

Mixed QCD-electroweak corrections and W -mass determination

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Delto, Jaquier, Melnikov, RR [hep-ph/1909.08428]

Buccioni, Caola, Delto, Jaquier, Melnikov, RR, [hep-ph/2005.10221]

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR [hep-ph/2009.10386, **hep-ph/2103.02671**]

Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile [hep-ph/2203.11237]

Precision calculations for Drell-Yan processes

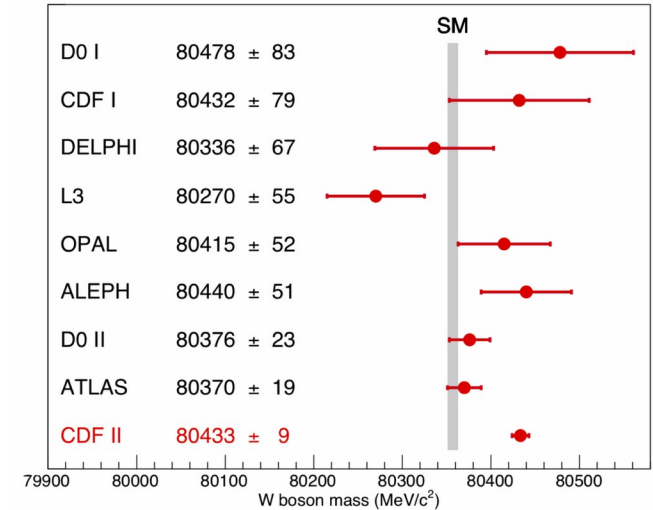
Milan, 21 November 2022



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- We are trying to measure a parameter of the SM with precision ~ 0.1 per mille at a *hadron collider*.
- **Extraordinary precision** demanded from theoretical predictions:
 - ✓ Fixed-order calculations – expansions in α_s and α .
 - ✓ Parton showers
 - ✓ Resummations
 - ✓ Non-perturbative effects
 - ✓ Numerical effects?
 - ✓ ...



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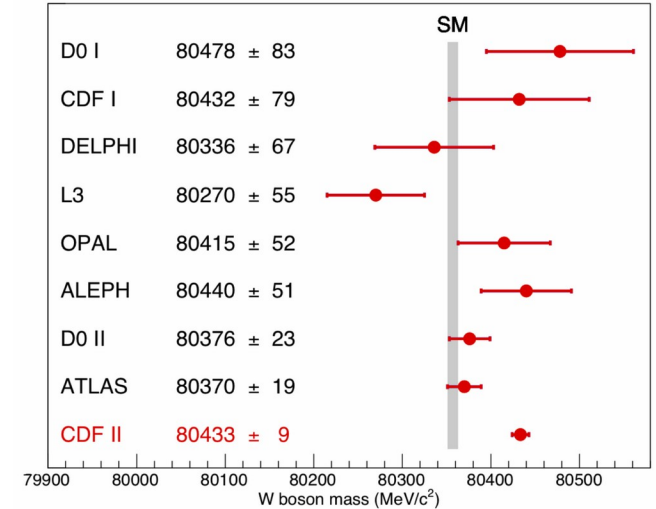
- ✓ Parton showers

- ✓ Resummations

- ✓ Non-perturbative effects

- ✓ Numerical effects?

- ✓ ...



$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0,0)} + \alpha_s \hat{\sigma}_{ij}^{(1,0)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2,0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(3,0)} + \dots + \alpha \hat{\sigma}_{ij}^{(0,1)} + \alpha_s \alpha \hat{\sigma}_{ij}^{(1,1)} + \dots$$

Mixed QCD-EW corrections

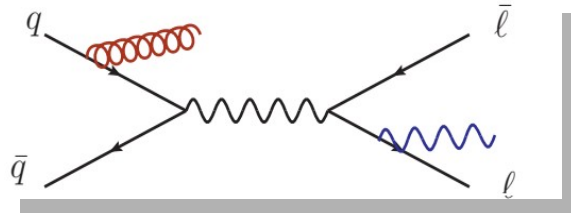
[Bonciani, Buccioni, Mondini, Vicini ('17); De Florian, Der, Fabre ('18); Delto, Jaquier, Melnikov, R.R. ('19); Bonciani, Buccioni, Rana, Triscari, Vicini ('19); Buccioni *et al.* ('20); Cieri, De Florian, Der, Mazzitelli ('20); Bonciani, Buccioni, Rana, Vicini ('20); Behring *et al.* ('20); Buonocore, Grazzini, Kallweit, Savoini, Tramontano ('21); Bonciani *et al.* ('21); Armadillo *et al.* ('22); Buccioni *et al.* ('22)]

- For W-mass determination, **offshell effects are secondary** → focus on **onshell** vector boson production $pp \rightarrow V \rightarrow \ell\bar{\ell}$

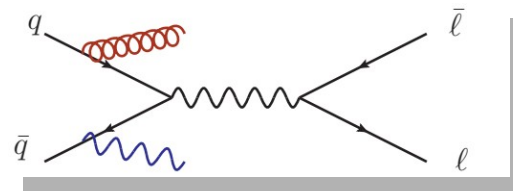
- Consider onshell vector boson production and decay $pp \rightarrow V \rightarrow \ell\bar{\ell}$
- In resonance region, QCD-EW corrections to production and decay processes can be treated separately.

