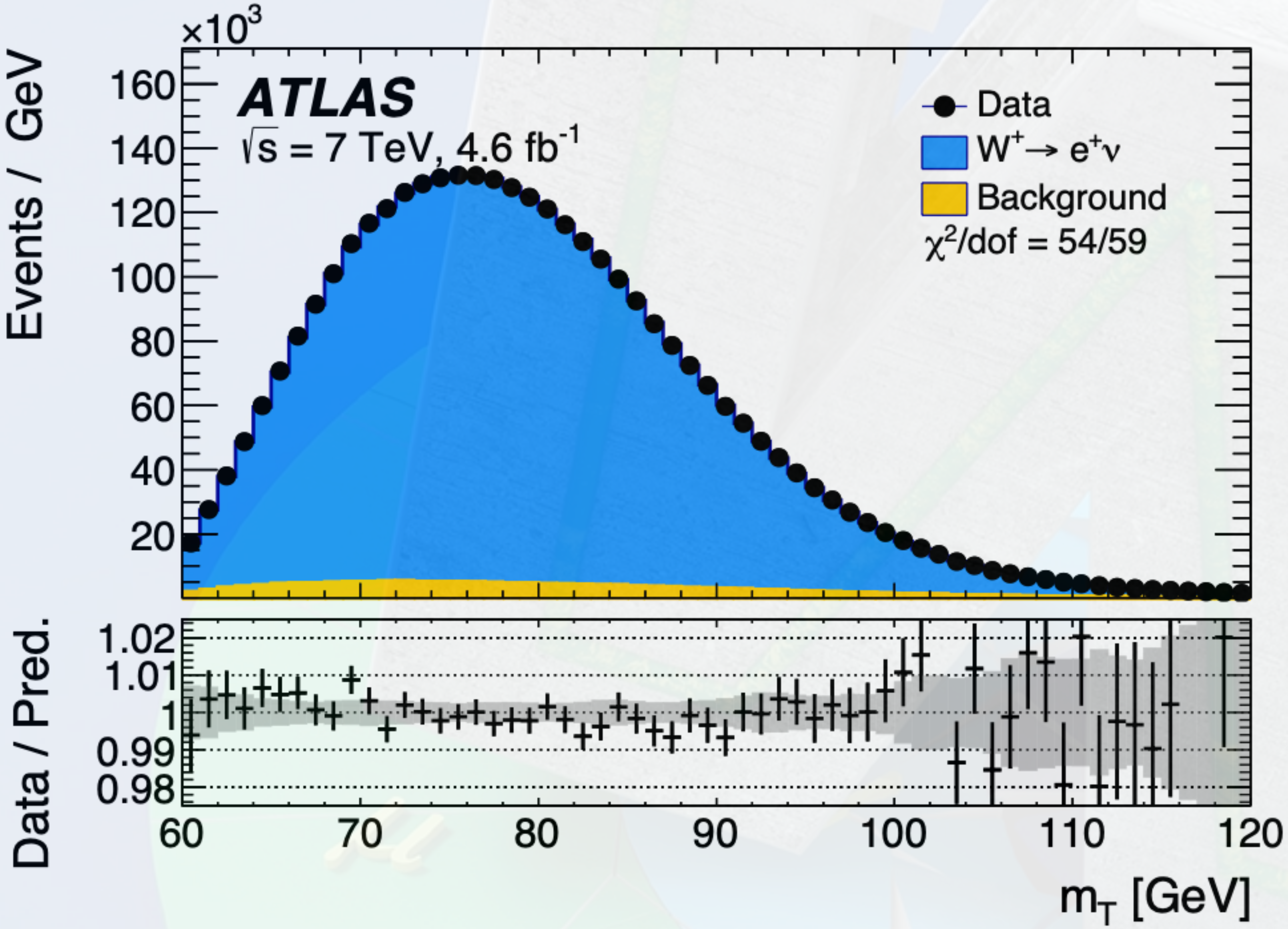


XC, Gehrmann, Glover, Huss, Yang, Zhu `22

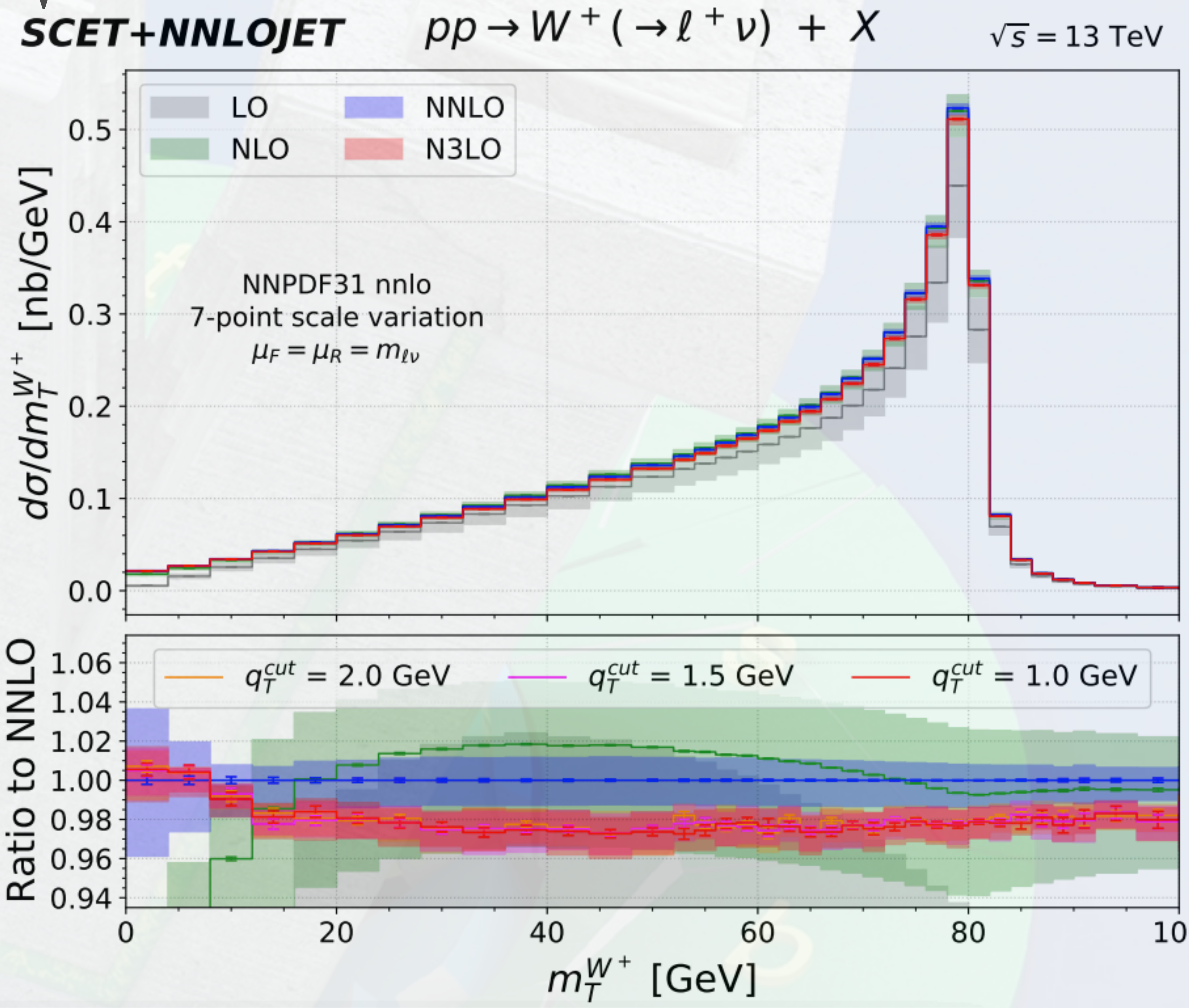
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^\nu)^2 - (\vec{p}_T^{\,l} + \vec{p}_T^{\,\nu})^2 = \sqrt{2E_T^l E_T^\nu\left(1 - cos\phi\right)}$$



ATLAS `17

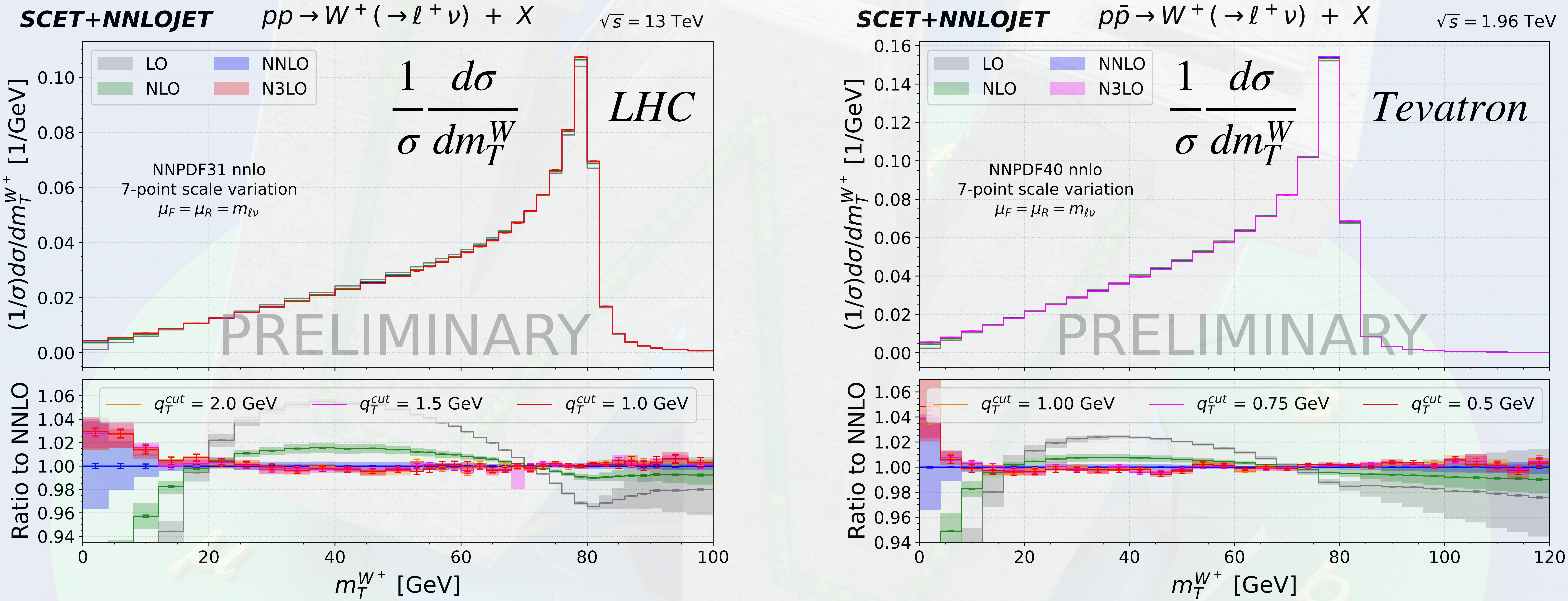


XC, Gehrmann, Glover, Huss, Yang, Zhu `22

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

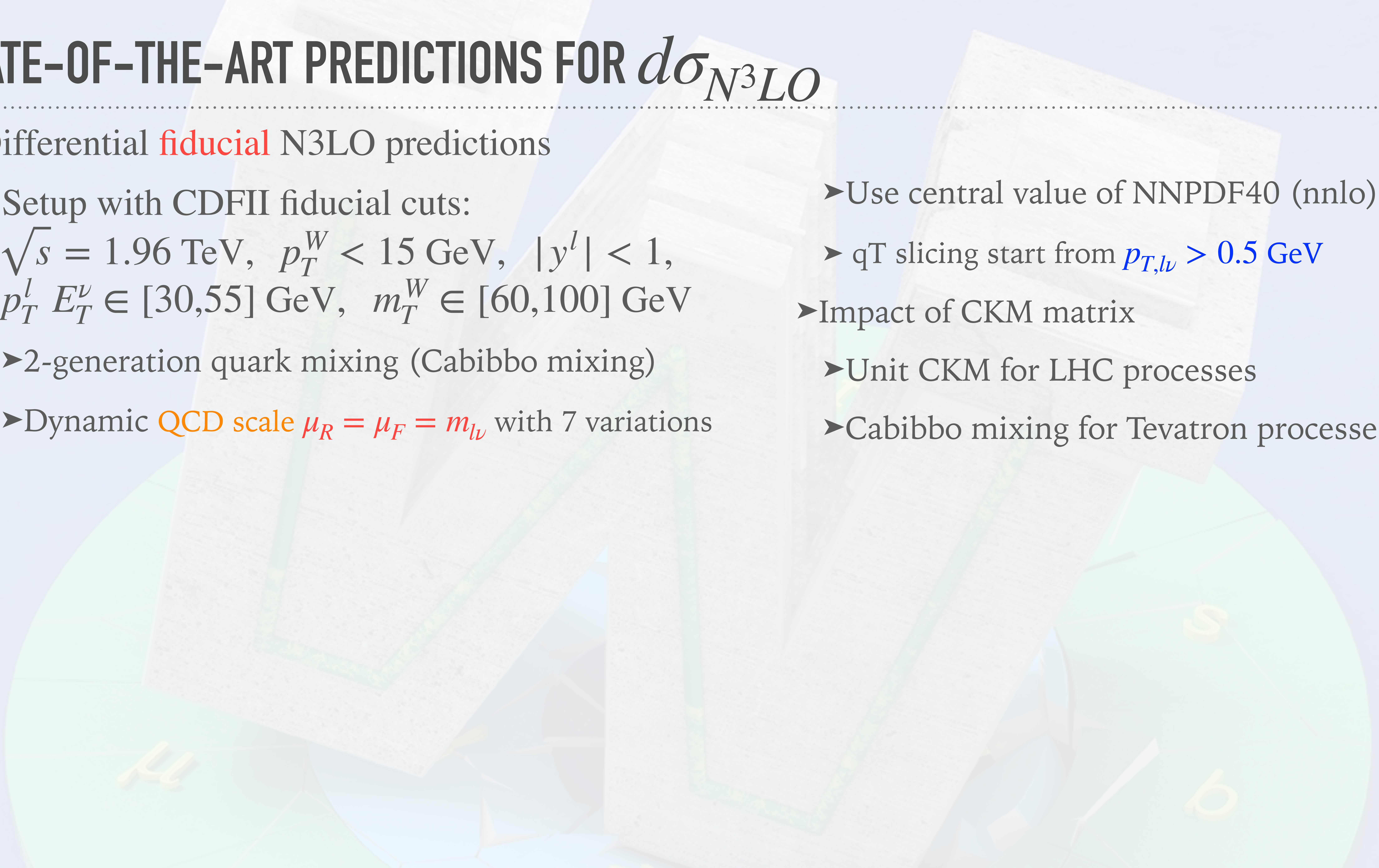
- Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^\nu)^2 - (\vec{p}_T^{\,l} + \vec{p}_T^{\,\nu})^2 = \sqrt{2E_T^l E_T^\nu (1 - cos\phi)}$$



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$



- Differential fiducial N3LO predictions
  - Setup with CDFII fiducial cuts:
    $\sqrt{s} = 1.96\ \text{TeV}, \quad p_T^W < 15\ \text{GeV}, \quad |y^l| < 1,$
    $p_T^l\ E_T^\nu \in [30{,}55]\ \text{GeV}, \quad m_T^W \in [60{,}100]\ \text{GeV}$
    - 2-generation quark mixing (Cabibbo mixing)
    - Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations
  - Use central value of NNPDF40 (nnlo)
  - qT slicing start from $p_{T,l\nu} > 0.5$ GeV
- Impact of CKM matrix
  - Unit CKM for LHC processes
  - Cabibbo mixing for Tevatron processes

