# Stars in modified gravity

**Aneta Wojnar**

Complutense University of Madrid

Dark Matter and Stars: Multi-Messenger Probes of Dark Matter and Modified Gravity 2023

# Motivation: why (sub-)stellar objects and modified gravity?

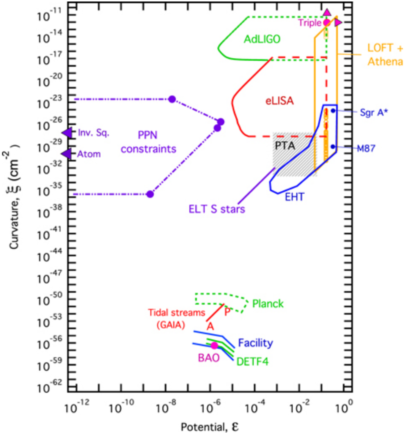
- MG affects physics of non-relativistic objects[1]
- We understand the physics of those objects a bit better than physics of neutron stars and black holes
- In stars and giant planets, all four interactions are taking place in the regimes of temperatures and pressures a bit better understood
- The biggest impact of MG seem to be mainly related to the age of the particular objects ("objects older" than the Universe, formation of the Solar System, age determination techniques)[2]
- We are/we will be equipped with more and more accurate data[3], e.g. Cosmic Vision 2015-2025, Voyage 2050, James Webb & Nancy Grace Roman Space Telescopes,...

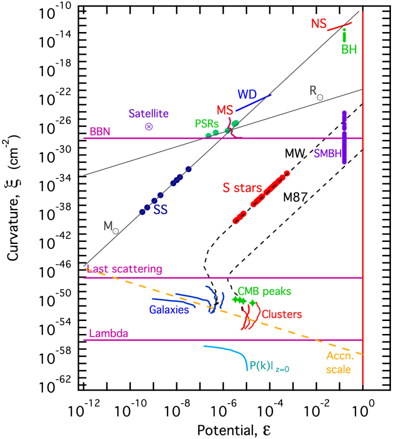
---

[1] G.J. Olmo, D. Rubiera-Gracia, AW, Stellar structure models in modified theories of gravity: lessons and challenges, Physics Reports 876 (2020); PRD 104 (2021) 2, 024045; A. Kozak, AW, Eur. Phys. J. C 81 (2021) 6, 492

[2] AW, PRD 103 (2021) 4, 044037; M. Benito, AW, PRD 103 (2021) 6, 064032; AW, PRD 104 (2021) 10, 104058; S. Kalita, L. Sarmah, AW, PRD 107 (2023) 4, 044072

[3] see current and near-future missions of ESA and NASA related to dwarf stars and (exo-)planets




Untested regime in the **the galaxy and stellar physics regime**. It could potentially hide the onset of corrections to GR (T. Baker et al 2015 ApJ 802, 63).

Curvature $\xi = (R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta})^{\frac{1}{2}} = \sqrt{48}\frac{GM}{r^3c^2}$

Potential $\varepsilon = \frac{GM}{rc^2}$

# Quantum gravity[4]

Observation 1:

Modifies Heisenberg uncertainty principle (GUP)

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\Big(1 + \text{modification}\Big)$$

or/and dispersion relation

$$E^2 + p^2\Big(1 + \text{modification}\Big) = m^2$$

[4] LQG, Doubly Special Relativity, String Theory, Noncommunative geometry,...

Observation 2:

The weighted phase space volume is modified ($D$ - dim of the phase space).

$$\frac{d^D\mathbf{x}d^D\mathbf{p}}{1+\text{modification}}$$

Consequence: modified partition function ($z = e^{\mu/k_BT}$)

$$\ln\mathcal{Z} = \frac{V}{(2\pi\hbar)^3}\frac{g}{\pm 1}\int \ln\left(1 \pm ze^{-E/k_BT}\right)\frac{d^3p}{1+\text{modification}}$$

Conclusion: Quantum Gravity modifies equations of state since

$$P = k_BT\frac{\partial}{\partial V}\ln\mathcal{Z},$$

$$n = k_BT\frac{\partial}{\partial\mu}\ln\mathcal{Z}\mid_{T,V},$$

$$U = k_BT^2\frac{\partial}{\partial T}\ln\mathcal{Z}\mid_{z,V}$$

Observation 3: MG as an effective theory derived from QG

# Gravity vs matter: motivation based on a number of indications

- Effective quantities: opacity[5], ...
- Modifications introduced by modified gravity to pressure[6]
- Chemical reactions rates depend on gravity[7]
- **Specific heat and crystallization depend on modified gravity**[8]
- Chemical potential depends on gravity[9]
- Elementary particle interactions modified by modified gravity (dependence of the metric on the local energy-momentum distributions[10]
- EoS depends on relativistic effects introduced by GR[11]
- Thermonuclear processes...?[12]
- **Fermi equation of state depends on (modified/quantum) gravity**[13]

---

[5] J. Sakstein, PRD 92 (2015) 124045; ...

