

# Electroweak corrections to Drell-Yan processes and $M_W$ determination

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# $M_W$ at hadron colliders

Template fit procedure:

- 1 measure a relevant distribution ( $P_T^l, M_T^{l\nu}$ )
- 2 generate several Monte Carlo samples with different  $M_W$  (templates)
- 3 the measured  $M_W$  corresponds to the sample that best fits to the data

Monte Carlo simulations are affected by perturbative (radiative corrections) and non-perturbative (i.e. PDFs,  $p_T^W$  modelling) theoretical uncertainties



these theory uncertainties propagate to  $M_W$  measurement

# Theoretical uncertainties in $M_W$ measurement: strategy

Theory uncertainties from weak and mixed QCD-EW corrs studied in Carloni Calame et al. arXiv:1612.02841

## 1 pseudodata

- Monte Carlo samples with a given theoretical accuracy
- play the role of experimental data

## 2 templates

- MC samples at NLO QCD+QCD-PS (or LO) generated for different values of  $M_W$
- will be fitted to the pseudodata

3  $\Delta M_W = M_W(\text{pseudodata}) - M_W(\text{fit output})$

# Theory uncertainties in $M_W$ measurement: event generators

- **HORACE** (Carloni Calame et al. hep-ph/0303102, hep-ph/050626)
  - MC event generator for DY
  - can generate events at NLO EW+QED-PS, and NLO EW+QED-PS+unresolved  $l^+l^-$  radiation
- **POWHEG-BOX-V2/W\_ew-BMNNP** (Barze et al. arXiv:1202.0465)
  - MC event generator for charged DY
  - can generate events at NLO QCD+QCD-PS and NLO (QCD+EW)+(QCD+QED)-PS
  - relies on external shower MC programs (i.e. PYTHIA, PYTHIA+PHOTOS)

# Theory unc. in $M_W$ measurement: shower MC tools)

- PYTHIA (Sjostrand et al. hep-ph/0603175; arXiv:0710.3820)
  - general purpose shower MC generator
  - can generate multiple QCD and QED radiation
  - used for ISR multiple QCD (and QED) radiation AND non-perturbative QCD effects
  - in some runs used for QED FSR (see later)
- PHOTOS (Barberio et al. CPC 66 (1991), CPC 79 (1994), Golonka et al. hep-ph/0506026)
  - general purpose shower MC generator
  - can generate multiple QED radiation off fermions (from  $W$  decay)
  - in some runs used for QED FSR (see later)
- HORACE (has its own implementation of QED PS algorithm)

# Theoretical uncertainties in $M_W$ measurement: notation

- NLO EW corrections:  $d\sigma = d\sigma_0 [1 + \delta_\alpha]$
- QED-PS: all order  $\gamma$  radiation in leading log approx.

$$d\sigma = d\sigma_0 \left[ 1 + \sum_{n=1}^{\infty} \delta'_{\alpha^n} \right]$$

- NLO EW+QED-PS:  $d\sigma = d\sigma_0 \left[ 1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$   
matching replaces first PS radiation with NLO real radiation
- HORACE NLO EW+QED-PS:  $d\sigma = d\sigma_0 \left[ 1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$
- POWHEG NLO (QCD+EW)+(QCD+QED)-PS:

$$d\sigma = d\sigma_0 \left[ 1 + \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1, n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n} \right]$$