[Dittmaier, Huss, Schwinn ('14, '15)]

QCD (production) x EW (decay)



QCD x EW (production)



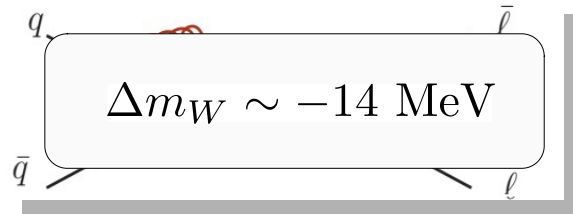
Non-factorizable
resonant contributions
(not shown) suppressed
by Γ_V/m_V

[Dittmaier, Huss, Schwinn ('14, '15)]

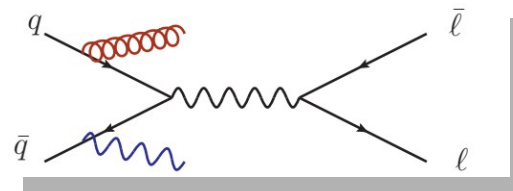
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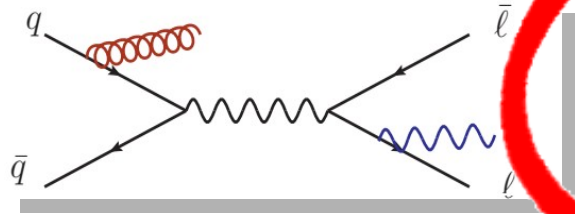
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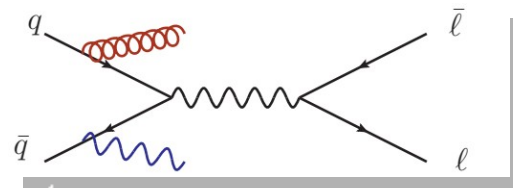
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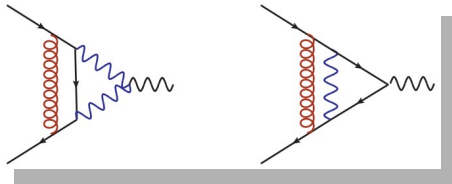
[Dittmaier, Huss, Schwinn ('14, '15)]

- Two major challenges in computing higher order corrections:

- Loop amplitudes
- Handling infrared singularities

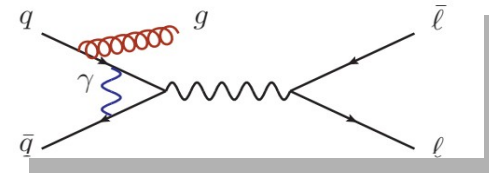
- Loop amplitudes:

- Two-loop form factors



- Known for Z production. [Kotikov, Kühn, Veretin ('08)]
- Computed for first time for W production. [Behring *et al.* ('20)]

- One-loop real-virtual amplitudes



- Standard one-loop programs, e.g. OpenLoops [Cascioli, Maierhöfer, Pozzorini ('12); Buccioni, Pozzorini, Zoller ('18); Buccioni *et al.* ('19)]

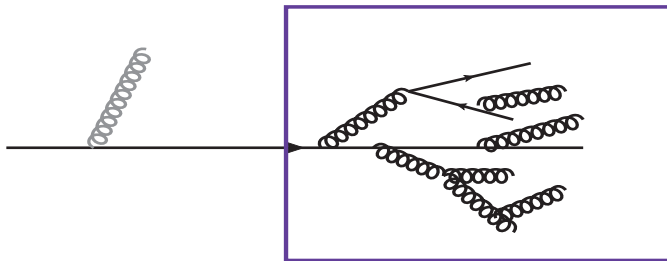
- Two major challenges in computing higher order corrections:
 1. Loop amplitudes
 2. Handling infrared singularities
- Infrared singularities with different origins appear simultaneously:
 - Virtual photons;
 - Virtual partons;
 - Unresolved real photons;
 - Unresolved real partons.
- Insight from NNLO QCD: treatment of IR singularities with non-trivial structure.

[Caola, Melnikov, R.R. ('17)]

[Caola, Melnikov, R.R. ('19); Asteriadis, Caola, Melnikov, R.R. ('19)]

[Delto, Frellesvig, Caola, Melnikov ('18); Delto, Melnikov ('19)]

- Extension of FKS subtraction to NNLO.
- Exploits **color coherence** of onshell, gauge-invariant amplitudes
 - Used in resummation & parton showers; not manifest in subtractions.

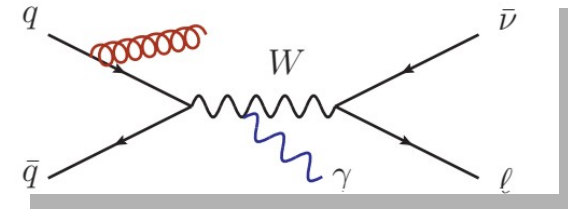


- Soft gluon cannot resolve details of collinear splittings; only sensitive to **total color charge**.

- No overlap between soft and collinear limits – treated **independently**:
 - Energies and angles **decouple**.
 - Regularize soft singularities first, then collinear singularities – **iterative subtraction** of divergences.
 - Overlapping **soft** singularities separated by **energy ordering**.
 - Overlapping **collinear** singularities separated using **partitioning** and **sectoring** of phase space.
 - Natural splitting by rapidity.
- Straightforward adaptation for mixed **QCD-EW** singularities. [Czakon ('10, '11)]

Consider onshell vector boson production $pp \rightarrow V \rightarrow \ell\bar{\ell}$

- Z production: subtractions proceed as “abelianized NNLO QCD”.
- W production: **qualitatively new feature** – photon radiated off W.
- Collinear limits regulated by W-mass, but **soft limit** is singular.
- Changes **form** of eikonal function in soft limit:



$$\text{Soft gluon} \rightarrow \text{Eik}_g(p_1, p_2; p_g) = \frac{2C_F(p_1 \cdot p_2)}{(p_1 \cdot p_g)(p_2 \cdot p_g)}$$

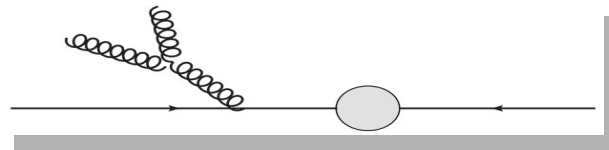
$$\begin{aligned} \text{Soft photon} \rightarrow \text{Eik}_\gamma(p_1, p_2, p_W; p_\gamma) &= Q_u Q_d \frac{2(p_1 \cdot p_2)}{(p_1 \cdot p_\gamma)(p_2 \cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W \cdot p_\gamma)^2} \\ &+ Q_W \left(Q_u \frac{2(p_W \cdot p_1)}{(p_W \cdot p_\gamma)(p_1 \cdot p_\gamma)} - Q_d \frac{2(p_W \cdot p_2)}{(p_W \cdot p_\gamma)(p_2 \cdot p_\gamma)} \right) \end{aligned}$$

➡ More complicated function, but method is **conceptually unchanged!**

Can make subtraction scheme **simpler**:

- NNLO QCD: soft limits of gluons **overlap** → introduced **energy ordering**.
- Mixed QCD-EW: soft limits of gluons and photons are **independent** → **no energy ordering** needed.
- Soft subtraction: **iterated NLO-like soft** limits.
- Genuine NNLO-like singularities in **collinear limits** → require **phase-space partitioning** and **sectoring**.

