Modified Gravity in Stellar Physics

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May 05, 2023

"DARK MATTER AND STARS"
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Based on

ApJ 884 95 (2019); JCAP 05 040 (2021); arXiv:2209.07389 [astro-ph.SR] (accepted in MNRAS); arXiv:2212.11620 [gr-qc]; ApJ 929 117 (2022)

Overview

- Modified gravity theories alter the hydrostatic equilibrium condition in the weak-field limit.
- Hydrostatic equilibrium condition is a crucial ingredient for deriving analytical formulas of stellar observables.

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- One can constrain such theories from astrophysical observations.
- In reality stellar and substellar objects are anisotropic: e.g., rotation and magnetic fields.

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- Such anisotropies also modify the hydrostatic equilibrium condition.

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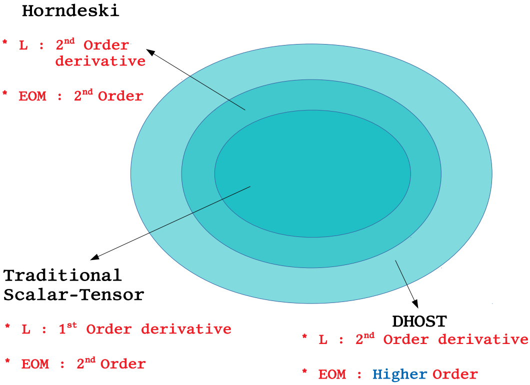
- Modified gravity + Anisotropy $\rightarrow$ stellar and substellar objects.

Modified gravity theories: Classification

- **Scalar-tensor theories** (STTs): One of the most popular avatars of modified gravity theories
- The scalar field is non-minimally coupled to gravity.



- DHOST: $\mathcal{L} = f(\phi, X)R + \sum_{i=1}^{5} A_i(\phi, X) L_i$

where $X \sim -\nabla^{\mu}\phi\nabla_{\mu}\phi$, L_i : quadratic in 2^{nd} order derivatives of ϕ.

Vainshtein Screening mechanism

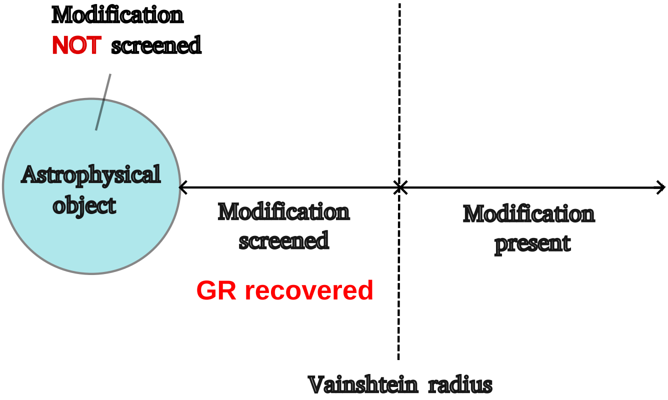


Figure: Partial breaking of Vainshtein mechanism in DHOST theories.

Modification in pressure balance equation

- In a particular subclass of DHOST theories beyond Horndeski:

$$\frac{dP_r}{dr} = -\rho G_N \frac{M}{r^2} - \frac{\Upsilon}{4}\rho G_N M'' \ .$$

Υ quantifies the deviation from standard gravity.

- Considering stellar pressure anisotropy $(P_r \neq P_\perp)$

$$\frac{dP_r}{dr} = -\rho G_N \frac{M}{r^2} - \frac{\Upsilon}{4}\rho G_N M'' + \Delta(r) \ , \ \ \text{with} \ \ \Delta(r) = \frac{2}{r}\left(P_\perp - P_r\right) \ .$$

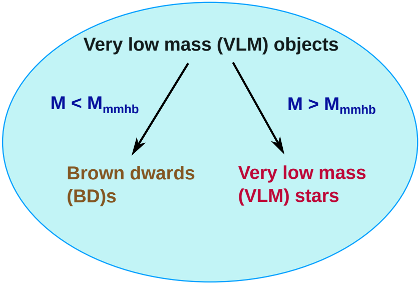
$\Uparrow$

All the above formulas are valid under the approximation of **spherical symmetry**.