# Mixed QCD-EW corrections (1)

$pp \rightarrow \mu^+ \nu_\mu$ , fit to  $M_T(\mu^+ \nu_\mu)$

Templates	Pseudodata	$M_W$ shifts (MeV)
1	LO POWHEG(QCD) NLO	$56.0 \pm 1.0$
2	LO POWHEG(QCD)+PYTHIA(QCD)	$74.4 \pm 2.0$
3	LO HORACE(EW) NLO	$-94.0 \pm 1.0$
4	LO HORACE (EW,QEDPS)	$-88.0 \pm 1.0$
5	LO POWHEG(QCD,EW) NLO	$-14.0 \pm 1.0$
6	LO POWHEG(QCD,EW) two-rad+PYTHIA(QCD)+PHOTOS	$-5.6 \pm 1.0$

	samples	$M_W$ shift (MeV)
$\sum_{m=1}^{\infty} \delta'_{\alpha_s^m \alpha^n}$	[6]-[5]	$8.4 \pm 1.4$ MeV
$\sum_{m=2}^{\infty} \delta'_{\alpha_s^m}$	[2]-[1]	$18.4 \pm 2.2$ MeV
$\sum_{n=2}^{\infty} \delta'_{\alpha^n}$	[4]-[3]	$6.0 \pm 1.4$ MeV

$$\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} = ([6]-[5])-([2]-[1])-([4]-[3]) = -16.0 \pm 3.0 \text{ MeV}$$

in agreement with the results of Dittmaier et al. 1511.08016 for the full  $\mathcal{O}(\alpha \alpha_S)$  corrections in pole approx. (-14 MeV)

# Mixed QCD-EW corrections (2)

see also next talk

- mixed QCD-EW corrections from POWHEG  $\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n}$ 
  - factorized approx
  - spurious H.O. effects ( $\text{PS} \times \Delta \times \overline{B}$ )
- another estimate including exact  $\mathcal{O}(\alpha \alpha_S)$  IS corrections in Behring et al.  
arXiv:2103.02671

$$m_W^{\text{meas}} = \frac{\langle p_{\perp}^{l,W} \rangle^{\text{meas}}}{\langle p_{\perp}^{l,Z} \rangle^{\text{meas}}} m_Z C_{\text{th}}, \quad C_{\text{th}} = \frac{m_W}{m_Z} \frac{\langle p_{\perp}^{l,Z} \rangle^{\text{th}}}{\langle p_{\perp}^{l,W} \rangle^{\text{th}}}, \quad \frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}}$$

- Full  $\mathcal{O}(\alpha \alpha_S)$  (arXiv:2102.12539, arXiv:2201.01754): it would be nice to study their impact on  $M_W$  extraction
  - per se
  - to asses the uncertainties coming from the factorized approach in MC generator at NLO QCD+NLO EW with PS matching (e.g. W\_ew-BMNNP in POWHEG)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$ Templates accuracy: NLO-QCD+QCD <sub>PS</sub> Pseudodata accuracy QED FSR				$M_W$ shifts (MeV)			
		$W^+ \rightarrow \mu^+ \nu$ $M_T$	$p_T^\ell$	$W^+ \rightarrow e^+ \nu(\text{dres})$ $M_T$	$p_T^\ell$		
1	NLO-QCD+(QCD+QED) <sub>PS</sub>	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2	
2	NLO-QCD+(QCD+QED) <sub>PS</sub>	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3	
3	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3	
4	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2	

■ dressed  $e$ : recombine  $\gamma$  with  $e$  if  $\Delta R(\gamma e) < 0.1$

■ bare  $\mu$ : corrections enhanced by  $\log(\frac{m_\mu^2}{Q^2})$

# Theory uncertainties from QED PS (1)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$ Templates accuracy: NLO-QCD+QCD <sub>PS</sub> Pseudodata accuracy			QED FSR	$M_W$ shifts (MeV)			
				$W^+ \rightarrow \mu^+ \nu$	$W^+ \rightarrow e^+ \nu(\text{dres})$	$M_T$	$p_T^\ell$
1	NLO-QCD+(QCD+QED) <sub>PS</sub>	Pythia		-95.2 $\pm$ 0.6	-400 $\pm$ 3	-38.0 $\pm$ 0.6	-149 $\pm$ 2
2	NLO-QCD+(QCD+QED) <sub>PS</sub>	PHOTOS		-88.0 $\pm$ 0.6	-368 $\pm$ 2	-38.4 $\pm$ 0.6	-150 $\pm$ 3
3	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	Pythia		-89.0 $\pm$ 0.6	-371 $\pm$ 3	-38.8 $\pm$ 0.6	-157 $\pm$ 3
4	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PHOTOS		-88.6 $\pm$ 0.6	-370 $\pm$ 3	-39.2 $\pm$ 0.6	-159 $\pm$ 2

