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Intermediate Mass-Ratio Inspirals with Dark Matter Minispikes

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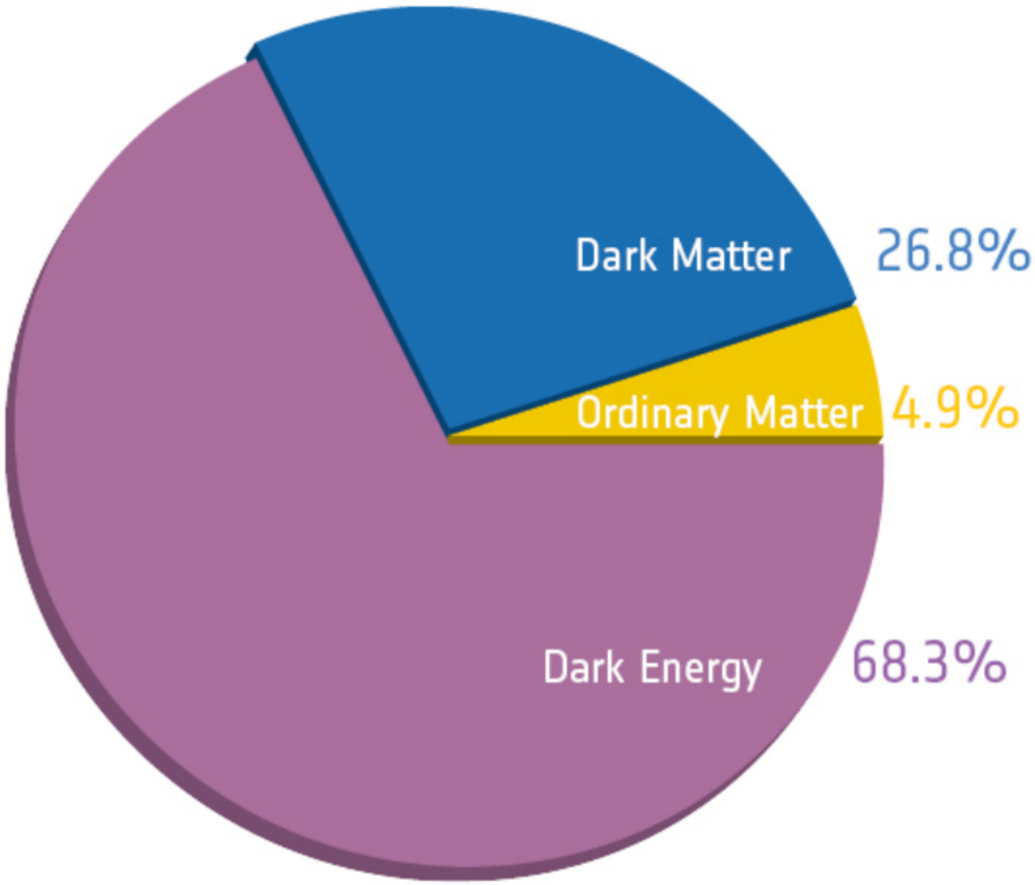
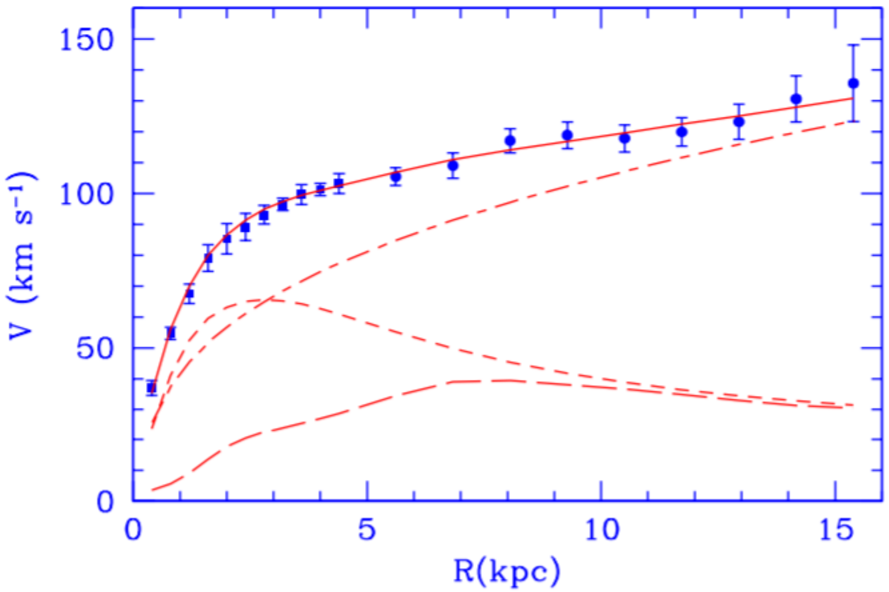
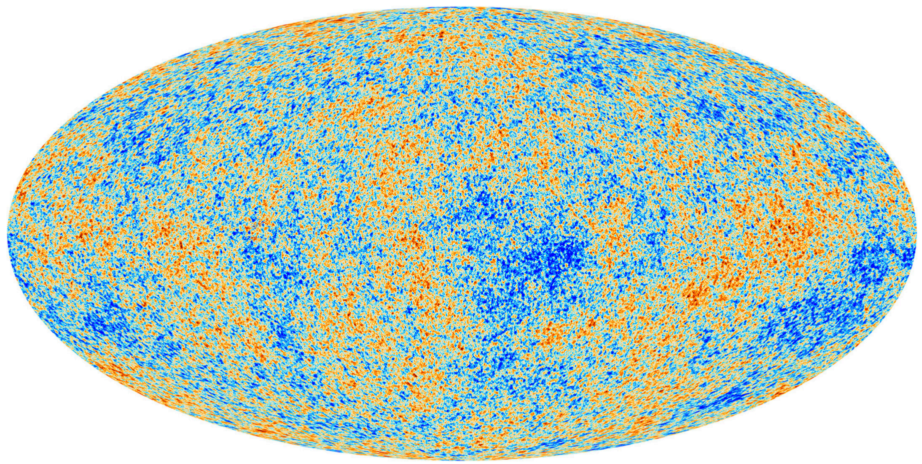
Dark Matter