- Fewer collinear limits, e.g.



disappears.

→ **Fewer sectors required.**

Can make subtraction scheme **simpler**:

- NNLO
- Mixed
- need
- Soft
- Gen
- sect
- Few
- Fe

Implemented this subtraction method to compute mixed QCD-EW corrections to production of onshell Z and W bosons.

[Delto, Jaquier, Melnikov, RR ('19);

Buccioni, Caola, Delto, Jaquier, Melnikov, RR, ('20)

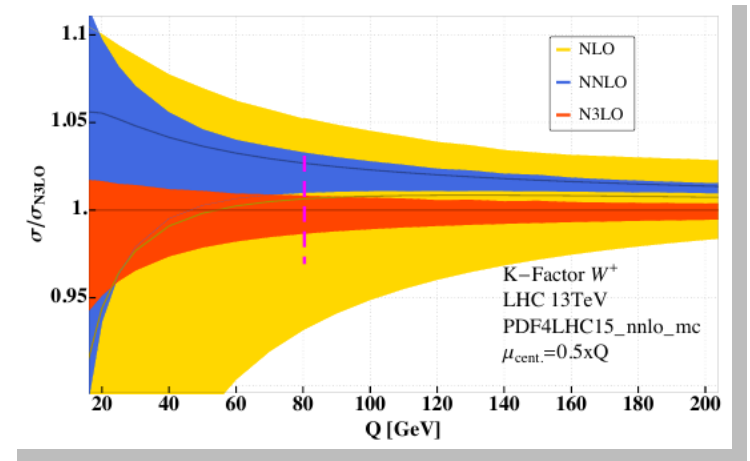
Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR ('20)]

Further extended method for dilepton production $pp \rightarrow Z/\gamma^* \rightarrow \ell^+ \ell^-$

[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile ('22)]

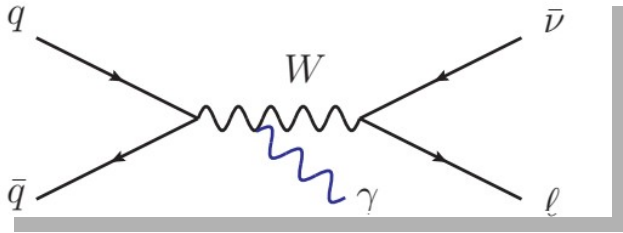
- **How could these corrections impact the measurement of the W -mass?**

- Aiming at **0.1 per mille** precision, but theoretical precision limited to \sim **1%**.
- Sources of theoretical uncertainty:
 - higher order corrections
 - subleading logs
 - pdfs
 - Non-perturbative effects
 - Quark masses
 - ...
- Use excellent **experimental** control of $pp \rightarrow Z \rightarrow \ell\bar{\ell}$ to tune generators and verify results.



[Duhr, Dulat, Mistlberger '20]

- Implicit assumption: higher-order corrections to W and Z production **strongly correlated**.
- Reasonable for QCD corrections:
 - Minor differences: pdfs, masses, helicity structures, ...
- **EW** corrections: **qualitatively different** – W charged, can radiate:



- Mixed QCD-EW corrections **potentially decorrelated**.
- Possible impact on W-mass measurements at desired precision.

Impact on W mass determination

- **Estimate** effect of QCD-EW corrections on W mass measurement, due to **decorrelations** between Z and W production.
- **Correlation** between **average transverse momentum** of leptons and **mass of boson**:

$$\frac{m_W}{m_Z} = \frac{\langle p_{T,l}^W \rangle}{\langle p_{T,l}^Z \rangle} \Rightarrow m_W^{\text{meas.}} = m_Z \frac{\langle p_{T,l}^{W,\text{meas.}} \rangle}{\langle p_{T,l}^{Z,\text{meas.}} \rangle} C_{\text{th.}}$$

- Theoretical correction: assume input masses, compute W-mass, and compare with input W-mass.

$$\Rightarrow C_{\text{th.}} = \frac{m_W^{\text{in}} \langle p_{T,l}^{Z,\text{th.}} \rangle}{m_Z^{\text{in}} \langle p_{T,l}^{W,\text{th.}} \rangle}$$

→ **estimate impact of decorrelations** in W and Z spectra from higher order corrections:

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

[Behring *et al.* ('21)].

Impact on W mass determination

Shifts in W-mass, inclusive:

- NLO EW: $\Delta m_W = 1 \text{ MeV}$
- QCD-EW: $\Delta m_W = -7 \text{ MeV}$

→ Impact of QCD-EW corrections **larger** than NLO EW:

- NLO EW corrections **suppressed** in G_μ scheme.
- NLO EW corrections **more correlated** between W and Z production.

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

$$\text{NLO EW: } \Delta m_W = -31 \text{ MeV} + 32 \text{ MeV}$$

$$\text{QCD-EW: } \Delta m_W = +54 \text{ MeV} - 61 \text{ MeV}$$

$$\sqrt{s} = 13 \text{ TeV}$$

G_μ scheme

$$m_Z = 91.1876 \text{ GeV}$$

$$m_W = 80.398 \text{ GeV}$$

$$m_t = 173.2 \text{ GeV}$$

$$m_H = 125 \text{ GeV}$$

$$G_F = 1.16339 \cdot 10^{-5} \text{ GeV}^{-2}$$

NNPDF31_luxQED

$$\mu_R = \mu_F = m_V/2$$

Shifts in W -mass: fiducial setup

- Inclusive setup: $\Delta m_W = -7$ MeV
 - “ATLAS” cuts: $\Delta m_W = -17$ MeV
 - “Tuned” cuts: $\Delta m_W = -1$ MeV
- Cuts can have **dramatic impact**: shifts vary by factor of ~ 20 .
- “ATLAS” cuts have **stronger cuts** on leptons from (lighter) W than from $Z \rightarrow$ decorrelation.
- QCD-EW shifts potentially **relevant for target precision of 8 MeV**.

