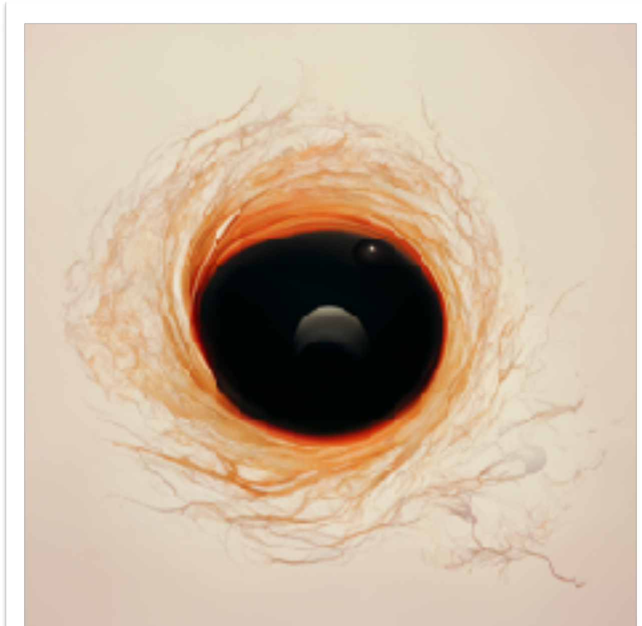
Dirty black hole binaries as probe of dark matter halos

@ Dark Matter and Stars, 3 May 2023

In collaboration with
V. Cardoso, F. Duque, K, Destounis, R. Panoso, E. Figueiredo



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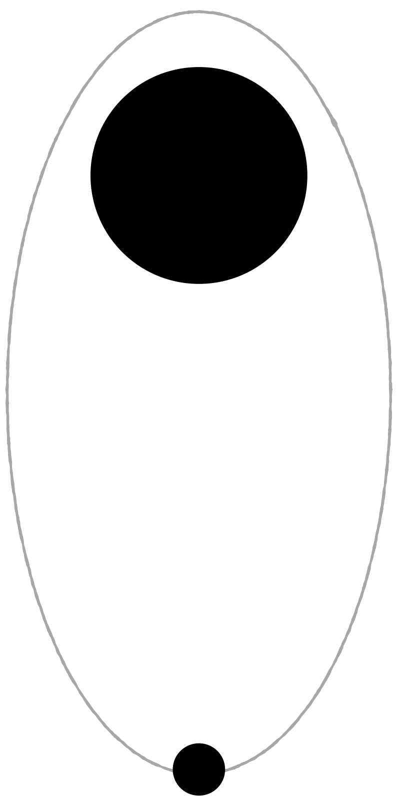
Milestones

- *Asymmetric binaries as natural (golden) laboratories to test fundamental physics*
  - *top-notch to study the (generic) environment in which binaries evolve*
- *How do we build a relativistic BH solutions embedded within a core of matter?*
  - *Are astrophysical observables affected by the non-vacuum background?*
  - *Which are the best observables to capture the properties of the environment?*



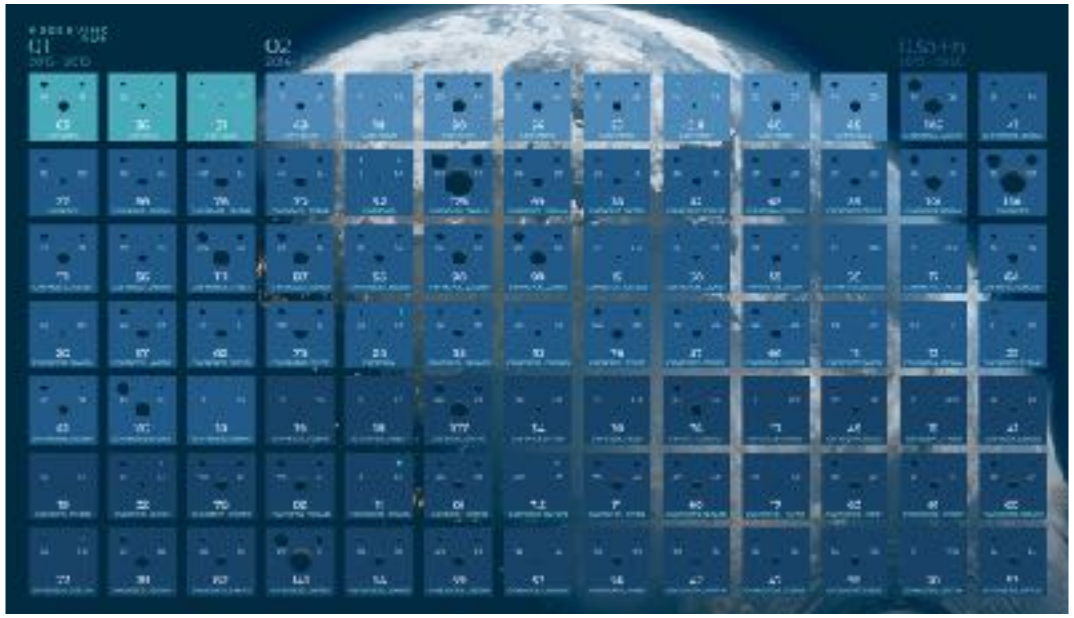
Why asymmetric binaries?

90+ events observed so far from LVK, spanning a relatively small interval of mass ratios $q \sim 1:30$

- *3G detectors are expected to beat down such value by several orders of magnitudes*

$$q \sim 10^{-6} - 10^{-7}$$

- *dynamics mostly dictated by **q**, with the duration of the inspiral & number of cycles growing as q decreases*



LVK GWTC 3 2111.03605

A discovery potential in 3 moves

1. *Slow inspiral phase which could allow to continuously observe AB for very long periods, from months to years*

2. *dynamical evolutions with an uncommon richness, with resonances, large eccentricities and off-equatorial orbits, etc.*

3. *astro-fundamental physics setups*

Extreme Mass Ratio Inspirals

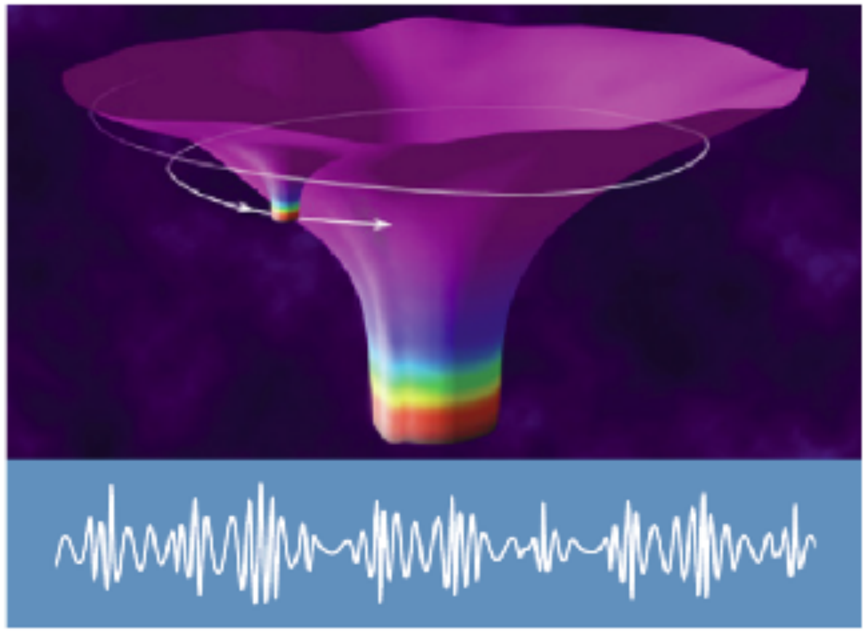
Binary systems with a stellar-mass body inspiralling into a massive black hole

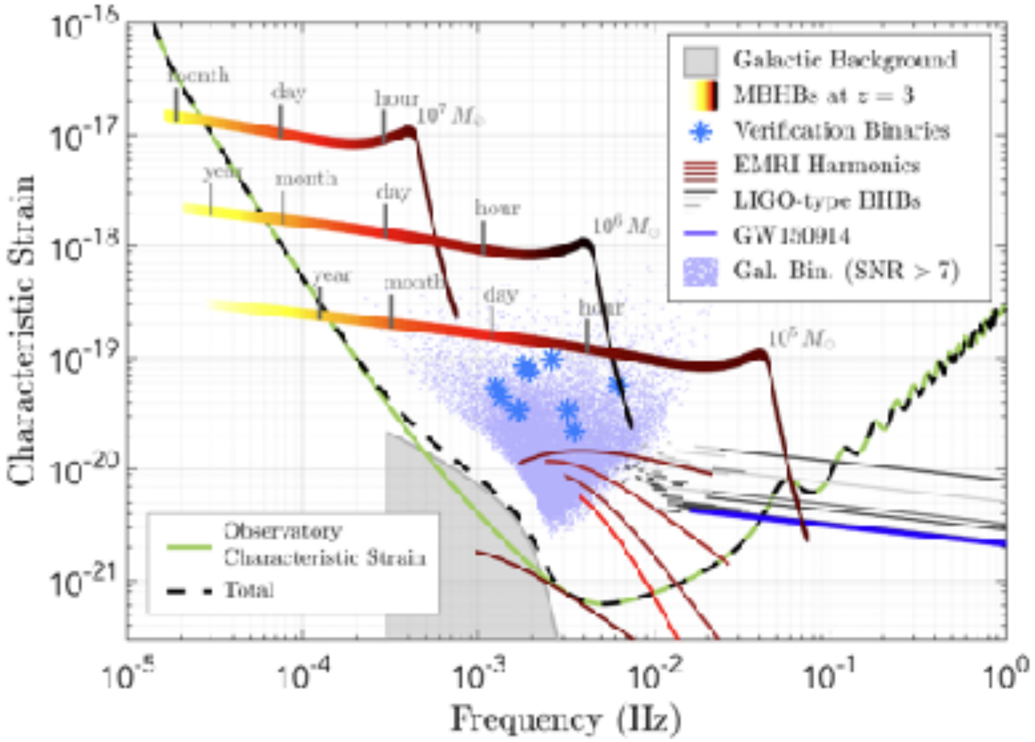
- *Primary with* $M \sim (10^4 - 10^8) M_\odot$

- *Secondary such that the mass ratio*

$$q = m_p/M \sim (10^{-6} - 10^{-3})$$

  - *Key point of theoretical description*





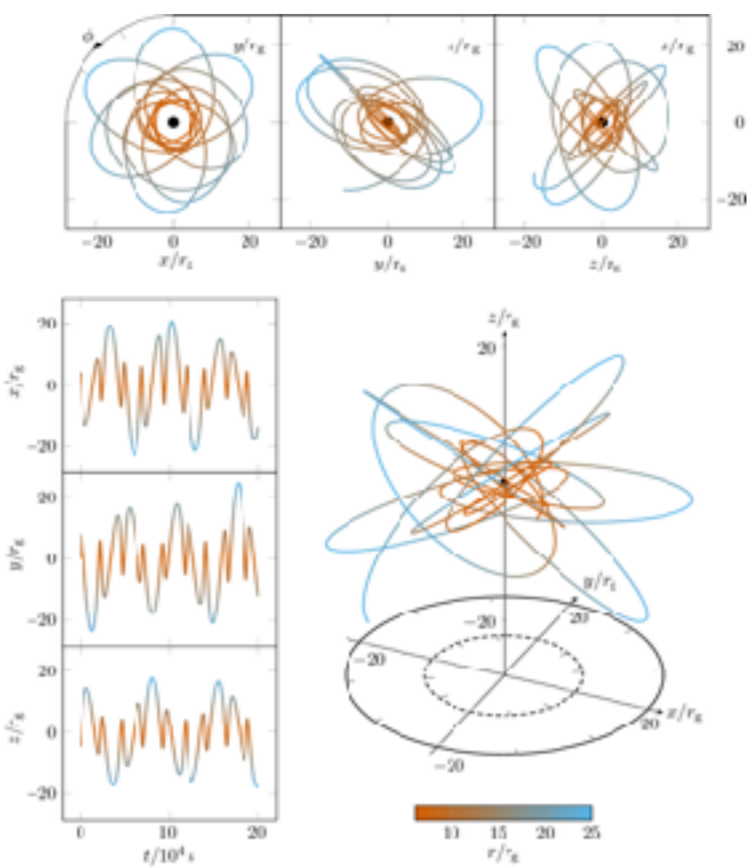
- *Emit GWs in the mHz band, golden targets for LISA*

Baker +, Astro2020 1907.06482 (2019)

EMRI in nuce

1 2

EMRIs provide a rich phenomenology, due to their orbital features



Berry +, Astro2020 1903.03686 (2019)

- *Non equatorial orbits*
- *Eccentric motion*
- *Resonances*
- *Complete* $\sim (10^4 - 10^5)$ *cycles before the plunge*

bless *and* ***disguise***

Tracking EMRIs for O(year) requires accurate templates

Very appealing to test fundamental & astro-physics

Precise space-time map and accurate binary parameters

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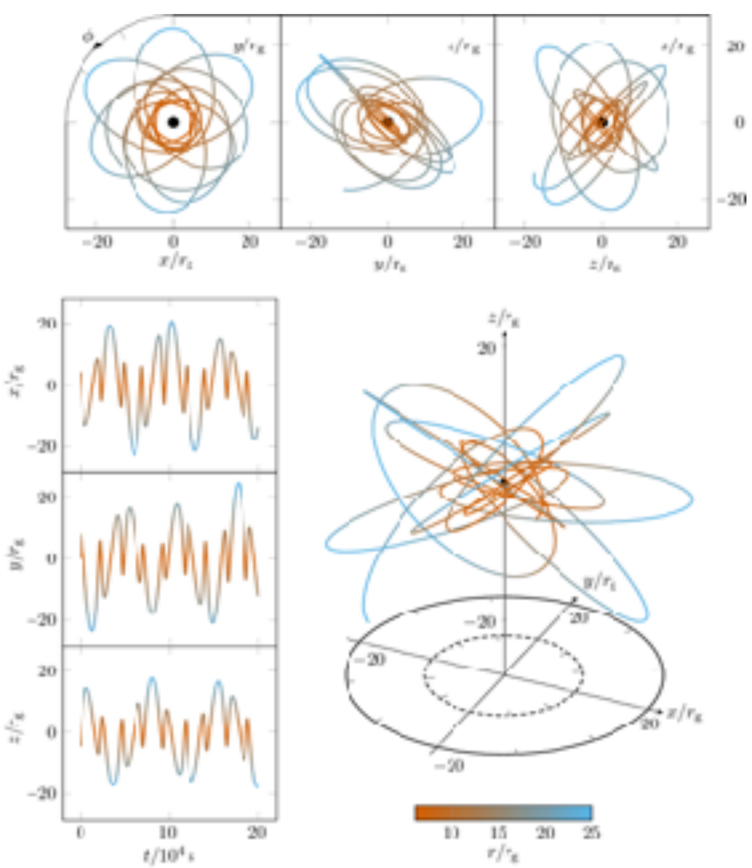

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bless and **disguise**

Tracking EMRIs for O(year) requires accurate templates

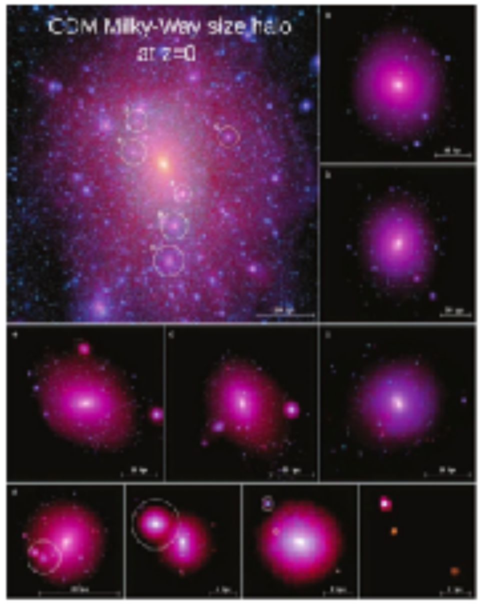
what are the features of the astrophysical environments in which binary systems evolve?

