Dark matter capture in celestial bodies: Effect of multi-scattering and light mediators

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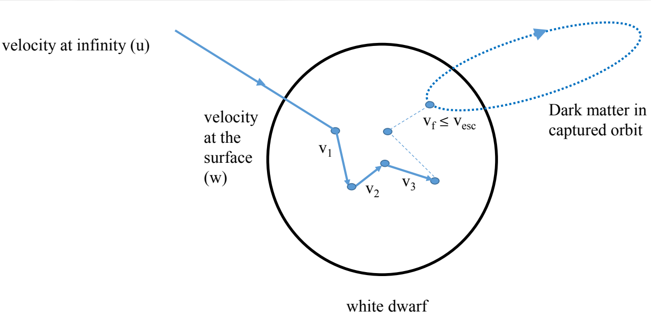
PART A: DM capture through multiple scatterings

- Dark matter capture, schematically:



- Number of scatterings inside a star is typically given by: $\sim R/\lambda_{\rm fs} = n\sigma R \simeq \sigma/R^2$
- It also depends on the mass of dark matter (kinematics).

Recap of capture by single scattering

- The capture rate is given by [1] :

$$C_1 = \sigma N_T n_{DM} \int_u \frac{f(u)du}{u} (u^2 + v_{\rm esc}^2) g_1(u)$$

- The fractional loss in kinetic energy lies in the interval:

$$0 \leq \frac{\Delta E}{E} \leq \beta \left(= 4 m_{DM} m_T / (m_{DM} + m_T)^2\right) \qquad (1)$$

- On the other hand, for capture we require:

$$\frac{\Delta E}{E} \geq \frac{m_{DM} w^2/2 - m_{DM} v_{\rm esc}^2/2}{m_{DM} w^2/2} = \frac{u^2}{w^2} \qquad (2)$$

- Hence, the probability of capture is given by:

$$g_1(u) = \frac{1}{\beta} \left\{ \left(\beta - \frac{u^2}{u^2 + v_{\rm esc}^2} \right) \right\} \Theta \left(\beta - \frac{u^2}{u^2 + v_{\rm esc}^2} \right)$$

[1] A. Gould (Astrophys. J. 321(1987) 571)

Generalising to multiple scatterings[2]

- The capture rate generalises to :

$$C_N = \underbrace{\pi R^2}_{\text{area of the object}} \times \underbrace{p_N(\tau)}_{\text{probability for } N \text{ collisions}} \times \underbrace{n_{\rm DM} \int \frac{f(u)du}{u} (u^2 + v_{\rm esc}^2)}_{\text{DM flux}} \times \underbrace{g_N(u)}_{\text{probability that } v_f \le v_{\rm esc} \text{ after } N \text{ collisions}} .$$

- The probability for a dark matter with optical depth $\tau = \frac{3\sigma N_T}{2\pi R^2}$ to participate in N actual scatters is given by Poisson(τ,N).

[2] Bramante, Delgado, Martin (Phys. Rev. D 96, 063002)

- Taking all incidence angle into account :

$$p_N(\tau) = 2\int_0^1 dy\, \frac{y e^{-y\tau}(y\tau)^N}{N!}$$

- Using $p_1(\tau) \sim \frac{2}{3}\tau$, we recover C_1 as expected.
- We know, $\Delta E = z\beta E$, where, $z = \cos^2\theta_{recoil}$.
- The capture probability g_N reduces to :

$$\begin{aligned} g_N(u) &= \int_0^1 dz_1 \int_0^1 dz_2 \ldots \int_0^1 dz_N\, \theta\left(v_{\rm esc} - \underbrace{w \prod_{i=1}^{N} (1 - z_i\beta)^{1/2}}_{\mathsf{v}_f} \right) \\ &= \frac{1}{\beta} \frac{v_{\rm esc}^2}{u^2 + v_{esc}^2} \left[\frac{1}{\beta} \log \frac{1}{1-\beta} \right]^{N-1} - \left(\frac{1}{\beta} - 1 \right) \end{aligned}$$