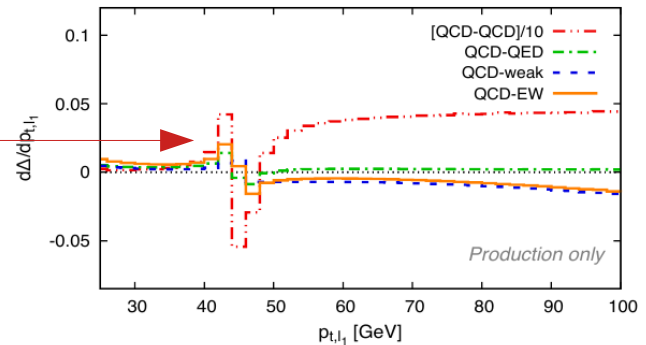
$$p_{T,\ell}^Z > 25 \text{ GeV}; |\eta_\ell^Z| < 2.4$$

$$\text{“ATLAS” cuts: } p_{T,\ell}^W > 30 \text{ GeV}; p_{T,\text{miss}}^W > 30 \text{ GeV}; |\eta_\ell^W| < 2.4.$$

$$\text{“Tuned” cuts: } p_{T,\ell}^W > 25.44 \text{ GeV}; p_{T,\text{miss}}^W > 25.44 \text{ GeV}; |\eta_\ell^W| < 2.4, \text{ such that } C_{\text{th.}} = 1 \text{ at LO.}$$

- These results are **estimates** of impact of QCD-EW corrections on W -mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W -mass measurements.
- Further investigations are **essential**:

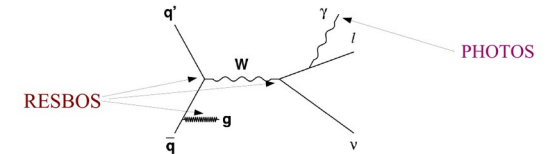
- These results are **estimates** of impact of QCD-EW corrections on W -mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W -mass measurements.
- Further investigations are **essential**:
 - What is the impact when using the **full transverse momentum spectrum**?
 - × Cannot be evaluated using purely fixed-order results due to **Sudakov shoulder**.
 - × Need to include **multiple photon radiation**.
 - × Match photon shower to QCDxEW?
- Can we gain further insight from using **higher moments** of lepton distribution?



[Buccioni, Caola, Delto, Jaquier, Melnikov, R.R. (2020)]

- These results are **estimates** of impact of QCD-EW corrections on W -mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W -mass measurements.
- Further investigations are **essential**:
 - How well are these captured with **standard experimental simulation tools**?
 - × (Some) Effects of simultaneous QCD and EW radiation included in experimental analyses through e.g. RESBOS+PHOTOS.
 - × Include **multiple emissions** but miss **virtual contributions**.
 - × Missing effects accounted for in uncertainties?

Generator-level Signal Simulation



- Generator-level input for W & Z simulation provided by RESBOS (C. Balazs & C.-P. Yuan, PRD56, 5558 (1997) and references therein), which
 - Calculates triple-differential production cross section, and p_T -dependent double-differential decay angular distribution
 - calculates boson p_T spectrum reliably over the relevant p_T range: includes tunable parameters in the non-perturbative regime at low p_T
- Multiple radiative photons generated according to PHOTOS (P. Golonka and Z. Was, Eur. J. Phys. C 45, 97 (2006) and references therein)

A. V. Kotwal, CERN, 4/21/22

[A. V. Kotwal, CERN EP Seminar 21/4/2022]

- These results are **estimates** of impact of QCD-EW corrections on W-mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are **essential**:
 - What is the impact on **other observables**?
 - ...

Dialogue between theorists and experimentalists is crucial!

- Calculation of mixed QCD-EW corrections to onshell vector boson production using nested soft-collinear subtraction scheme.
- (Analogous calculation exists for dilepton production).
- Estimated impact on measurement of W-mass at LHC ~ 10 MeV.
 - Looked at **decorrelation** in ratios of average lepton transverse momentum.
 - **Strongly cut-dependent.**
 - **Potentially relevant** for target uncertainty of 0.1 per mille.
 - More refined analysis is required, and discussions are encouraged!

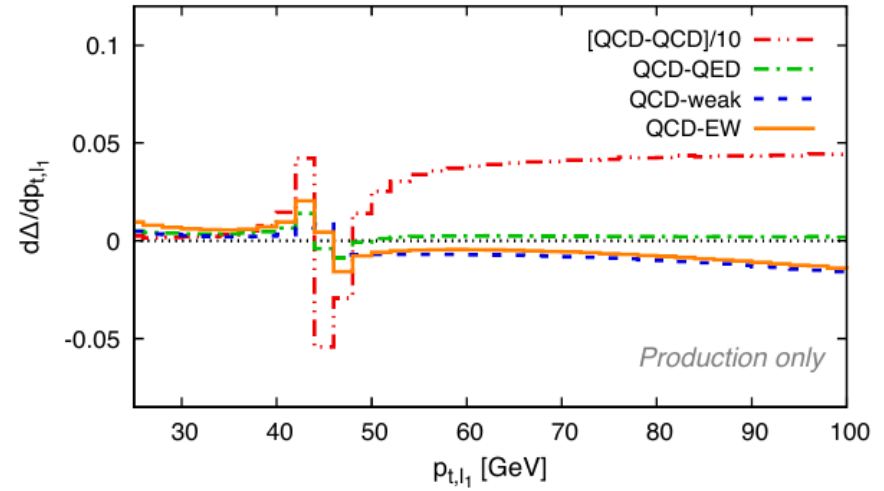
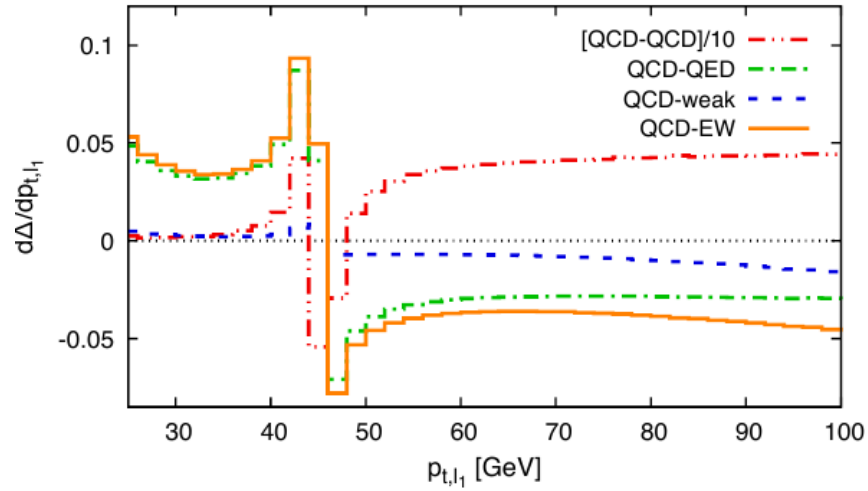