- $$(P_\perp - P_r) = \beta(r) P_r(r) \ , \quad \beta(r) = \tau (r/r_0)^2$$
  - $\tau \rightarrow$ Anisotropy parameter: Tunes the strength of anisotropy.
  - $r_0 \rightarrow$ Appropriate length scale, depends on the stellar modeling.

H. Heintzmann and W. Hillebrandt, Astron. & Astrophys, 38, 51 (1974).

Very Low Mass objects (Polytropes with $n = 1.5$)



- In isotropic Newtonian case: $M_{\rm mmhb} \sim 0.080 M_\odot$.
- Over-massive BDs for $+$ve Υ, τ,
- Lowest mass main-sequence stars $= 0.093 M_\odot \implies \boldsymbol{\Upsilon \leq 1.6}$ (isotropic).

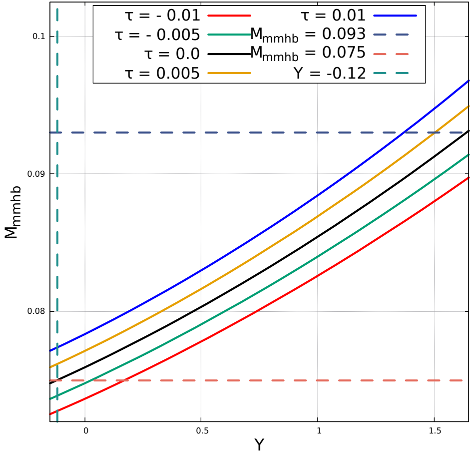


Figure: $M_{\rm mmhb}$ (in units of $M_\odot$) as a function of Υ for different τ.

G. Chabrier and I. Baraffe, Ann. Rev. Astron. Astrophys. 38 337-377 (2000).

White dwarfs (Polytropes with $n = 3.0$)

$$M = 4\pi \int_0^R \rho(r) r^2 dr = -4\pi r_c^3 \rho_c \xi_R^2 \theta'(\xi_R) \ ,$$

- The value of M for $\Upsilon = 0 = \tau$ is the Chandrasekhar mass limit $M_{\rm CH} = 1.4 M_\odot$.
- For $\tau = 0$, $\Upsilon = 1.6 \implies M = \mathbf{2.1} M_\odot$.
- For $\tau = 0.01$, $\Upsilon = 1.6 \implies M = \mathbf{2.6} M_\odot$.
- Super-Chandrasekhar white dwarfs have been reported in the past.

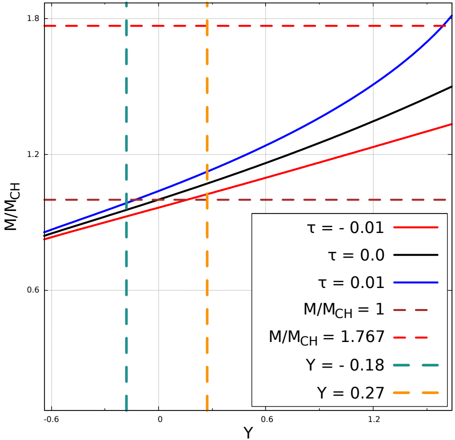
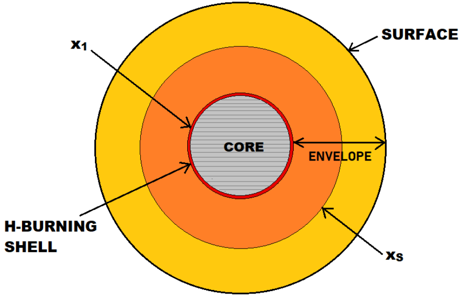


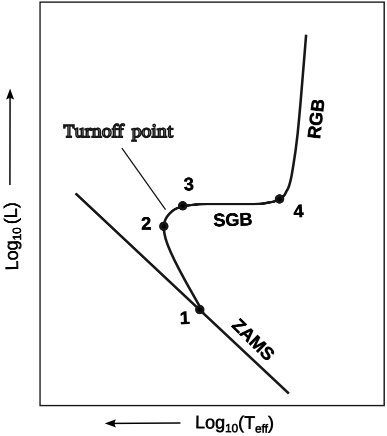
Figure: Mass of WD (in units of $M_{\rm CH}$) as a function of Υ for different τ values.