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential fiducial N3LO predictions
  - Setup with CDFII fiducial cuts:
    $\sqrt{s} = 1.96$ TeV, $p_T^W < 15$ GeV, $|y^l| < 1$,
    $p_T^l\ E_T^\nu \in [30,55]$ GeV, $m_T^W \in [60,100]$ GeV
    - 2-generation quark mixing (Cabibbo mixing)
    - Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations
  - Use central value of NNPDF40 (nnlo)
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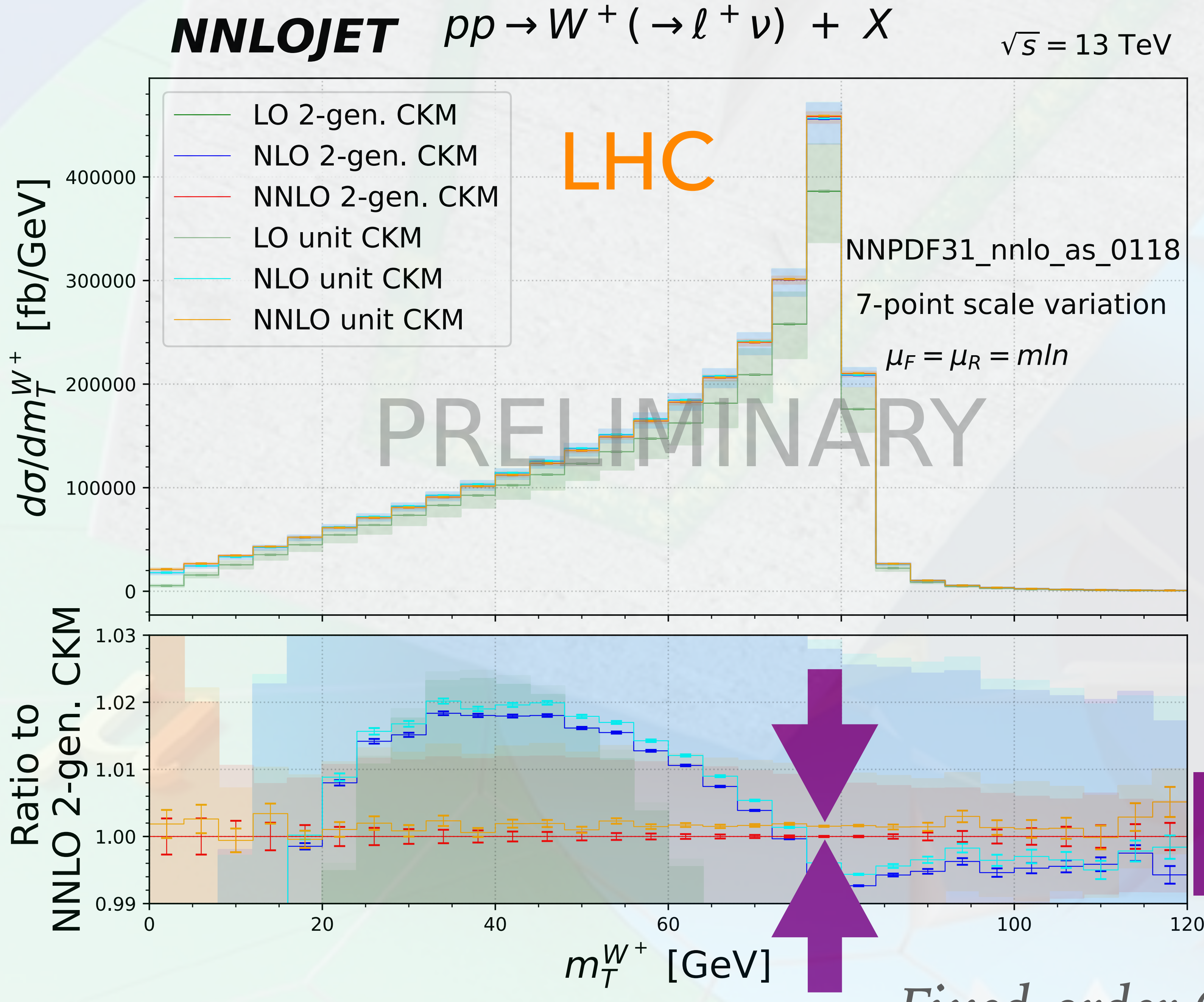
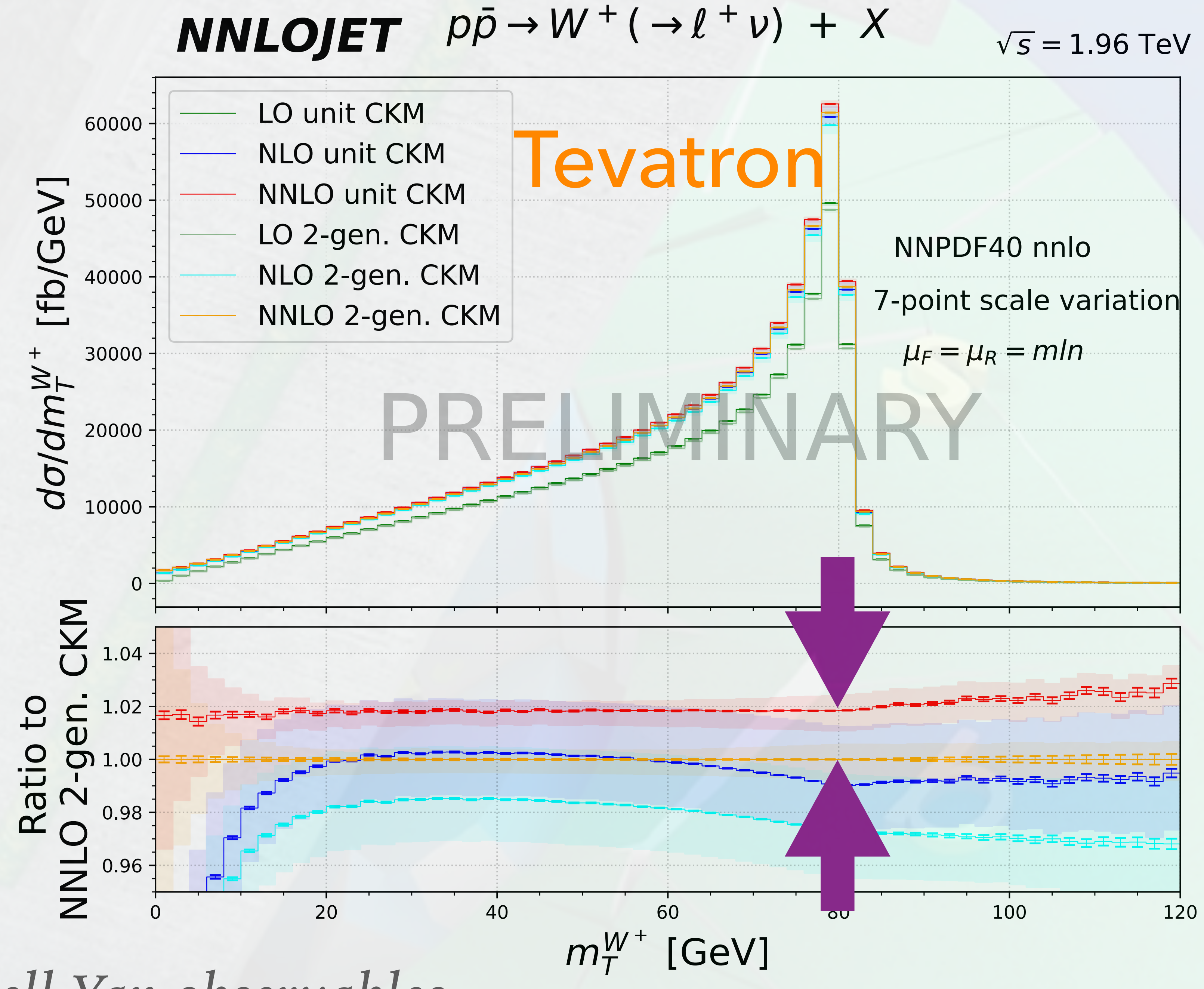
  - Cabibbo mixing for Tevatron processes

XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation



±0.2%

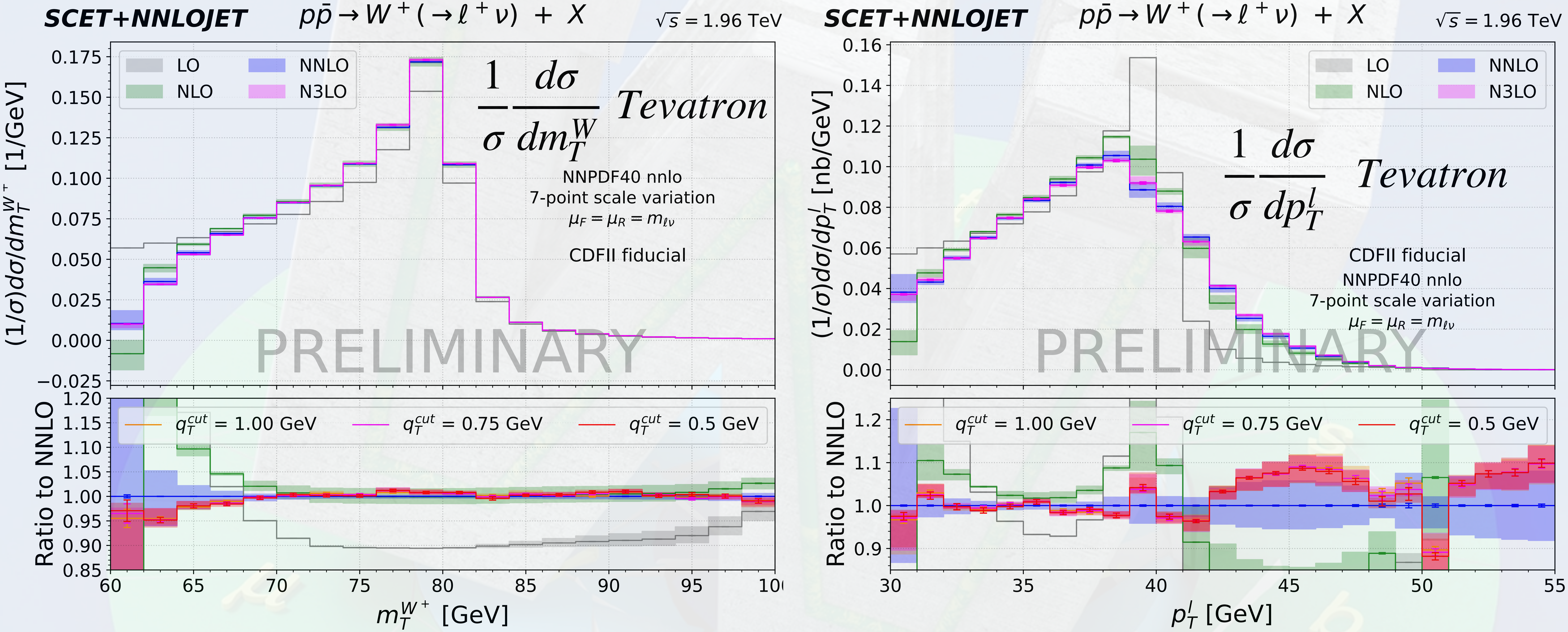
$$\frac{d\sigma}{dm_T^W}$$



±2%

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential fiducial N3LO predictions for charged current production



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

WHAT ARE WE MISSING?

- ➤Realistic theory uncertainty estimation

$$\frac{1}{\sigma}\frac{d\sigma}{dm_T^W}$$ @Tevatron with fiducial cuts

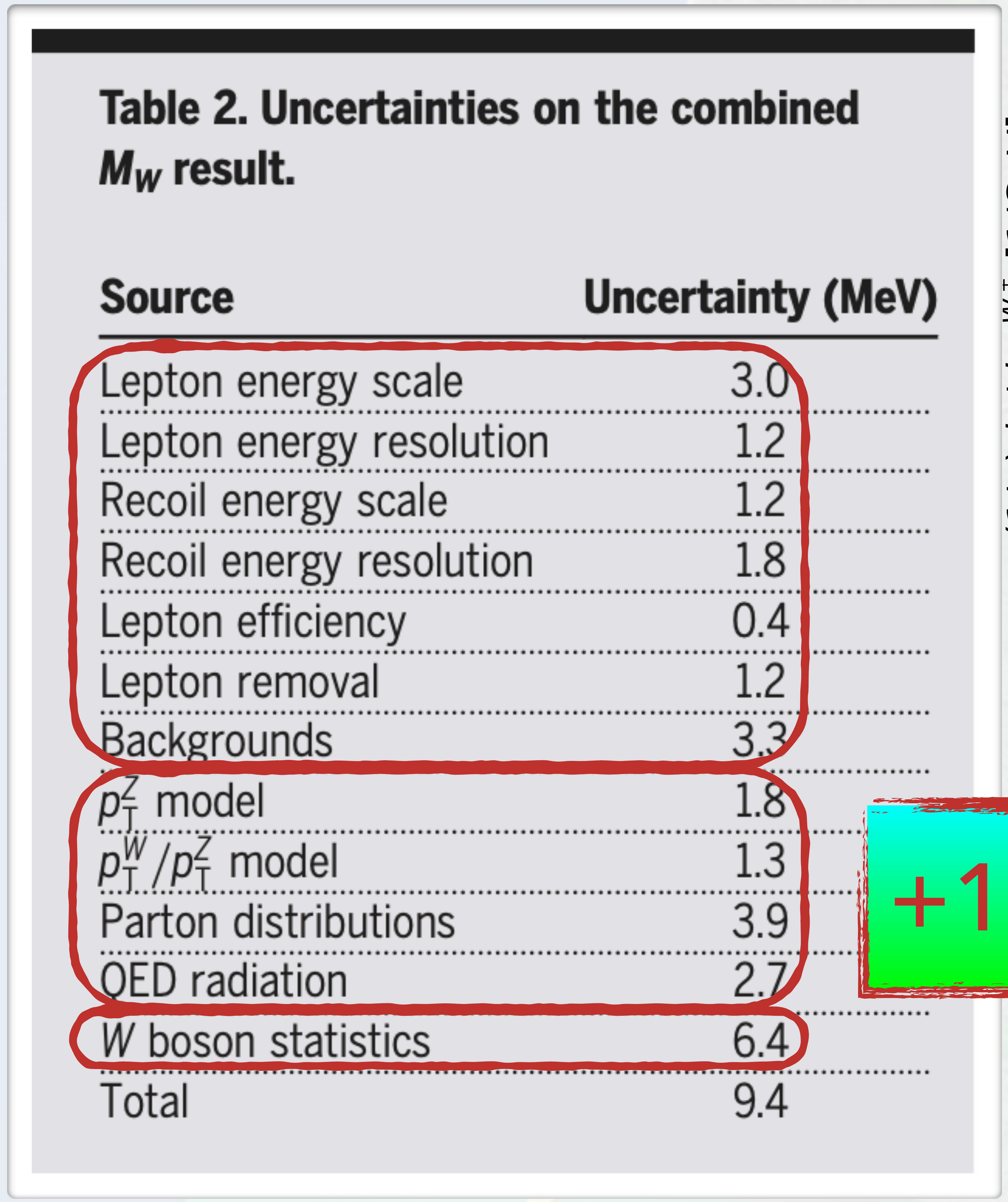
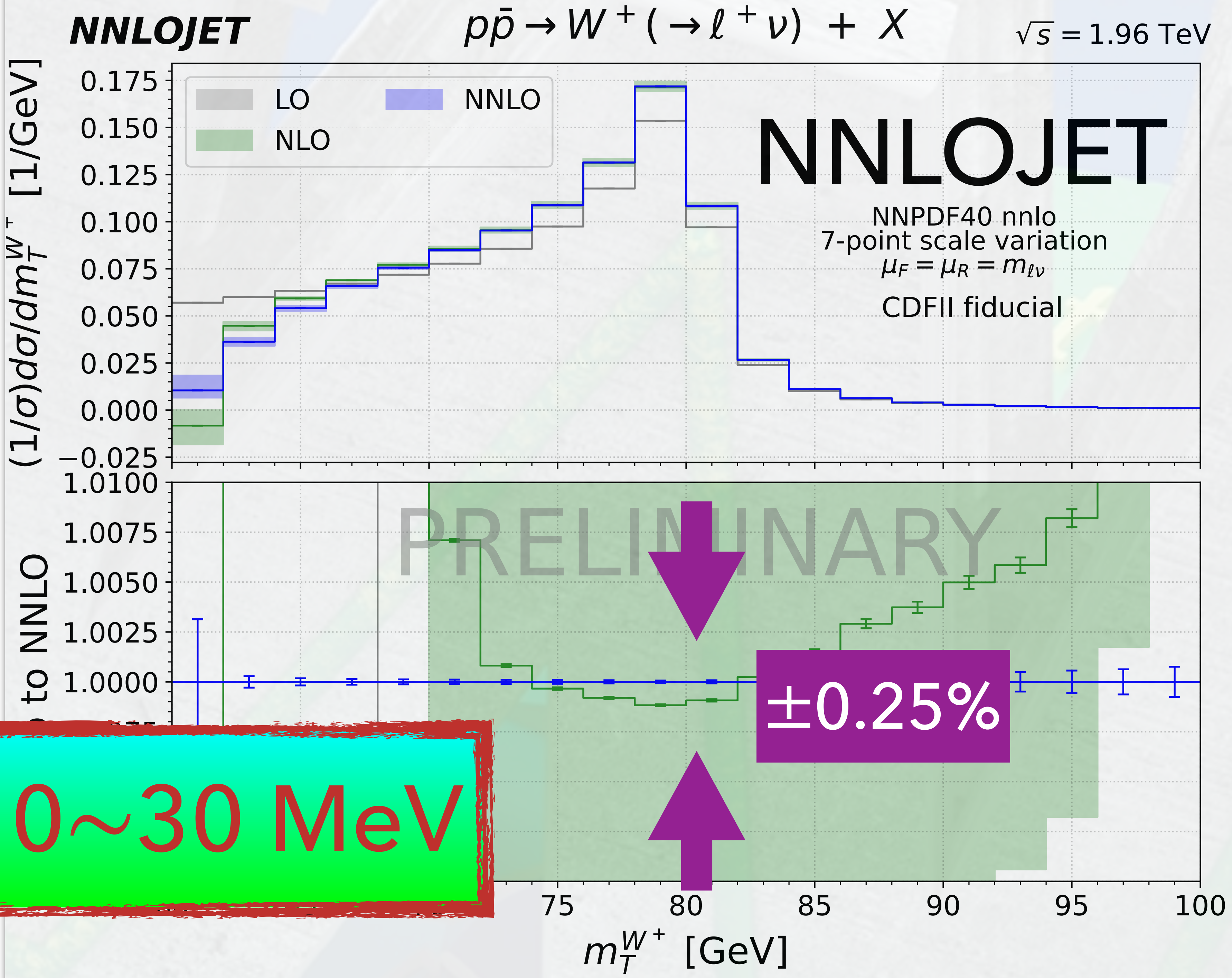