THANK YOU FOR YOUR ATTENTION

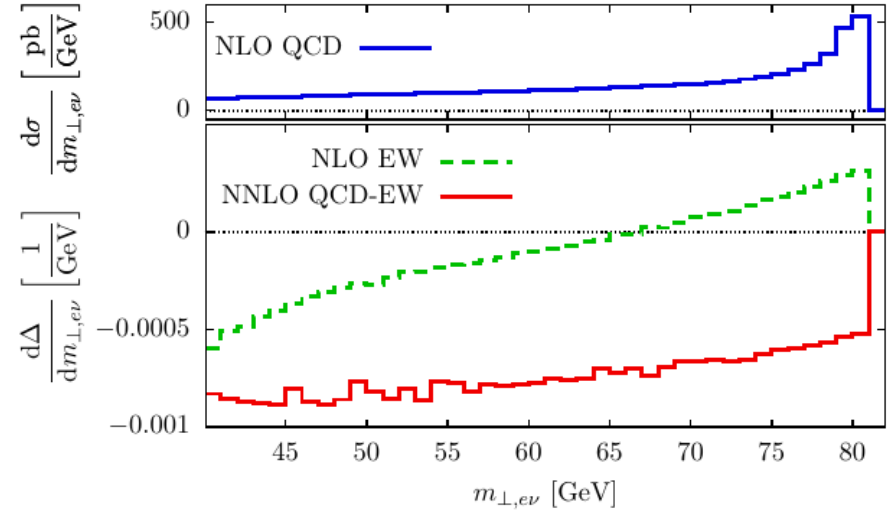
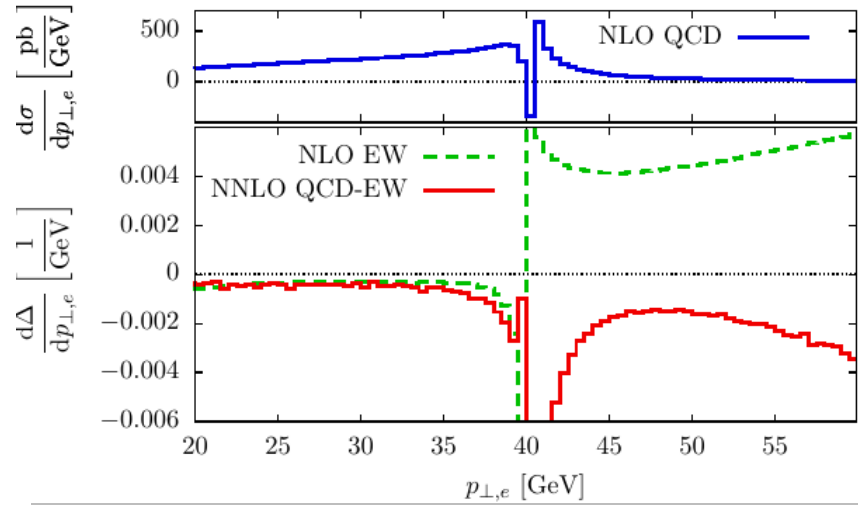


BACKUP SLIDES

QCD-EW corrections to Z boson production



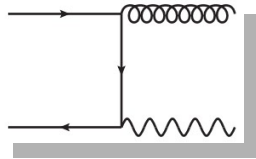
QCD-EW corrections to W boson production



Higher order corrections contain IR singularities from **soft and/or collinear radiation**.

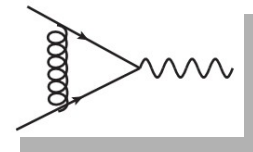
- **Real corrections**

- **Integrate** over phase space of radiated parton:


$$\longrightarrow \int |\mathcal{M}|^2 F_J d\phi_g \text{ diverges}$$

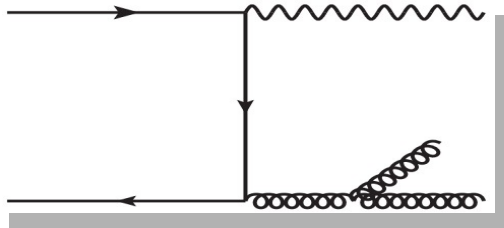
- **Virtual corrections**

- **Explicit** IR singularities from loop integration


$$\longrightarrow \mathcal{M}_{1\text{-loop}} = \frac{c_{-2}}{\epsilon^2} + \frac{c_{-1}}{\epsilon} + c_0$$

- Singularities **unphysical**, guaranteed to cancel in sum (KLN theorem).
- Cancellation only manifest after integrating over full phase space of emitted parton:
→ **lose kinematic information**.