Modified gravity theory in Composite stellar model

Q: What happens when a low mass star depletes central H completely ?

A: Develops a **core-envelope structure** with **hydrogen burning shell**.





F. Hoyle and M. Schwarzschild, Astrophys. J. Supp. 2 (1955) 1.

Results from numerical analysis

- Overall gravity weakens with increase in Υ:
  - The stellar radius increases.
  - The effective temperature decreases.
  - The luminosity decreases.
- Near the core, gravity strengthens with increase in Υ:
  - The core radius decreases.
  - The temperature increases at the core-envelope junction.
  - The density increases at the core-envelope junction.
- Rule of thumb:

$$\frac{dP(r)}{dr} = -\frac{G_N M(r)}{r^2}\rho(r) \; - \frac{\Upsilon}{4} G_N \rho(r) \left[8\pi r \rho + 4\pi r^2 \frac{d\rho}{dr}\right]$$

$\Longrightarrow$ higher Υ strengthens gravity, near the stellar center, although it weakens gravity far away from it.

R. Saito, et al., JCAP 06 (2015) 008.

Effect on the Schönberg-Chandrasekhar (SC) limit

Q: How long the hydrogen shell burning continues ?

A: Untill the core can support the pressure of overlying envelope.

- SC limit $\rightarrow$ Max core mass fraction.
- SC limit $= f(\alpha)$. (isotropic & Newtonian)
- SC limit $= f(\alpha, \tau, \Upsilon)$.
  - Positive τ, $\Upsilon \Rightarrow$ SC limit decreases.
  - Negative τ, $\Upsilon \Rightarrow$ SC limit increases.
- Higher SC limit $\Rightarrow$ Increased lifetime of H shell burning phase.

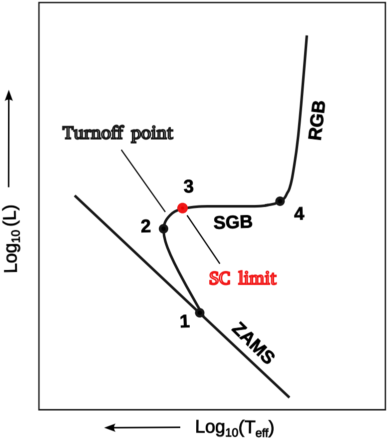


Figure: Schematic SC limit.

Analytic formalism: slow rotation in MG theories

- Equations of mechanical equilibrium

$$\frac{\partial P}{\partial r} = -\rho \frac{\partial \Phi}{\partial r} + \rho \Omega^2 r (1-\mu^2) \, , \qquad \frac{\partial P}{\partial \mu} = -\rho \frac{\partial \Phi}{\partial \mu} - \rho \Omega^2 r^2 \mu$$

- Modified Poisson's equation:
{beyond-Horndeski, Palatini $f(R)$, EiBI gravity}

$$\nabla^2 \Phi = 4\pi G_N \rho + L\Phi_{mod}(r,\mu)$$

$$\Downarrow$$

- Modified Lane-Emden equation (MLEE)

$$\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left(\xi^2 \frac{\partial \Theta}{\partial \xi}\right) + \frac{1}{\xi^2}\frac{\partial}{\partial \mu}\left((1-\mu^2)\frac{\partial \Theta}{\partial \mu}\right) = -\Theta^n + \mathbf{v} + g_{mod}(\xi,\mu)$$

- Dimensionless variables and modification terms

$$\rho = \rho_c \Theta^n, \qquad r = r_c \xi \qquad \text{with} \qquad r_c^2 = \frac{K(n+1)\rho_c^{(\frac{1}{n}-1)}}{4\pi G_N}$$

$$\mathbf{v} = \frac{\mathbf{\Omega^2}}{\mathbf{2\pi G_N \rho_c}}, \qquad g_{mod} = -\frac{L\Phi_{mod}}{4\pi G_N \rho_c}$$

Analytic formalism: Forms of functions

- We assume the following form for Θ

$$\Theta(\xi,\mu) = \theta(\xi) + v\Psi(\xi,\mu)$$

$$\Psi(\xi,\mu) = \psi_0(\xi) + \sum_{j=1}^{\infty} A_j \psi_j(\xi) P_j(\mu)$$

- Similarly, we need to make the following choice for g_{mod}

$$g_{mod}(\xi,\mu) = g_{mod0}(\xi) + v\tilde{g}_{mod}(\xi,\mu),$$

$$\tilde{g}_{mod}(\xi,\mu) = \bar{\tilde{g}}_{mod}(\xi) + \sum_{j=1}^{\infty} \bar{\bar{\tilde{g}}}_{modj}(\xi) P_j(\mu)$$

Chandrasekhar, S., (communicated by Milne, E., A.) 1933, MNRAS, **93**, 5, 390.