(Credit: ESA and the Planck Collaboration)



Rotation Curve of M33
(E. Corbelli & P. Salucci, 2000)



Cosmic Microwave Background
(Credit: ESA and the Planck Collaboration)

Dark Matter Minispikes around an Intermediate Mass Black Hole

density profile

$$\rho_{\rm DM}(r) = \begin{cases} \rho_{\rm sp}\left(\dfrac{r_{\rm sp}}{r}\right)^{\alpha}, & r_{\rm min} \le r \le r_{\rm sp} \\ 0, & r \le r_{\rm min} \end{cases}$$

$$M = 10^3 M_\odot$$

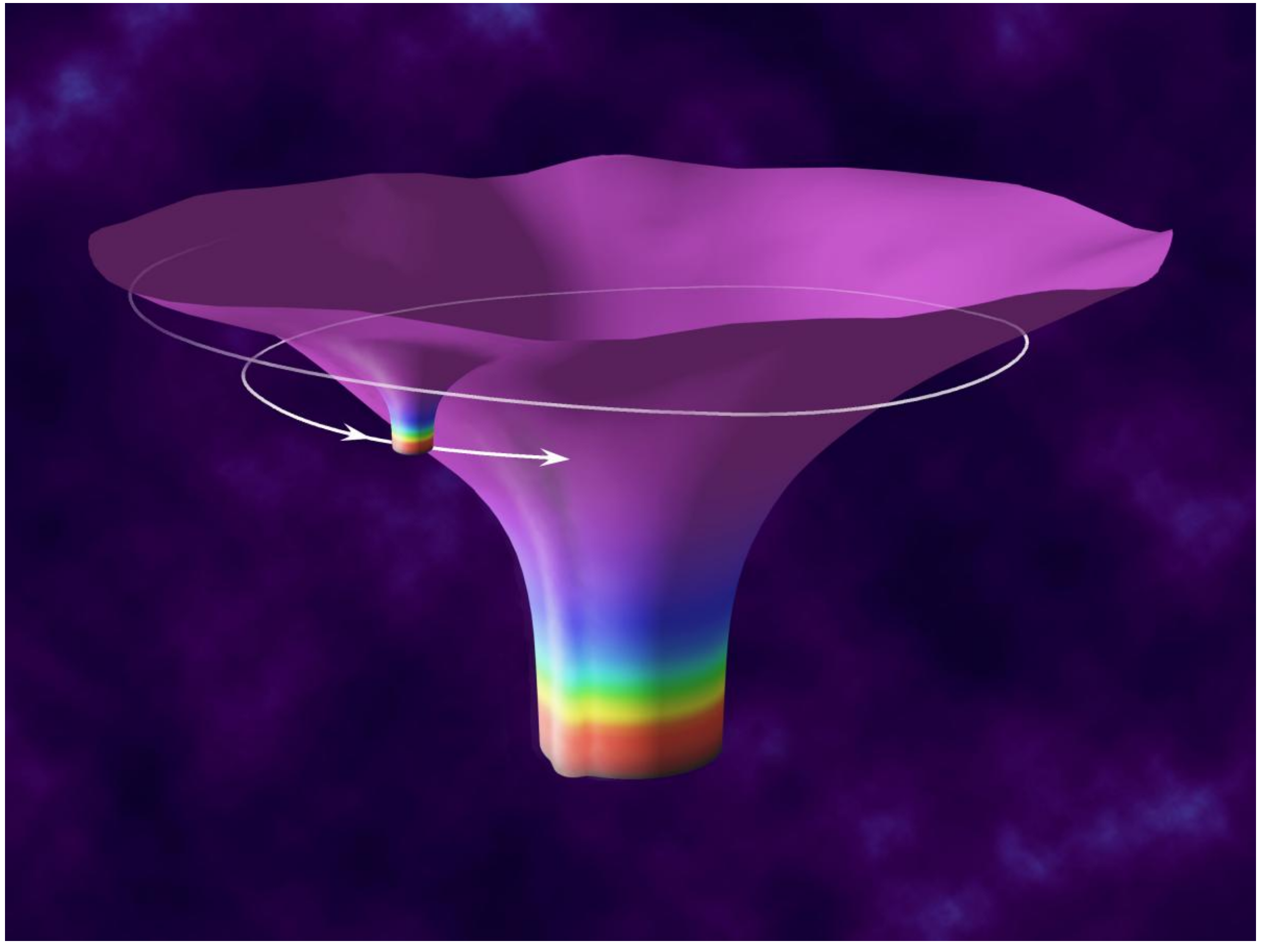
$r_{\rm min} = r_{\rm ISCO} = 3R_{\rm s}$ $\quad$ $r_{\rm sp} = 0.54$ pc