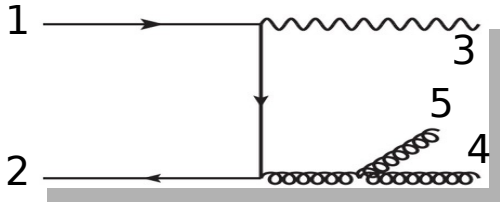
Consider **double-real** emissions in vector boson production: $q\bar{q} \rightarrow V + g + g$



Singularities arise when:

- *Either* gluon or *both* gluons \rightarrow **soft**.
 - *Either* gluon or *both* gluons \rightarrow **collinear** to either initial state quark.
 - Gluons \rightarrow **collinear** to each other.
- Any combination of above - **overlapping singularities**.
 - Can approach each limit in different ways.
 - Need to separate the singularities.
 - Multiple approaches: q_T , N-jettiness, antennas, STRIPPER, CoLoRFuNNLO, Projection-to-Born, **nested soft-collinear subtraction**, geometric subtraction, local analytic subtraction, unsubtraction, Loop-Tree Duality, ...

- Consider partonic process $q(p_1)\bar{q}(p_2) \rightarrow V(p_3)g(p_4)g(p_5)$



- Define

$$F_{LM}(1, 2, 4, 5) = \text{dLips}_V |\mathcal{M}(1, 2, 4, 5, V)|^2 \mathcal{F}_{\text{kin}}(1, 2, 4, 5, V)$$

$$\longrightarrow 2s \cdot \text{d}\sigma^{\text{RR}} = 1/2! \int [\text{d}g_4][\text{d}g_5] F_{LM}(1, 2, 4, 5) \quad [\text{d}g_i] = \frac{\text{d}^{d-1}p_i}{(2\pi)^d 2E_i} \theta(E_{\text{max}} - E_i)$$

- Overlapping **double-soft** and **single-soft** singularities:

$$E_4, E_5 \rightarrow 0; \quad E_4 \rightarrow 0; \quad E_5 \rightarrow 0.$$

- Order energies: $E_4 > E_5$ \rightarrow soft singularities: either **double soft** or **g_5 soft**.

$$\begin{aligned} \longrightarrow 2s \cdot d\sigma^{\text{RR}} &= 1/2! \int [dg_4][dg_5] F_{LM}(1, 2, 4, 5) \\ &= \int [dg_4][dg_5] \theta(E_4 - E_5) F_{LM}(1, 2, 4, 5) \equiv \langle F_{LM}(1, 2, 4, 5) \rangle \end{aligned}$$

- **Regulate** the soft singularities:

$$\begin{aligned} \langle F_{LM}(1, 2, 4, 5) \rangle &= \langle \mathcal{S} F_{LM}(1, 2, 4, 5) \rangle + \langle (I - \mathcal{S}) F_{LM}(1, 2, 4, 5) \rangle \\ &= \langle \mathcal{S} F_{LM}(1, 2, 4, 5) \rangle + \langle S_5 (I - \mathcal{S}) F_{LM}(1, 2, 4, 5) \rangle \quad \text{Double- and single-soft counterterms} \\ &\quad + \langle (I - S_5)(I - \mathcal{S}) F_{LM}(1, 2, 4, 5) \rangle. \end{aligned}$$

Soft-subtracted term - still has
(overlapping) collinear
singularities

\mathcal{S} : Extracts double-soft limit

S_5 : Extracts $E_5 \rightarrow 0$ limit

Separate overlapping collinear limits in two stages:

1. Introduce **phase-space partitions** $1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}$.

$$C_{42}w^{14,15} = C_{52}w^{14,15} = 0 \quad \longrightarrow \quad w^{14,15} \text{ contains } C_{41}, C_{51}, C_{45}$$

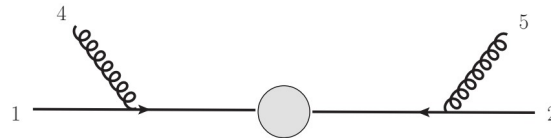
Triple collinear
partition



and

$$C_{42}w^{14,25} = C_{51}w^{14,25} = C_{45}w^{14,25} = 0 \quad \longrightarrow \quad w^{14,25} \text{ contains } C_{41}, C_{52}$$

Double collinear partition



2. Sector decomposition to remove remaining overlapping singularities in triple collinear partitions.

- Define **angular ordering** to separate singularities.

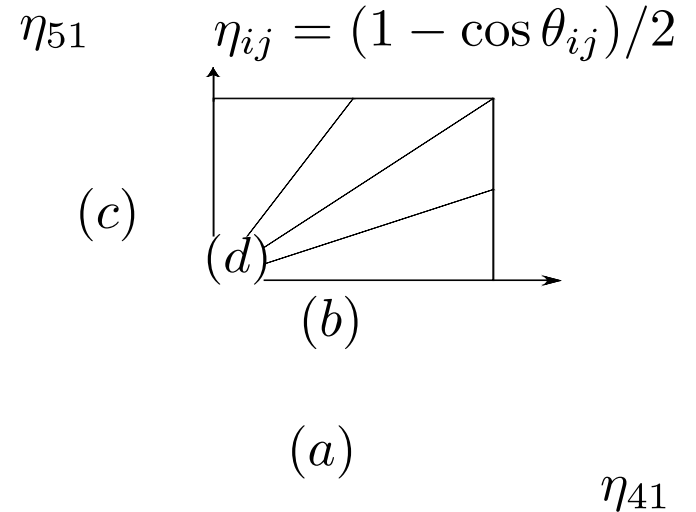
$$\begin{aligned}
 1 &= \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) \\
 &+ \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \\
 &\equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.
 \end{aligned}$$

- Thus the limits are

$$\theta^{(a)} : C_{51} \quad \theta^{(b)} : C_{45}$$

$$\theta^{(c)} : C_{41} \quad \theta^{(d)} : C_{45}$$

- Achieved using angular phase-space parametrization [Czakon ('10, '11)].