- The expression for $g_N(u)$ differs from the analogous expression in a previous work by Bramante et al.[3], where z_i was replaced by its average value of $1/2$

$$g_{\mathrm{N}}^{\mathrm{approx}}(u) = \Theta\left(v_{\mathrm{esc}} \prod_{i=1}^{N} \left(1 - \frac{1}{2}\beta\right)^{-1/2} - (u^2 + v_{\mathrm{esc}}^2)^{1/2}\right).$$

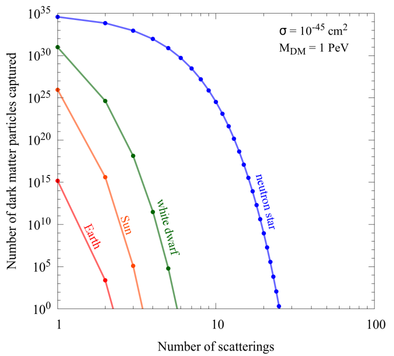
- $g_N(u) \leq 1$ and $g_N(u) \geq 0$ gives a lower and an upper limit on u:

$$v_{\mathrm{esc}}^2 \left[\left(\tfrac{1}{\beta}\log\tfrac{1}{1-\beta}\right)^{N-1} - 1\right] \leq u^2 \leq v_{\mathrm{esc}}^2 \left[\tfrac{1}{1-\beta}\left(\tfrac{1}{\beta}\log\tfrac{1}{1-\beta}\right)^{N-1} - 1\right]$$

[3] Bramante, Delgado, Martin (Phys. Rev. D 96, 063002)

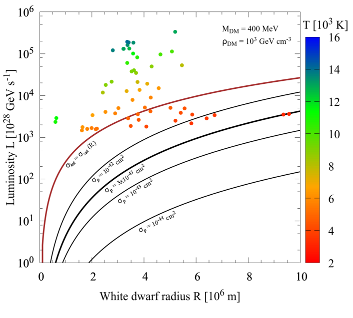
Number of scatterings allowed

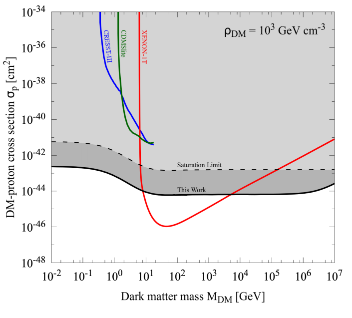
- Total capture rate: $C_{\rm tot} = \sum C_N$.
- $\tau_{lifetime} \times C_N$ is an integer
  $\Rightarrow$ we truncate at that N for which $\tau_{lifetime} \times C_N < 1$.



Evolution of the captured dark matter

- Time evolution of number dark matter particles inside the star is given by: $dN/dt = C_{\rm tot} - C_{\rm ann}\, N^2$.
- $L_{\rm DM} = M_{\rm DM}(C_{\rm ann}\, N^2) \overset{\text{Equil.}}{=} M_{\rm DM}\, C_{\rm tot} = f(\sigma_p, M_{DM}, R_{WD}, \rho_{DM}^{halo})$.
- For WD and NS, $L \simeq 4\pi\, \sigma_{SB}\, R_{WD}^2\, T_{\rm eff}^4$. [4]





[4] Data taken from McCullough & Fairbarn (Phys.Rev.D 81 (2010)083520)

PART B: Effect of light mediators in DM capture [5]

- Remember, that the capture probability via single scattering was given by:

$$g_1(u) = \frac{1}{\beta}\left\{\left(\beta - \frac{u^2}{u^2+v_{\text{esc}}^2}\right)\right\}\Theta\left(\beta - \frac{u^2}{u^2+v_{\text{esc}}^2}\right) \quad (3)$$

- **Assumption** $\Rightarrow$ the distribution of $\Delta E/E$ is uniform.
- The distribution of θ_{CM} in the CM frame is: $p(\Omega_{CM}) = \frac{1}{\sigma}\frac{d\sigma}{d\Omega_{CM}}$.
- Using $\theta_{recoil} = \pi/2 - \theta_{CM}/2$ we can hence find the distribution of θ_{recoil} and consequently the distribution of $\Delta E/E$ where $\Delta E/E = \beta\cos^2\theta_{\text{recoil}} \equiv \beta z$.
- Hence, $\frac{d\sigma}{d\Omega} = \text{const} \Leftrightarrow$ an uniformly distributed $\Delta E/E$.