Dirty & asymmetric BBHs

3

GW sources evolve embedded in a variety of gas/matter contents/fields, which may leave detectable imprints on GW

- *Can we infer properties on the environment in which binaries evolve?*
- *Are vacuum waveform models safe?*



V. Springel et al., Mon. Not. Roy. Astron. 391 (2008)

G. Bertone et al., Nature 562, 7725 (2008)

MBH and inspirals evolve in DM-rich environment, within galaxies

IMIR/EMRI can assemble in accretion disks

→ *particle physics laboratories*

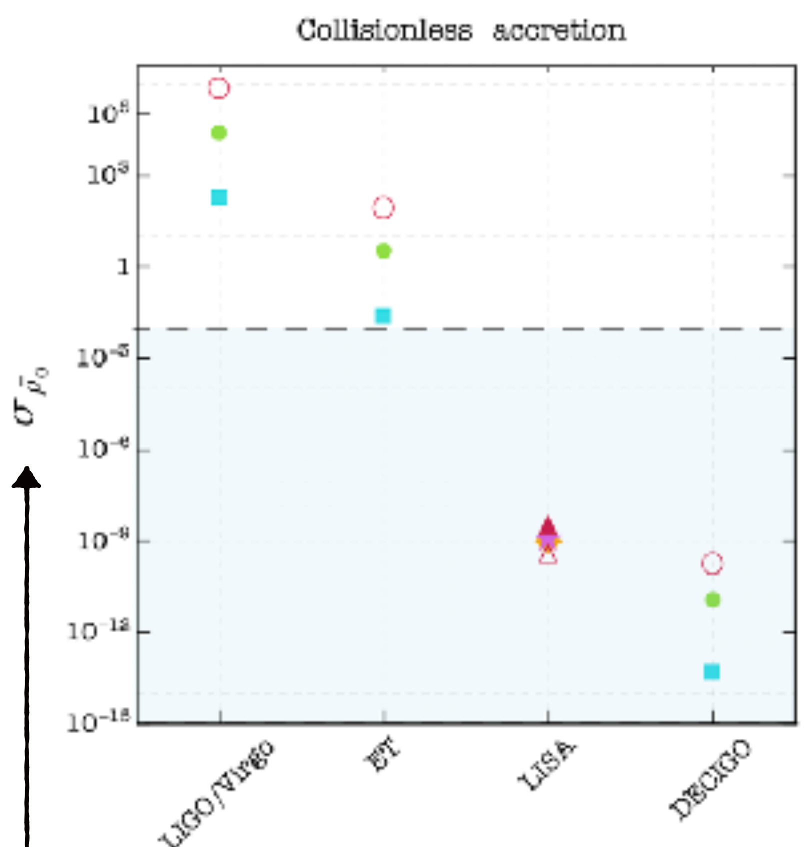
Why EMRIs?

Constraints on the environment's density from different effects/sources/detectors

V. Cardoso & A. M., A&A 644, A147 (2020)

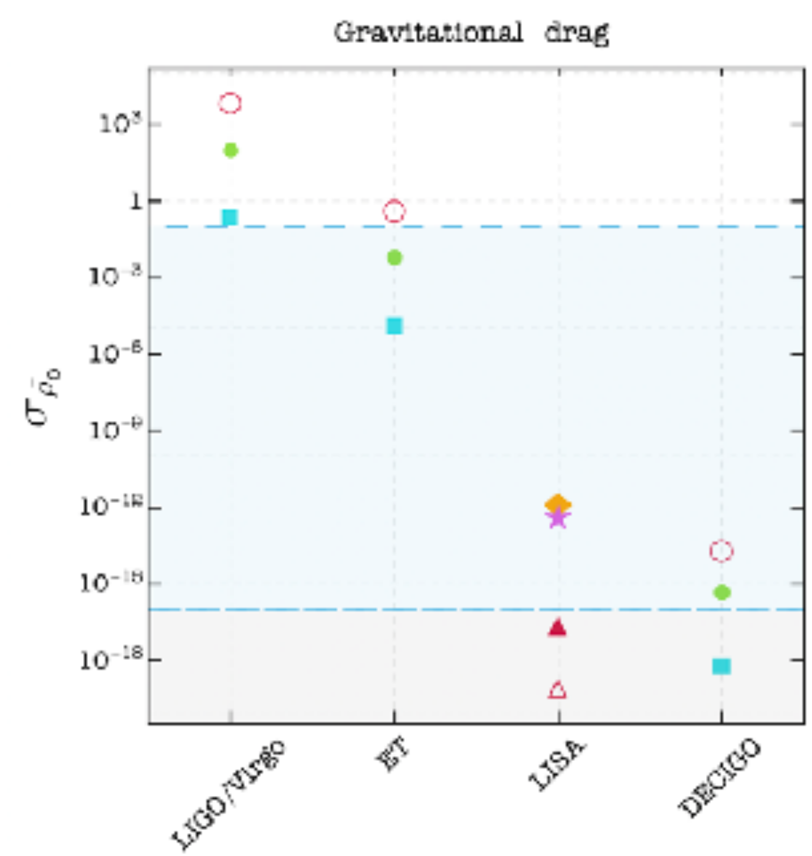
Useful (extrapolated) lessons from comparable mass binaries

- *environmental effects typically contribute at low frequencies*



smallest value different from vacuum

density of medium normalised to H_2O



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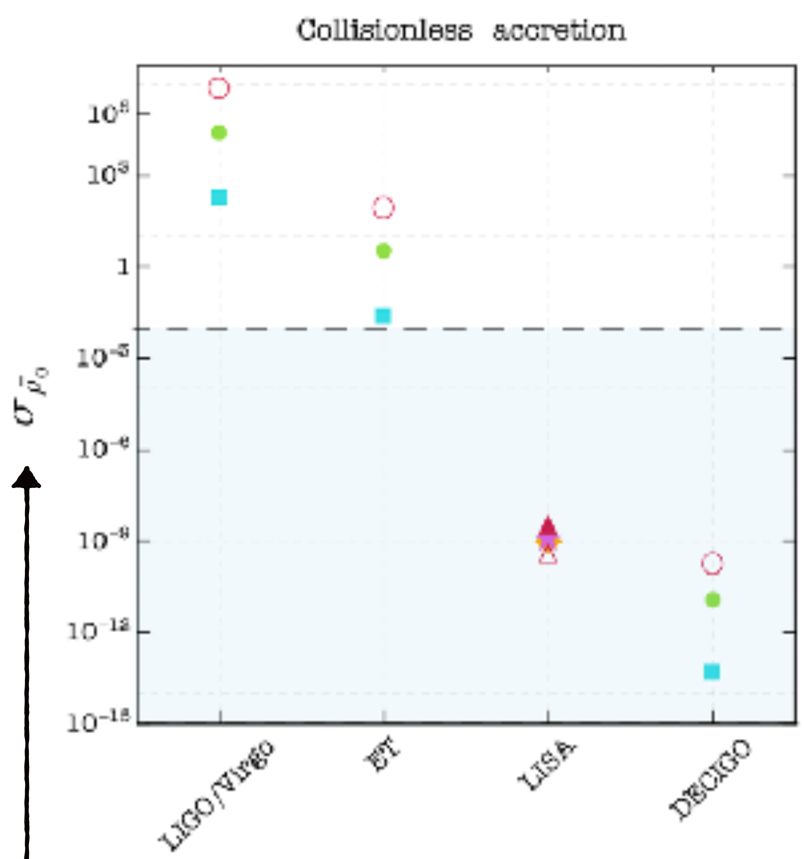

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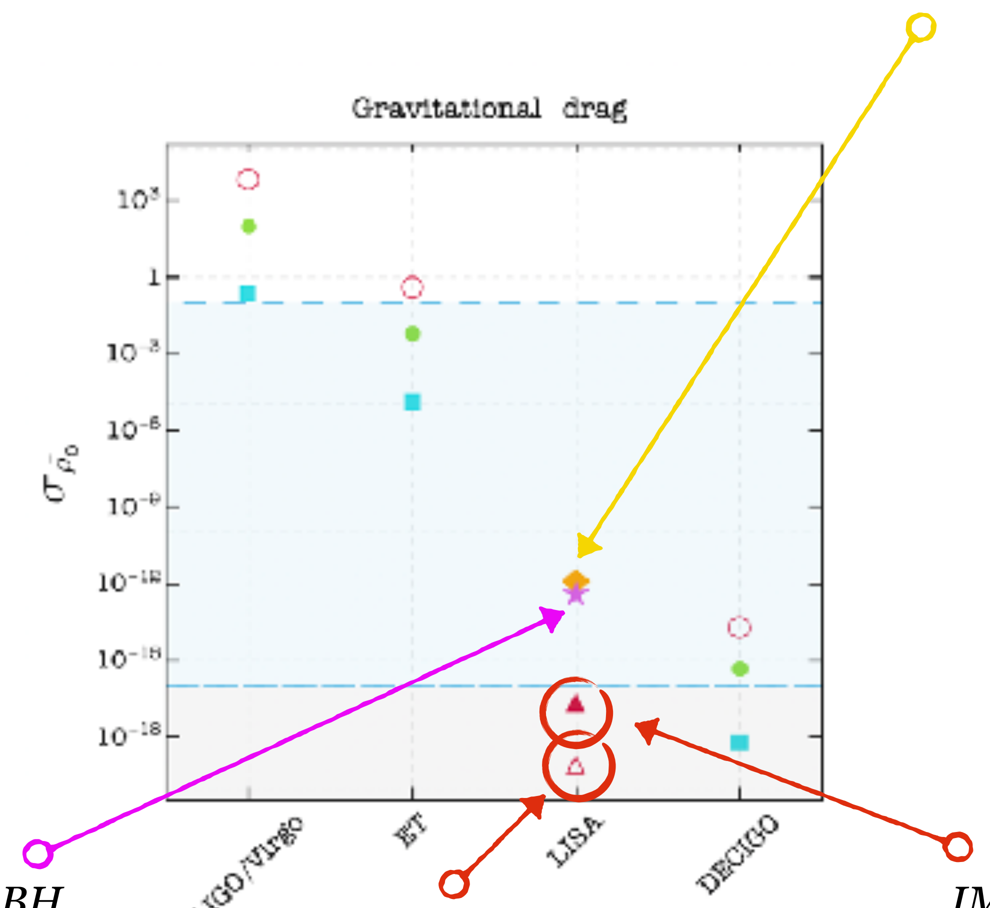


$(10^4\text{-}5\times10^3)M_\odot$ IBBH

MBBH $(10^6\text{-}5\times10^5)M_\odot$

EMRI $(10^5\text{-}10)M_\odot$

IMRI $(10^4\text{-}10)M_\odot$

Asymmetric binaries work (really) well

What goes into an EMRI?

Using EMRI observations for precision astrophysics is a complex task

- *The **S**elf-**F**orce program in (vacuum GR) is @ work for more than two decades for second order waveforms*

  Barack & Pound, Rep. Prog. Phys. 82 016904 (2018)

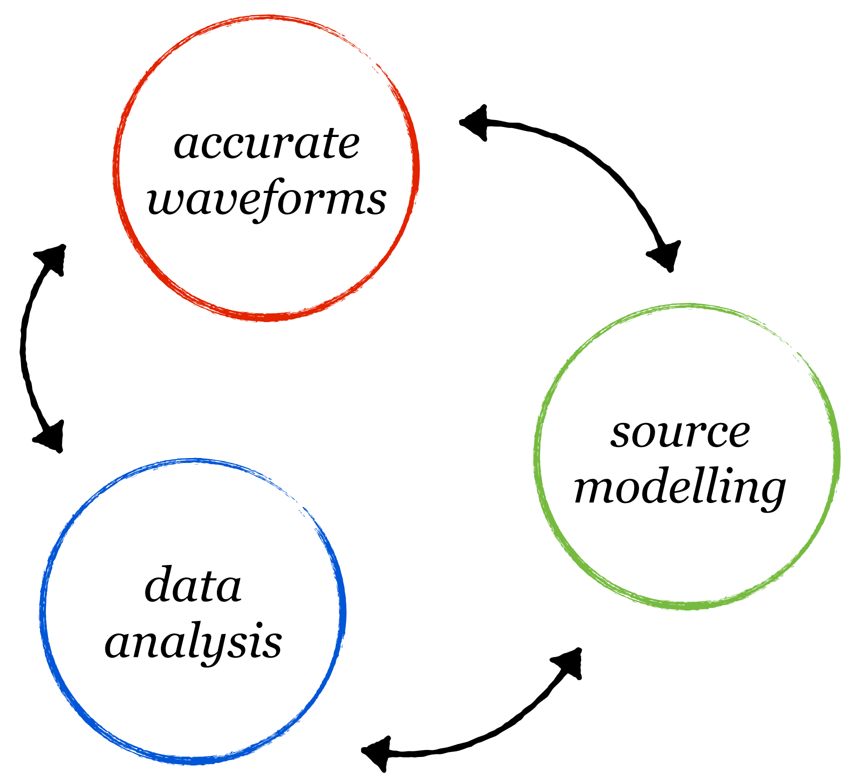
- *Complexity of calculations beyond standard vacuum BBH grows very (very fast)*
  - *Couplings with extra fields*

  A.M. et al. Nature Astronomy 6, 4 464-470 (2022)
  V. Cardoso et al., Phys. Rev. D 105, L061501 (2022)

- *Even the background solutions are poorly characterised (spin?)*

- *Generation of (fast) waveform models*

  Katz +, PRD 104 064047 (2021)



EMRI in vacuum

How do we study EMRI in vacuum (GR)?

