Anisotropic stars made of exotic matter in light of the complexity factor formalism

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Outline

- Introduction
- Relativistic stars in General Relativity
- Anisotropic matter: Complexity factor
- Results and Discussion
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Introduction

1- Celestial bodies are not always made of isotropic fluid only!

2- Anisotropic stars can be made, at least theoretically, of exotic matter (as dark matter).

3- To study anisotropic stars, usually it is necessary to consider a concrete form of the anisotropic factor (problem!)

4- To bypass that, we will consider the complexity factor formalism to close the system and obtain the physical properties of such anisotropic stars.

Relativistic stars in General Relativity

In absence of cosmological constant term, the classical Einstein field equations acquire the simple form:

$$G_\mu^\nu = 8\pi G T_\mu^\nu,$$

The covariant energy-momentum tensor can be expressed in local Minkowski coordinates as

$$T_\nu^\mu = \{\rho, P_r, P_\perp, P_\perp\},$$

and the resulting field equations take the form:

Relativistic stars in General Relativity

$$\rho = -\frac{1}{8\pi}\left[-\frac{1}{r^2} + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right)\right],$$

$$P_r = -\frac{1}{8\pi}\left[\frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)\right],$$

$$P_\perp = \frac{1}{32\pi}e^{-\lambda}\left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r}\right),$$

where the derivatives with respect to r are denoted by primes.

Relativistic stars in General Relativity

We can combine the last equations to produce the generalized Tolman-Opphenheimer-Volkoff equation, i.e.

$$-\frac{1}{2}\nu'\left(\rho+P_r\right)-P_r'+\frac{2}{r}\left(P_\perp-P_r\right)=0.$$

And removing the ν' - dependence, we finally obtain the familiar equation

$$P_r'=-\frac{\left(m+4\pi P_r r^3\right)}{r\left(r-2m\right)}\left(\rho+P_r\right)+\frac{2}{r}\left(P_\perp-P_r\right),$$

Relativistic stars in General Relativity

In what follow, we will rewrite the energy-momentum tensor in a more convenient way:

$$T^{\mu}_{\nu} = \rho u^{\mu} u_{\nu} - P h^{\mu}_{\nu} + \Pi^{\mu}_{\nu},$$

we set the four-velocity and the four acceleration as follow:

$$u^{\mu} = (e^{-\frac{\nu}{2}}, 0, 0, 0), \qquad a^{\alpha} = u^{\alpha}_{;\beta} u^{\beta},$$

Where the unique non--vanishing component of the acceleration is

$$a_1 = -\nu'/2,$$

Relativistic stars in General Relativity

Notice that the set $\{\Pi^{\mu}_{\nu}, \Pi, h^{\mu}_{\nu}, s^{\mu}, P\}$ is taken according to

$$\Pi^{\mu}_{\nu} = \Pi\left(s^{\mu}s_{\nu} + \frac{1}{3}h^{\mu}_{\nu}\right),$$

$$\Pi = P_{\perp} - P_{r},$$

$$h^{\mu}_{\nu} = \delta^{\mu}_{\nu} - u^{\mu}u_{\nu},$$

$$s^{\mu} = (0, e^{-\frac{\lambda}{2}}, 0, 0),$$

$$P \equiv \frac{1}{3}\left(P_{r} + 2P_{\perp}\right),$$

Relativistic stars in General Relativity

with the properties

$$s^{\mu} u_{\mu} = 0, \qquad s^{\mu} s_{\mu} = -1.$$

The problem should be supplemented using certain boundary conditions on the surface

$$e^{\nu_{\Sigma}} = 1 - \frac{2M}{R},$$

$$e^{-\lambda_{\Sigma}} = 1 - \frac{2M}{R},$$

$$[P_r]_{\Sigma} = 0.$$

Anisotropic matter: Complexity factor

The complexity factor describes the influence of the local anisotropy of pressure and density inhomogeneity on the Tolman mass

$$m_T = (m_T)_\Sigma \left(\frac{r}{r_\Sigma}\right)^3 + r^3 \int_r^{r_\Sigma} \frac{e^{(\nu+\lambda)/2}}{\tilde{r}} Y_{TF} d\tilde{r},$$

where

$$Y_{TF} = 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r \tilde{r}^3 \rho' d\tilde{r},$$