- difference between QED-PS in PHOTOS and PYTHIA at  $\mathcal{O}(\alpha)$
- $\text{PHOTOS} \propto \frac{1}{1 - \beta \cos \theta_{l\gamma}}$
- $\text{PYTHIA-QED} \propto \frac{1}{p_T^\gamma}$
- 32 MeV ( $p_T$ )/ 7 MeV ( $M_T$ ) effect for bare  $\mu$

# Theory uncertainties from QED PS (2)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$ Templates accuracy: NLO-QCD+QCD <sub>PS</sub>			$M_W$ shifts (MeV)			
	Pseudodata accuracy	QED FSR	$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
			$M_T$	$p_T^\ell$	$M_T$	$p_T^\ell$
1	NLO-QCD+(QCD+QED) <sub>PS</sub>	PYTHIA	-95.2 $\pm$ 0.6	-400 $\pm$ 3	-38.0 $\pm$ 0.6	-149 $\pm$ 2
2	NLO-QCD+(QCD+QED) <sub>PS</sub>	PHOTOS	-88.0 $\pm$ 0.6	-368 $\pm$ 2	-38.4 $\pm$ 0.6	-150 $\pm$ 3
3	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PYTHIA	-89.0 $\pm$ 0.6	-371 $\pm$ 3	-38.8 $\pm$ 0.6	-157 $\pm$ 3
4	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PHOTOS	-88.6 $\pm$ 0.6	-370 $\pm$ 3	-39.2 $\pm$ 0.6	-159 $\pm$ 2

- first QED radiation generated by POWHEG
- difference between QED-PS in PHOTOS and PYTHIA at  $\mathcal{O}(\alpha^2)$

## Theory uncertainties from QED PS (3)

Theory uncertainties from QED PS estimated from the difference in the shifts from PYTHIA and PHOTOS

- might be an overestimate (photon spectrum suggests that PHOTOS is more accurate)
- comparison should be extended to other QED PS: HERWIG, SHERPA, ...
- how would the shifts change if a QED PS beyond LL accuracy was used?

# non-log QED, weak and mixed EW-QCD contributions (1)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$ Templates accuracy: NLO-QCD+QCD <sub>PS</sub>			$M_W$ shifts (MeV)			
	Pseudodata accuracy	QED FSR	$W^+ \rightarrow \mu^+ \nu$	$p_T^\ell$	$W^+ \rightarrow e^+ \nu(\text{dres})$	$p_T^\ell$
1	NLO-QCD+(QCD+QED) <sub>PS</sub>	PYTHIA	$-95.2 \pm 0.6$	$-400 \pm 3$	$-38.0 \pm 0.6$	$-149 \pm 2$
2	NLO-QCD+(QCD+QED) <sub>PS</sub>	PHOTOS	$-88.0 \pm 0.6$	$-368 \pm 2$	$-38.4 \pm 0.6$	$-150 \pm 3$
3	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PYTHIA	$-89.0 \pm 0.6$	$-371 \pm 3$	$-38.8 \pm 0.6$	$-157 \pm 3$
4	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PHOTOS	$-88.6 \pm 0.6$	$-370 \pm 3$	$-39.2 \pm 0.6$	$-159 \pm 2$

- impact of non-log QED, weak and mixed EW-QCD contributions
- different effects for PHOTOS or PYTHIA (different non-log QED terms)
- more stable results for  $M_T$  (less sensitive to mixed EW-QCD corrections)