- The asymmetric character introduces a natural parameter to study the problem in perturbation theory $q = m_p/M \ll 1$

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta}$$

$$G_{\mu\nu} = T^{\mathrm{p}}_{\mu\nu} = 8\pi m_{\mathrm{p}} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy^{\alpha}_p}{d\lambda} \frac{dy^{\beta}_p}{d\lambda} d\lambda$$

leading → adiabatic

Regge-Wheleer-Zerilli (Schwarzschild background)

Teukolsky (Kerr background)

- The solution determines the phase evolution

adiabatic — first post-adiabatic

$$\phi(t) = \phi_{\mathrm{diss}-1} + \dots$$

$\phi_{\mathrm{diss}-1}$ → $\mathcal{O}(1/q)$

$\dots$ → $\mathcal{O}(1)$

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adiabatic: $\mathcal{O}(1/q)$

first post-adiabatic: $\mathcal{O}(1)$

error needed $\ll 1$ *radian*

The perturbation scheme

For the gravitational sector

$$h_{\alpha\beta} = h^{\rm pol}_{\alpha\beta} + h^{\rm ax}_{\alpha\beta}$$

$(-1)^\ell$ ← $h^{\rm pol}_{\alpha\beta}$, $h^{\rm ax}_{\alpha\beta}$ → $(-1)^{\ell+1}$

$$\mathbf{h} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \mathcal{A}^{(0)}_{\ell m}\mathbf{a}^{(0)}_{\ell m} + \mathcal{A}^{(1)}_{\ell m}\ \mathbf{a}^{(1)}_{\ell m} + \mathcal{A}_{\ell m}\mathbf{a}_{\ell m} + \mathcal{B}^{(0)}_{\ell m}\ \mathbf{b}^{(0)}_{\ell m} + \mathcal{B}_{\ell m}\ \mathbf{b}_{\ell m} + \mathcal{Q}^{(0)}_{\ell m}\ \mathbf{c}^{(0)}_{\ell m} + \mathcal{Q}_{\ell m}\ \mathbf{c}_{\ell m}$$
$$+ \mathcal{D}_{\ell m}\ \mathbf{d}_{\ell m} + \mathcal{G}_{\ell m}\ \mathbf{g}_{\ell m} + \mathcal{F}_{\ell m}\ \mathbf{f}_{\ell m}.$$

$$\mathbf{b}_{\ell m} = \frac{n_\ell r}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y^{\ell m}_{,\theta} & Y^{\ell m}_{,\phi} \\ 0 & Y^{\ell m}_{,\theta} & 0 & 0 \\ 0 & Y^{\ell m}_{,\phi} & 0 & 0 \end{pmatrix}$$

- 7 **polar** components + 3 **axial** harmonics
  - For a spherically symmetric background the 2 families decouple
- In vacuum GR using the Regge-Wheeler-Zerilli gauge the components reduce to **1** axial and **1** polar functions

Regge & Wheeler, PRD 108, 1063 (1957)
Zerilli, PRD 2, 2141 (1970)

The wave equations

2 master equations for 2 perturbations

$$e^{-\lambda} = 1 - 2M/r$$
$$\Lambda = \ell(\ell+1)/2 - 1$$

$$\frac{d^2 R_{\ell m}}{dr_\star^2} + \left[\omega^2 - e^{-\lambda}\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)\right] R_{\ell m} = J_{\rm ax}$$

→ *info on the orbital setup* — ***R**egge-**W**heeler*

$$\frac{d^2 Z_{\ell m}}{dr_\star^2} + \left[\omega^2 - \frac{18M^3 + 18M^2 r\Lambda + 6Mr^2\Lambda^2 + 2r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)}\right] Z_{\ell m} = J_{\rm pol}$$

***Z**erilli*

- *Perturbations are need to compute the GW fluxes…*

$$\dot{E}^{\pm}_{\rm grav} = \frac{1}{64\pi}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell}\frac{(\ell+2)!}{(\ell-2)!}\left(\omega^2|Z^{\pm}_{\ell m}|^2 + 4|R^{\pm}_{\ell m}|^2\right)$$

- *… which drive the orbital evolution*

orbital radius ←

$$\frac{dr(t)}{dt} = -\dot{E}\frac{dr}{dE_{\rm orb}} \quad , \quad \frac{d\Phi(t)}{dt} = \frac{M^{\frac{1}{2}}}{r_p^{3/2}}$$

→ *orbital phase*

The wave equations

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info on the orbital setup

Regge-Wheeler

$$\frac{d^2 Z_{\ell m}}{dr_\star^2} + \left[\omega^2 - \frac{18M^3 + 18M^2 r\Lambda + 6Mr^2\Lambda^2 + 2r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)}\right] Z_{\ell m} = J_{\mathrm{pol}}$$

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Regge-**W**heeler

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Zerilli

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orbital phase

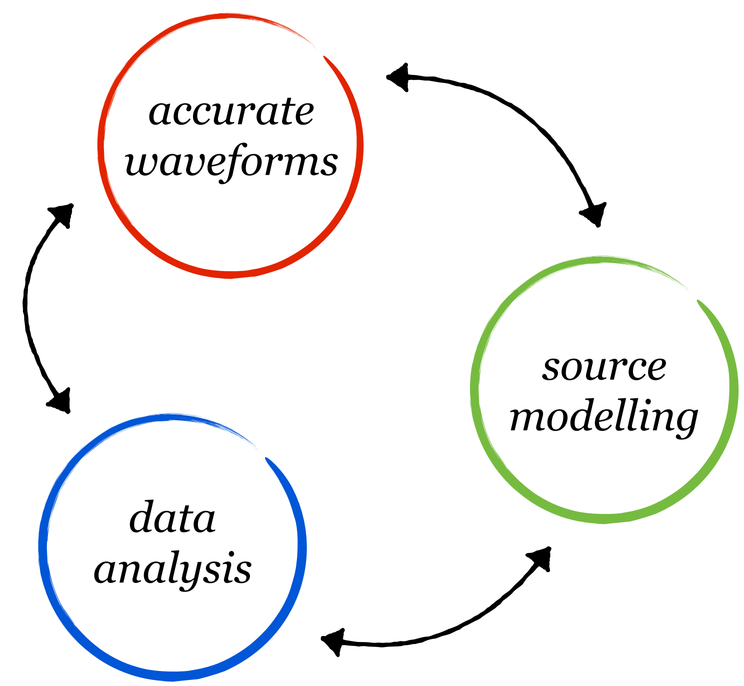
How do we move on?

The landscape of calculations for asymmetric binaries is relatively virgin

- *Analyses within comparable-mass, adding Newtonian terms to the orbital energy or the GW fluxes of the vacuum BBH baseline*

- *Inclusion of relativistic corrections to leading Newtonian terms show relevance of a fully relativistic descriptions*

  → *larger waveform dephasings*



- *lack of fully relativistic solutions able to describe BHs within a medium*

  → *need of background geometry to do our perturbative job*

  → *allows to study couplings between grav/fluid perturbations*

  → *impact on on QNMs, GW fluxes, particle motion*

L. Sberna +, PRD 106, 064056 (2022)
L. Speri + 2207.10086
A. Toubiana +, PRL 126, 101105 (2021)
A. Coogan + PRD 105, 043009 (2022)
N. Speeney+ PRD 106, 044027 (2022)
...

The Background

The Einstein cluster prescription

V. Cardoso +, PRD Lett. 105, L061501, (2022)
V. Cardoso +, PRL 129, 241103, (2022)

- Reduces matter distribution to average anisotropic stress energy tensor

$$\langle T^{\mu\nu}\rangle = \frac{n}{m_p}\langle P^\mu P^\nu\rangle \longleftrightarrow T^\mu{}_\nu = \mathrm{diag}(-\rho, 0, p_t, p_t)$$

A. Einstein, Annals Math. 40 (1939)

- Spherical symmetry

$$ds^2 = -a(r)\,dt^2 + \frac{dr^2}{1-2m(r)/r} + r^2 d\Omega^2$$

- Choose mass profile: we focused on DM rich environments

$$m(r) = M_{\rm BH} + \frac{Mr^2}{(a_0+r)^2}\left(1-\frac{2M_{\rm BH}}{r}\right)^2$$

← Hernquist profile

L. Hernquist, The Astrroph. Journal 356 (1990)

- Generalized to any density profiles with radial dependence

E. Figueiredo, A.M. V. Cardoso, 2303.08183
Jusufi, arXiv:2202.00010
Igata et al, arXiv:2202.00202
Konoplyaet al., The Astrroph. Journal 933 (2020)

The Background

- *Solve the Einstein's fields equations sourced by the halo stress-energy tensor*

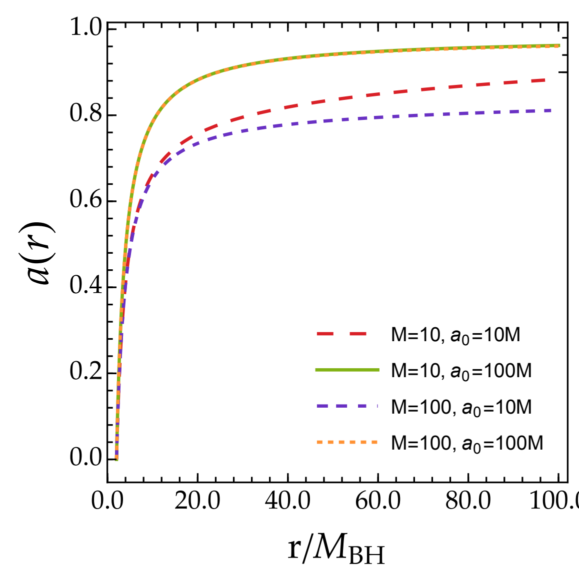
$$\rho(r) = \frac{2M(a_0 + 2M_{\rm BH})(1 - 2M_{\rm BH}/r)}{4\pi r(r + a_0)^3}$$

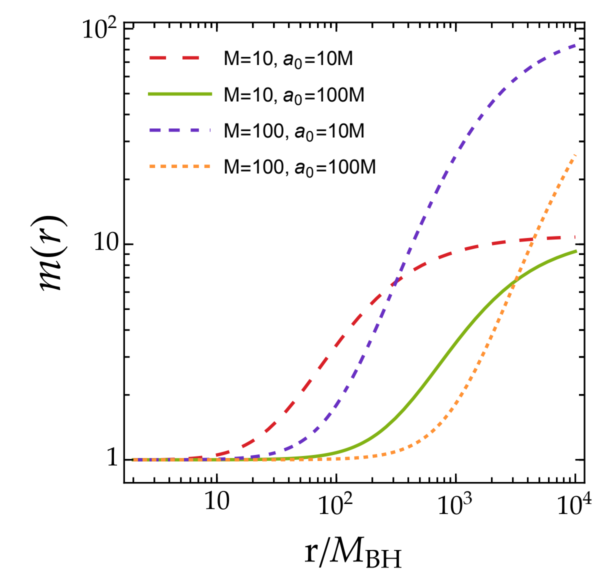
$$a(r) = \left(1 - \frac{2M_{\rm BH}}{r}\right) e^{\Gamma}$$

$$\Gamma = -\pi\sqrt{\frac{M}{\xi}} + 2\sqrt{\frac{M}{\xi}}\arctan\frac{r + a_0 - M}{\sqrt{M\xi}}$$

$$\xi = 2a_0 - M + 4M_{\rm BH}$$

- *To mimic galaxy observations* $a_0 \gtrsim 10^4 M$ → $M_{\rm BH} \ll M \ll a_0$
- *Asymptotically flat*
- *Horizon at* $r = 2M_{\rm BH}$





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$$e^{\Gamma} \xrightarrow[r \to 2M_{\rm BH}]{} 1 - 2M/a_0$$ ← *Redshift factor*

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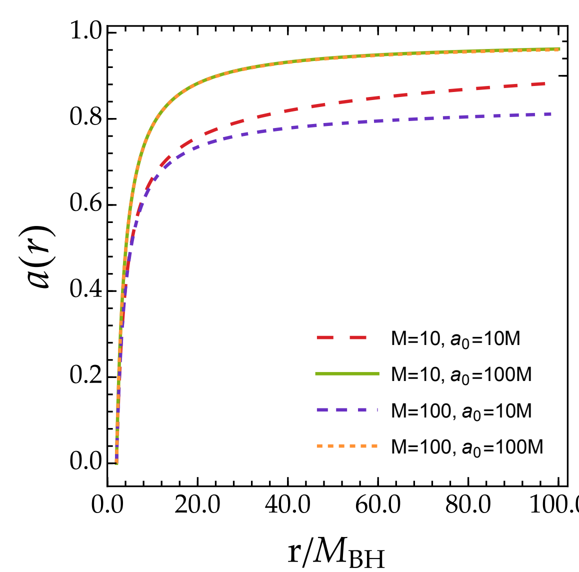

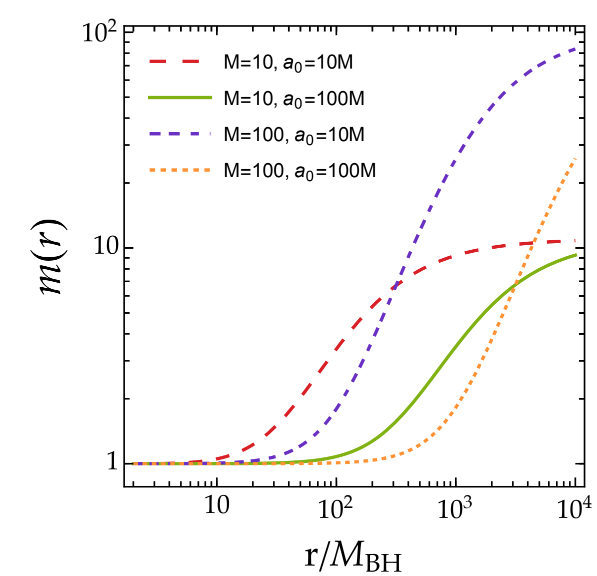

Geodesics

Generic halo configuration affects the orbital properties of massive and massless particles

$$r_{\rm LR} \simeq 3M_{\rm BH}\left(1+\frac{MM_{\rm BH}}{a_0^2}\right) \longrightarrow M_{\rm BH}\Omega_{\rm LR} \simeq \frac{1}{3\sqrt{3}}\left(1-\frac{M}{a_0}+\frac{M(M+18M_{\rm BH})}{6a_0^2}\right)$$

$$r_{\rm ISCO} \simeq 6M_{\rm BH}\left(1-\frac{32MM_{\rm BH}}{a_0^2}\right) \longrightarrow M_{\rm BH}\Omega_{\rm ISCO} \simeq \frac{1}{6\sqrt{6}}\left(1-\frac{M}{a_0}+\frac{M(M+396M_{\rm BH})}{6a_0^2}\right)$$