$\rho_{\rm sp} = 226 M_\odot/{\rm pc}^3$ $\quad$ $2.25 \le \alpha \le 2.5$

H. Zhao & J. Silk, 2005
K. Eda, et al. 2013

Intermediate Mass-Ratio Inspiral $(M/\mu = 10^2 \sim 10^4)$

$$M(10^3 M_\odot) + \mu(10 M_\odot)$$



Four Major Factors

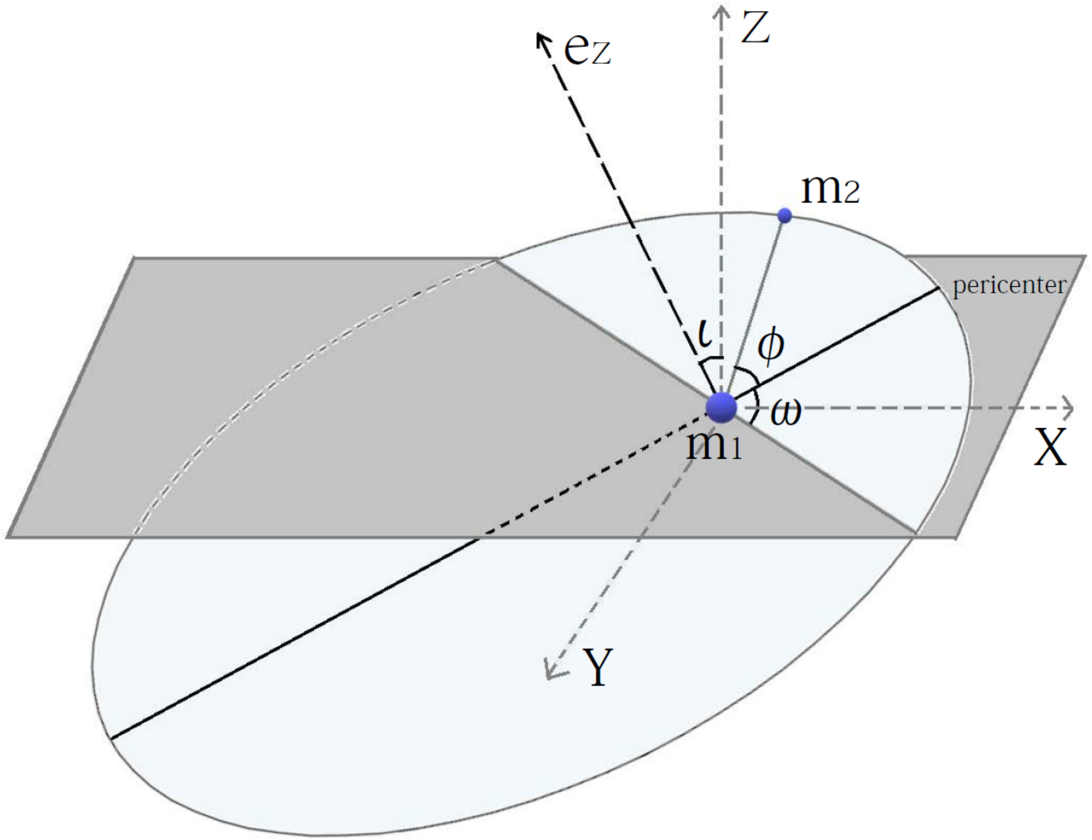
- Gravity from DM
- Dynamic Friction
- Accretion
- Radiation Reaction

Osculating Orbit

$$\mathbf{a} = -\frac{Gm}{r^2}\mathbf{n} + \mathbf{f}$$

perturbative force

$$\mathbf{f} = \mathscr{R}\,\mathbf{n} + \mathscr{S}\,\mathbf{k} + \mathscr{W}\,\mathbf{e}_z$$



$$\frac{dp}{dt} = 2\sqrt{\frac{p^3}{Gm}}\frac{1}{1+e\cos\phi}\mathscr{S}$$

$$\frac{de}{dt} = \sqrt{\frac{p}{Gm}}\left[\sin\phi\,\mathscr{R} + \frac{2\cos\phi + e(1+\cos^2\phi)}{1+e\cos\phi}\mathscr{S}\right]$$

$$\frac{d\omega}{dt} = \frac{1}{e}\sqrt{\frac{p}{Gm}}\left[-\cos\phi\,\mathscr{R} + \frac{2+e\cos\phi}{1+e\cos\phi}\mathscr{S} - e\cot\iota\,\frac{\sin(\omega+\phi)}{1+e\cos\phi}\mathscr{W}\right]$$

$$\frac{d\phi}{dt} = \sqrt{\frac{Gm}{p^3}}\left(1+e\cos\phi\right)^2 + \frac{1}{e}\sqrt{\frac{p}{Gm}}\left[\cos\phi\,\mathscr{R} - \frac{2+e\cos\phi}{1+e\cos\phi}\sin\phi\,\mathscr{S}\right]$$

Gravity of DM minispikes

$$\mathbf{f}_{\mathrm{G}} = -\frac{4\pi G\rho_{\mathrm{sp}} r_{\mathrm{sp}}^{\alpha}}{(3-\alpha) r^{\alpha-1}}\mathbf{n}$$