Table 2. Uncertainties on the combined M_W result.

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

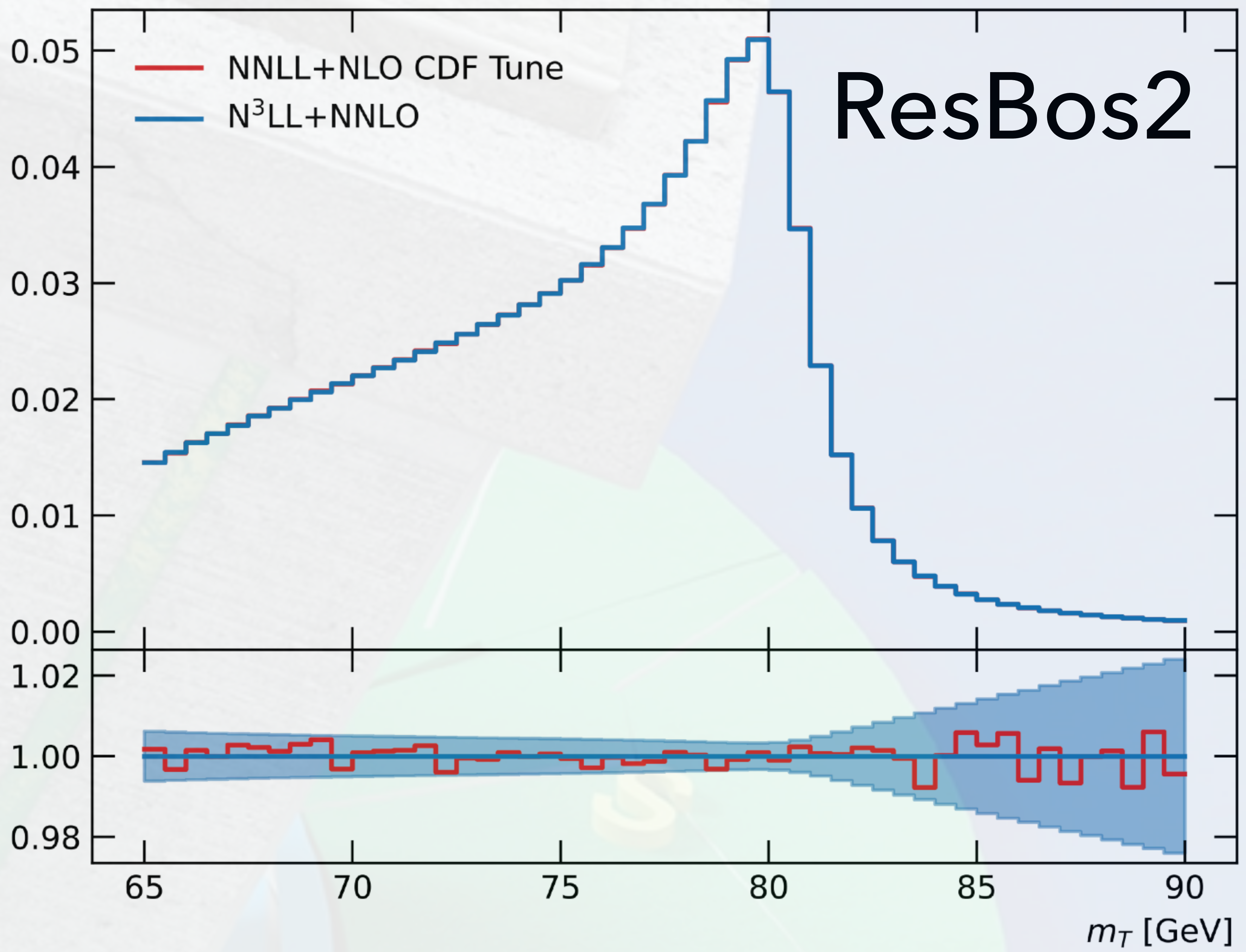
+10~30 MeV

CDFII uncertainty budget



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

- ➤Blue band: envelope of correlated scale variations in the ratio @NNLO

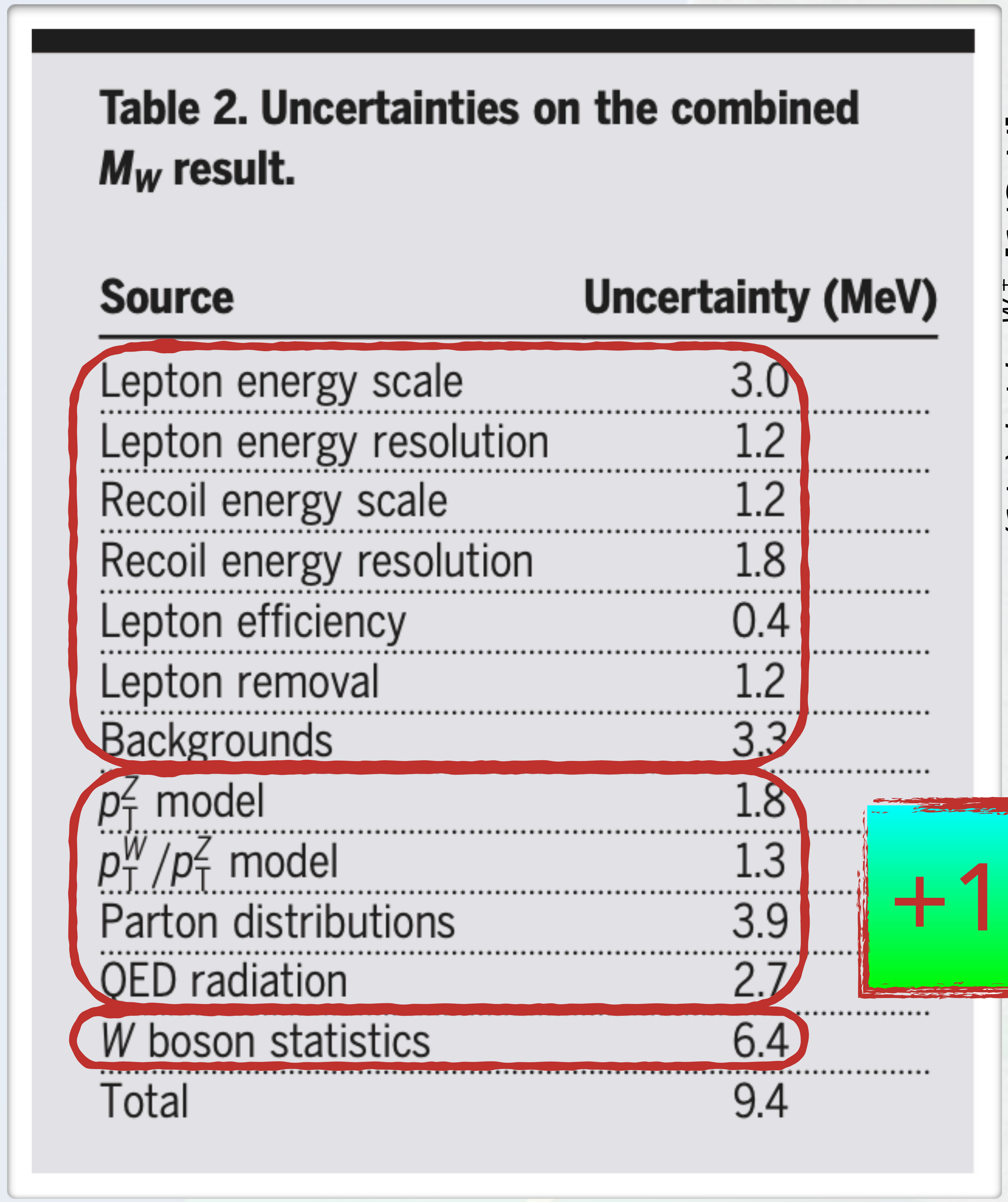


Isaacson, Fu and Yuan`22

- ➤Blue band: CDFII statistical uncertainties

WHAT ARE WE MISSING?

- ➤Realistic theory uncertainty estimation



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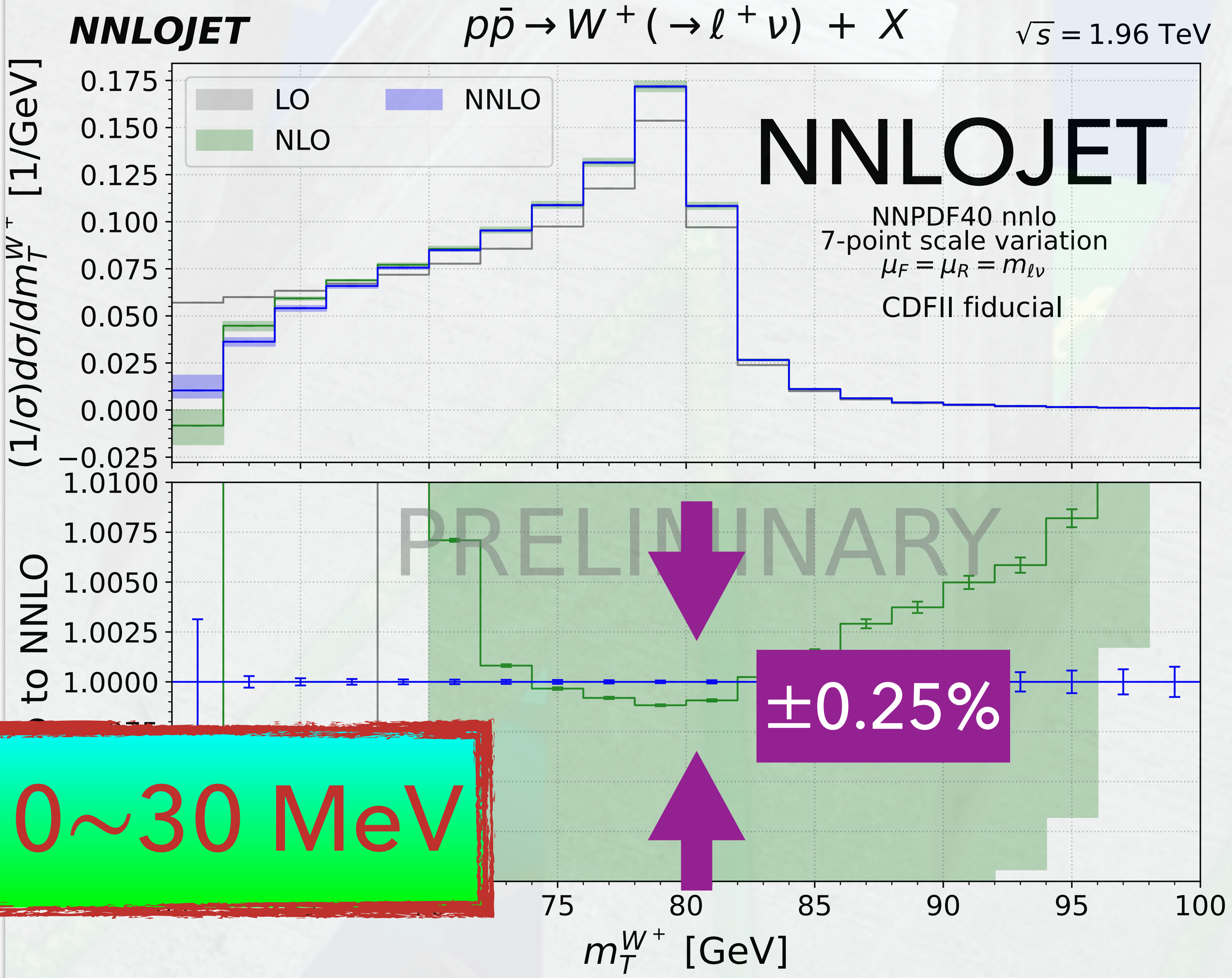
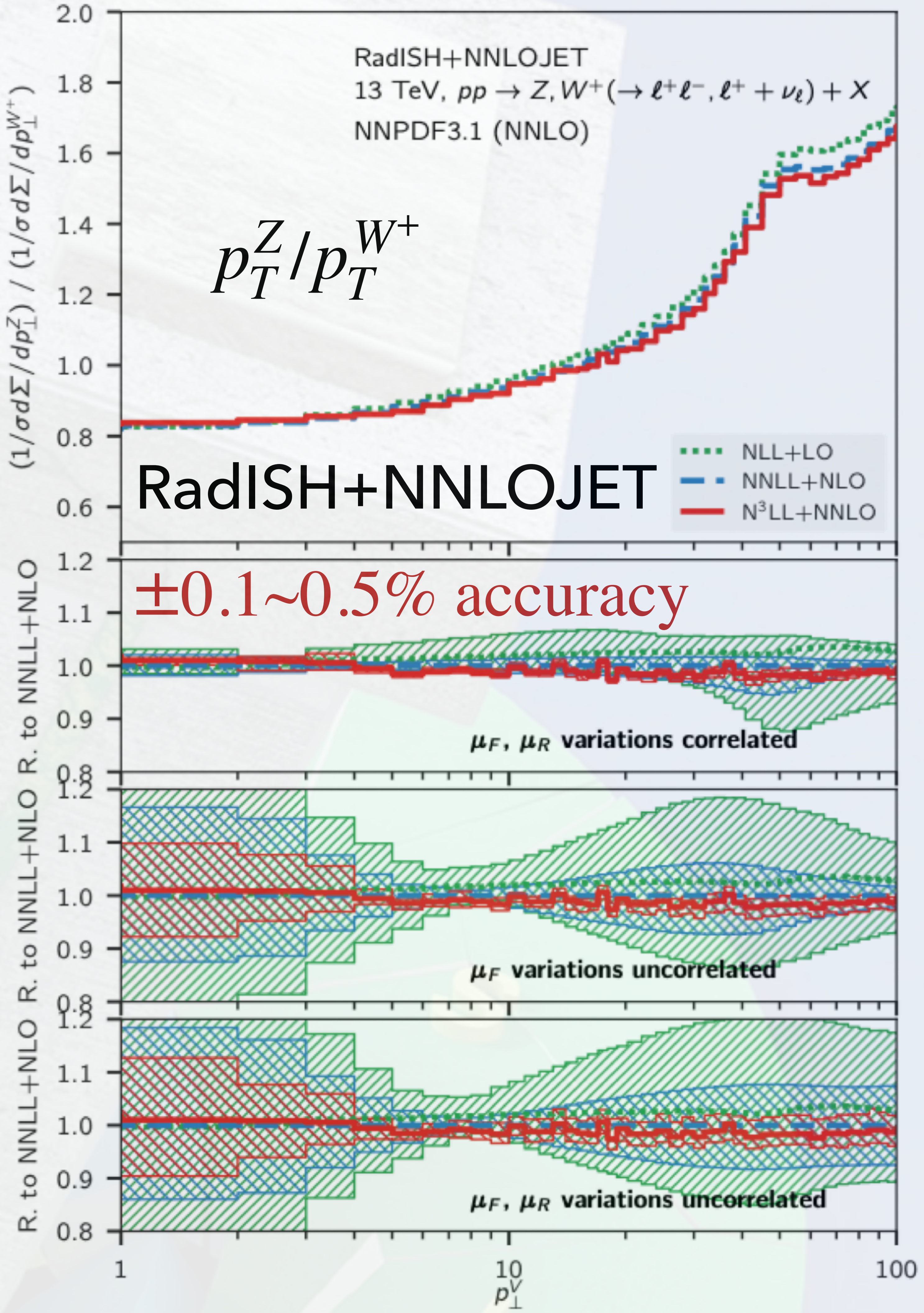
CDFII uncertainty budget