- *At the leading order the halo only redshifts the dynamics*
- *BH shadows gets correction of the order* $M^2/a_0^2 \lesssim 10^{-8}$ *(light-ring tests are safe)*

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Redshift factor

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Redshift factor

non-linear corrections

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Dirty perturbations

EMRI (and more) evolving within environments

- Consider linear perturbations of a BH+halo background induced by the small secondary $\mathcal{G}_{\mu\nu} = 8\pi(T_{\mu\nu} + T^{p}_{\mu\nu})$

grav-sector

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h^{\rm ax}_{\alpha\beta} + h^{\rm pol}_{\alpha\beta}$$

fluid-sector

$$u_\mu = u^0_\mu + u^1_\mu \qquad \rho = \rho_0 + \rho_1$$

$$p_t = p^0_t + p^1_t \qquad p_r = p^1_r$$

- Decompose $h_{\alpha\beta}$ and (u_μ, p, ρ) in tensor, vector, scalar spherical harmonics
  - e.g. for the pressure field

$$p^1_t(t,r,\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \delta p_{t,\ell m}(t,r) Y_{\ell m}(\theta,\phi)$$

- Go to the Fourier space, replace into the field's equation and solve ODEs
  - For BH-halo configurations this 'simple' reduction works only partially
  - We have extra equations for matter quantities

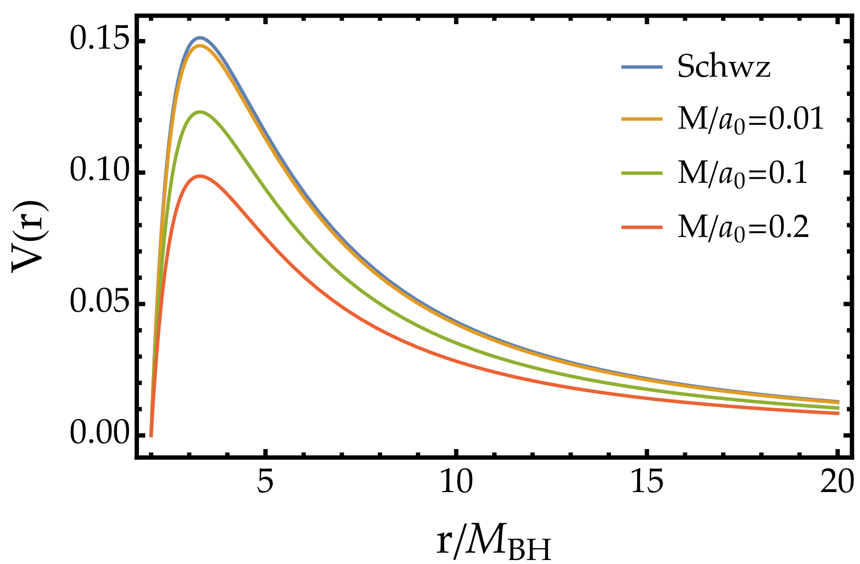
BH & halo: axial modes

How does the halo change the axial perturbations of the BH?

- *Same functional form but…*

$$\frac{d^2 R_{\ell m}}{dr_\star^2} + [\omega^2 - V^{\rm ax}]R_{\ell m} = J_{\rm ax} \qquad V^{\rm ax} = \frac{a(r)}{r^2}\left[\ell(\ell+1) - \frac{6m(r)}{r} + m'(r)\right]$$

- *Homogenous and in-homogenous problems provide the set up to study QNM and EMRI dynamics*



Change in the scattering potential due to the halo compactness

- *The halo affects the structure of the potential, as well the boundary conditions of the wave propagation at the horizon and at infinity*
- *axial modes are not couple to fluid perturbations*

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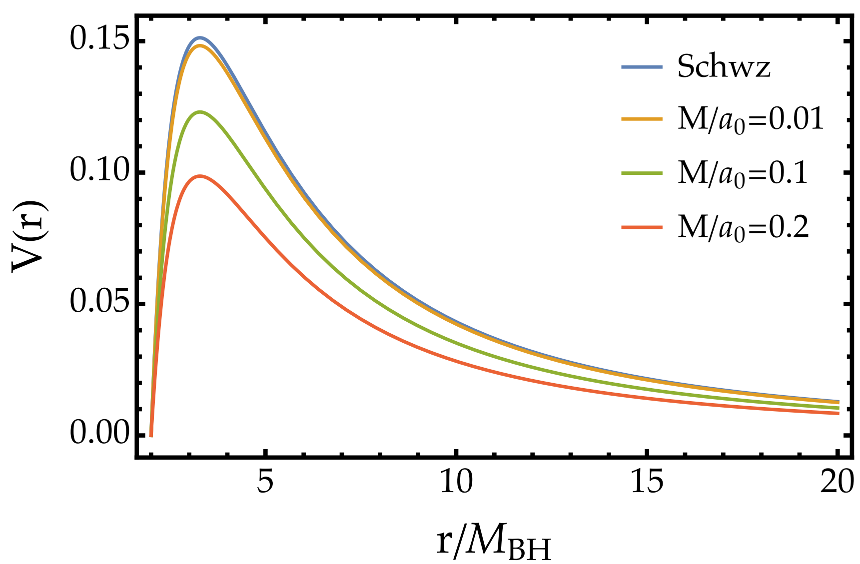
$$\frac{d^2 R_{\ell m}}{dr_\star^2} + [\omega^2 - V^{\rm ax}]R_{\ell m} = J_{\rm ax} \qquad V^{\rm ax} = \frac{a(r)}{r^2}\left[\ell(\ell+1) - \frac{6m(r)}{r} + m'(r)\right]$$

- *Homogenous and in-homogenous problems provide the set up to study QNM and EMRI dynamics*



Change in the scattering potential due to the halo compactness

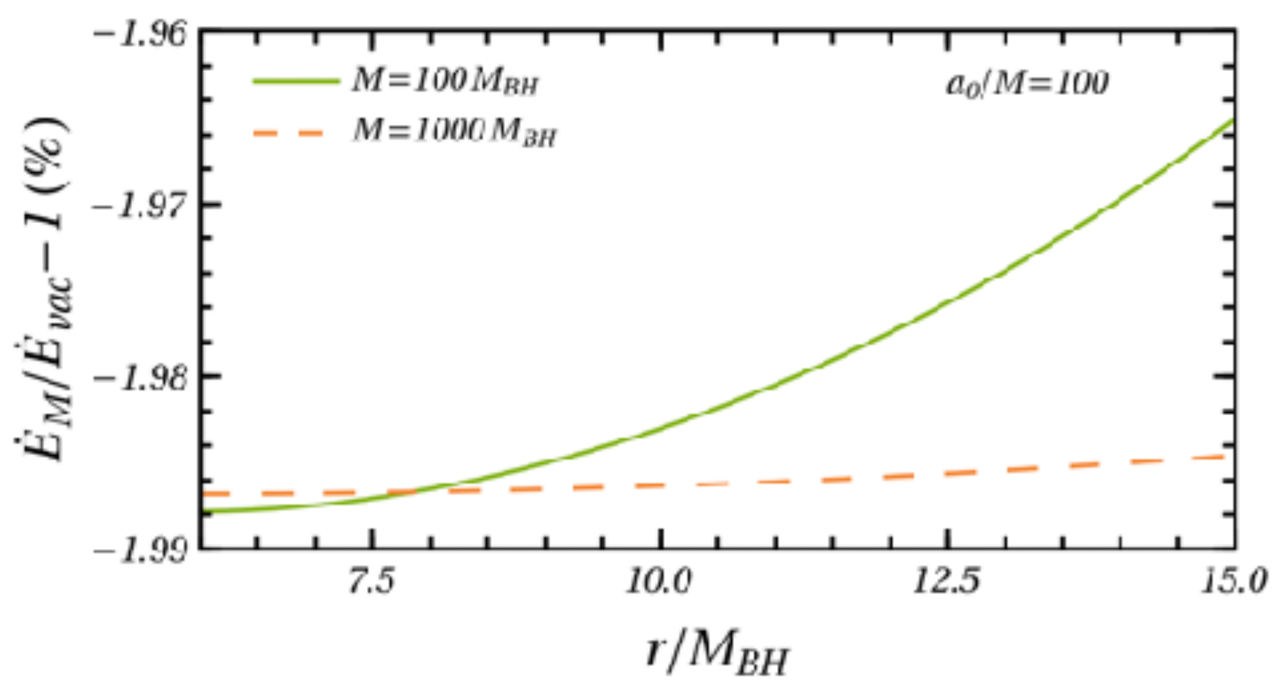
- *The halo affects the structure of the potential, as well the boundary conditions of the wave propagation at the horizon and at infinity*
- *axial modes are not couple to fluid perturbations*

BH & halo EMRI: axial modes

The halo properties affect the GW emission and hence the EMRI inspiral evolution (already) at adiabatic level

V.Cardoso +, PRD Lett. 105, L061501, (2022)

Relative change in the axial flux v.s. vacuum



- *The difference with vacuum grows with M*
- *Suppression as M/a_o decreases*
- *Difference << 1% as $M/a_0 \sim 10^{-3}$*

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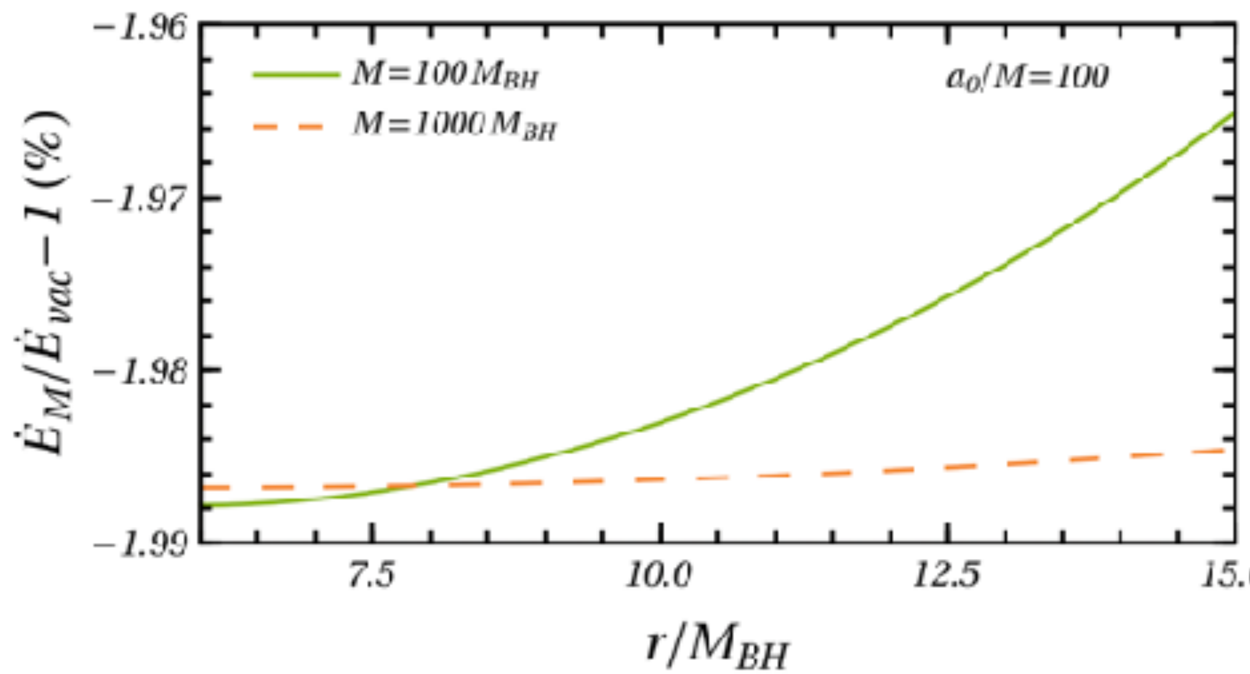

1 year observation *before the plunge*

~ 500 radians

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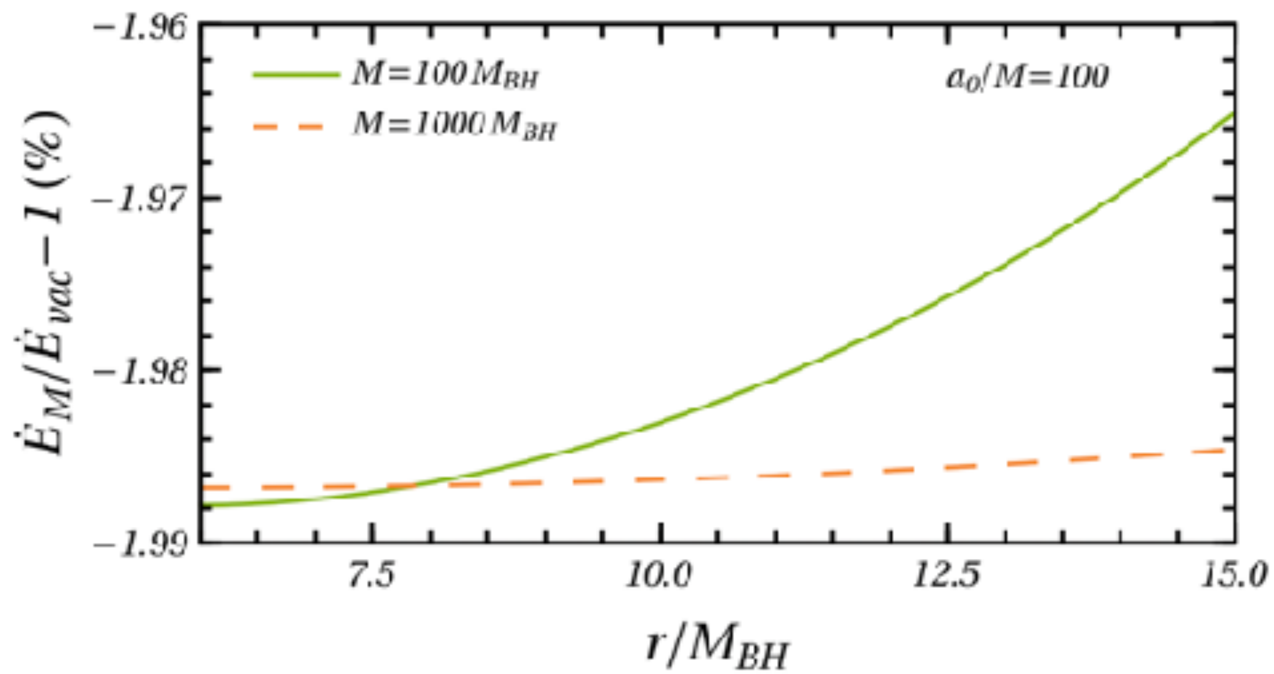

1 year observation before the plunge

~ 500 radians

look promising but...