Anisotropic matter: Complexity factor

To define the complexity factor, we need to perform the orthogonal decomposition of the Riemann tensor

$$Y_{\alpha\beta} = R_{\alpha\gamma\beta\delta}u^{\gamma}u^{\delta},$$

$$Z_{\alpha\beta} = R^{*}_{\alpha\gamma\beta\delta}u^{\gamma}u^{\delta} = \frac{1}{2}\eta_{\alpha\gamma\epsilon\mu}R^{\epsilon\mu}{}_{\beta\delta}u^{\gamma}u^{\delta},$$

$$X_{\alpha\beta} = R^{*}_{\alpha\gamma\beta\delta}u^{\gamma}u^{\delta} = \frac{1}{2}\eta_{\alpha\gamma}{}^{\epsilon\mu}R^{*}_{\epsilon\mu\beta\delta}u^{\gamma}u^{\delta},$$

where

$$R^{*}_{\alpha\beta\gamma\delta} = \frac{1}{2}\eta_{\epsilon\mu\gamma\delta}R_{\alpha\beta}{}^{\epsilon\mu},$$

Anisotropic matter: Complexity factor

We rewrite last expressions in term of the physical variables as follow

$$Y_{\alpha\beta} = \frac{4\pi}{3}(\rho + 3P)h_{\alpha\beta} + 4\pi\Pi_{\alpha\beta} + E_{\alpha\beta},$$

$$Z_{\alpha\beta} = 0,$$

$$X_{\alpha\beta} = \frac{8\pi}{3}\rho h_{\alpha\beta} + 4\pi\Pi_{\alpha\beta} - E_{\alpha\beta}.$$

where

$$E_{\alpha\beta} = C_{\alpha\gamma\beta\delta}u^{\gamma}u^{\delta},$$

Anisotropic matter: Complexity factor

We rewrite last expressions in term of the physical variables as follow

$$E_{\alpha\beta} = E\left(s_\alpha s_\beta + \frac{1}{3}h_{\alpha\beta}\right),$$

$$E = -\frac{e^{-\lambda}}{4}\left[\nu'' + \frac{\nu'^2 - \lambda'\nu'}{2} - \frac{\nu' - \lambda'}{r} + \frac{2(1-e^\lambda)}{r^2}\right],$$

The following properties must be satisfied:

$$E^\alpha_{\ \alpha} = 0, \quad E_{\alpha\gamma} = E_{(\alpha\gamma)}, \quad E_{\alpha\gamma}u^\gamma = 0,$$

Anisotropic matter: Complexity factor

Finally, last tensors can be represented in term of alternative scalar functions, i.e.,

$$X_T = 8\pi\rho,$$

$$X_{TF} = \frac{4\pi}{r^3}\int_0^r \tilde{r}^3 \rho' d\tilde{r},$$

$$Y_T = 4\pi(\rho + 3P_r - 2\Pi),$$

$$Y_{TF} = 8\pi\Pi - \frac{4\pi}{r^3}\int_0^r \tilde{r}^3 \rho' d\tilde{r},$$

Anisotropic matter: Complexity factor

Demanding a vanishing complexity factor, we can obtain the anisotropic factor by solving

$$\Pi(r) = \frac{1}{2r^3} \int_0^r \tilde{r}^3 \rho'(\tilde{r}) d\tilde{r}.$$

Given that a profound comprehension of the idea of complexity is still under construction, the connection between any equation of state and the definition of complexity factor is still missing.

Anisotropic matter: Complexity factor

In what follow, we will take a Chaplyin equation-of-state to close the system

$$P_r = A^2\rho(r) - B^2\rho(r)^{-1},$$

and with help of a vanishing complexity factor, we solve the problem numerically.

Also, for comparison, we will check our solution vs the more conventional approach (i.e., assuming a concrete form of the anisotropic factor).