# non-log QED, weak and mixed EW-QCD contributions (2)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$ Templates accuracy: NLO-QCD+QCD <sub>PS</sub> Pseudodata accuracy			M <sub>W</sub> shifts (MeV)			
	QED FSR		$W^+ \rightarrow \mu^+ \nu$	$W^+ \rightarrow e^+ \nu(\text{dres})$		
			$M_T$	$p_T^\ell$	$M_T$	$p_T^\ell$
1	NLO-QCD+(QCD+QED) <sub>PS</sub>	PYTHIA	-95.2 $\pm$ 0.6	-400 $\pm$ 3	-38.0 $\pm$ 0.6	-149 $\pm$ 2
2	NLO-QCD+(QCD+QED) <sub>PS</sub>	PHOTOS	-88.0 $\pm$ 0.6	-368 $\pm$ 2	-38.4 $\pm$ 0.6	-150 $\pm$ 3
3	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PYTHIA	-89.0 $\pm$ 0.6	-371 $\pm$ 3	-38.8 $\pm$ 0.6	-157 $\pm$ 3
4	NLO-(QCD+EW)+(QCD+QED) <sub>PS two-rad</sub>	PHOTOS	-88.6 $\pm$ 0.6	-370 $\pm$ 3	-39.2 $\pm$ 0.6	-159 $\pm$ 2

ATLAS collaboration at LHC use as templates  
 POWHEG -QCD+PYTHIA -QCD+PHOTOS

uncertainties from  
 non-log QED,  
 weak,  
 mixed QCD-EW corr.

$\Delta M_W (\text{MeV})$ bare muons			
	QED FSR model	$M_T$	$p_T^\ell$
LHC	PYTHIA	+6.2 $\pm$ 0.8	+29 $\pm$ 4
	PHOTOS	-0.6 $\pm$ 0.8	-2 $\pm$ 4

## non-log QED, weak and mixed EW-QCD contributions (3)

- QED, WEAK, and mixed effects inevitably have an interplay with IS QCD effects (e.g.  $\text{PS} \times \overline{B}$ )
- in our simulation we only used PYTHIA8 for ISR QED and QCD shower and non-perturbative effects with a default PYTHIA tuning
- how do the shifts change if we use another shower MC, say HERWIG?
- how do the estimates change when changing the PYTHIA tune? (having in mind the ATLAS procedure of tuning PYTHIA to reproduce the  $Z p_T$  data)
- how do the shifts change if we use another description of IS effects, say for instance RESBOS like in TEVATRON analyses?

# Higher order effects: pair radiation (1)



same order as 2  $\gamma$  radiation

Unresolved pair radiation can be included in the Sudakov through the running<sup>1</sup> of  $\alpha$

$$\alpha \Rightarrow \alpha(s) = \begin{cases} \alpha / \left( 1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2} \right) & \text{electrons only} \\ \alpha / \left( 1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2} - \theta(s - m_\mu^2) \frac{\alpha}{3\pi} \ln \frac{s}{m_\mu^2} \right) & \text{electrons + muons} \end{cases}$$

pp $\rightarrow W^+$ , $\sqrt{s} = 14$ TeV		shifts (MeV)			
Templates accuracy: LO		$M_W$	$p_T^\ell$	$W^+ \rightarrow e^+ \nu$	
Pseudo-data accuracy				$M_T$	$p_T^\ell$
1	HORACE FSR-LL	-89 $\pm$ 1	-97 $\pm$ 1	-179 $\pm$ 1	-195 $\pm$ 1
2	HORACE FSR-LL + Pairs	-94 $\pm$ 1	-102 $\pm$ 1	-182 $\pm$ 2	-199 $\pm$ 1

$\Delta M_W(\mu^+ \nu) \sim 5 \pm 1$  MeV (from  $\mu$ ) and  $\sim 3 \pm 2$  MeV (from  $e$ )