- The difference with vacuum grows with M
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The redshift strikes back

Series expansion for low compactness $M/a_0 \ll 1$

$$V^{\rm ax} \approx \left(1 - \frac{2M}{a_0}\right) V^{\rm ax}_{\rm Schw} \qquad J^{\rm ax}_{\ell m} \approx \mu \left(1 - \frac{3M}{a_0}\right) J^{\rm ax,Schw}_{\ell m} \qquad \frac{dr}{dr_\star} \approx \left(1 - \frac{M}{a_0}\right) \frac{dr}{dr_{\star,\rm Schw}}$$

$$\frac{d^2 R_{\ell m}}{dr^2_{\star,\rm Schw}} + \left[\left[\omega\left(1 + \frac{M}{a_0}\right)\right]^2 - V^{\rm ax}_{\rm Schw}\right] R_{\ell m} = \mu \left(1 - \frac{M}{a_0}\right) J_{\rm ax,Schw}$$

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$$\Omega_p \to \tilde{\Omega}_p = \Omega_p\left(1 - \frac{M}{a_0}\right)$$

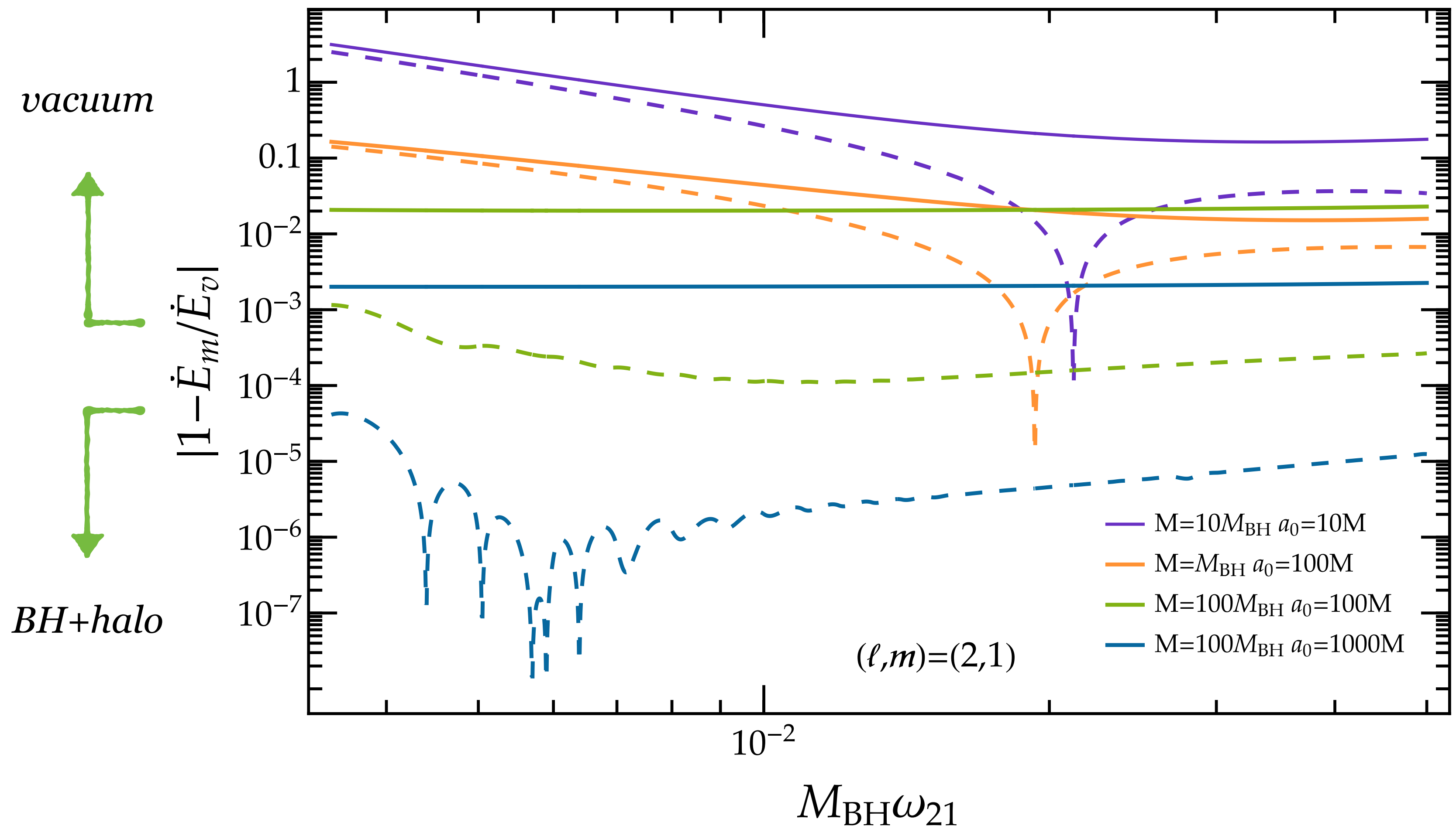
$$\mu \to \tilde{\mu} = \mu\left(1 + \frac{M}{a_0}\right)$$

Equivalent to a vacuum solution
with rescaled parameters
(redshift of the BH mass scale)

The redshift strikes back

(2,1) axial flux emitted by an EMRI on circular motion

- *fluxes tend to be smaller in the presence of the halo*



- *Redshifted quantities drastically reduce the discrepancy for realistic halos*
- *Unless new effects pop up in the polar sector, the halo seems undetectable*

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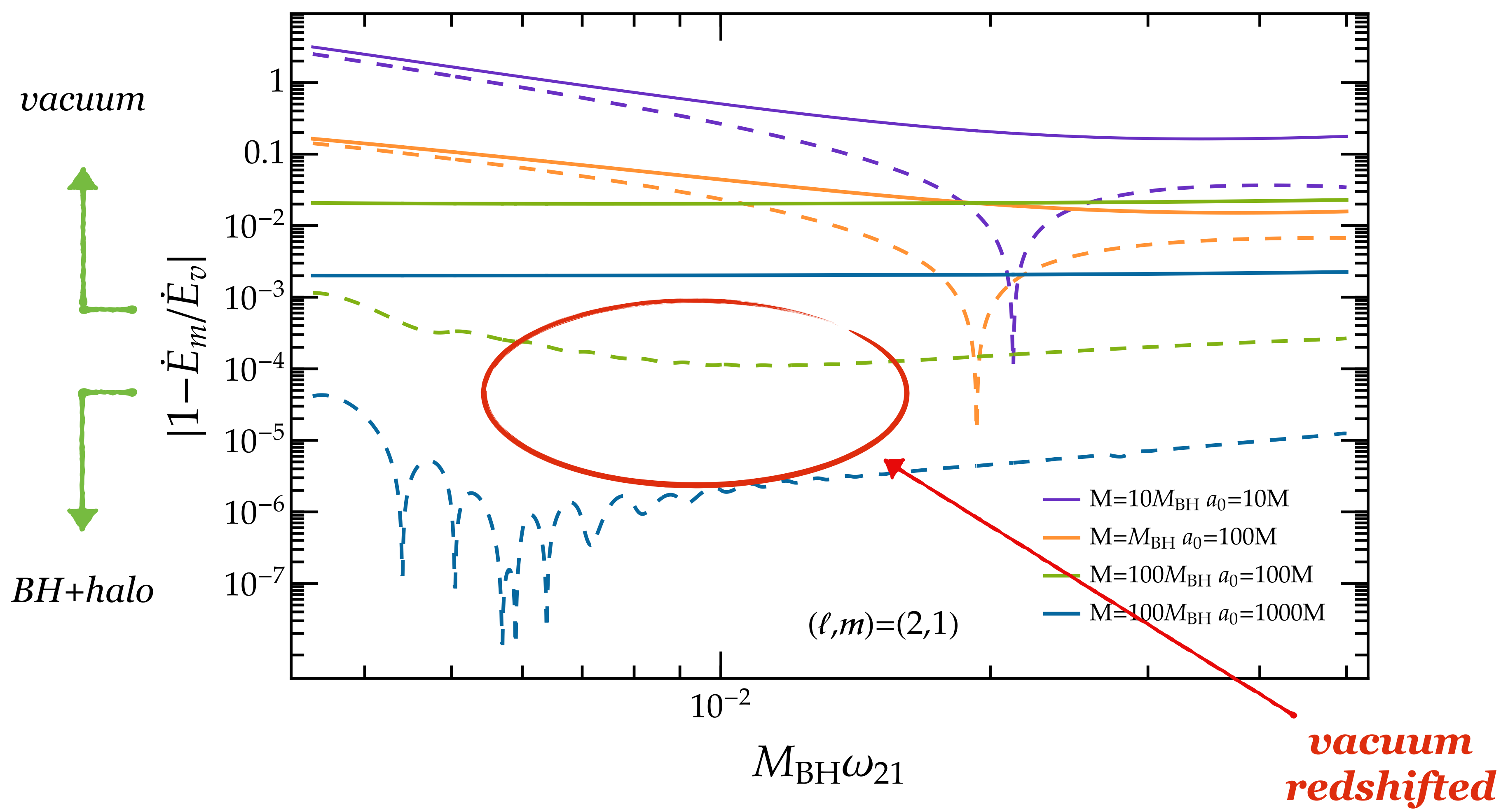

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More general than it looks

E. Figueiredo, A. M., V. Cardoso, 2303.08183

Approach extended to generic density profiles

- Developed a fully numerical approach to treat any $\rho(r)$
- applied to new DM models

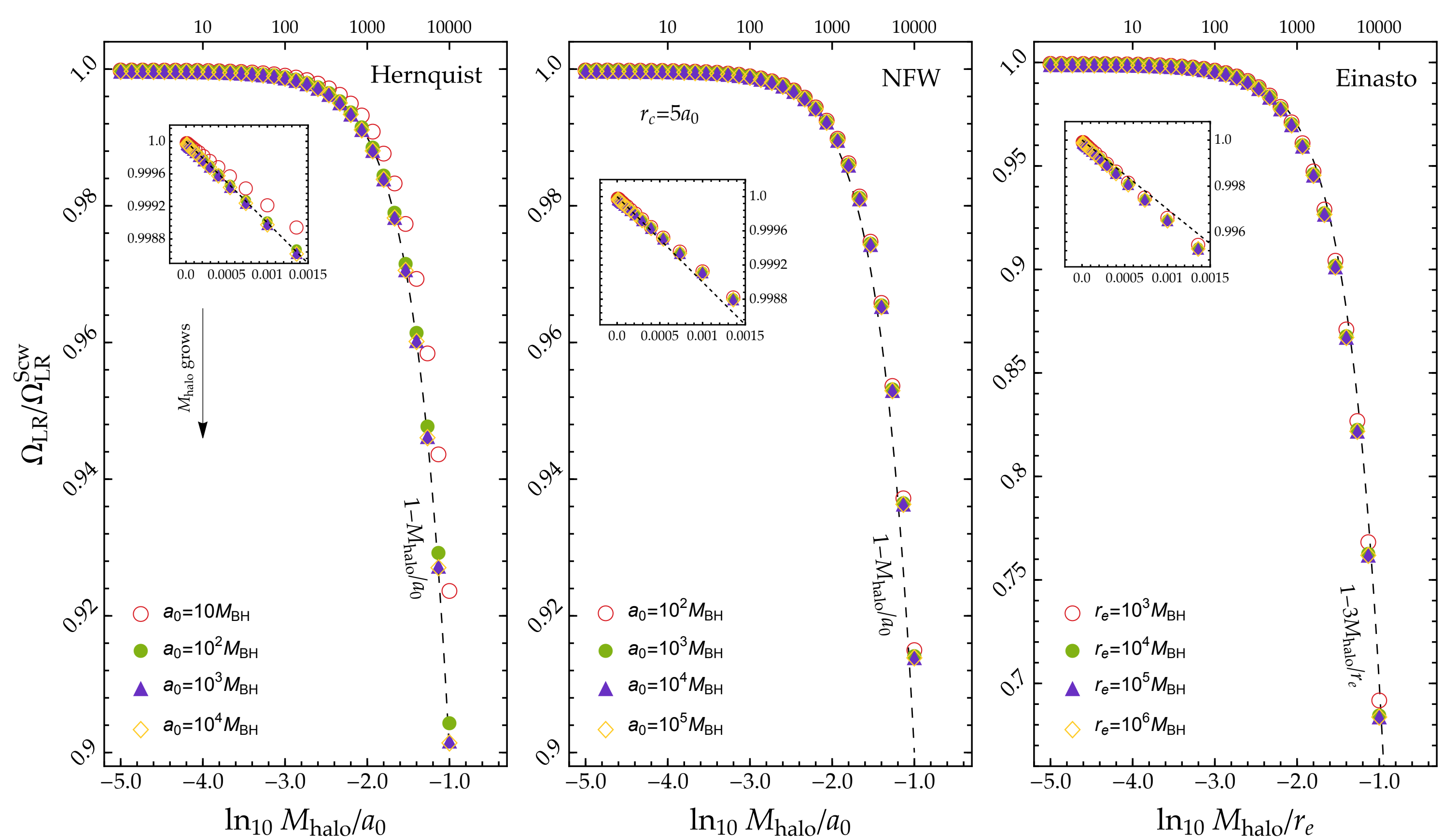
$$\rho(r) = \rho_0 (r/a_0)^{-\gamma}[1 + (r/a_0)^{\alpha}]^{(\gamma-\beta)/\alpha}$$

Hernquist & Navarro-Frenk-White

$$\rho(r) = \rho_e \exp\left\{-d_n[(r/r_e)^{1/n} - 1]\right\}$$

Einasto

geodesic frequency @ Light Ring



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Hernquist & Navarro-Frenk-White *Einasto*