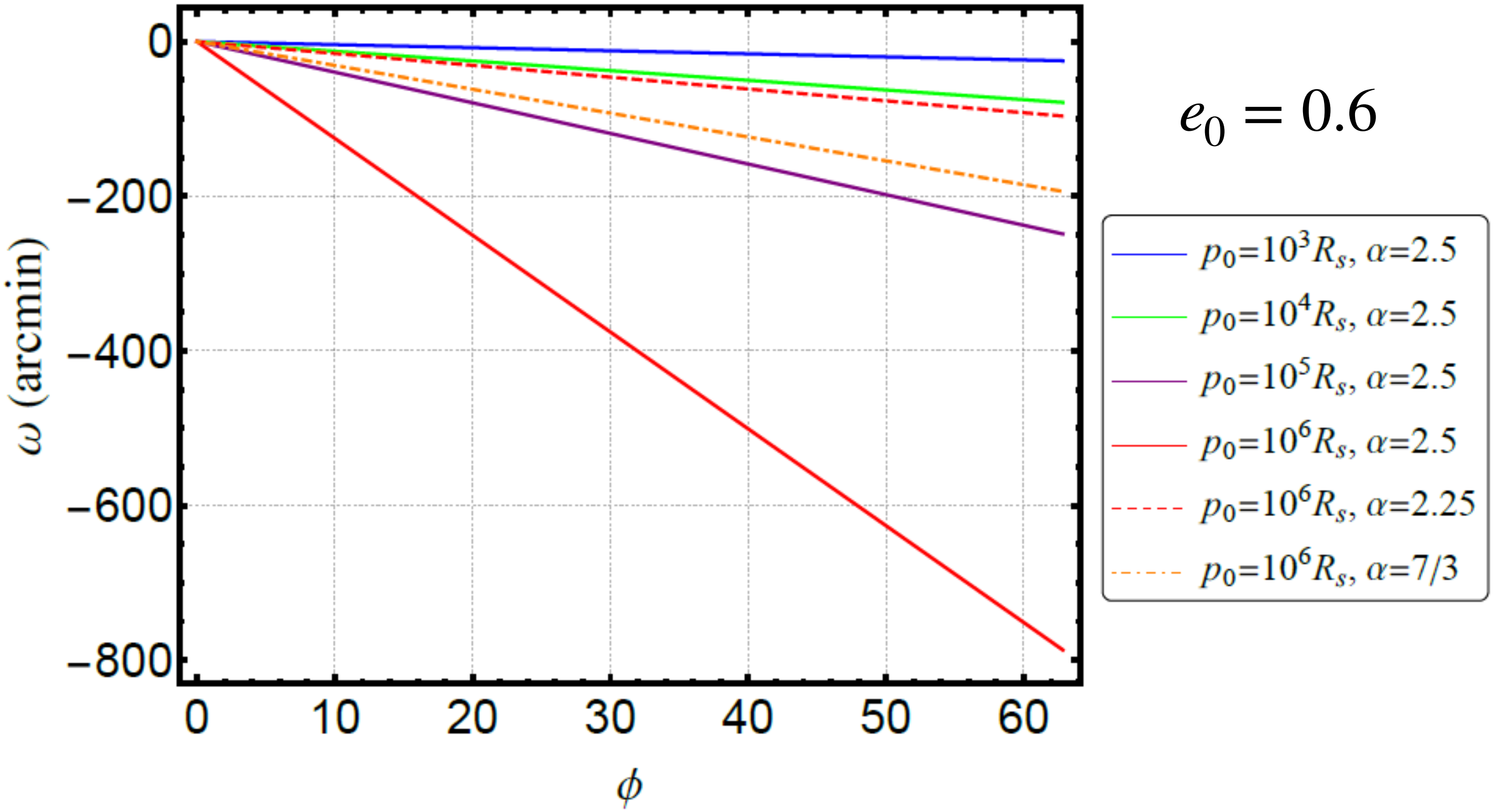
$\Delta p = 0$ conservative

$\Delta e = 0$

$$\Delta\omega_{\mathrm{DM}} = \frac{4\pi p^{3-\alpha}\rho_{\mathrm{sp}} r_{\mathrm{sp}}^{\alpha}}{(3-\alpha) M_{\mathrm{eff}}} W_{\mathrm{DM}}(e)$$

$$W_{\mathrm{DM}}(e) = \int_0^{2\pi} \cos\phi (1 + e\cos\phi)^{\alpha-3} e^{-1}\, d\phi$$

precession



$e_0 = 0.6$

N. Dai, et al. 2022

Dynamic Friction

V. Cardoso, et al. 2020

$$\mathbf{f}_{\rm DF} = -\frac{4\pi G^2 \mu \rho_{\rm DM} I_v}{v^3}\mathbf{v}$$

$$\left\langle \frac{dp}{d\phi} \right\rangle_{\rm DF} = -\frac{4\mu\rho_{\rm sp} r_{\rm sp}^{\alpha} I_v}{M^2} p^{4-\alpha} g(e) < 0$$

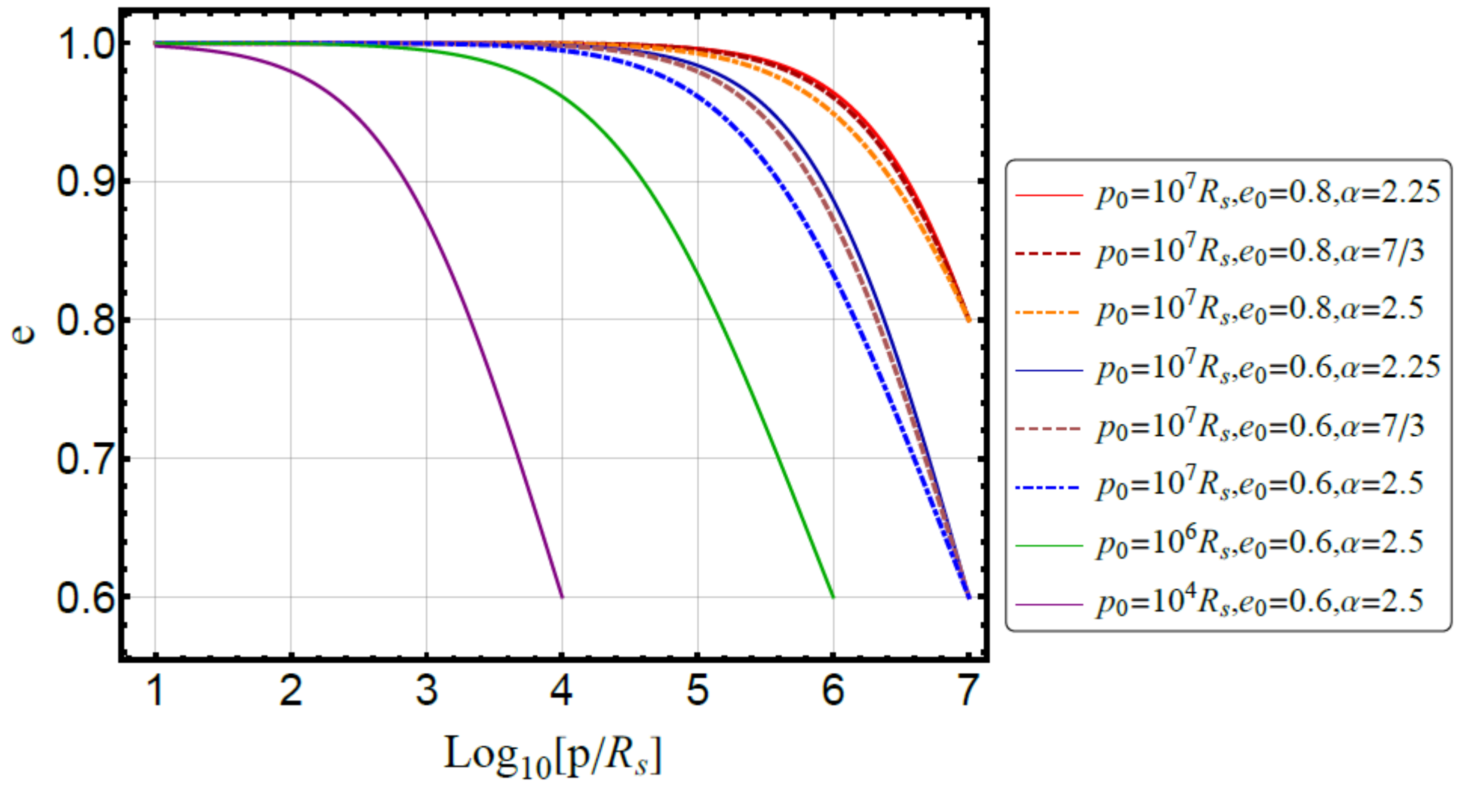
$$\left\langle \frac{de}{d\phi} \right\rangle_{\rm DF} = -\frac{4\mu\rho_{\rm sp} r_{\rm sp}^{\alpha} I_v}{M^2} p^{3-\alpha} f(e) > 0$$

$$\left\langle \frac{d\omega}{d\phi} \right\rangle_{\rm DF} = 0$$

dissipative

$$g(e) = \int_0^{2\pi} \frac{d\phi}{(1+2e\cos\phi+e^2)^{3/2}(1+e\cos\phi)^{2-\alpha}}$$

$$f(e) = \int_0^{2\pi} \frac{(\cos\phi+e)d\phi}{(1+2e\cos\phi+e^2)^{3/2}(1+e\cos\phi)^{2-\alpha}}$$