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

- ➤Blue band: envelope of correlated scale variations in the ratio @NNLO



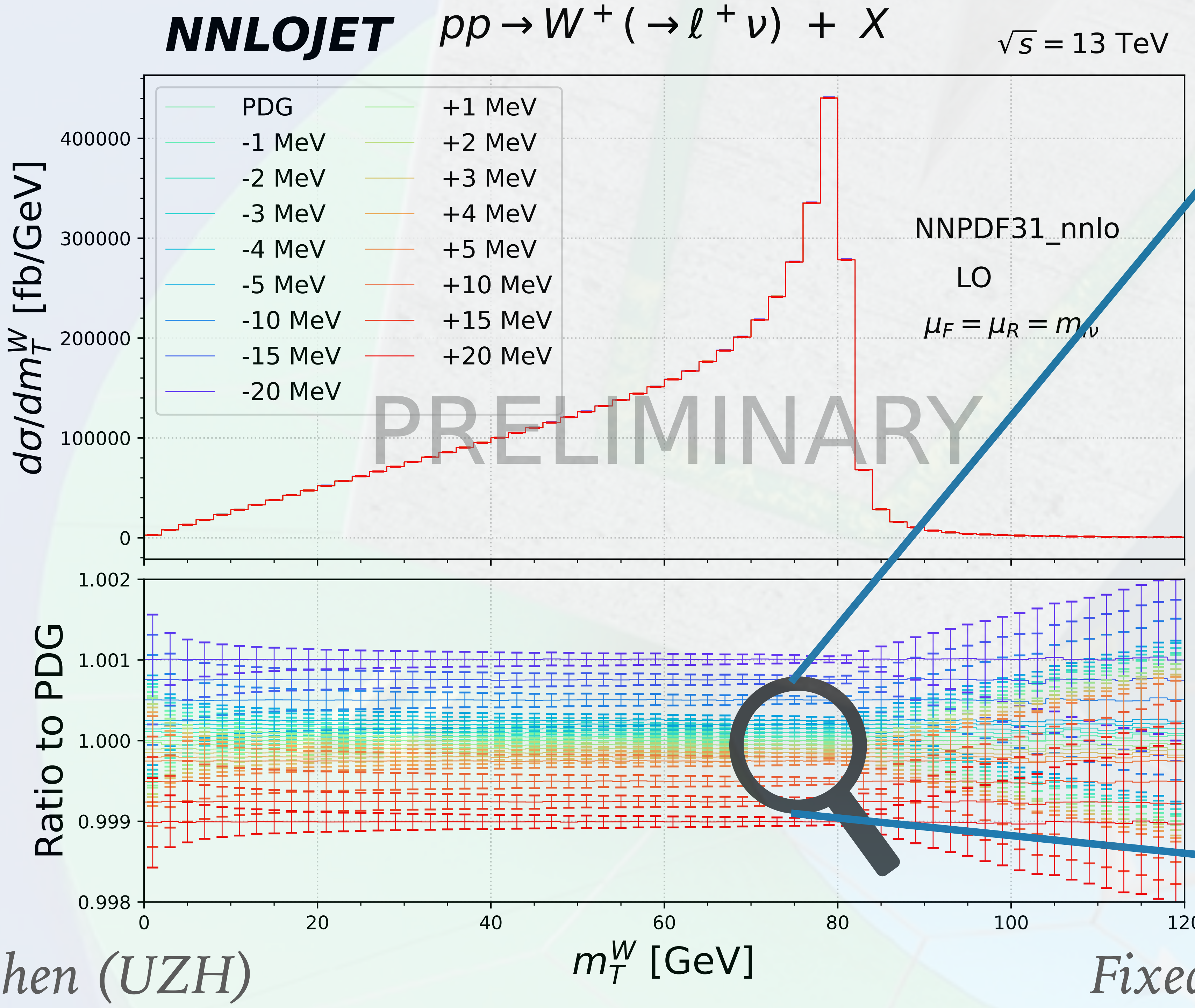
Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker `19

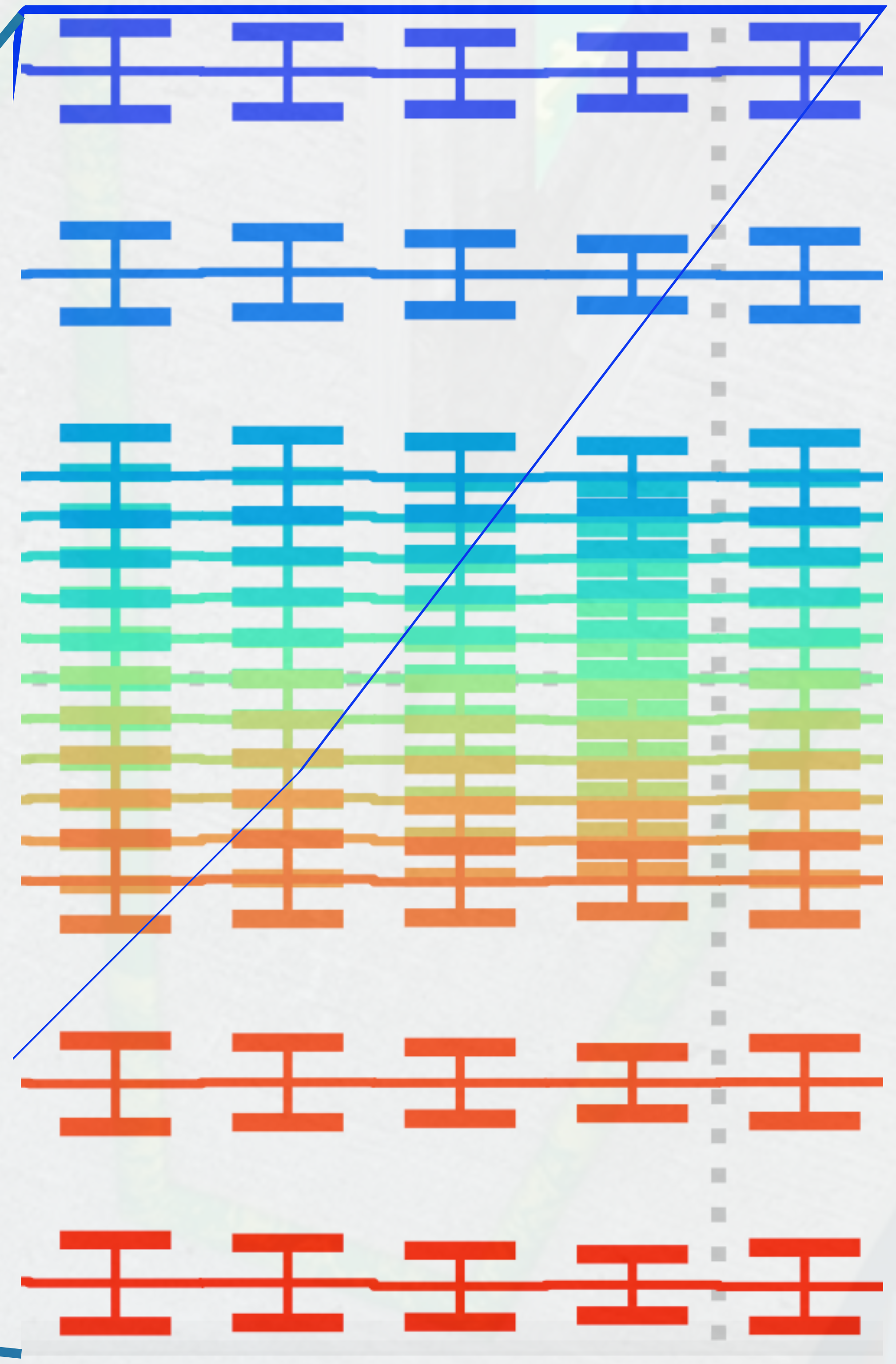
WHAT ARE WE MISSING?

- Numerical error during template generation

Method A: event-by-event self reweighing: $w(m_{l\nu}, m_W, m_W^{\text{ref}}) = \dfrac{(m_{l\nu}^2 - m_W^2)^2 + m^4\Gamma_W^2/m_W^2}{(m_{l\nu}^2 - m_W^{\text{ref}^2})^2 + m^4\Gamma_W^2/m_W^{\text{ref}^2}}$

  - Analytical reweighing factor without interpolating error.
  - MC error of reference propagates to all m_W choices.
  - New fiducial cuts → new calculation.