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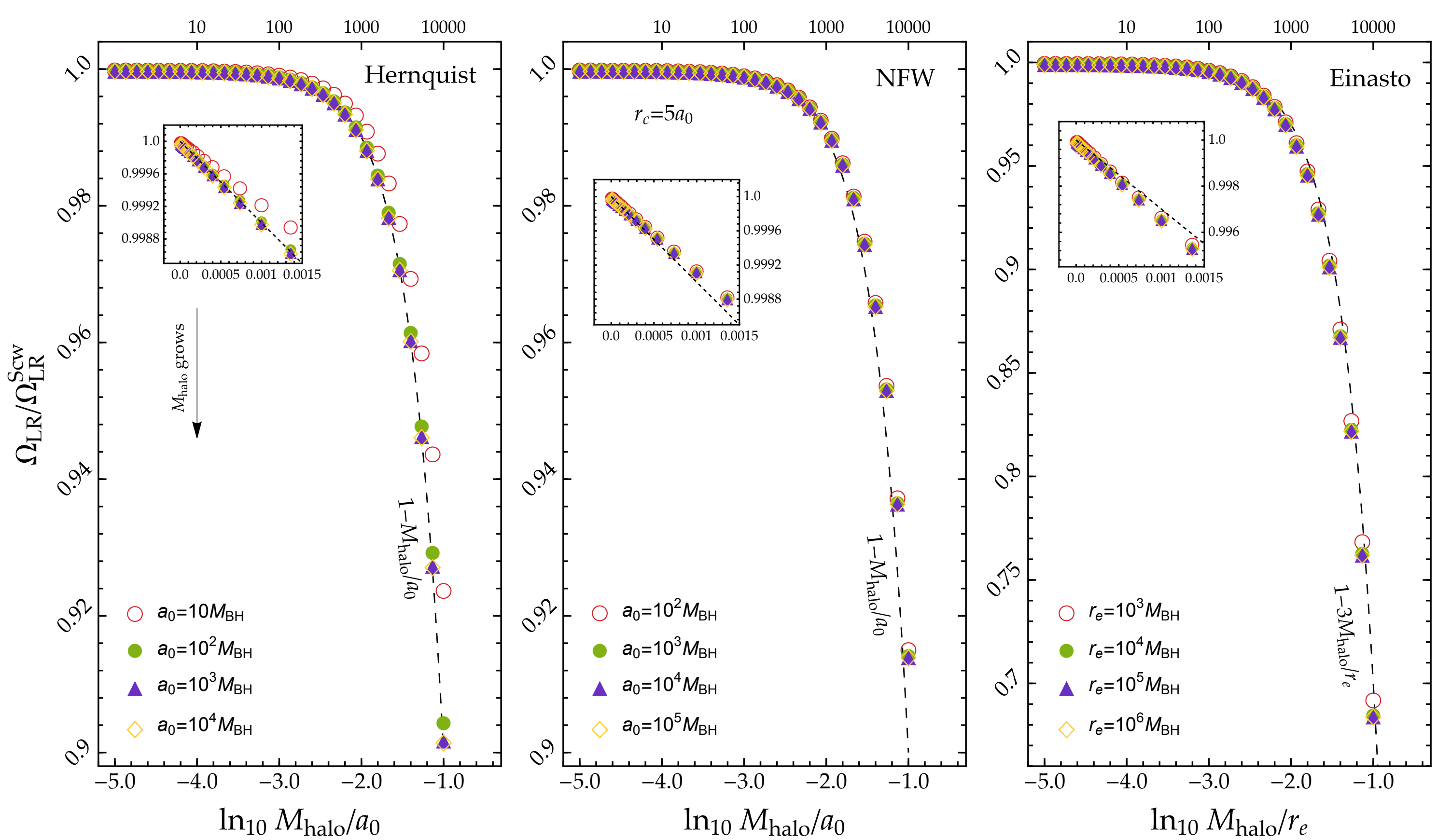
universal behaviour for $M/a_0 \lesssim 10^{-3}$



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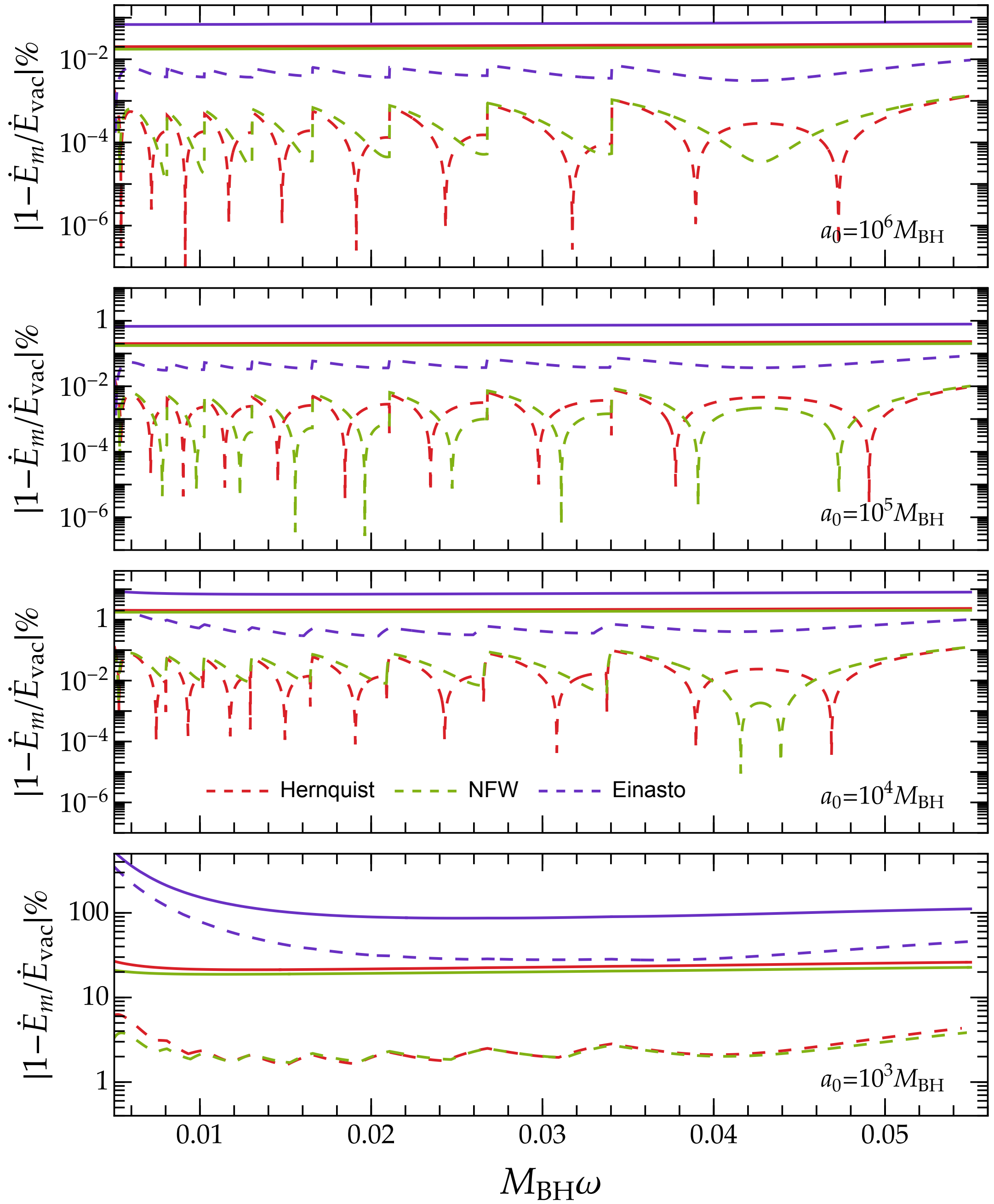
More general than it looks

Axial fluxes from EMRIs on circular orbits

E. Figueiredo, A.M., V. Cardoso, 2303.08183



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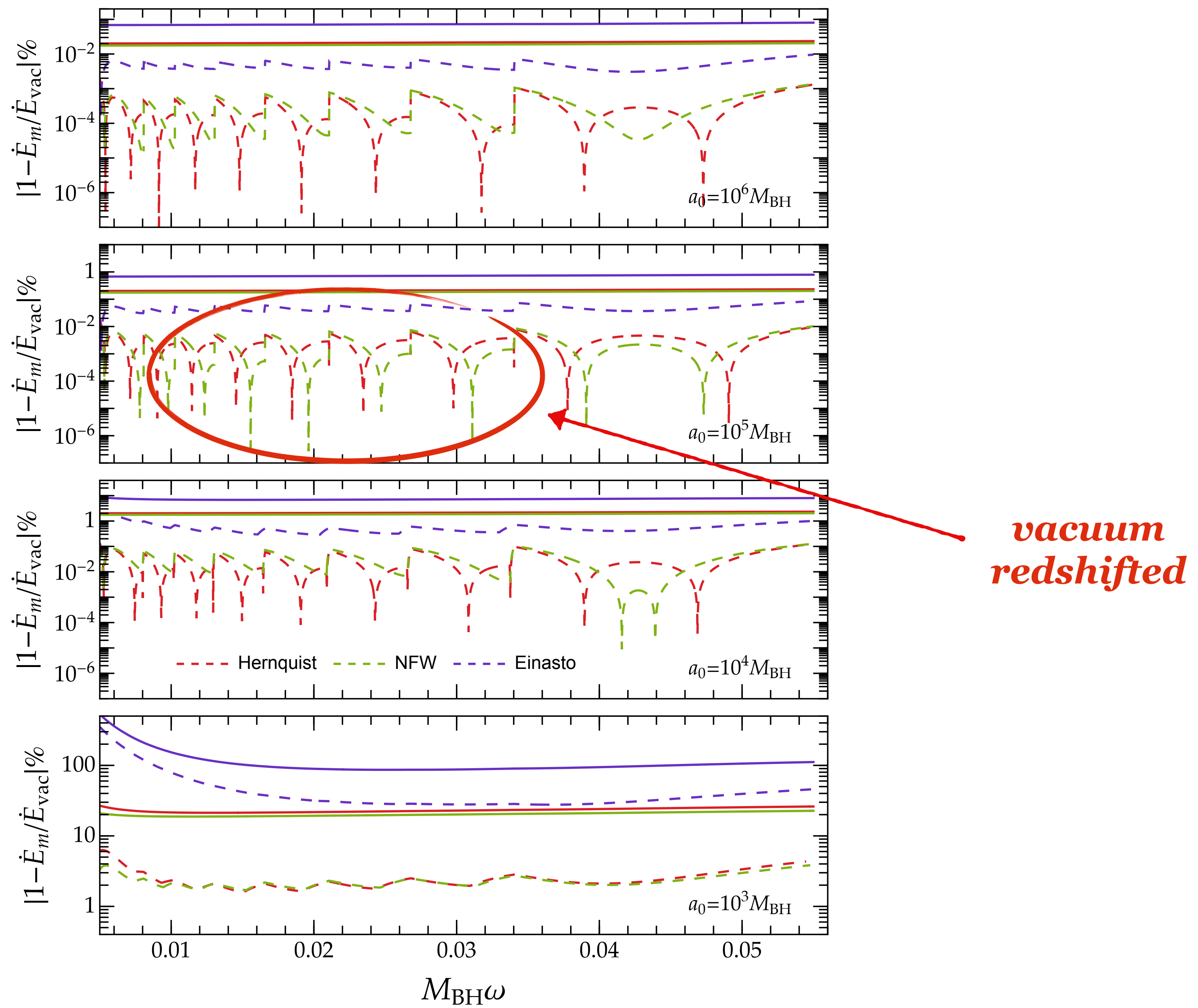

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BH & halo EMRI: polar modes

Polar sector is more challenging due (more variables &) couplings between matter and metric components

V. Cardoso.+, PRL 129, 241103, (2022)

- *System of 5 coupled differential equations for* $\vec{V} = (H_1, H_0, K, W, \delta\rho)$

$$\frac{d\vec{V}}{dr} = \mathbf{A}\vec{V} = \vec{S}$$

- *The radial/tangential speeds of sound enter the modes*

$$\delta p_{r,\ell m} = c_{s_r}^2 \delta\rho_{\ell m}$$
$$\delta p_{t,\ell m} = c_{s_t}^2 \delta\rho_{\ell m}$$

$M \to 0$

ℓ	m	$\dot{E}_\infty$ (Finite Diff)	$\dot{E}_\infty$ ($\mathrm{FD_{inh}}$)
2	2	1.7068011e-4	1.706(3)e-4
3	3	2.5489538e-5	2.547(3)e-5
4	4	4.7351723e-6	4.726(0)e-6