<sup>1</sup>alternative implementation: N. Davidson et al arXiv:1011.0937

## Higher order effects: pair radiation (2)

$pp \rightarrow W^+, \sqrt{s} = 14 \text{ TeV}$		shifts (MeV)			
Templates accuracy: LO		$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu$	
Pseudo-data accuracy		$M_T$	$p_T^\ell$	$M_T$	$p_T^\ell$
1	HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
2	HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1

- pair corrections estimated using HORACE: no interplay with QCD effects possible
- similar strategy can be implemented in POWHEG (actually is already there...)  
⇒ one could repeat the study within POWHEG-BOX-V2/W\_ew-BMNNP to see what are the changes in the shifts due to the presence of QCD pert and non-pert effects

# NNLO uncertainty: input parameter schemes (1)

scheme choice = choice of the 3 independent EW params

- all choices formally equivalent at a given order in P.T.
- numerical differences in predictions from missing H.O. terms

⇒ difference in predictions from different schemes at a given order can be taken as an estimate of the theory uncertainty from missing H.O.

However....

- not conclusive: basically impossible to consider all possible choices of IPS
- might be over-estimate: we might consider some schemes as “more precise” than others
  - parametric uncertainties
  - perturbative convergence
  - ...

## NNLO uncertainty: input parameter schemes (2)

- $\alpha(0)$ ,  $M_W$  and  $M_Z$
- $G_\mu$ ,  $M_W$  and  $M_Z$  to be preferred in the CC DY
- we can define

$$\begin{aligned}\alpha_\mu^{tree} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2 \vartheta \\ \alpha_\mu^{1l} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2 \vartheta (1 - \Delta r)\end{aligned}$$

The expressions for the cross section differ at  $\mathcal{O}(\alpha^2)$

$$\begin{aligned}\alpha_0 &: \quad \sigma = \alpha_0^2 \sigma_0 + \alpha_0^3 (\sigma_{SV} + \sigma_H), \\ G_\mu \text{ I} &: \quad \sigma = (\alpha_\mu^{tree})^2 \sigma_0 + (\alpha_\mu^{tree})^2 \alpha_0 (\sigma_{SV} + \sigma_H) - 2\Delta r (\alpha_\mu^{tree})^2 \sigma_0, \\ G_\mu \text{ II} &: \quad \sigma = (\alpha_\mu^{1l})^2 \sigma_0 + (\alpha_\mu^{1l})^2 \alpha_0 (\sigma_{SV} + \sigma_H)\end{aligned}$$

# NNLO uncertainty: input parameter schemes (3)

- potentially effects on  $M_W$  because of the different sharing among different photon multiplicities

		$p\bar{p} \rightarrow W^+, \sqrt{s} = 1.96 \text{ TeV}$ Templates accuracy: LO	$M_W$ shifts (MeV) $W^+ \rightarrow \mu^+ \nu$	
	Pseudodata accuracy	Input scheme	$M_T$	$p_T^\ell$
1	HORACE NLO-EW	$\alpha_0$	-101±1	-117±2
2		$G_\mu - I$	-112±1	-130±1
3		$G_\mu - II$	-101±1	-117±1
4	HORACE NLO-EW+QED-PS	$\alpha_0$	-70±1	-81±1
5		$G_\mu - I$	-72±2	-83±1
6		$G_\mu - II$	-72±1	-82±2

- differences present at NLO, after matching with higher orders, become much smaller

$$\Delta M_W \sim 2 \text{ MeV} \pm 1 - 2 \text{ MeV}$$

# NNLO uncertainty: input parameter schemes (4)

- uncertainties from IPS choice evaluated with HORACE: no interplay with QCD
- how do the shifts from different IPS change in the presence of QCD effects?

order	$G_\mu$	$a(0)$	$\delta(G_\mu - a(0)) \text{ (%)}$
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

the LO + NLO-EW result would suffer of only 0.55% spread;