Analytic formalism: Complete solution of MLEE

- The complete analytic form of the solution Θ is obtained as follows:

$$\Theta(\xi,\mu) = \theta(\xi) + v\Big[\psi_0(\xi) + \Big\{-\frac{5}{6}\frac{\xi_1^2}{[3\psi_2(\xi_1) + \xi_1\psi_2'(\xi_1)]}\Big\}\psi_2(\xi)P_2(\mu)\Big]$$

$$\downarrow$$

$$A_2$$

- The component functions satisfy the following equations

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\Big(\xi^2\frac{\partial\theta}{\partial\xi}\Big) = -\theta^n + g_{mod0}(\xi)$$

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\Big(\xi^2\frac{\partial\psi_0}{\partial\xi}\Big) = -n\theta^{n-1}\psi_0 + 1 + \bar{\tilde{g}}_{mod}(\xi)$$

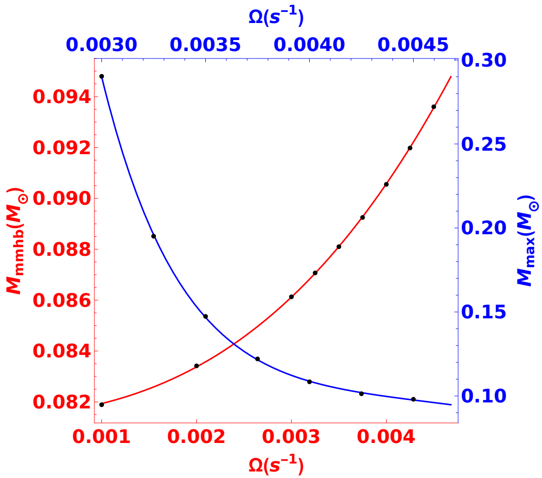
$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\Big(\xi^2\frac{\partial\psi_2}{\partial\xi}\Big) = \Big(\frac{6}{\xi^2} - n\theta^{n-1}\Big)\psi_2 + \frac{\bar{\bar{\tilde{g}}}_{mod2}(\xi)}{A_2}$$

- Boundary conditions:

$$\theta(0) = 1,\ \theta'(0) = 0;\ \ \psi_0(0) = 0,\ \psi_0'(0) = 0;\ \ \psi_2(0) = 0,\ \psi_2'(0) = 0.$$

Rapid uniform rotation in VLM objects

- For a given Ω we obtain a transition mass range $\mathbf{M}_{\mathrm{mmhb}}(\boldsymbol{\Omega}) \leq \mathbf{M} \leq \mathbf{M}_{\max}(\boldsymbol{\Omega})$
- M_{mmhb} increases with Ω.
- $M_{\max}$ decreases with Ω.
- The transition mass range gradually decreases with Ω and reduces to a single point at a particular $\boldsymbol{\Omega}_{\max} \sim \mathbf{0.0047 s^{-1}}$ $(22\ \mathrm{mins})$.
- Model solutions for VLM objects do not exist beyond this maximal rotation.



Blue curve → $M_{\max}(\Omega)$
Red curve → $M_{\mathrm{mmhb}}(\Omega)$.

Summary

- The hydrostatic pressure balance equation gets altered in presence of small stellar pressure anisotropy in modified gravity theories.
- Therefore, retaining spherical symmetry we obtained modifications in the stellar and substellar observables.
- We compared such modified predictions with observational data to put bounds on the modified gravity parameter.
- We then relaxed the spherical symmetry to incorporate slow stellar rotation in any modified gravity theory in general.
- We finally studied rapid rotation in VLM objects and obtained important limits.

Thank You All !