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Bondi-Hoyle Accretion

H. Bondi & F. Hoyle, 1944
C. Macedo, et al. 2013

$$\dot{\mu} = 4\pi G^2 \lambda \frac{\mu^2 \rho_{\mathrm{DM}}}{(v^2 + c_s^2)^{3/2}}$$

$$\mathbf{f}_{\mathrm{a}} \simeq -\frac{4\pi G^2 \mu \lambda \rho_{\mathrm{DM}}}{v^3}\mathbf{v}$$

dissipative

similar to dynamic friction

Radiation Reaction

T. Damour & N. Deruelle, 1981

$$\mathbf{f}_{\mathrm{GW}} = \frac{8}{5}\frac{G^2 M \mu}{c^5 r^3}\left[\left(3v^2 + \frac{17}{3}\frac{Gm}{r}\right)\dot{r}\mathbf{n} - \left(v^2 + 3\frac{Gm}{r}\right)\mathbf{v}\right]$$

$$\left\langle \frac{dp}{d\phi} \right\rangle_{\mathrm{GW}} = -\frac{8}{5}\eta\frac{(Gm)^{5/2}}{c^5 p^{3/2}}\left(8 + 7e^2\right) < 0$$

$$\left\langle \frac{de}{d\phi} \right\rangle_{\mathrm{GW}} = -\frac{8}{5}\eta\frac{(Gm)^{5/2}}{c^5 p^{5/2}}\left(\frac{304}{24}e + \frac{121}{24}e^3\right) < 0$$