$M = 10 M_{\mathrm{BH}}$ $\quad M/a_0 = 0.1$

$c_{s_r} = 0.8$

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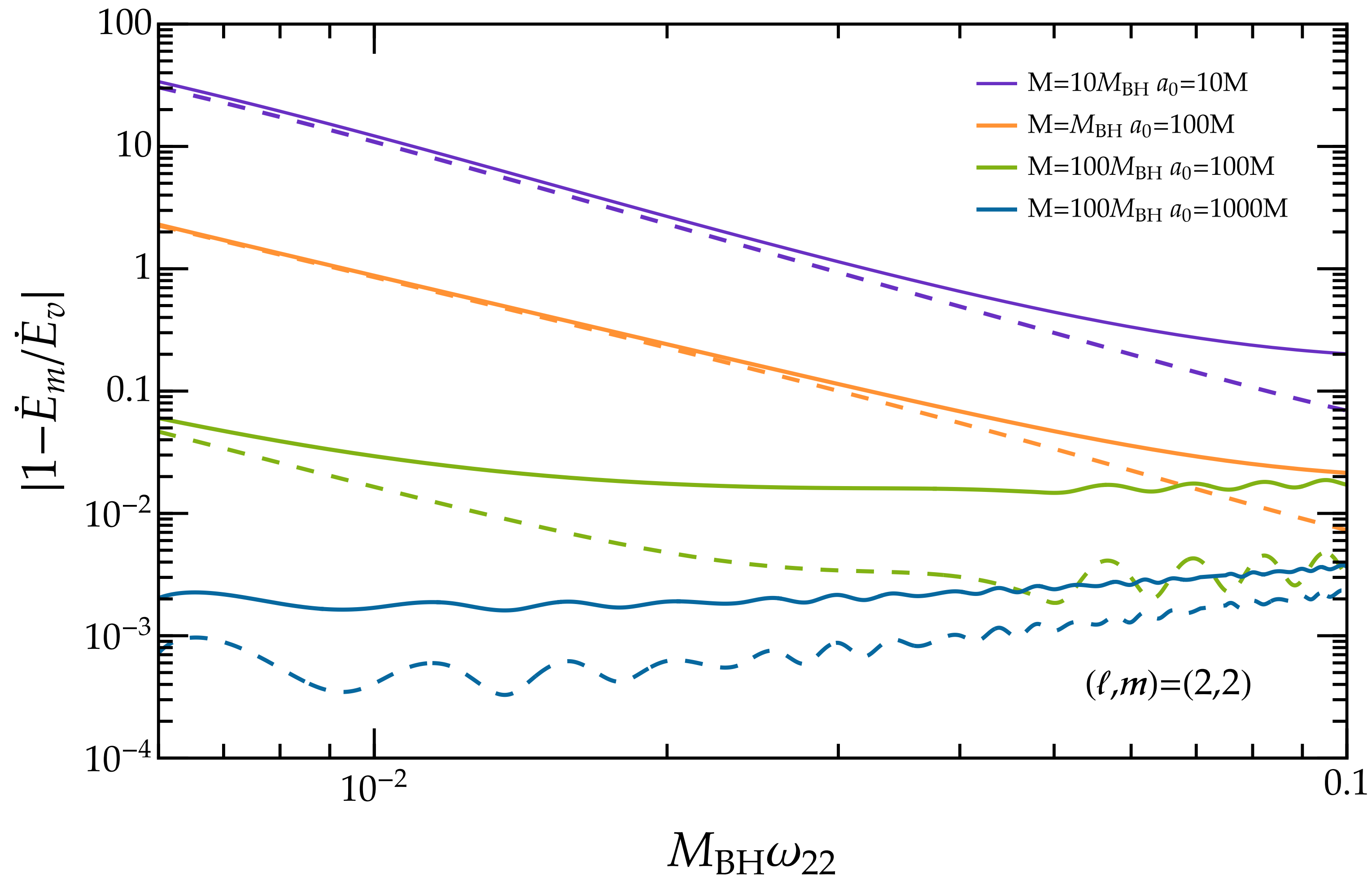
BH & halo EMRI: polar modes

(2,2) polar flux emitted by an EMRI on circular motion



- *Redshift rescaling not enough to take into account shift in the fluxes*
- *generation and propagation affected by deviations due to the coupling between polar modes and the fluid*
- *deviations seem "promising" in terms of detectability*

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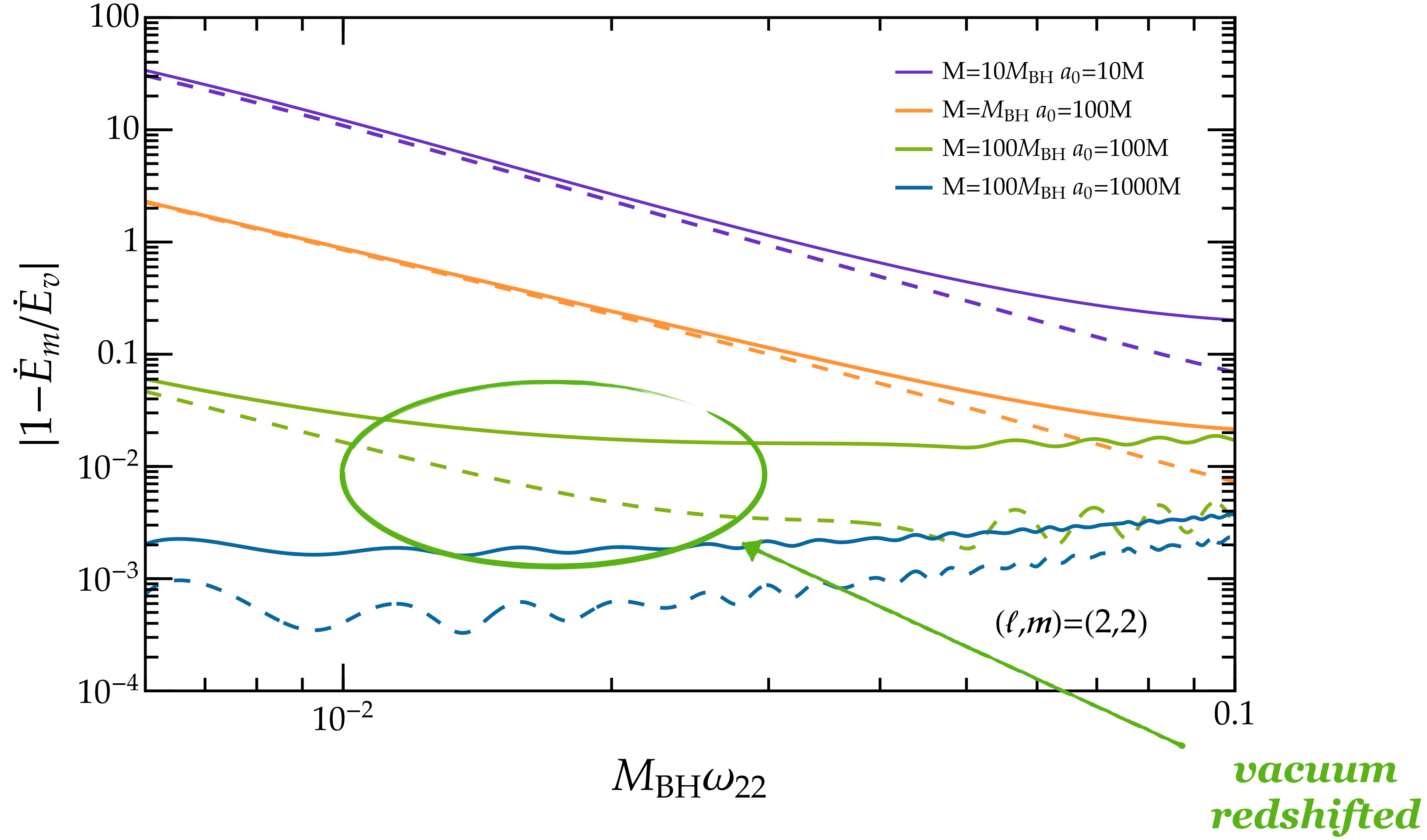

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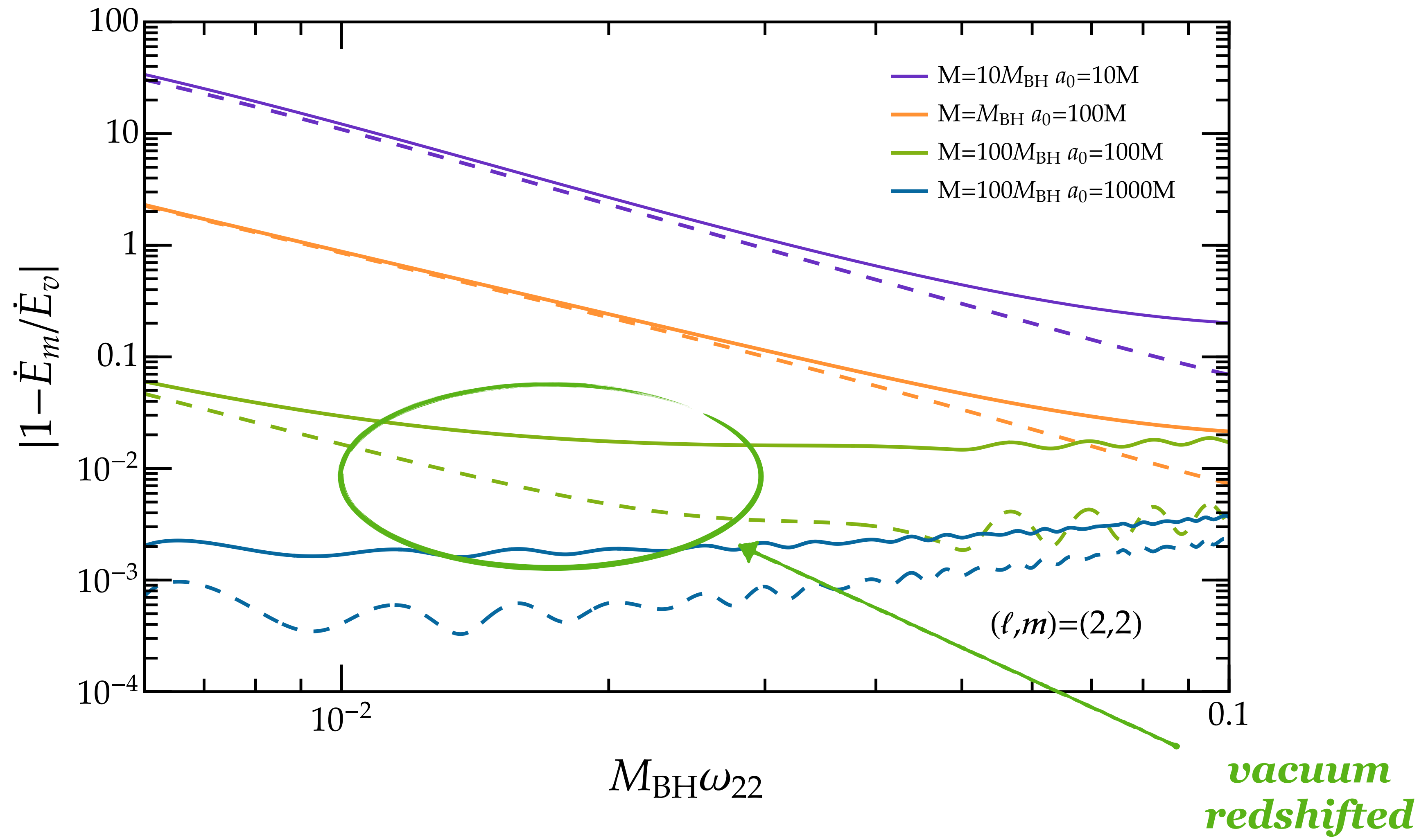
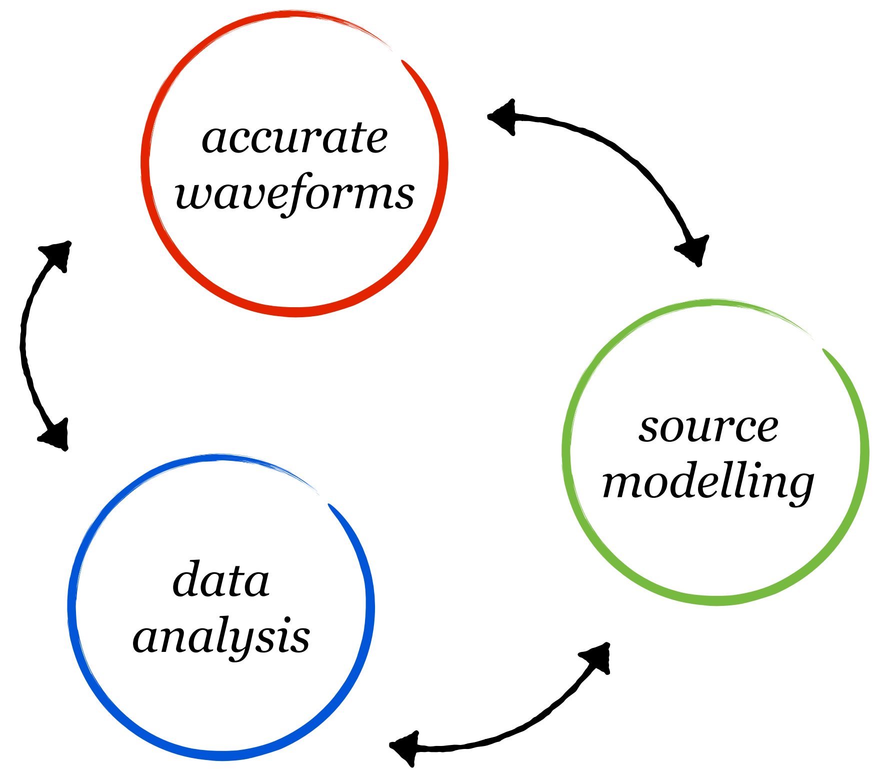


- *Redshift rescaling not enough to take into account shift in the fluxes*
- *generation and propagation affected by deviations due to the coupling between polar modes and the fluid*
- *deviations seem "promising" in terms of detectability* → *new waveform models to build*

EMRI in dirty backgrounds

First steps towards fully relativistic description

- *Ab-initio calculation of EMRI evolution with a non-vacuum BBH background*
- *Changes in the emitted gravitational wave fluxes due to the environment's properties*
- *GW propagation and generation can be strongly affected by coupling between polar modes and the the fluid*



But

- *Rotating background for (more) realistic astrophysical BHs*
- *Waveform generation*
- *Detectability of halo parameters*
- *Degeneracy with beyond-GR modifications*

Back up

Wave equation (homogeneous)

*Solving the homogeneous problem allows to study the dirty BH **Q**uasi-**N**ormal-**M**ode spectrum*

$$\frac{d^2\psi_{\ell m}}{dr_\star^2} + [\omega^2 - V]\psi_{\ell m} = 0$$

- *Solve for the homogeneous part with suitably boundary conditions*

horizon

$$\psi_{\ell m}^{(-)} \sim e^{-i\omega r_\star}$$

(ingoing)

infinity

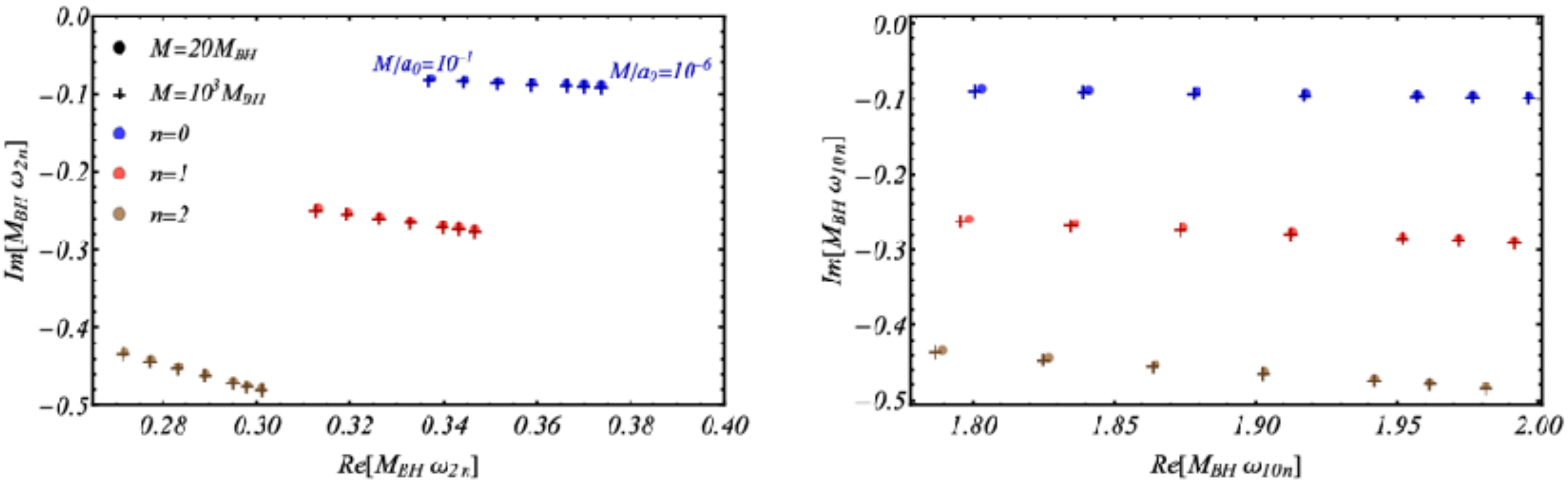
$$\psi_{\ell m}^{(+)} \sim e^{+i\omega r_\star}$$