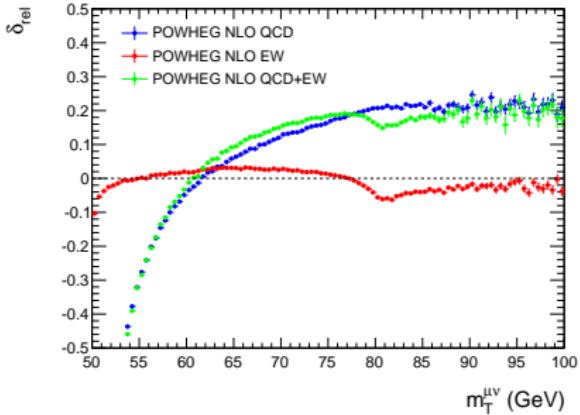
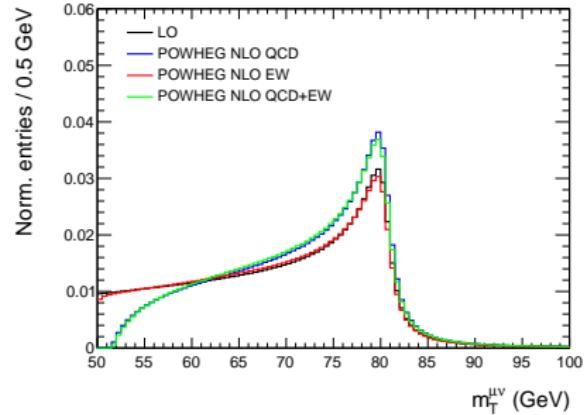
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence ( $\rightarrow 0.88\%$ )

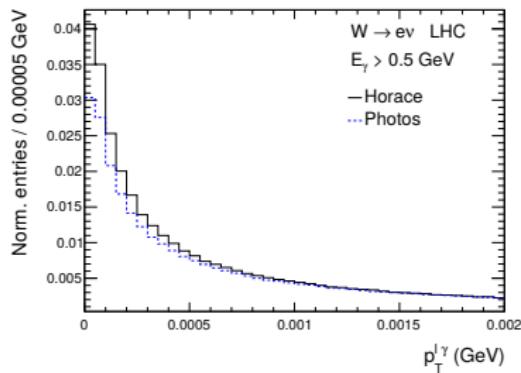
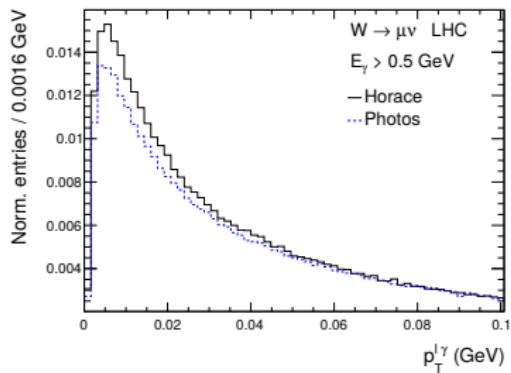
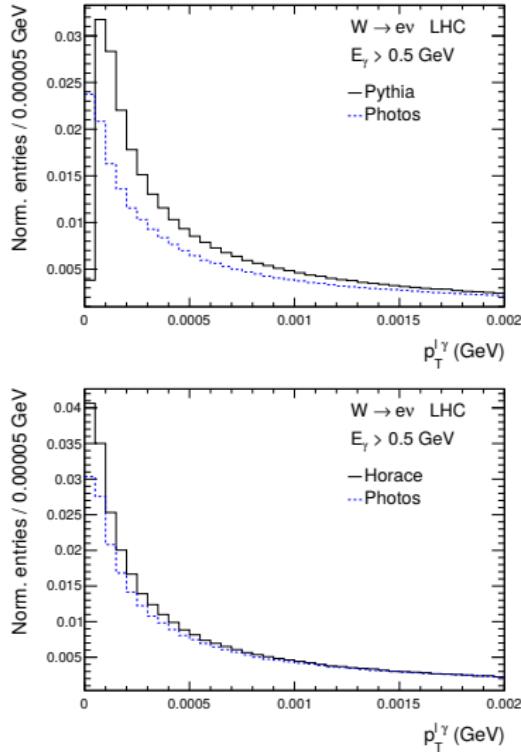
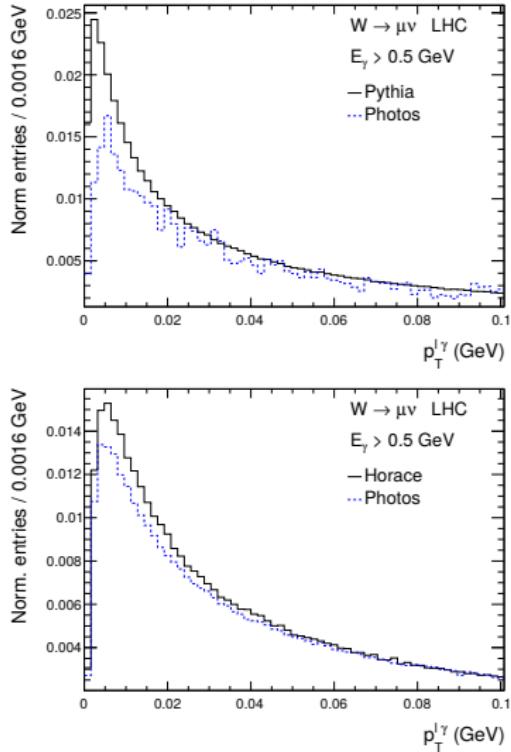
- one could estimate the H.O. corrections including the universal fermionic corrections connected to  $\Delta\alpha$  and  $\Delta\rho$