NNLOJET

W@LO $\pm 0.0005\,\%$ XS error

4k CPU hours

$d\sigma/dm_T^W > \pm 0.003\,\%$ error

ΔMeV ~ 23 fb

$\Delta\sigma \sim 17$ fb

$3\Delta\sigma/\Delta\text{MeV} =$

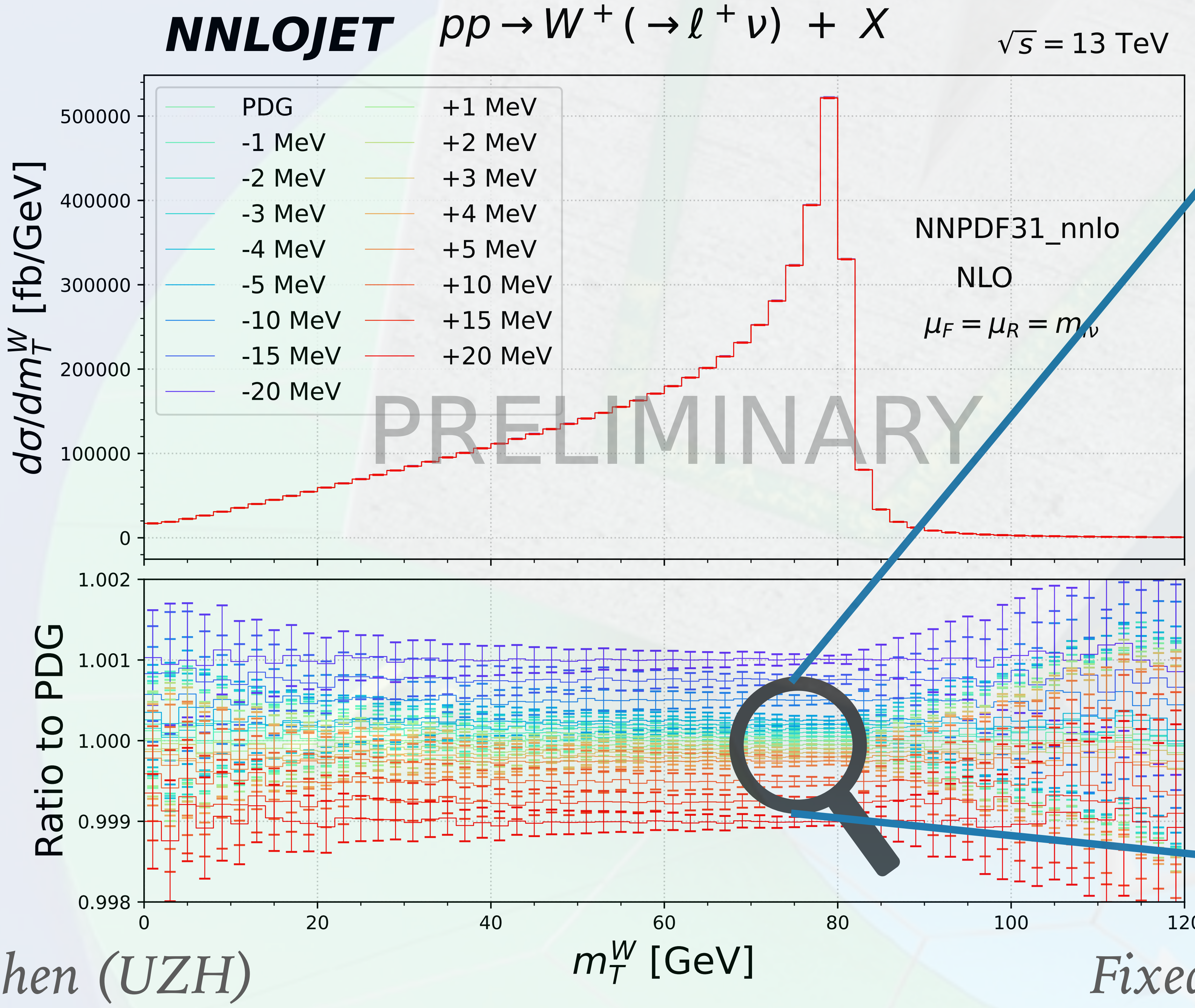
±2.2 MeV MC error

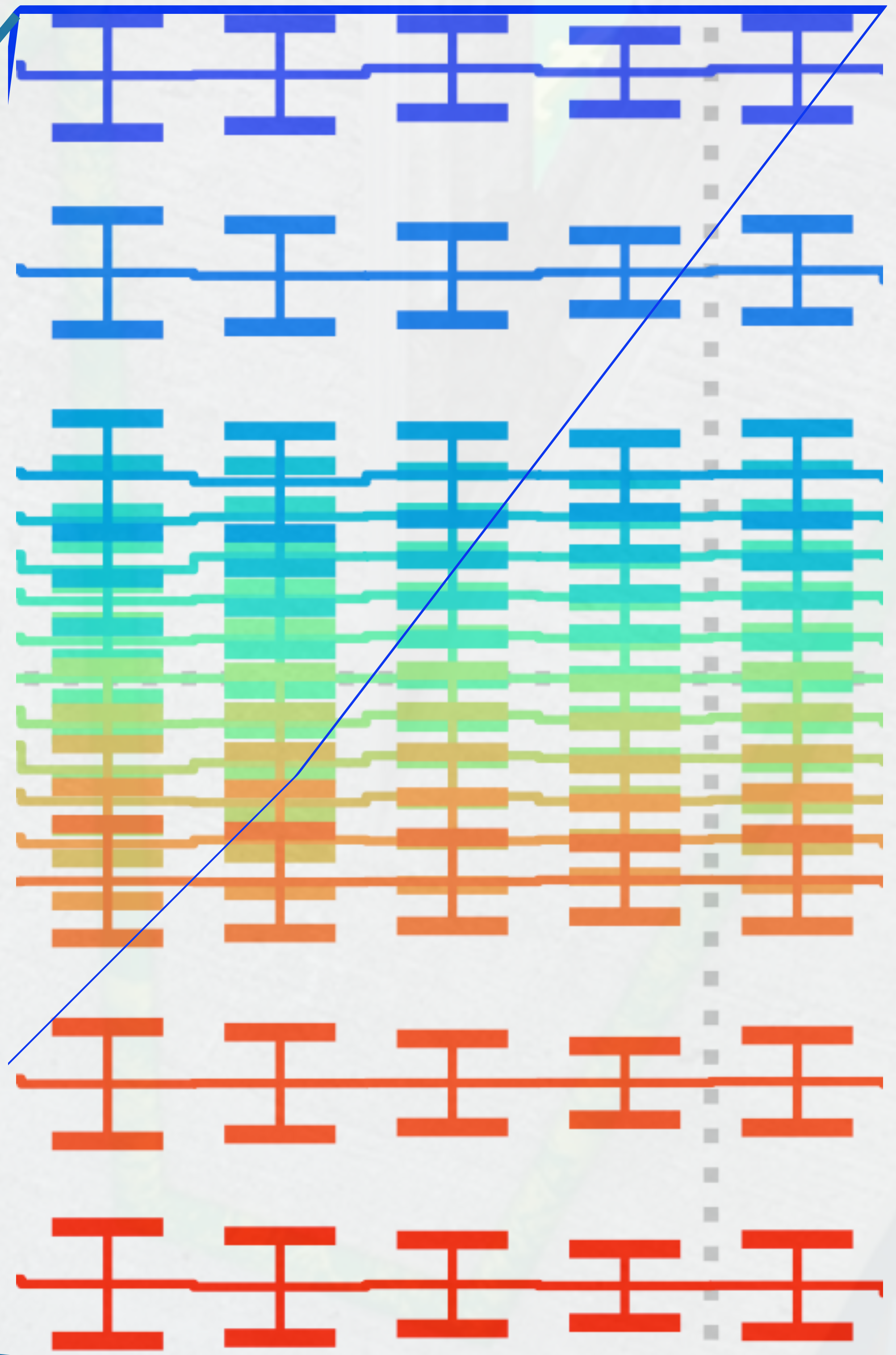
WHAT ARE WE MISSING?

➤Numerical error during template generation

Method A: event-by-event self reweighing: $$w(m_{l\nu}, m_W, m_W^{\mathrm{ref}}) = \frac{(m_{l\nu}^2 - m_W^2)^2 + m^4\Gamma_W^2/m_W^2}{(m_{l\nu}^2 - m_W^{\mathrm{ref}^2})^2 + m^4\Gamma_W^2/m_W^{\mathrm{ref}^2}}$$

- ➤ Analytical reweighing factor without interpolating error.
- ➤ MC error of reference propagates to all m_W choices.
- ➤ New fiducial cuts → new calculation.





NNLOJET

W@NLO ±0.0009 % XS error

8k CPU hours

$d\sigma/dm_T^W > \pm 0.005\,\%$ error

Δ MeV ~ 26 fb

$\Delta\sigma \sim 24$ fb

$3\Delta\sigma/\Delta\mathrm{MeV} =$

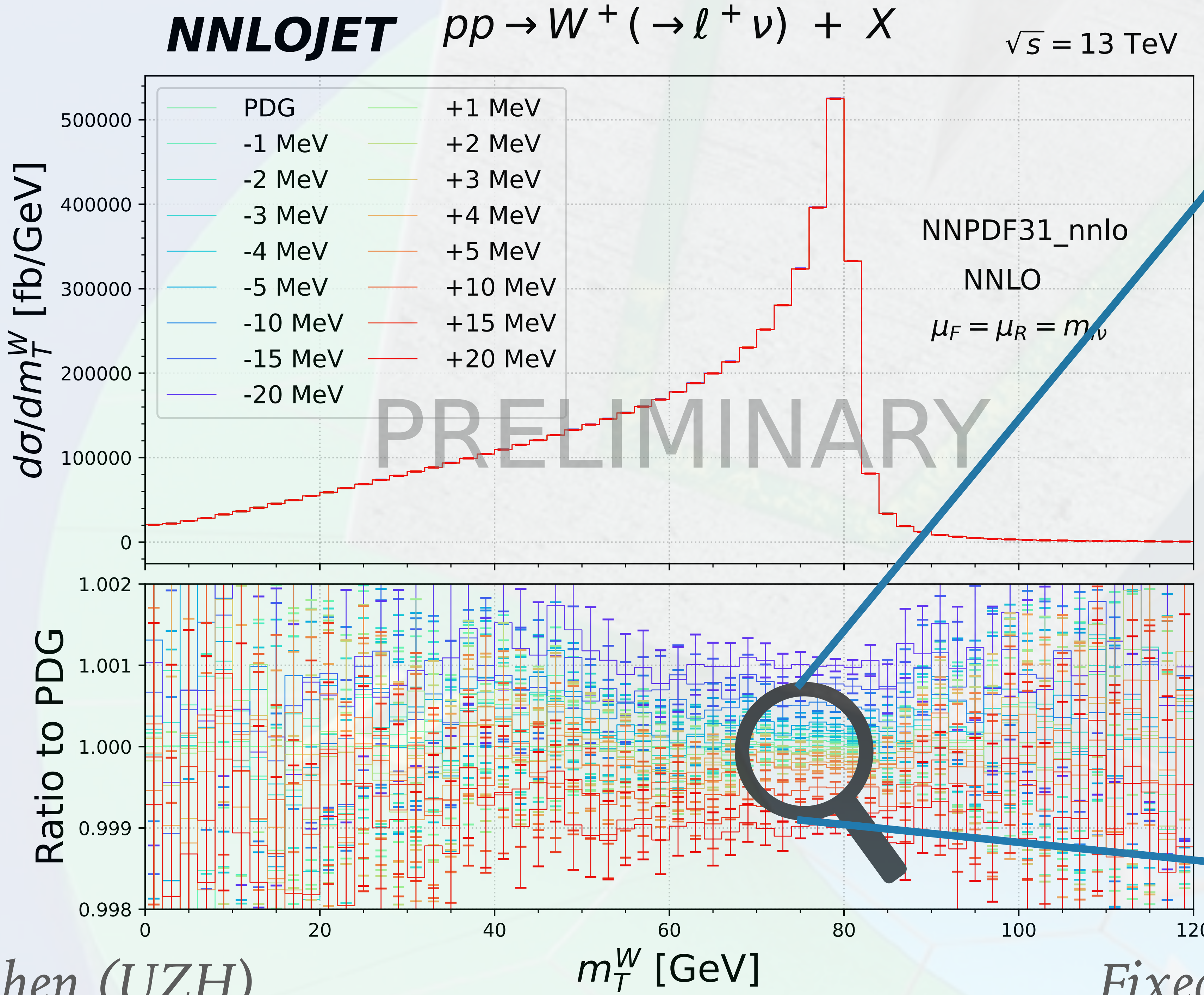
±2.8 MeV MC error

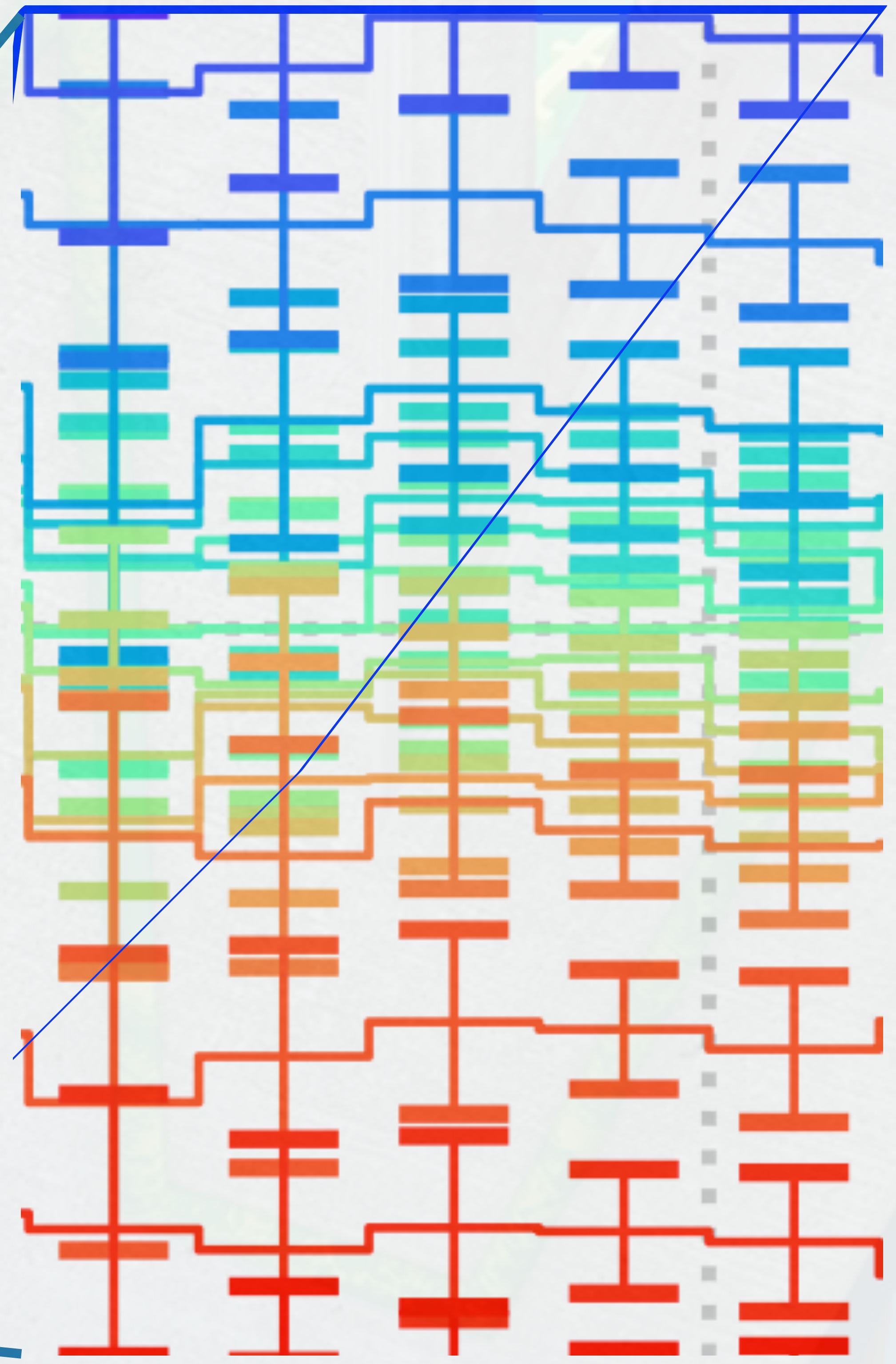
WHAT ARE WE MISSING?

➤Numerical error during template generation

Method A: event-by-event self reweighing: $$w(m_{l\nu}, m_W, m_W^{\mathrm{ref}}) = \frac{(m_{l\nu}^2 - m_W^2)^2 + m^4\Gamma_W^2/m_W^2}{(m_{l\nu}^2 - m_W^{\mathrm{ref}^2})^2 + m^4\Gamma_W^2/m_W^{\mathrm{ref}^2}}$$

- ➤ Analytical reweighing factor without interpolating error.
- ➤ MC error of reference propagates to all m_W choices.
- ➤ New fiducial cuts → new calculation.





NNLOJET

W@NNLO $\pm 0.0013\,\%$ XS error

35k CPU hours

$d\sigma/dm_T^W > \pm 0.008\,\%$ error

Δ MeV ~ 32 fb

$\Delta\sigma$ ~ 39 fb

$3\Delta\sigma/\Delta$MeV =

±3.7 MeV MC error

Is it comptable with $q_T^{cut}, \tau^{cut}, s_{ij}^{cut}$?

Cross check Alekhin, Kardos, et. al. `21

WHAT ARE WE MISSING?