(outgoing)

- *Find the system eigenvalues (the QNM frequencies) which render the two solutions dependent*

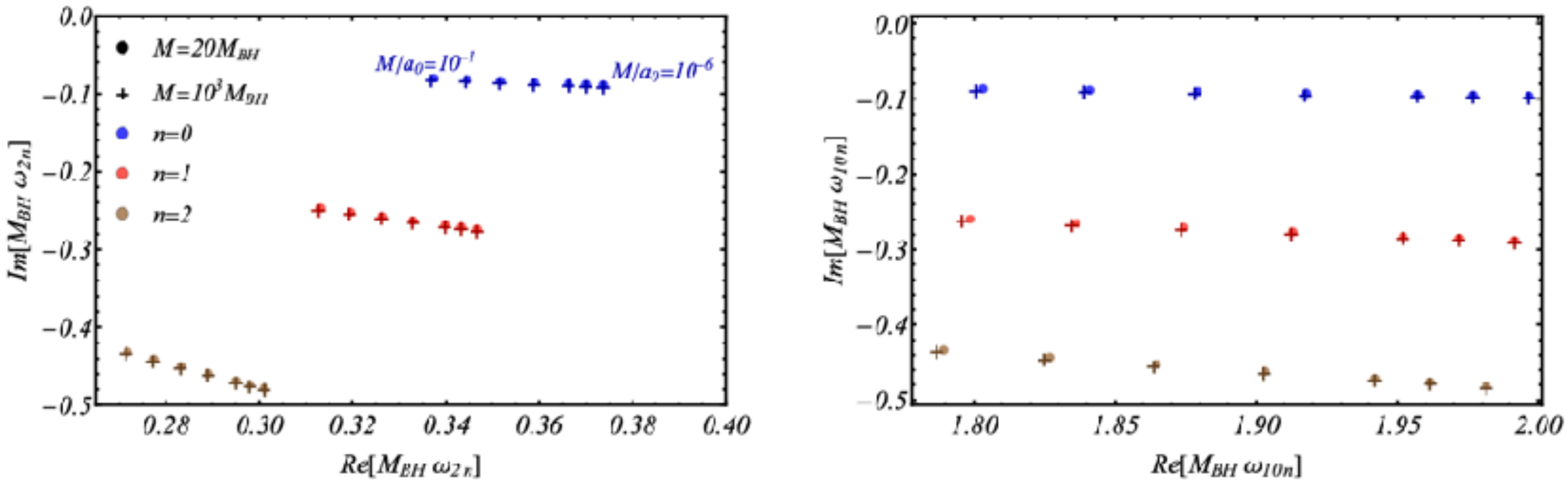
$$\psi_{\ell m}^{(-)}\psi_{\ell m}^{'(+)} - \psi_{\ell m}^{(+)}\psi_{\ell m}^{'(-)} = 0$$

Quasi Normal Modes



- Both real and imaginary part decrease as M/a_0 increases
- Very little dependence on M

Quasi Normal Modes

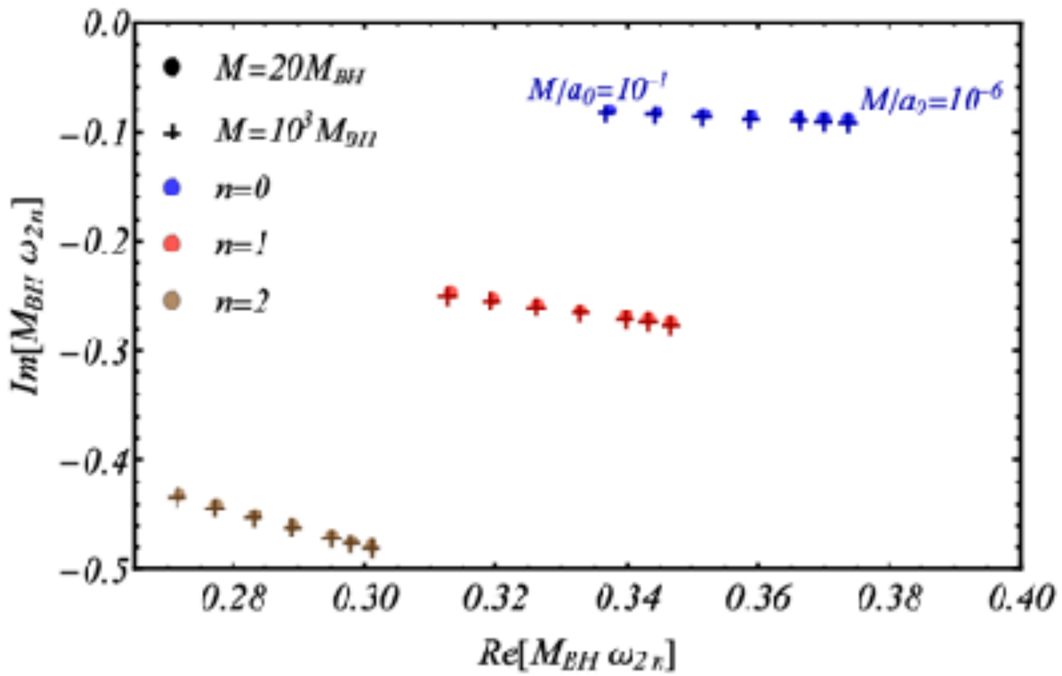


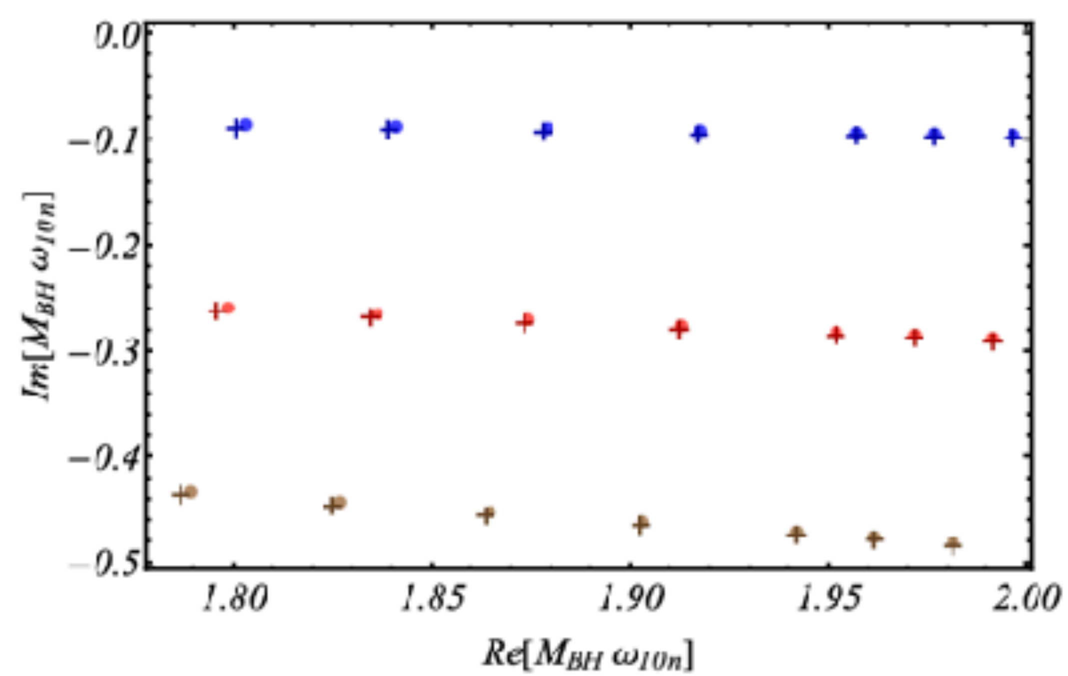
- *Both real and imaginary part decrease as M/a_o increases*
- *Very little dependence on M*

redshift

effect

Quasi Normal Modes



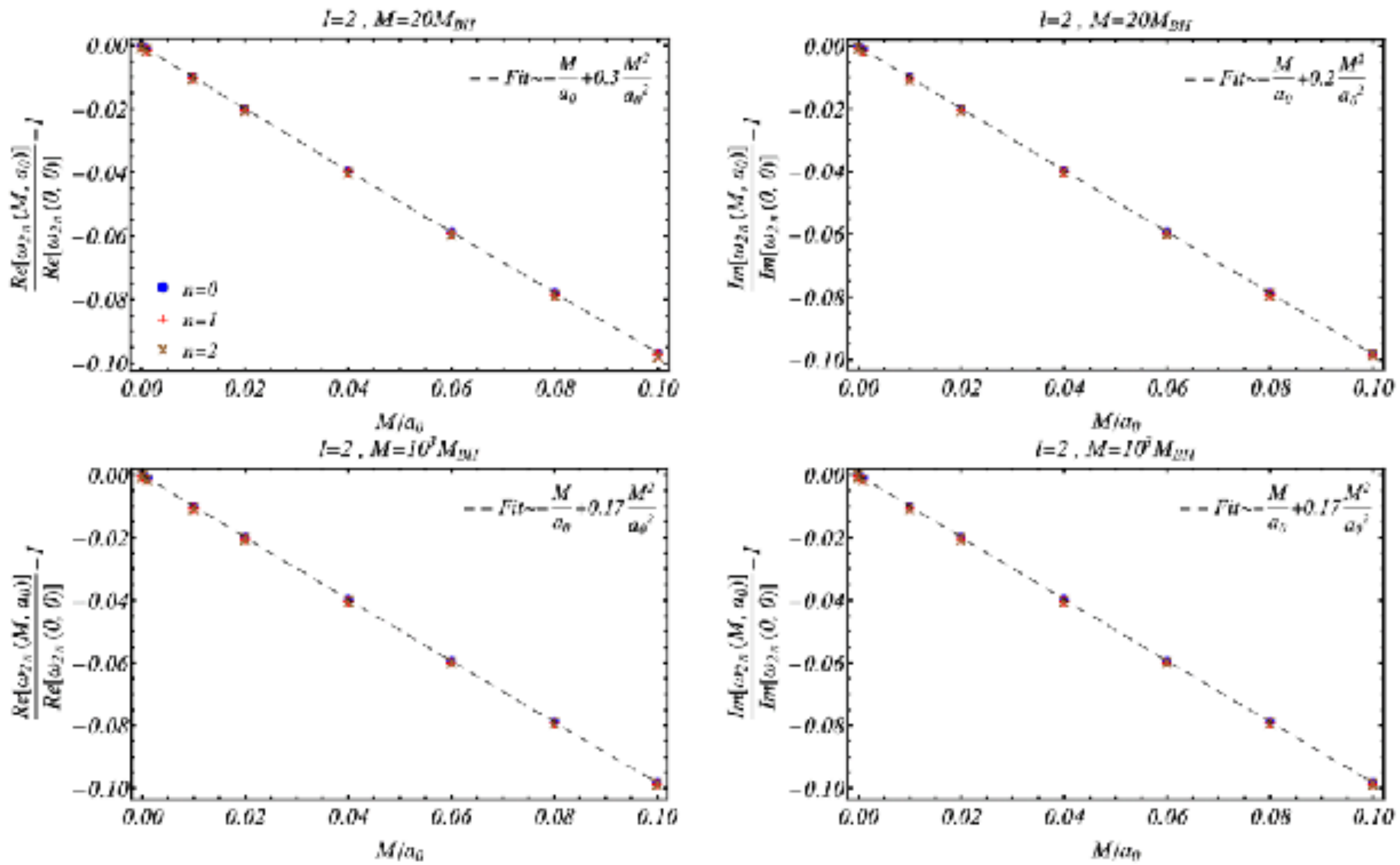


- *Both real and imaginary part decrease as M/a_0 increases* ← *redshift effect*
- *Very little dependence on M*
- *In the eikonal limit* $\omega_{\rm QNM} = \Omega_{\rm LR}\ell - i(n+1/2)|\lambda|$
- *For small compactness*

$$\frac{\Omega_{\rm LR}}{\Omega_{\rm LR}^{\rm vac}} \simeq 1 - \frac{M}{a_0} - 0.17\frac{M^2}{a_0} \qquad\qquad \frac{\lambda}{\lambda^{\rm vac}} \simeq 1 - \frac{M}{a_0} - 0.17\frac{M^2}{a_0}$$

Quasi Normal Modes

QNM behaviour as a function of the halo compactness



- *The QNMs have a clear light-ring interpretation*
- *Linear and subdominant corrections agree with the analytic scaling of frequencies and damping times*

Wave equation (inhomogeneous)

EMRI evolution by solving the full inhomogeneous problem

$$\frac{d^2\psi_{\ell m}}{dr_*^2} + \left[\omega^2 - V\right]\psi_{\ell m} = J_{\ell m}$$

- *Solve for the homogeneous part with suitably boundary conditions*

horizon

$\psi_{\ell m}^{(-)} \sim e^{-i\omega r_\star}$

(ingoing)

infinity

$\psi_{\ell m}^{(+)} \sim e^{+i\omega r_\star}$

(outgoing)

- *Integrate over the source term for the full solution*

$$\psi_{\ell m}^{\pm} \equiv \lim_{r_\star \to \pm\infty} \psi_{\ell m}(r_*) = e^{\pm i\omega r_\star} \int_{-\infty}^{\infty} \frac{\psi_{\ell m}^{(\mp)} J}{W} dr_\star$$

- *For circular orbits the integral greatly simplifies because of* $\delta(r - r_p)$