## Possible points to discuss

- approximated factorized mixed QCD-EW corrections VS  $\alpha\alpha_S$  corrections: theory uncertainties in MC generators at NLO QCD+NLO EW matched to QED and QCD PS
- estimate of QED, weak, and mixed corrections: how do they change when changing the IS QCD modeling? (use different shower MCs, different tunes,...)
- uncertainties from QED PS (H.O. qed): comparing other QED showers
- impact of pair radiation: possible interplay with IS QCD modeling
- estimate of missing NNLO EW corrections: input parameter scheme variations? Interplay with QCD? Other possible estimates?

# Backup Slides





HORACE

$$d\sigma^\infty = F_{SV} \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \left( \prod_{i=0}^n F_{H,i} \right) |\mathcal{M}_{n,LL}|^2 d\Phi_n$$

POWHEG

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{\left[ d\Phi_{rad} \theta(k_T - p_T^{min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1}) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

taken from 1701.07240

W-boson charge Kinematic distribution	$W^+$		$W^-$		Combined	
	$p_T^\ell$	$m_T$	$p_T^\ell$	$m_T$	$p_T^\ell$	$m_T$
$\delta m_W$ [MeV]						
Fixed-order PDF uncertainty	13.1	14.9	12.0	14.2	8.0	8.7
AZ tune	3.0	3.4	3.0	3.4	3.0	3.4
Charm-quark mass	1.2	1.5	1.2	1.5	1.2	1.5
Parton shower $\mu_F$ with heavy-flavour decorrelation	5.0	6.9	5.0	6.9	5.0	6.9
Parton shower PDF uncertainty	3.6	4.0	2.6	2.4	1.0	1.6
Angular coefficients	5.8	5.3	5.8	5.3	5.8	5.3
Total	15.9	18.1	14.8	17.2	11.6	12.9

Table 3: Systematic uncertainties in the  $m_W$  measurement due to QCD modelling, for the different kinematic distributions and W-boson charges. Except for the case of PDFs, the same uncertainties apply to  $W^+$  and  $W^-$ . The fixed-order PDF uncertainty given for the separate  $W^+$  and  $W^-$  final states corresponds to the quadrature sum of the CT10nnlo uncertainty variations; the charge-combined uncertainty also contains a 3.8 MeV contribution from comparing CT10nnlo to CT14 and MMHT2014.