- Numerical error during template generation

Method B: grid with all D.O.F.:

$$\frac{d\sigma}{dm_{l\nu}dp_Tdy}\left[(1+\cos^2\theta)+\sum_{i=0}^{7}A_i f_i(\theta,\phi)\right]$$

ResBos2 approach for W@NNLO

- Numerically challenging for D.O.F = 11 (may drop $A_{5,6,7}$ for being very small)
- MC error of each grid bin + interpolation error cross bins (prefer fine granularity)
- Once $A_i(p_T, y, m_{l\nu})$ available, no new calculation is needed for different fiducial cuts

Z+J @ NNLO

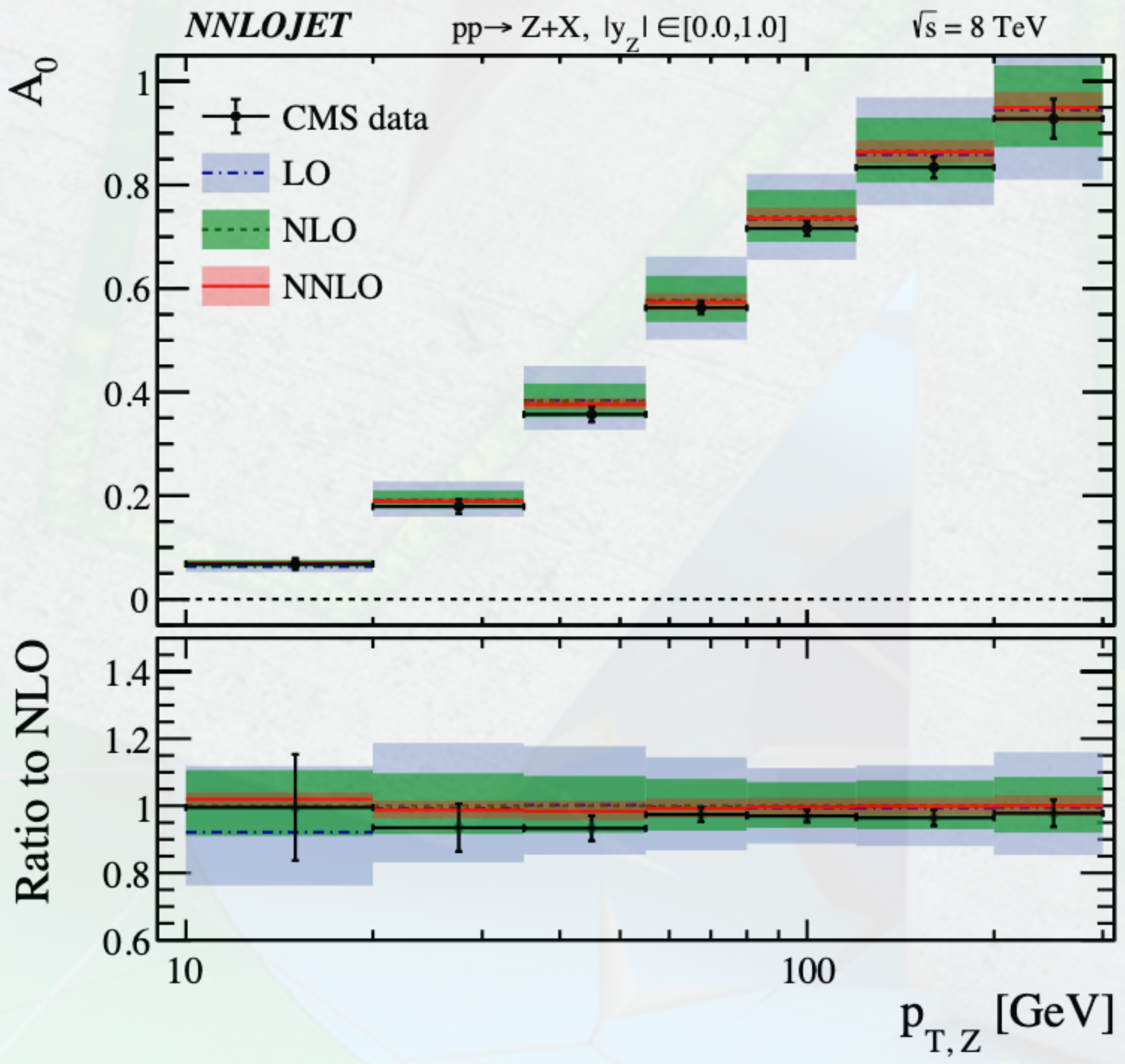
$A_0(p_T, y)$

$|y| < 1$

Inclusive in $m_{l\nu}$

Smallest bin @ 10 GeV

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss `17



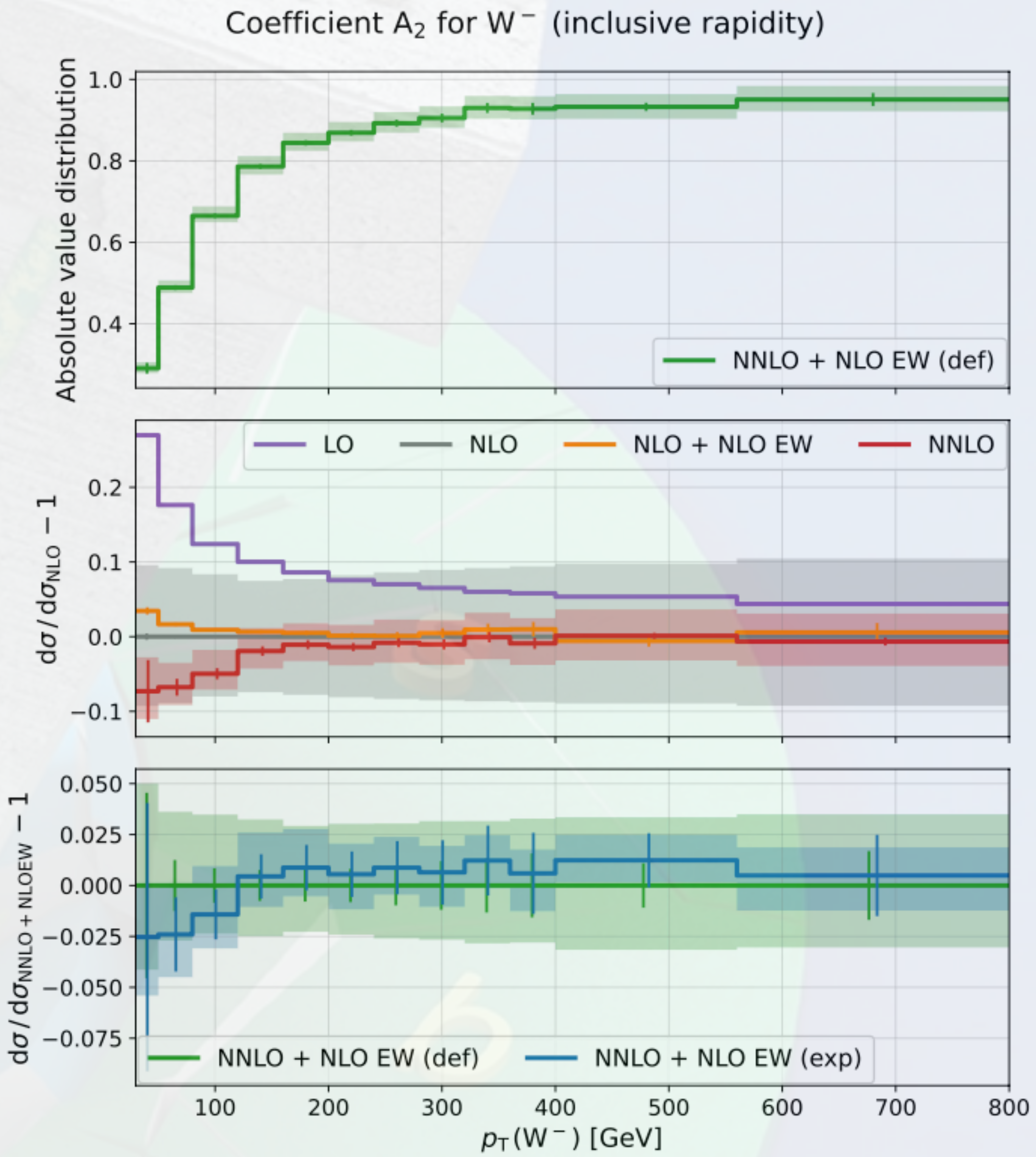
W+J @ NNLO

$A_2(p_T)$

Inclusive in y, $m_{l\nu}$

Smallest bin ~ 20 GeV

Pellen, Poncelet, Popescu, Vitos `22



WHAT ARE WE MISSING?

- Light and Heavy quark effects
  - Full CKM mixing vs. Cabibbo mixing
    - 5-flavour scheme Cabibbo mixing affects only PDF

      $f_u f_{\bar{d}} \to |V_{ud}|^2 f_u f_{\bar{d}} + |V_{us}|^2 f_u f_{\bar{s}}$

      $f_c f_{\bar{s}} \to |V_{cs}|^2 f_c f_{\bar{s}} + |V_{cd}|^2 f_c f_{\bar{d}}$

      $f_q f_Q \to f_q f_Q$ for q and Q not from a SU(2) doublet
    - Full CKM mixing

      $f_u f_{\bar{d}} \to |V_{ud}|^2 f_u f_{\bar{d}} + |V_{us}|^2 f_u f_{\bar{s}} + |V_{ub}|^2 f_u f_{\bar{b}}$

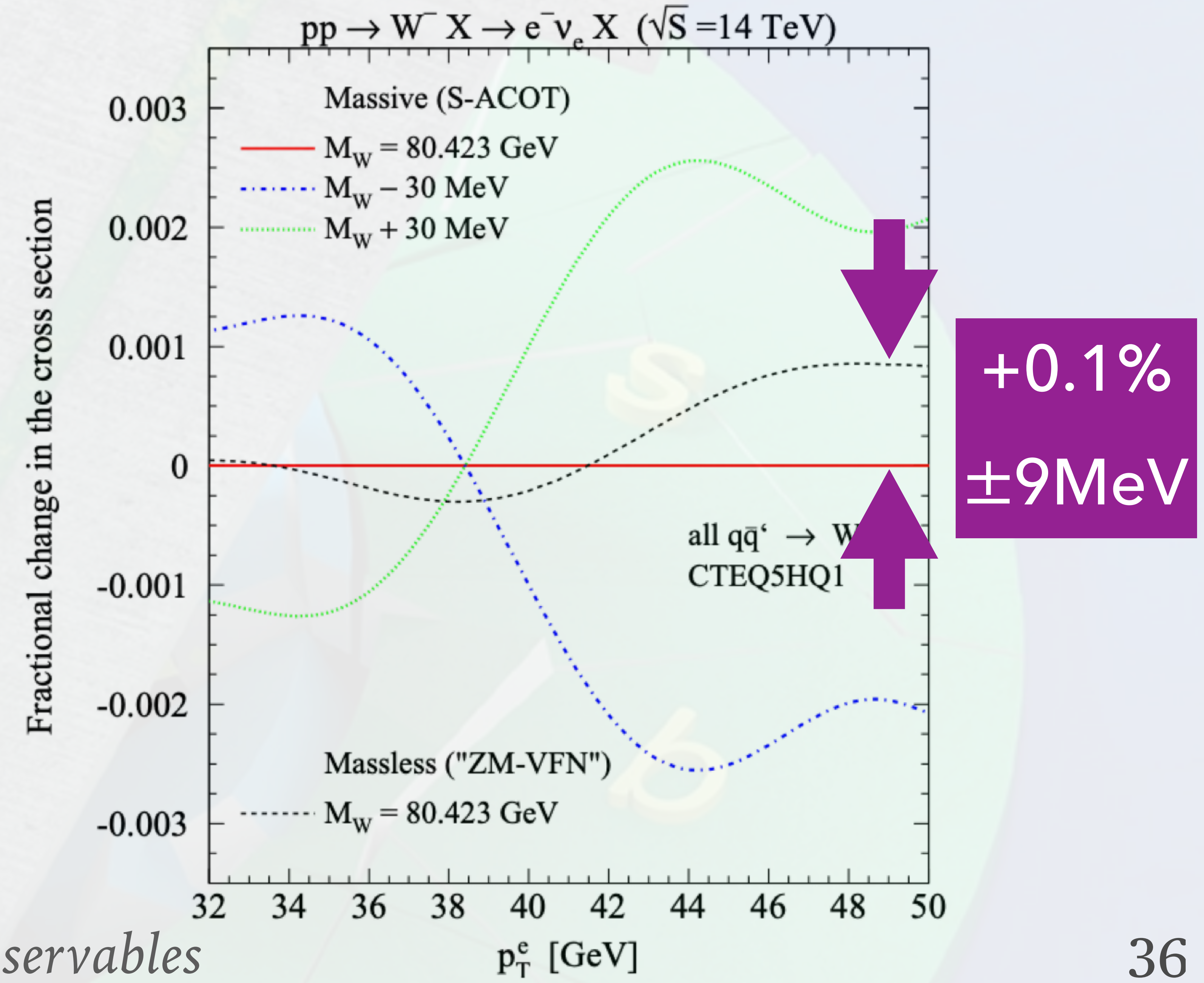
      $f_c f_{\bar{s}} \to |V_{cs}|^2 f_c f_{\bar{s}} + |V_{cd}|^2 f_c f_{\bar{d}} + |V_{cb}|^2 f_c f_{\bar{b}}$

      $f_q f_Q \to f_q f_Q$ for one of q, Q $\in [u, c]$

      $f_q f_Q \to f_q f_Q \left[\left(|V_{uq}|^2 + |V_{cq}|^2 \right) \otimes \sigma + |V_{tq}|^2 \otimes \sigma(m_t) \right]$

      for one of q, Q $\in [d, s, b]$
      - $|V_{tb}|^2$ requires W + top production
  - Sea quark mixing → flat K factor, final top could be special

- Light quark mass effects require through study
  - Retain full quark mass dependence in FO, PDF and resummation: GM-VFN scheme. Collins `98
  - Reasonably good approximation in S-ACOT scheme @NLO+NLL indicates 9 MeV (LHC) and 3 MeV (Tevatron) shift of m_W.



Berge, Nadolsky, Olness `05

CONCLUSION AND OUTLOOK

- The determination of W boson mass requires delicate treatment and thorough understanding of experiment and theory uncertainties.
- Theoretical uncertainties at 0.1% level is required to achieve 10 MeV accuracy in m_W
- Best predictions for CC DY production at N3LO QCD achieves 1% accuracy.
- Thorough study of resummation schemes, transition region profiling, non-perturbative effects indicates few % extra error.
- Choices of correlated, uncorrelated, MHO uncertainty analysis can make theoretical uncertainties artificial small at 0.1% level.
- Require collective efforts to reliably estimate theory uncertainties and to reduce it.
- As the LHC entering precision era, fascinating progress is ahead to understand the m_W puzzle. Many uncertainties and opportunities!