$$\left\langle \frac{d\omega}{d\phi} \right\rangle_{\mathrm{GW}} = 0$$

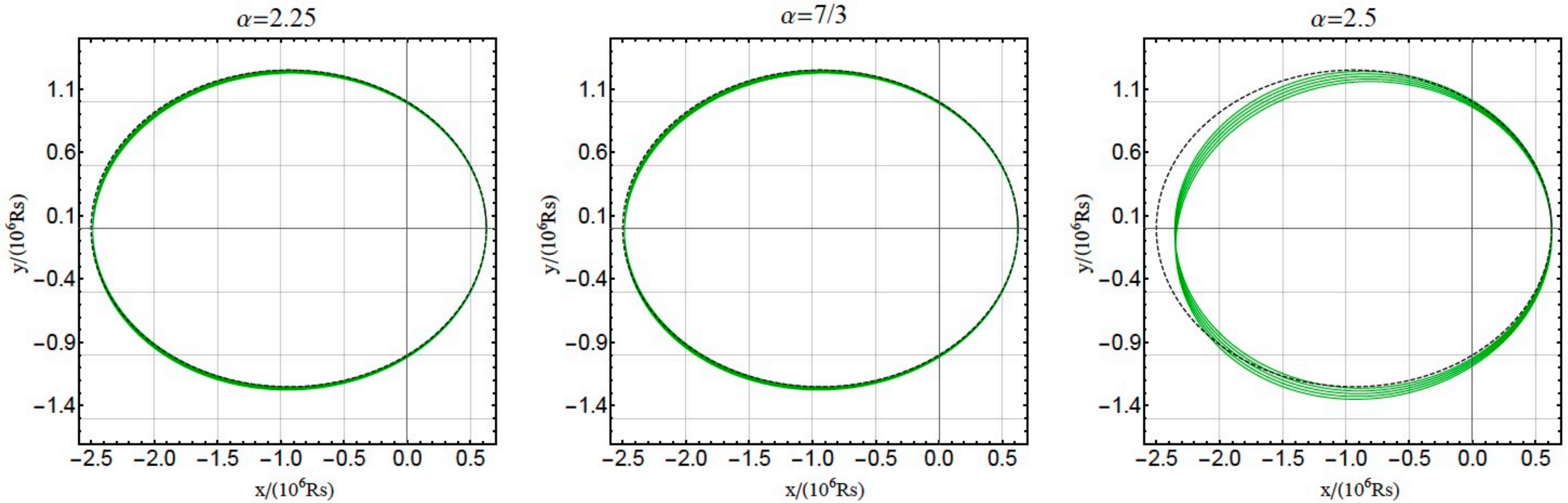
dissipative

Large Orbital Distance

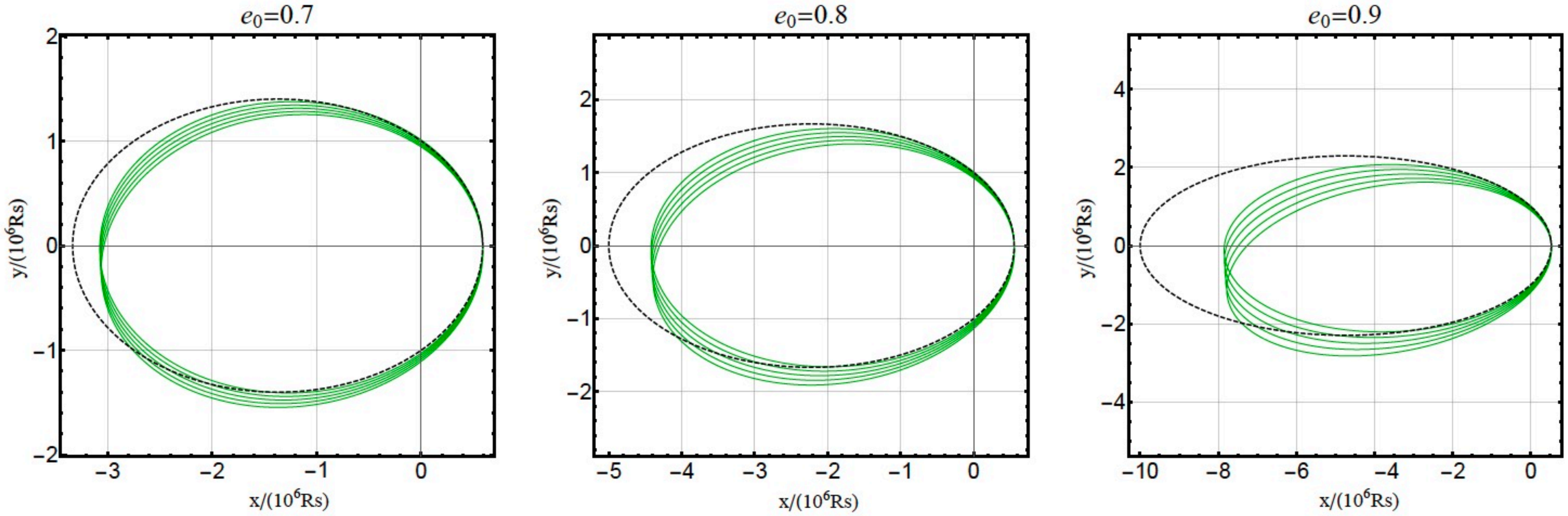
$$p_0 = 10^6 R_s$$

period~100 years

$e_0 = 0.6$



$\alpha = 2.5$



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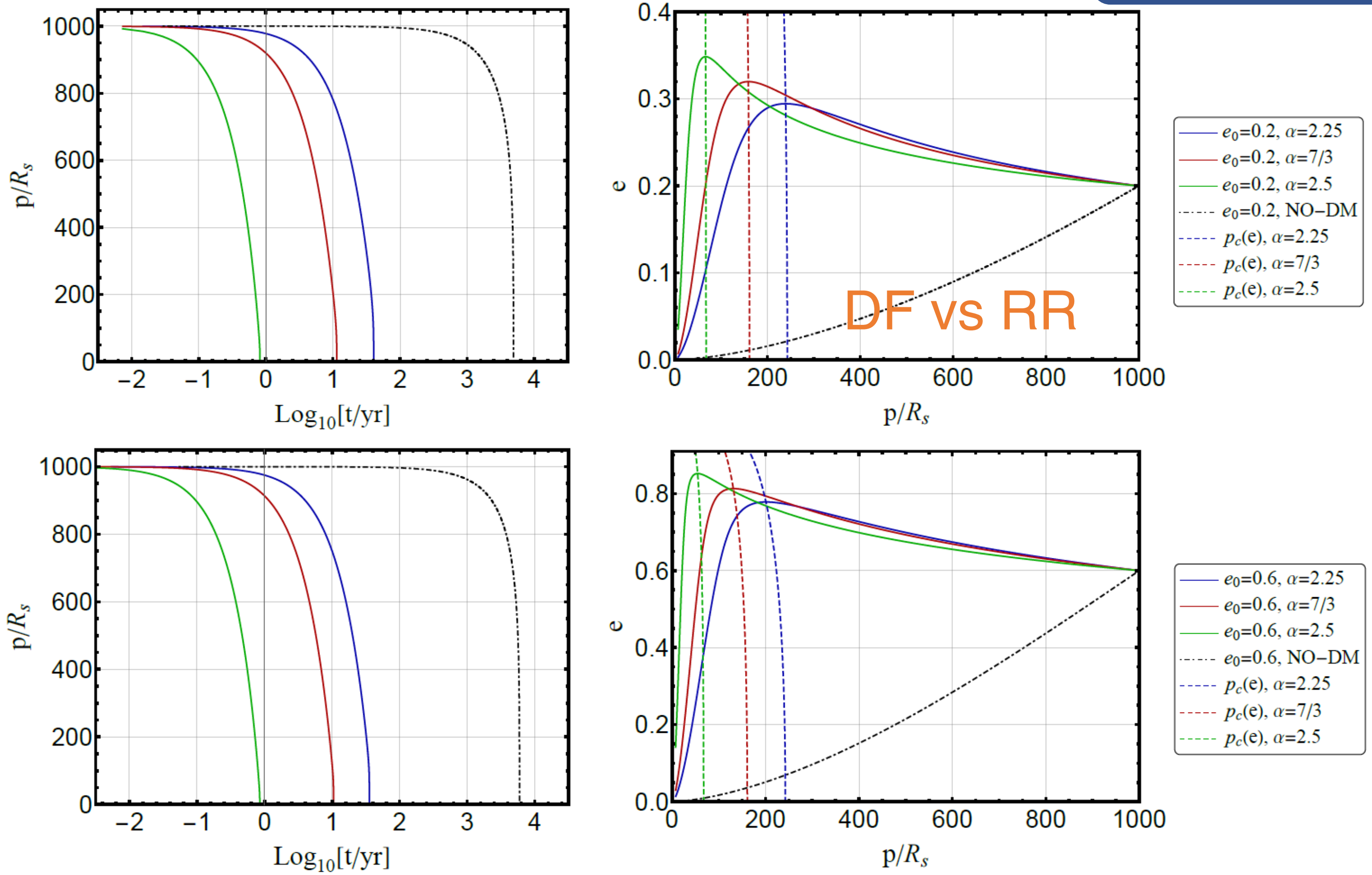
Small Orbital Distance

$$p_0 = 10^3 R_s$$

$p : 10^3 R_s \rightarrow 10 R_s$

orbital evolution



evolution time (yr)

e	No DM	$\alpha = 2.25$	$\alpha = 7/3$	$\alpha = 2.5$
0	4829	41.0	11.5	0.813
0.2	4901	40.4	11.4	0.815
0.4	5178	38.6	11.1	0.826
0.6	5928	35.6	10.5	0.848
0.8	8354	30.3	9.5	0.879
0.9	12625	25.5	8.4	0.898