Separates collinear limits - subtract **iteratively** from soft-regulated term

$$\langle (I - S_5)(I - \mathcal{S})F_{LM}(1, 2, 4, 5) \rangle =$$

$$\langle F_{LM}^{s_r c_s}(1, 2, 4, 5) \rangle + \langle F_{LM}^{s_r c_t}(1, 2, 4, 5) \rangle + \langle F_{LM}^{s_r c_r}(1, 2, 4, 5) \rangle$$

(Soft-regulated) single and triple collinear counterterms.

Fully subtracted term - finite

Integrate four singular counterterms

$$\langle \mathcal{S}F_{LM}(1, 2, 4, 5) \rangle \quad \langle S_5(I - \mathcal{S})F_{LM}(1, 2, 4, 5) \rangle \quad \langle F_{LM}^{s_r c_s}(1, 2, 4, 5) \rangle \quad \langle F_{LM}^{s_r c_t}(1, 2, 4, 5) \rangle$$

over **unresolved** phase space :

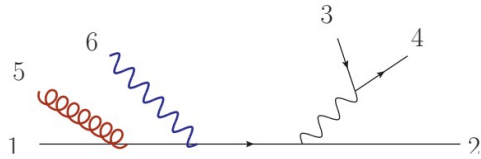
- **cancel** IR poles against loop amplitudes;
- Finite remainder: **subtraction counterterm**.

$$q(p_1)\bar{q}(p_2) \rightarrow e^-(p_3)e^+(p_4)g(p_5)\gamma(p_6)$$

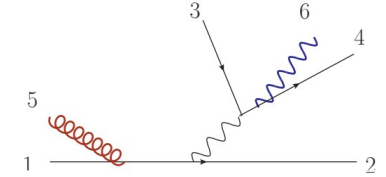
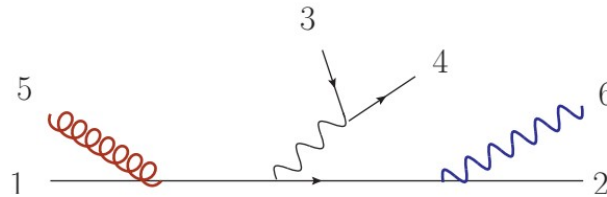
- Partitioning:

$$1 = w^{15,16} + w^{25,26} + w^{15,26} + w^{16,25} + w^{15,36} + w^{15,46} + w^{25,36} + w^{15,46}$$

- Triple collinear sectors



- Double collinear sectors

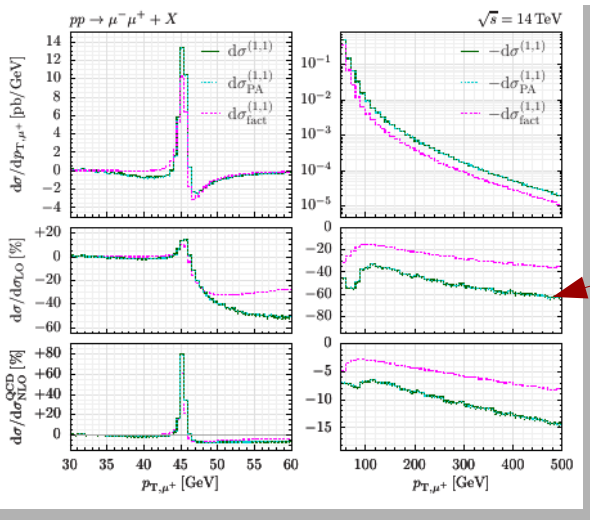


- Additional partitions have only double collinear limits

➤ ~ NLO x NLO – simple!

- First mixed QCD-EW corrections to dilepton production presented by [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini ('21)].
- Two-loop amplitudes evaluated with help of semi-analytic method [cf. Armadillo *et al.* ('22)]
- IR singularities regulated by qT subtractions as implemented in MATRIX.

[Grazzini, Kallweit, Wiesemann ('17)]



- Results for **massive leptons** (collinear singularities regulated by mass):
 - Fiducial cross section increased by **0.5%** relative to LO.
 - Larger impact at high- p_T : **-60%** correction
 - High invariant mass: correction \sim **-1.5%**.
 - **Factorized approximation** works well at Jacobian peak, fails at higher p_T .

LO Corrections $\sim \mathcal{O}(\alpha_s^i \alpha^j)$

$\sigma[\text{fb}]$	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	-49.907	44.60	-16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		-32.66	1.03
$q\gamma$			-0.305		-0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	-47.046	13.88	-15.71

- LHC 13.6 TeV
- NNPDF31_nnlo_as_0118_luxqed
- G_μ input scheme for EW parameters.
- Massless leptons, clustered with photons if $\Delta R_{\ell\gamma} < 0.1$ (“lepton jets”)
- $\mu_R = \mu_F = \mu = m_{\ell\ell}/2$
- $m_{\ell\ell} > 200$ GeV
- $p_{T,\ell^\pm} > 30$ GeV
- $|y_{\ell^\pm}| < 2.5$
- $\sqrt{p_{T,\ell^+} p_{T,\ell^-}} > 35$ GeV

- Fiducial cross section to **NNLO QCD + NLO EW**:

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%} \text{ fb}$$

- **Theoretical uncertainty:**

- Vary scale μ by factor of 2 in either direction.
- Change input scheme for EW parameters to $\alpha(m_Z)$ -scheme.
- Take envelope of these results.