CONCLUSION AND OUTLOOK

- The determination of W boson mass requires delicate treatment and thorough understanding of experiment and theory uncertainties.
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Thank You for Your Attention

PRECISION PREDICTIONS IN CDFII

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson Ph.D. thesis`17
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

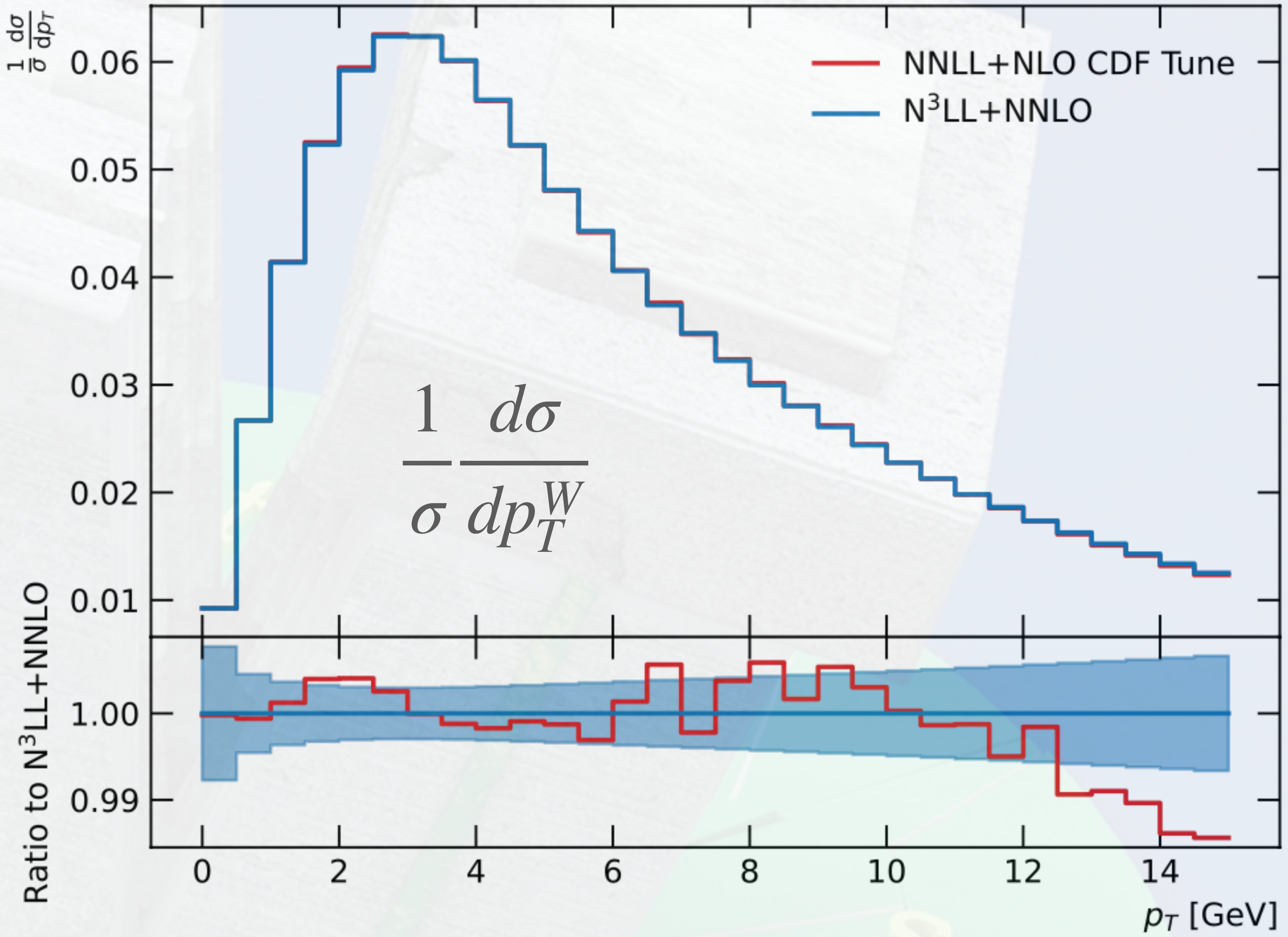
    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

$$(1+\cos^2\theta) + \frac{1}{2}A_0\left(1-3\cos^2\theta\right) + A_1 \sin 2\theta \cos\phi + A_2 \sin^2\theta \cos 2\phi$$
$$+A_3 \sin\theta \cos\phi + A_4 \cos\theta + A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta \sin\phi$$

Isaacson, Fu and Yuan`22

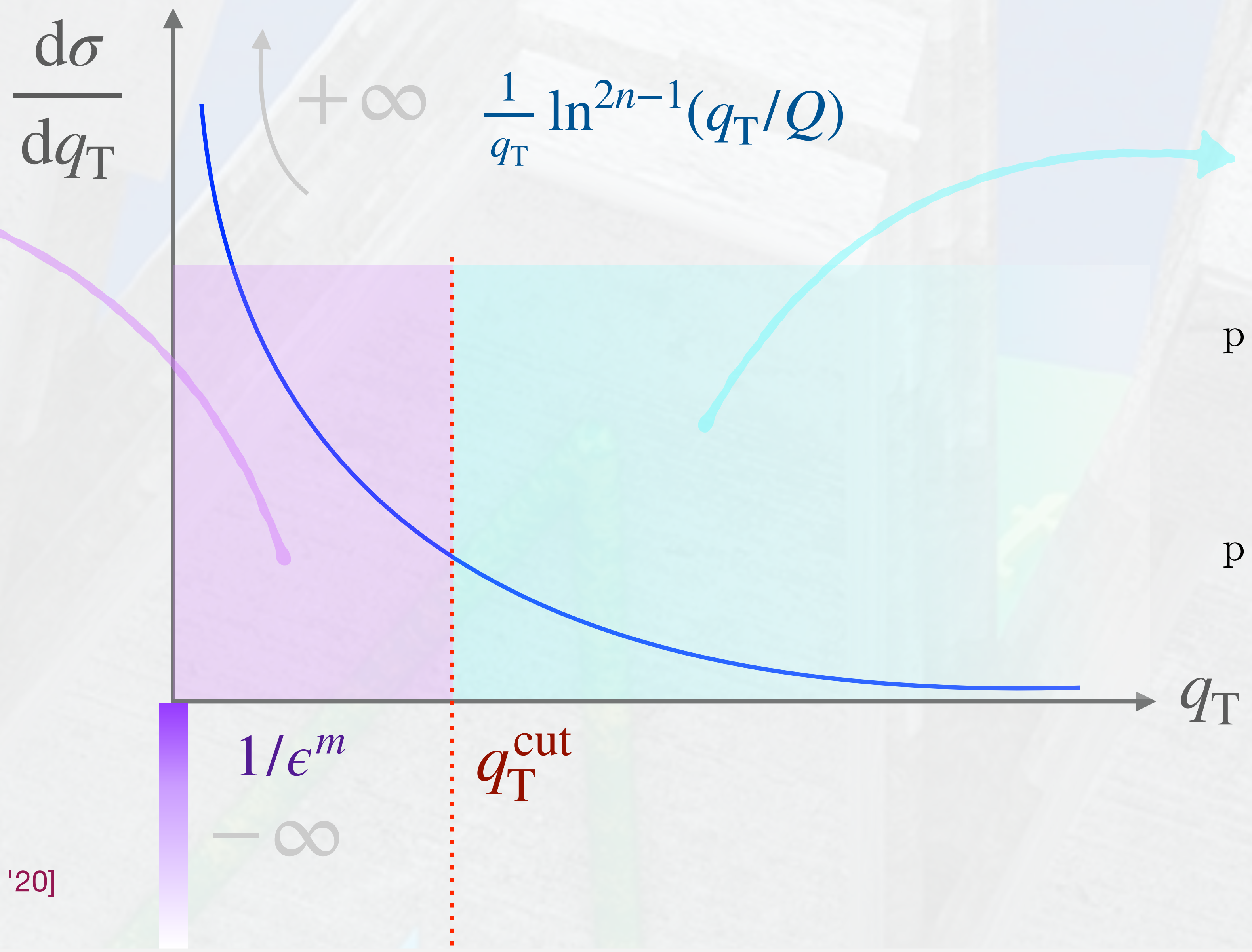


- Pseudo data: NNLO+N3LL p_T^Z with global fit
- Fit g2, α_s in NLO+NNLL p_T^Z to pseudo data
- Use fitted g2, α_s in NLO+NNLL W templates

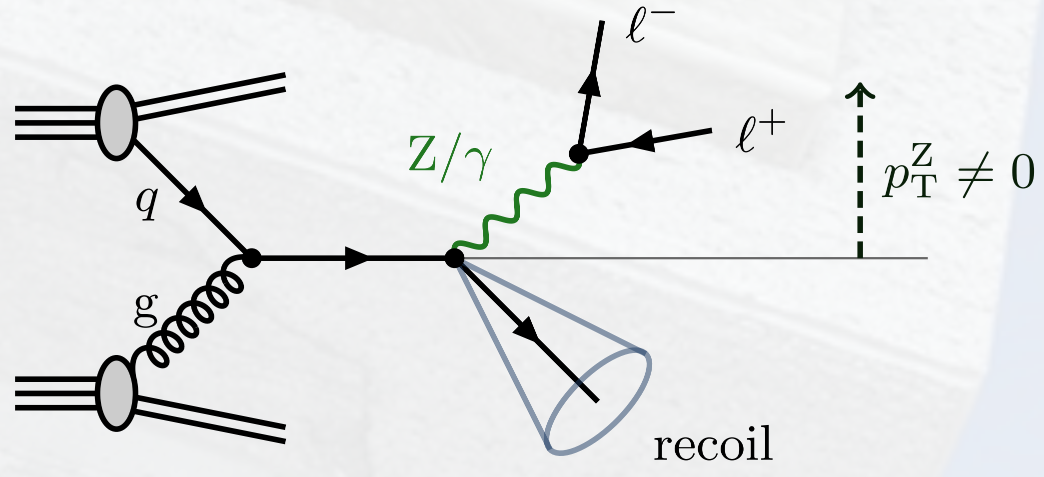
q_T SUBTRACTION @ N^{3}LO

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$
    [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$
    [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$
    [Luo, Yang, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$d\sigma^V_{\text{N}^3\text{LO}} = d\sigma^V_{\text{N}^3\text{LO}}\Big|_{q_T<q_T^{\text{cut}}} + d\sigma^V_{\text{N}^3\text{LO}}\Big|_{q_T>q_T^{\text{cut}}}$$

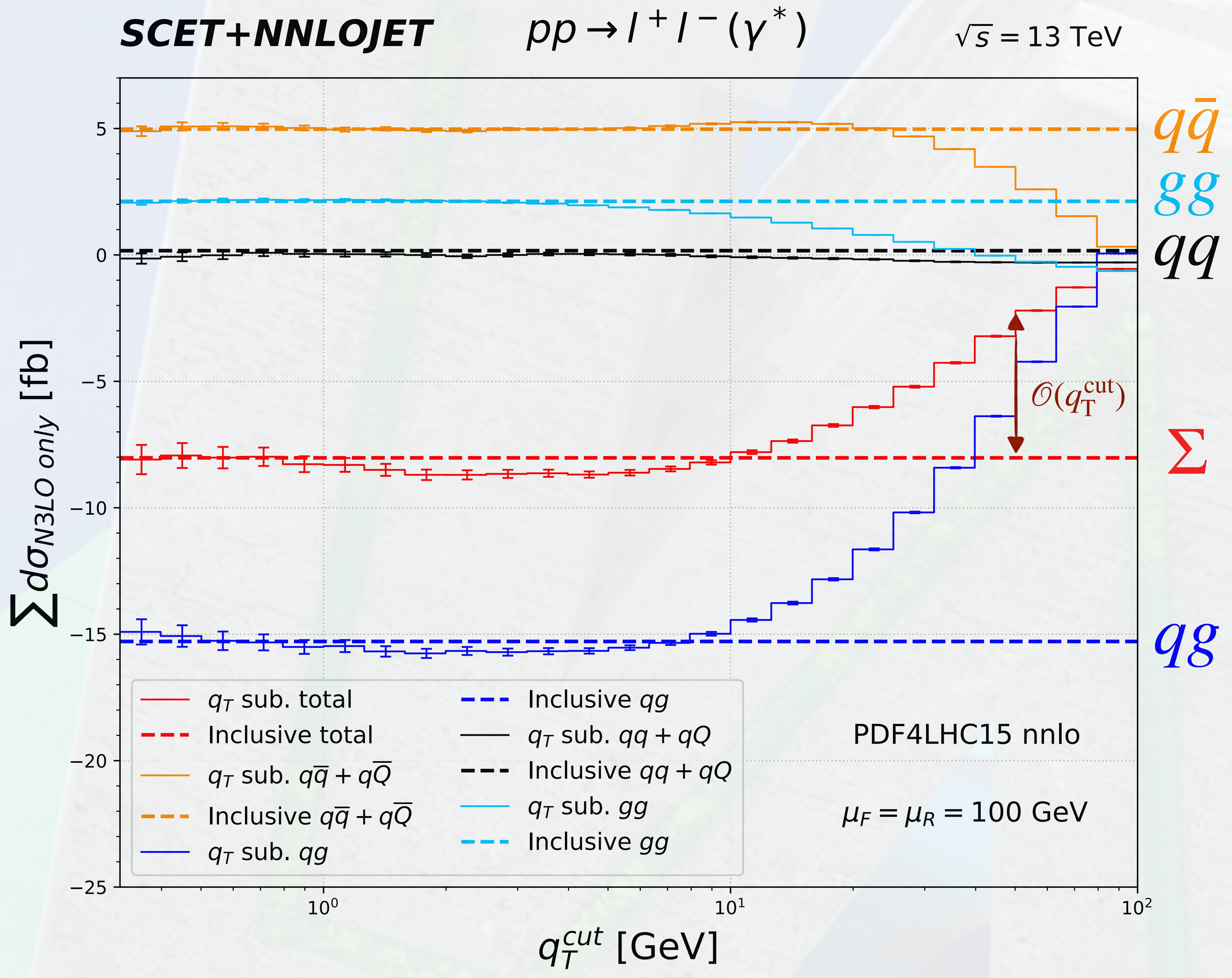
$$= \mathcal{H}^V_{\text{N}^3\text{LO}} \otimes d\sigma^V_{\text{LO}} + \left[d\sigma^{V+\text{jet}}_{\text{NNLO}} - d\sigma^{V,\text{CT}}_{\text{N}^3\text{LO}}\right]_{q_T>q_T^{\text{cut}}} + \mathcal{O}\left((q_T^{\text{cut}}/Q)^n\right)$$

Competing interests: q_T^{cut} as small as possible ⇜⇝ q_T^{cut} as large as possible

↪ suppress power corrections ↪ numerical stability & efficiency

INCLUSIVE $pp \to \gamma^*$ @ N³LO

[Chen, Gehrmann, Glover, AH, Yang Zhu '21]



Fixed order	$\sigma_{pp\to\gamma^*}$ (fb)		
LO	$339.62^{+34.06}_{-37.48}$		
NLO	$391.25^{+10.84}_{-16.62}$		
NNLO	$390.09^{+3.06}_{-4.11}$		
N³LO	$382.08^{+2.64}_{-3.09}$ [14]		
N³LO only	$q_T^{cut}=0.63$ GeV	$q_T^{cut}\to 0$ fit	[14]
qg	−15.32(32)	−15.34(54)	−15.29
$q\bar{q}+q\bar{Q}$	+5.06(12)	+5.05(12)	+4.97
gg	+2.17(6)	+2.19(6)	+2.12
$qq+qQ$	+0.09(13)	+0.09(17)	+0.17
Total	−7.98(36)	−8.01(58)	−8.03

- validation against analytic result (⇠) [Duhr, Dulat, Mistlberger '20]
- fully independent calculation
  - ⇝ confirmation of large negative N³LO corrections (−2% & outside of NNLO band)

THE PROJECTION-TO-BORN METHOD — MASTER FORMULA

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}\sigma_{F,\,\mathrm{inc.}}^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \left\{ \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}}\bigg|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

observables projected to Born
fully local counter term

- real-emission phase space: $\mathrm{d}\Phi_{\mathrm{H}+n}$

  $p_a + p_b \to p_{\mathrm{H}} + k_1 + k_2 + \ldots + k_n$

- projection to Born: $\mathrm{d}\widetilde{\Phi}_{\mathrm{H}}$

  $\tilde{p}_a + \tilde{p}_b \to \tilde{p}_{\mathrm{H}} \qquad (\tilde{p}_a = \xi_a p_a, \quad \tilde{p}_b = \xi_b p_b)$

  on-shell: $\tilde{p}_{\mathrm{H}}^2 \equiv p_{\mathrm{H}}^2 = M_{\mathrm{H}}^2 \quad \Rightarrow \quad \xi_a\,\xi_b = \dfrac{2p_ap_b - 2(p_a+p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_ap_b}$

  rapidity: $\tilde{y}_{\mathrm{H}} \equiv y_{\mathrm{H}} \quad \Rightarrow \quad \xi_a/\xi_b = \dfrac{2p_bp_{\mathrm{H}}}{2p_ap_{\mathrm{H}}}$

  ↪ decay products: $p_{\mathrm{H}} \to p_1 + \ldots + p_m \qquad (p_i^\mu \to \tilde{p}_i^\mu = \Lambda^\mu{}_\nu\, p_i^\nu)$