Results and Discussion

We will use the following numerical values:

Case # 1:

$$A = \sqrt{0.400}, \qquad B = 0.230 \times 10^{-3} km^{-2}.$$

Case # 2:

$$A = \sqrt{0.425}, \qquad B = 0.215 \times 10^{-3} km^{-2}.$$

Case # 3:

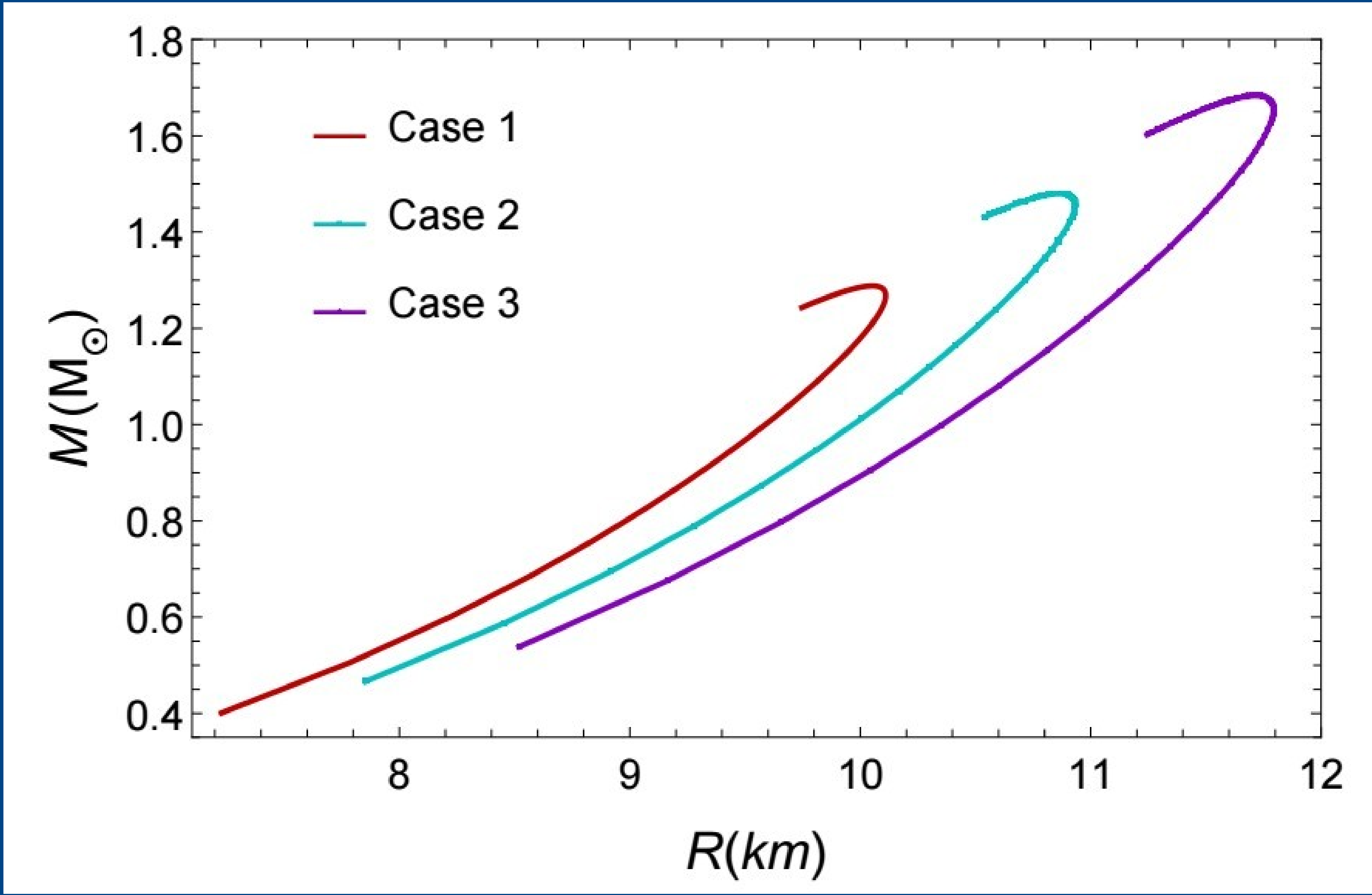
$$A = \sqrt{0.450}, \qquad B = 0.200 \times 10^{-3} km^{-2}.$$