- **Mixed QCD-EW** corrections ($\sim -1\%$) **comparable** to theoretical uncertainty.
- Including mixed QCD-EW corrections **decreases** theoretical uncertainty (mainly through decreasing dependence on EW input scheme)

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%} \text{ fb}$$

(*) Uncertainties from pdfs not included

- At high energies, EW corrections dominated by universal [Sudakov logarithms](#).
- Look at fiducial cross section in 4 mass windows:

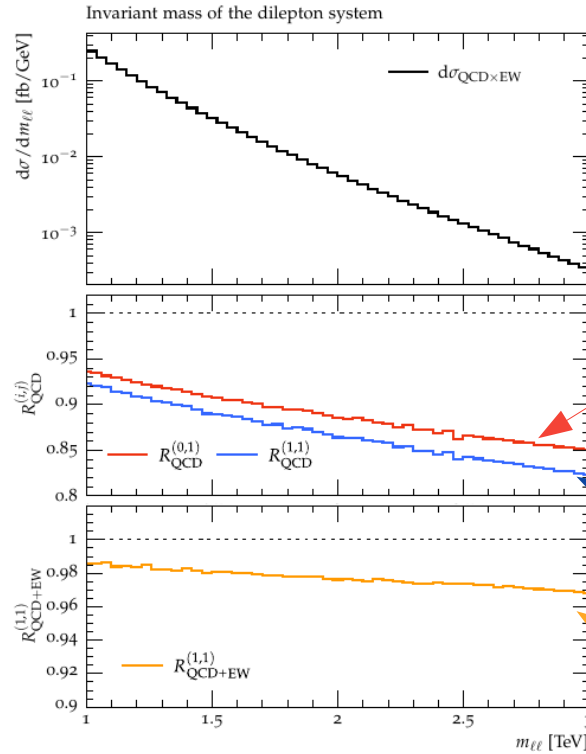
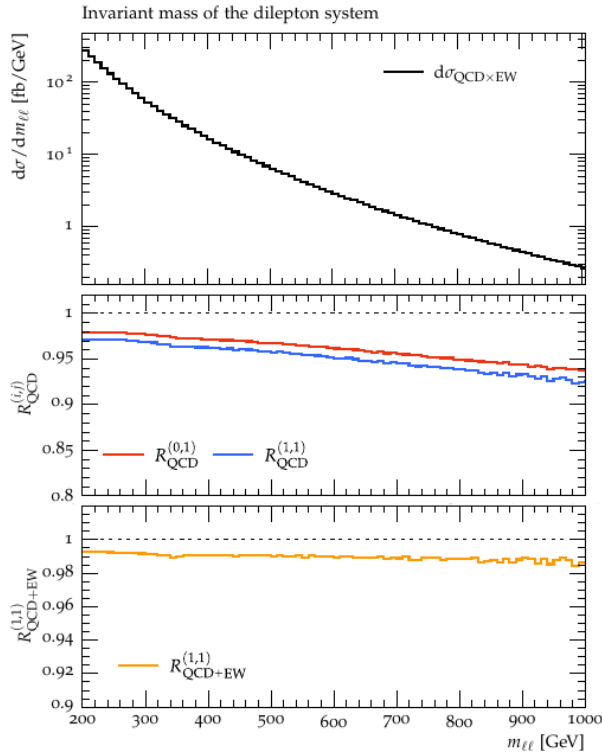
$$\Phi^{(1)} : 200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV},$$

$$\Phi^{(2)} : 300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV},$$

$$\Phi^{(3)} : 500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV},$$

$$\Phi^{(4)} : 1.5 \text{ TeV} < m_{\ell\ell} < \infty.$$

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma_{\text{fact.}}^{(1,1)}$	$\sigma_{\text{QCD}\times\text{EW}}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734	$1392.6^{+0.75\%}_{-0\%}$
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321	$427.1^{+0.41\%}_{-0.02\%}$
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137	$91.98^{+0.22\%}_{-0.14\%}$
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946	$9.500^{+0\%}_{-0.97\%}$



$$R_{\text{QCD}}^{(0,1)} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)}}$$

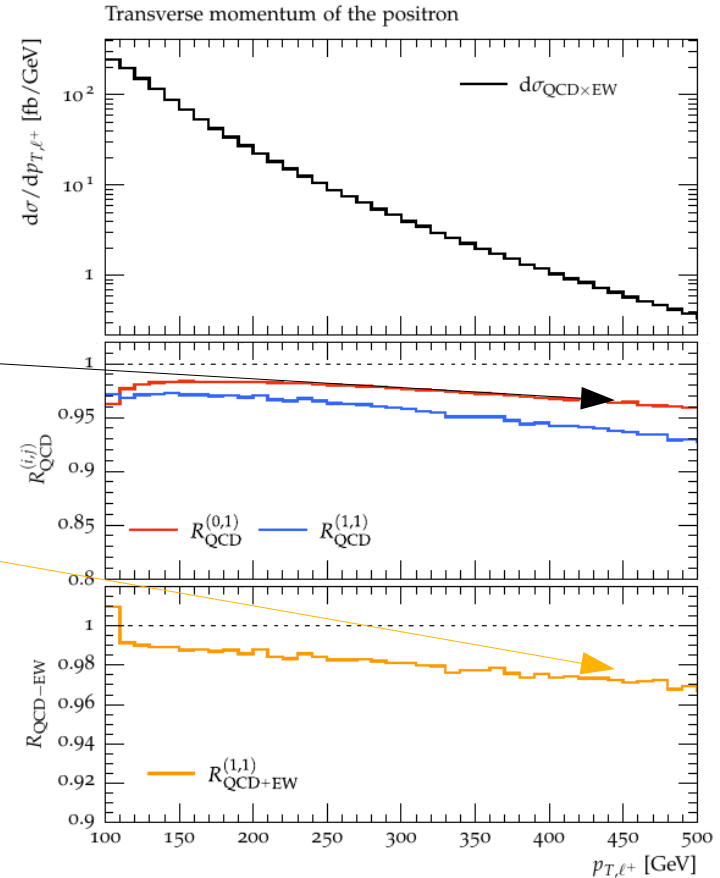
$$R_{\text{QCD}}^{(1,1)} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(1,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)}}$$

$$R_{\text{QCD+EW}}^{(1,1)} = R_{\text{QCD}}^{(1,1)} / R_{\text{QCD}}^{(0,1)}$$

$$= \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(1,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}$$

Similar pattern for p_{T,ℓ^+} :

- NLO EW and QCD-EW corrections become more important at high transverse mass.
- QCD-EW corrections have **slightly stronger dependence** on transverse momentum compared to NLO EW corrections.
- Reach $\sim -3\%$ at $p_{T,\ell^+} \simeq 500$ GeV



- **NLO EW** corrections to angular and rapidity distributions show **minor shape changes**.
- Mixed **QCD-EW** corrections very flat.