  $\Lambda^\mu{}_\nu(p_{\mathrm{H}}, \tilde{p}_{\mathrm{H}}) = g^\mu{}_\nu - \dfrac{2(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^\mu (p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})_\nu}{(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^2} + \dfrac{2\tilde{p}_{\mathrm{H}}^\mu p_{\mathrm{H},\nu}}{p_{\mathrm{H}}^2}$

◉ sub-divergences
- dealt with F+jet @ $\mathrm{N}^{k\text{-}1}\mathrm{LO}$

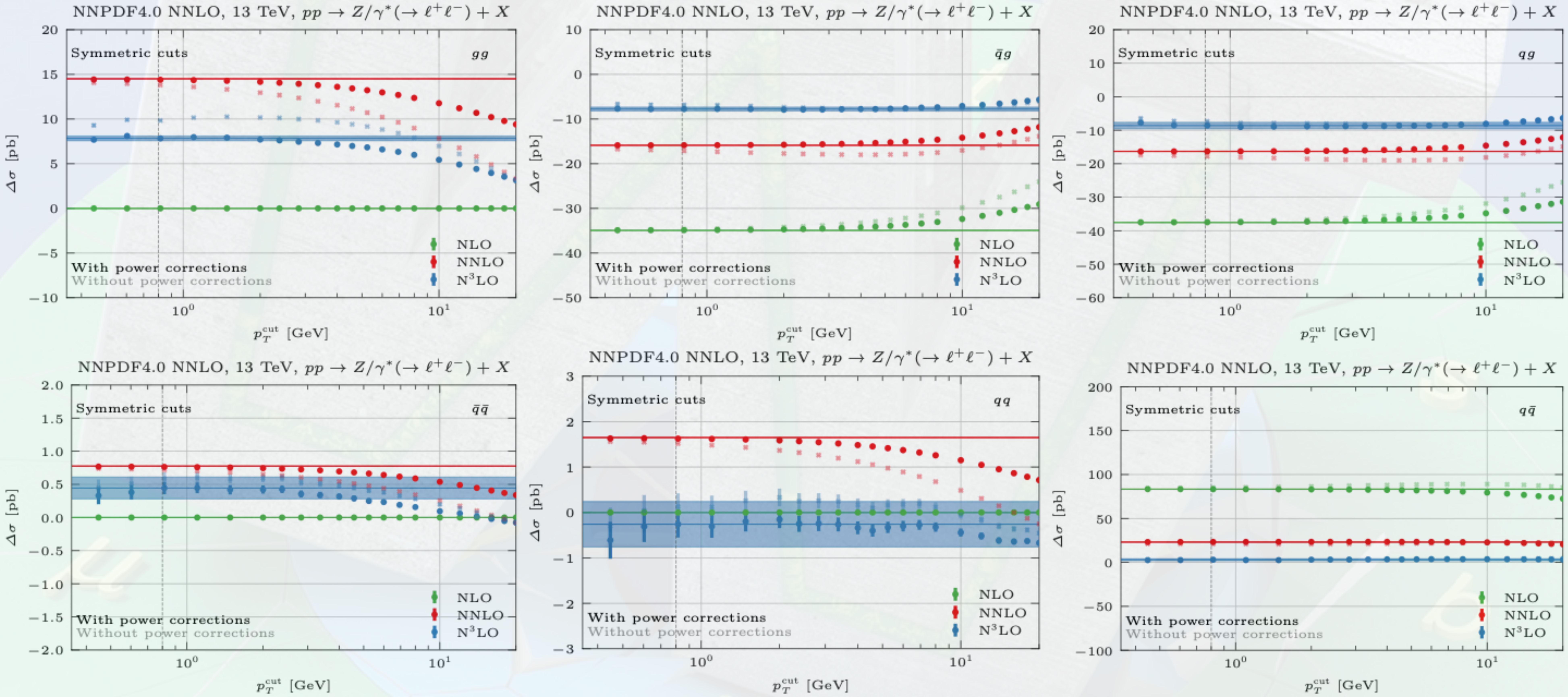
◉ $\mathrm{N^kLO}$ divergences
- fully local prescription
- P2B (“ideal” subtraction)

BACKUP SLIDES

- Differential N3LO predictions for neutral current production with fiducial cuts
  - Resum all order contributions at N3LL using RadISH and matched to N3LO

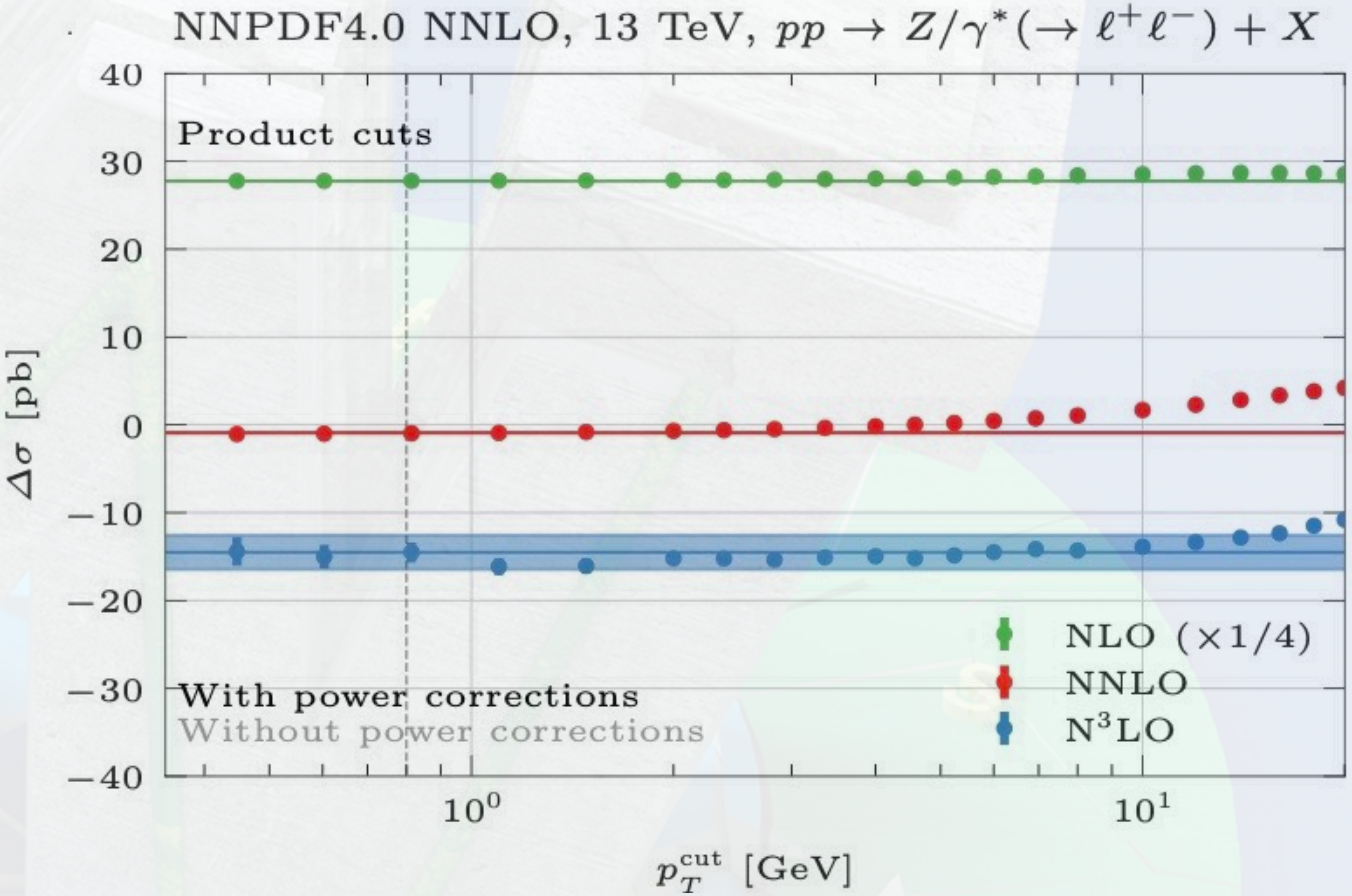
Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$	$\mathrm{N}^k\mathrm{LO}$	$\mathrm{N}^k\mathrm{LO}+\mathrm{N}^k\mathrm{LL}$
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

XC, Gehrmann, Glover, Huss,
Monni, Rottoli, Re, Torrielli `22



BACKUP SLIDES

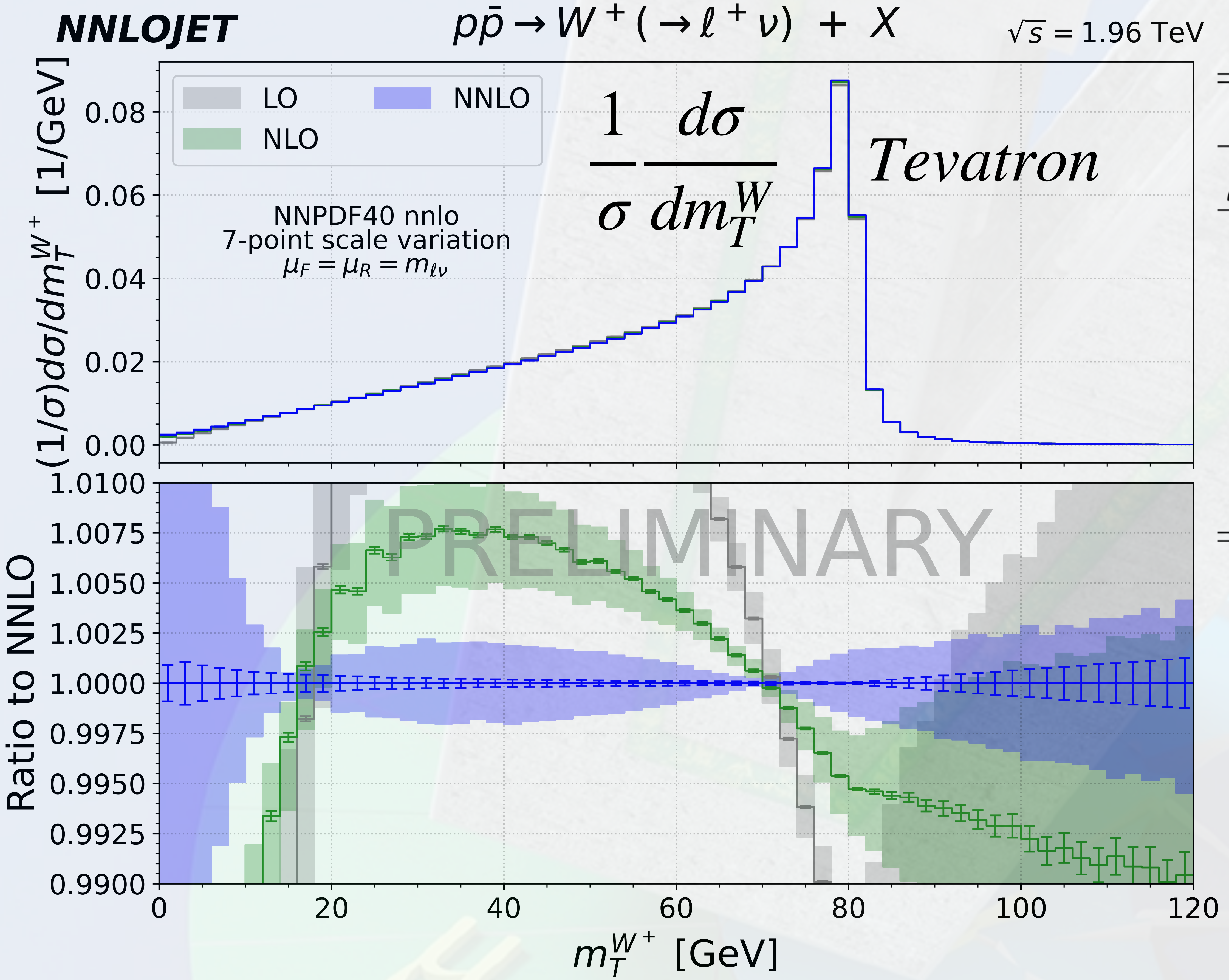
- Differential N3LO predictions for neutral current production with fiducial cuts
  - Apply ATLAS fiducial cuts at 13 TeV
    - Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll2}}$
    - $m_{ll} \in [66{,}116]$ GeV, $|\eta^{l^\pm}| < 2.5$
    - Symmetric cuts: $|p_T^{l^\pm}| > 27$ GeV
      Introduce power correction at $\mathcal{O}(q_T^{cut}/m_{ll})$
    - Solution:
      - Apply Lorentz Boost below q_T^{cut}
        Buonocore, Rottoli, Kallweit, Wiesemann `21
        Camarda, Cieri, Ferrera `21
      - Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27$ GeV
        Salam, Slade `21 $\min\{p_T^{l^+}, p_T^{l^-}\} > 20$ GeV
  - Typical fiducial cuts for m_T^V, p_T^V in DY production
    - Large log terms appear in $p_T^l \sim m_V/2$, $m_T^V \sim 2 \times \min[p_T^l]$, $p_T^V \sim 0$



XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22

WHAT ARE WE MISSING?

- Realistic theory uncertainty estimation



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

$$R^{(n)}_{W^+W^-} = \sigma^{W^+}/\sigma^{W^-}$$

	NLO		NNLO		N^{3}LO	
μ^{cent}	m_W	$m_W/2$	m_W	$m_W/2$	m_W	$m_W/2$
A	$1.342^{+0.10\%}_{-0.08\%}$	$1.342^{+0.07\%}_{-0.05\%}$	$1.348^{+0.12\%}_{-0.10\%}$	$1.349^{+0.15\%}_{-0.11\%}$	$1.350^{+0.05\%}_{-0.06\%}$	$1.350^{+0.04\%}_{-0.05\%}$
A′	$1.343^{+0.13\%}_{-0.16\%}$	$1.344^{+0.10\%}_{-0.21\%}$	$1.349^{+0.13\%}_{-0.09\%}$	$1.351^{+0.33\%}_{-0.13\%}$	$1.350^{+0.02\%}_{-0.03\%}$	$1.350^{+0.01\%}_{-0.09\%}$
B	$1.342^{+8.82\%}_{-8.08\%}$	$1.342^{+12.9\%}_{-11.4\%}$	$1.348^{+2.26\%}_{-2.31\%}$	$1.349^{+2.24\%}_{-2.27\%}$	$1.350^{+2.21\%}_{-2.14\%}$	$1.350^{+2.21\%}_{-2.14\%}$
B′	$1.343^{+5.28\%}_{-7.40\%}$	$1.344^{+8.09\%}_{-8.97\%}$	$1.349^{+1.85\%}_{-2.63\%}$	$1.351^{+2.21\%}_{-2.24\%}$	$1.350^{+2.60\%}_{-2.25\%}$	$1.350^{+4.65\%}_{-2.70\%}$
C	$1.342^{+0.99\%}_{-0.99\%}$	$1.342^{+0.58\%}_{-0.58\%}$	$1.349^{+0.52\%}_{-0.52\%}$	$1.349^{+0.53\%}_{-0.53\%}$	$1.350^{+0.15\%}_{-0.15\%}$	$1.350^{+0.11\%}_{-0.11\%}$

Duhr, Dulat, Mistlberger `20

- A: $\sigma^{W^+}_{FO}/\sigma^{W^-}_{FO}$ truncate at fixed order
- A’: $\sigma^{W^+}_{FO}/\sigma^{W^-}_{FO}$ expand in α_s then truncate
- B, B’: A, A’ with uncorrelated scale variations
- C: $\pm\left|1 - R^{(n)}_{w^+w^-}/R^{(n-1)}_{w^+w^-}\right|$