Results and Discussion

1- Complexity Factor Formalism

$$\Pi(r) = \frac{1}{2r^3} \int_0^r \tilde{r}^3 \rho'(\tilde{r}) d\tilde{r}.$$

Results and Discussion



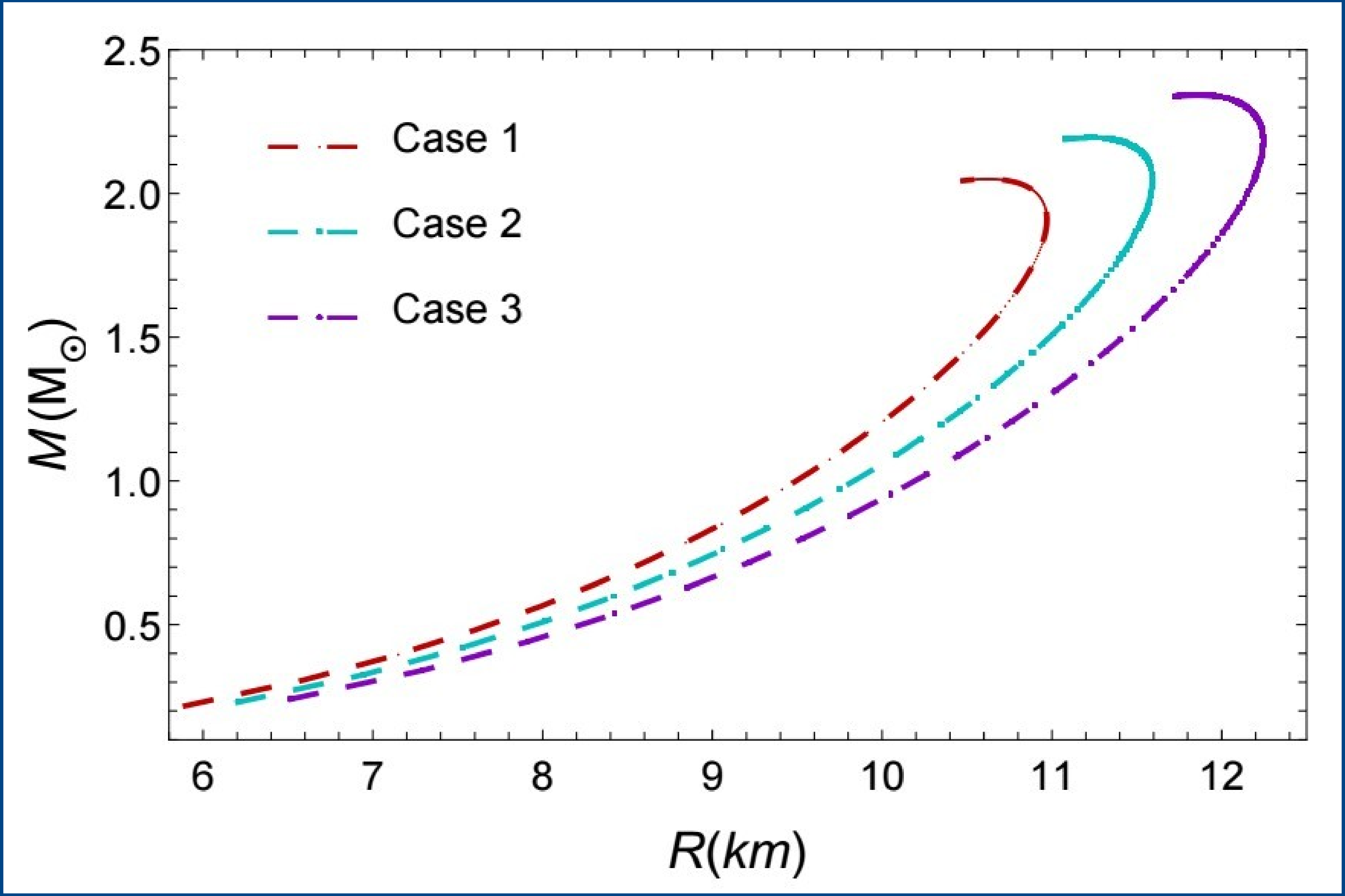
Results and Discussion

2- Conventional Approach

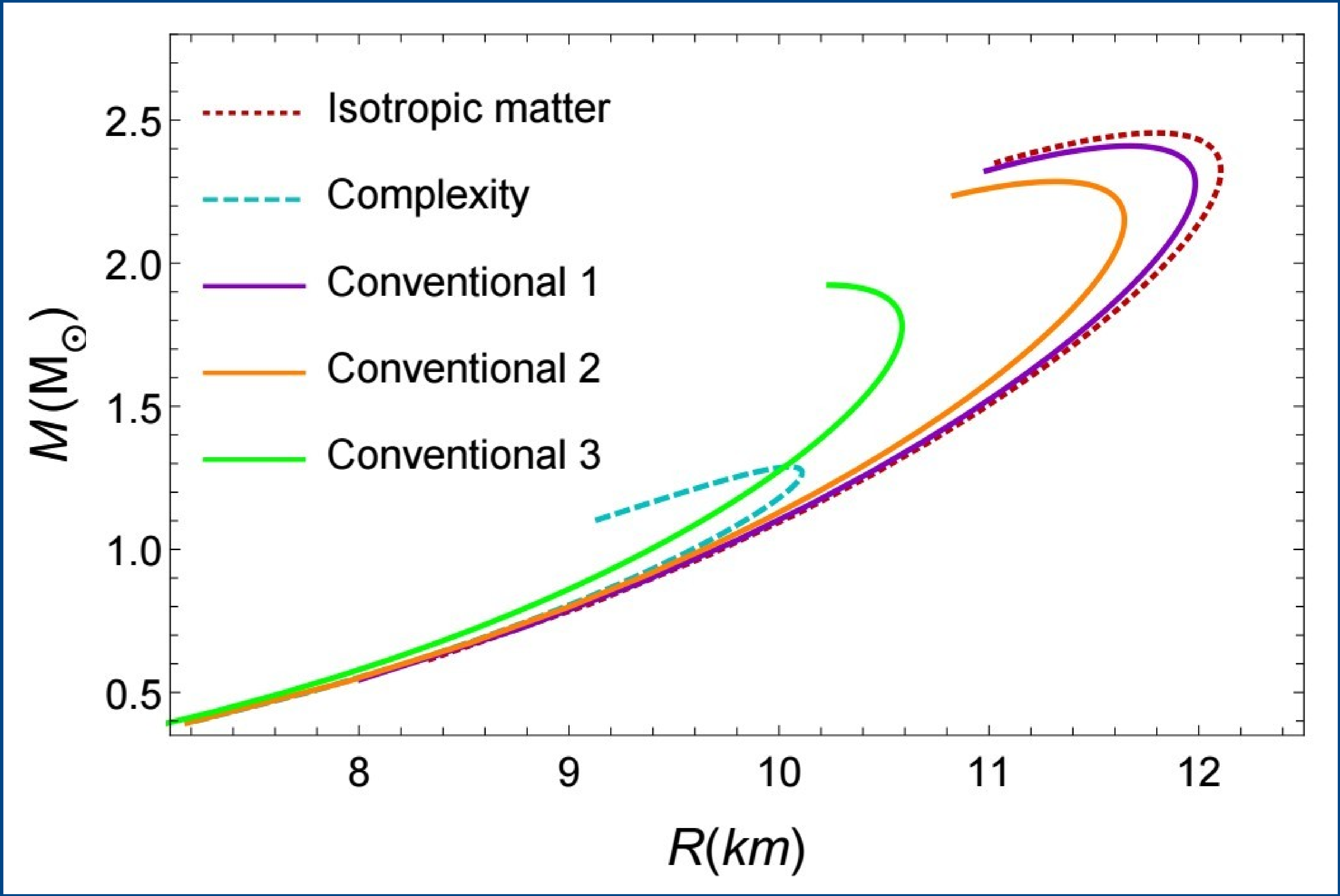
$$\Pi(r) = -\left(\frac{r}{a}\right)^2 \rho(r).$$

$$a \equiv a_{\mathrm{large}} = 30 \text{ km}.$$

Results and Discussion



Results and Discussion



Take home message

The complexity factor formalism predicts smaller and lighter objects compared to the conventional approach, despite both cases having a negative anisotropic factor.