acceleration

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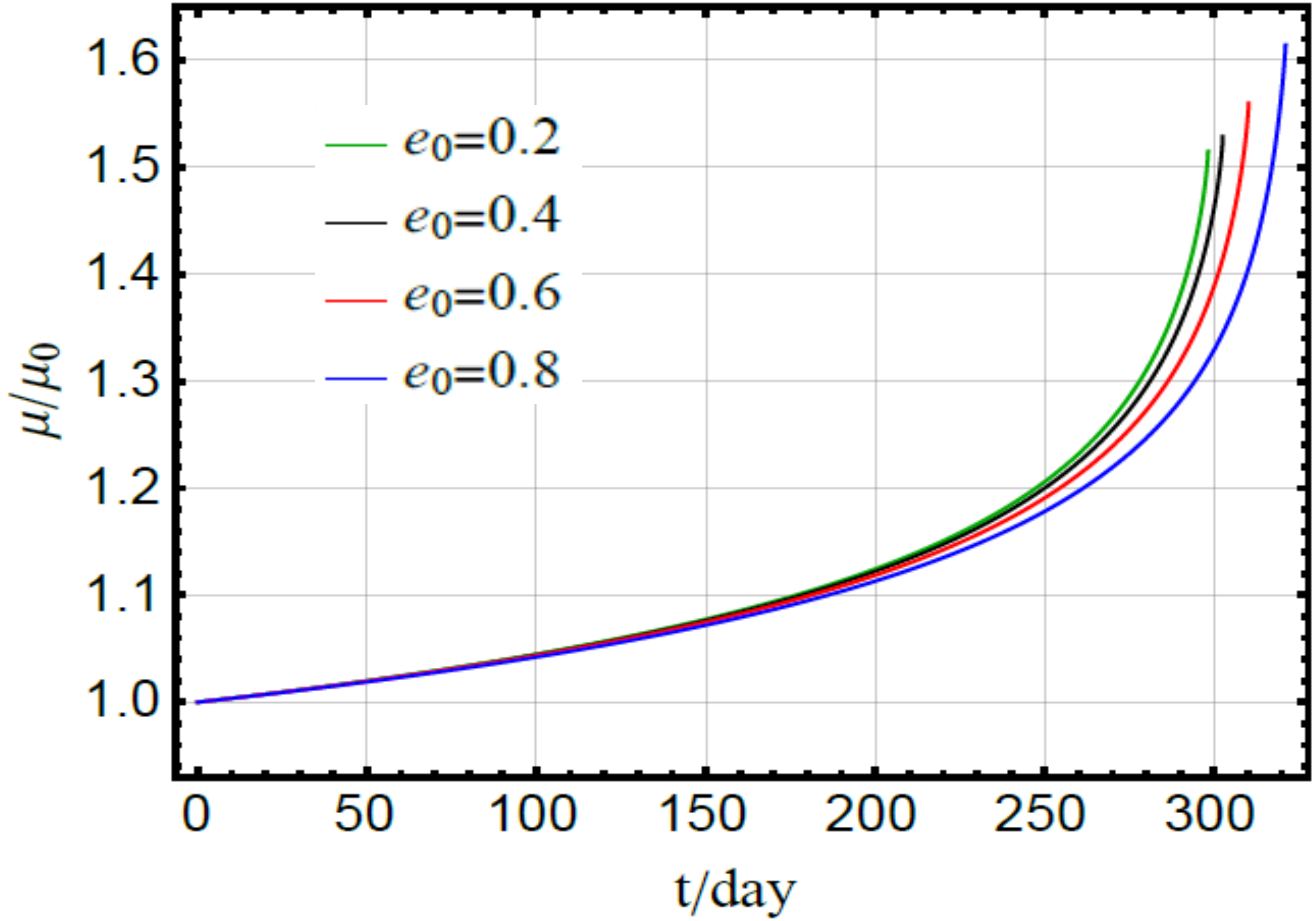
Small Orbital Distance

$$p_0 = 10^3 R_s$$

accretion

$$\alpha = 2.5$$



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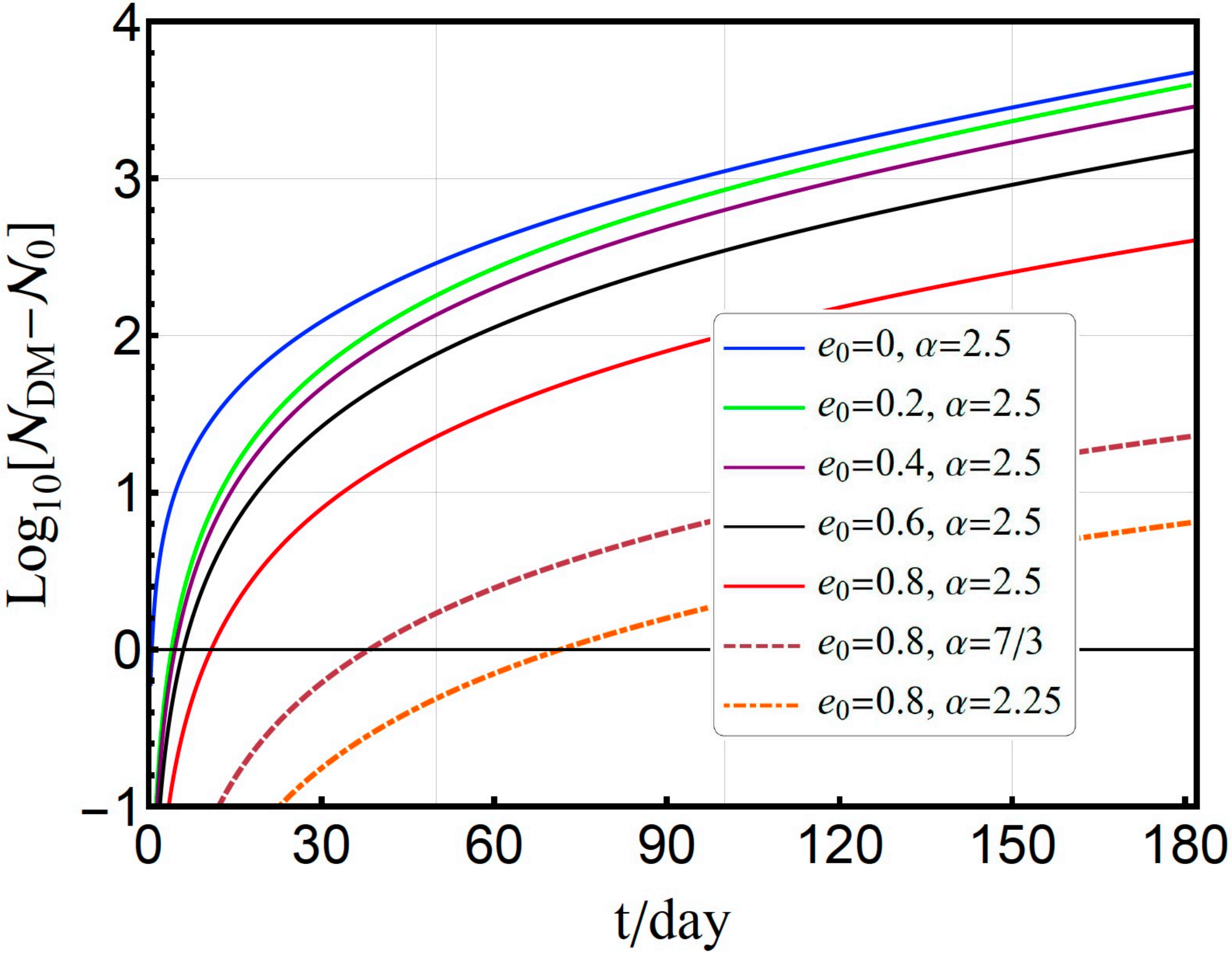
Small Orbital Distance