[5] Also, see previous works on q-dependent capture by A. Vincent

- For DM-nucleon interaction via a Yukawa potential (in non-relativistic Born regime) we have:

$$\frac{d\sigma}{d\Omega_{\rm CM}} = \frac{4\mu^2\alpha^2}{\left(4\mu^2 v_{\rm rel}^2 \sin^2(\theta_{\rm CM}/2) + m_\phi^2\right)^2}\,.$$

- Following the prescription described in the previous slide, the energy loss distribution $s(z)$ is given by [6]

$$s(z) = \frac{m_\phi^2\left(4\mu^2 v_{\rm rel}^2 + m_\phi^2\right)}{\left(4\mu^2 v_{\rm rel}^2 z + m_\phi^2\right)^2}\,.$$

- Here, we will restrict ourselves to the regime of capture by single scattering.

[6] B. Dasgupta, AG, A. Ray (JCAP10(2020)023)

- The generalized capture probability by single scatter is thus given by:

$$g_1(u) = \int_0^1 dz\,\Theta\left(v_{\rm esc} - v_{\rm f}\right) s(z)$$

where $v_{\rm f} = \sqrt{(u^2 + v_{\rm esc}^2)(1 - z\beta)}$ = the final velocity of DM.

- Using the analytical expressions we thus have:

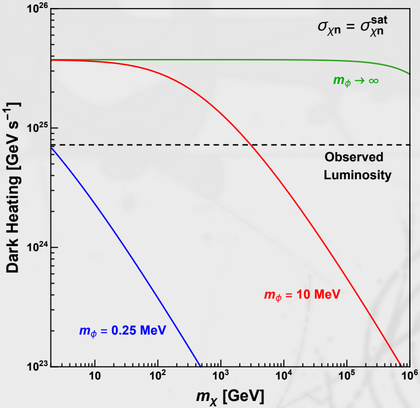
$$g_1(u) = \frac{m_\phi^2 \left(1 - \frac{1}{\beta}\frac{u^2}{u^2 + v_{\rm esc}^2}\right)}{\left(m_\phi^2 + \frac{4\mu^2 u^2}{\beta c^2}\right)} \Theta\left(v_{\rm esc}\sqrt{\frac{\beta}{1-\beta}} - u\right) .$$

- The capture rate takes the form:

$$C = \frac{\rho_\chi}{m_\chi} \int \frac{f(u)du}{u} (u^2 + v_{\rm esc}^2) N_{\rm n}\,{\rm Min}\left[\sigma_{\chi {\rm n}}, \sigma_{\chi {\rm n}}^{\rm sat}\right] g_1(u).$$

- $\sigma_{\chi {\rm n}}^{\rm sat} = \pi R^2 / N_{\rm n}$ = geometrical limit of DM-nucleon cross section $\sim 2 \times 10^{-45}\,{\rm cm}^2$ (for typical NS parameters).

- Taking a neutron star with $T = 1950$ K, we calculate $L_{\rm obsv}$ by using Stefan-Boltzmann law as before ($R_{\rm NS} = 10.6$ Km).
- The DM halo density is taken to be 0.4 GeV/cc.
- We can rule out choice of parameters for which $L_{\rm theory} \geq L_{\rm obsv}$.



- No constraints can be inferred for $m_\phi \leq 0.25$ MeV.

Take home messages

- **Part A:** We improved upon the previous treatment of multiple scattering by (a) deriving an analytical expression for the capture probability along with (b) a better understanding of about the actual number of scatterings that can actually take place.
- Using actual WD data from the M4 globular cluster, in both high and low mass regime we derive the most stringent constraint available till date in σ_P-$M_{\rm DM}$ plane.
- **Part B:** We generalized the treatment of DM capture inside celestial objects for arbitrary mediator masses.
- For mediators lighter than 0.25 MeV, constraints via heating of NS are washed out.

THANK YOU