orbital cycles

$$\mathcal{N} = \frac{1}{2\pi}\int_{t_i}^{t_f} f(t)\,dt$$

$p_0 = 10^3 R_s$



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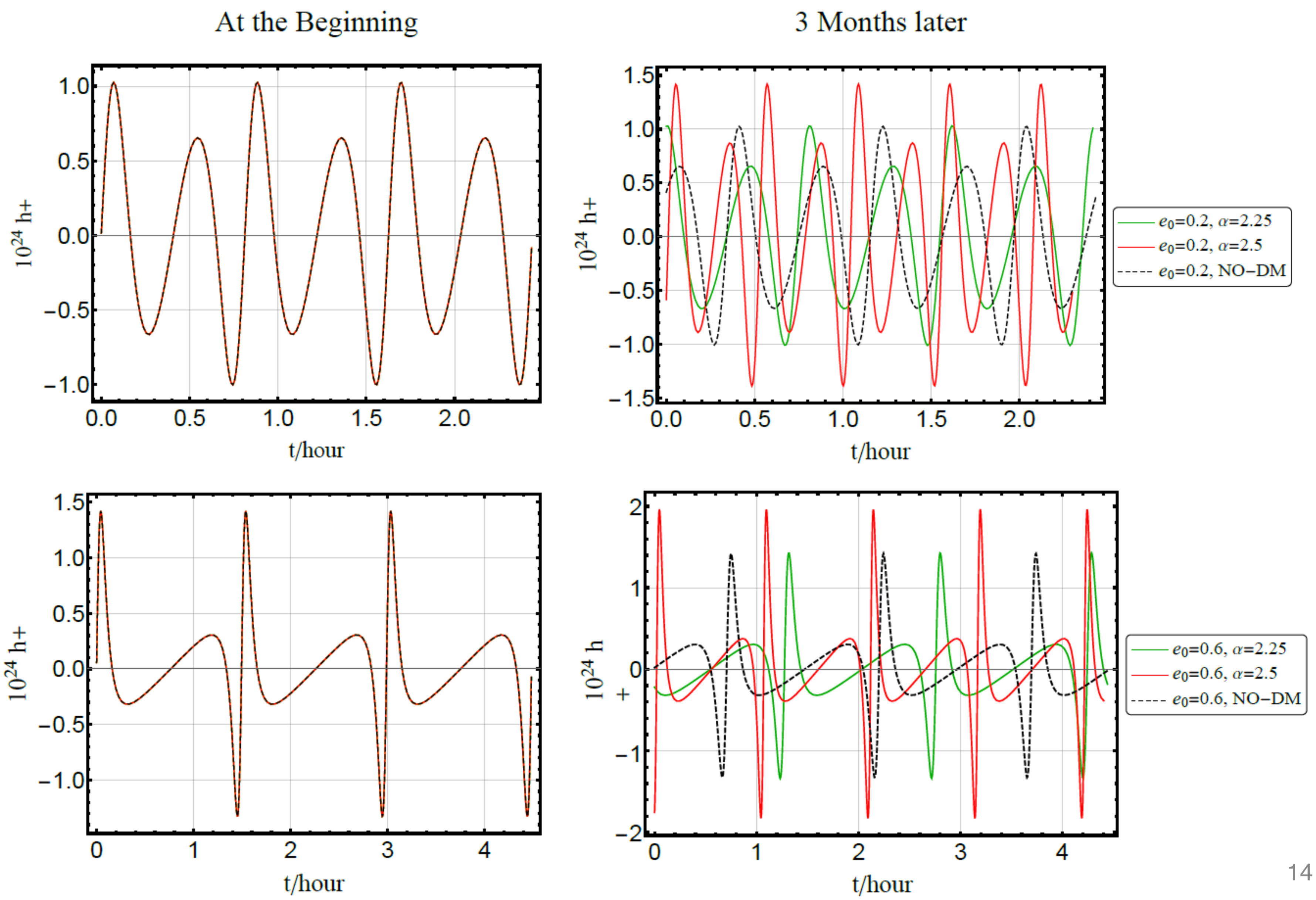
Small Orbital Distance

$$p_0 = 10^3 R_s$$

GW waveform



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Small Orbital Distance

one-year evolution before $10R_S$

$\alpha = 7/3$

$D_L = 100\text{Mpc}$

e_0	SNR_0	SNR_D	Mismatch	$D_{\max}$
0.2	34.13	42.97	0.99992	358.1
0.4	34.19	47.74	0.99944	397.8
0.6	34.44	36.71	0.99965	305.9

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$$\text{Mismatch} = 1 - \max_{t_c,\phi_c} \frac{(\tilde{h}_0, \tilde{h}_{\text{DM}})}{\sqrt{(\tilde{h}_0, \tilde{h}_0)(\tilde{h}_{\text{DM}}, \tilde{h}_{\text{DM}})}}$$

Summary

- The existence of dark matter significantly affects the evolution of IMRIs
- How to model these environmental effects will be essential in the future

Thank you for listening