[6] H-Ch. Kim, PRD 89 (2014) 064001

[7] P. Lecca, J. Phys.: Conf. Ser. 2090 (2021) 012034

[8] S. Kalita, L. Sarmah, AW, PRD 107 (2023) 4, 044072

[9] I.K. Kulikov, P.I. Pronin, Int. J. Theor. Phys. 34, (1995) 9

[10] A.D.I Latorre, G.J. Olmo, M. Ronco, PRB 780, 294 (2018)

[11] G.M. Hossain, S. Mandal, JCAP 02 (2021) 026; PRD 104 (2021) 123005

[12] J. Sakstein, PRD 92 (2015) 124045; AW, PRD 103 (2021) 4, 044037; M. Guerrero, AW, in preparation

[13] AW, PRD 107 (2023) 4, 044025; A. Pachol, AW, arXiv:2304.08215

# Non-relativistic equations of modified gravity

Modified Poisson equation

$$\nabla^2\Phi \approx \frac{\kappa}{2}(\rho + \text{modification})$$

For spherical-symmetric spacetime the gravitational potential

$$\Phi(r) = -\frac{GM}{r} - 4\pi G \int_r^R \Big(\rho(r)r + \text{modification}\Big)dr.$$

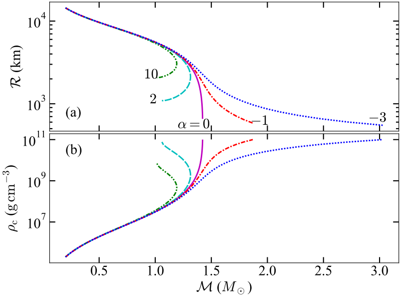
The hydrostatic equilibrium equation

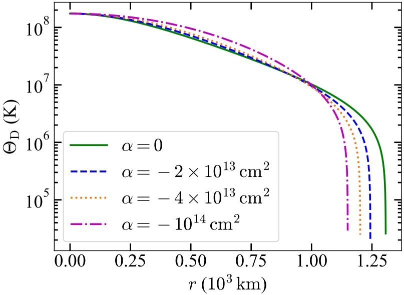
$$\frac{d\Phi}{dr} = -\rho^{-1}\frac{dP}{dr},$$

+ matter description (EoS, temperature dependence,...)

+ eventual equations for additional fields

# White dwarfs and modified gravity[14]




Left panel: with the use of the Chandrasekhar equation of state. Right panel: Variation of Debye temperature inside a modified gravity inspired carbon WD with different $\alpha$ for $\rho_c = 10^{10}\,\mathrm{g\,cm^{-3}}$.

Debye temperature: $$\Theta_D = 0.174 \times 10^4 \frac{2Z}{A} \sqrt{\rho}$$

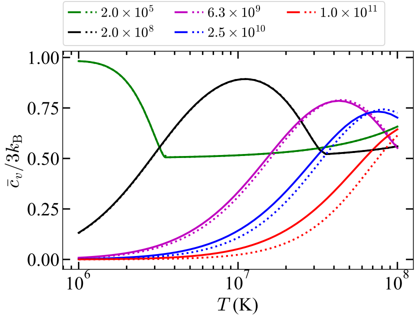
Mean specific heat: $$\bar{c}_v = \frac{1}{\mathcal{M}} \int_0^{\mathcal{M}} \left[ \frac{3}{2} \frac{k_B \pi^2}{3} Z \frac{k_B T}{\epsilon_F} + 9 k_B \left( \frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D / T} \frac{x^4 e^x}{(e^x - 1)^2} dx \right] dm$$

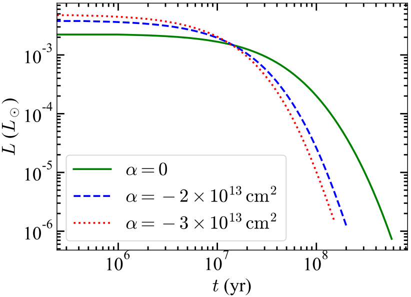
Cooling equation: $$L = \frac{3 k_B \mathcal{M}}{A m_H} \left( - \frac{\bar{c}_v}{3 k_B} + \rho_s q \frac{1}{\mathcal{M}} \frac{dm}{dr} \frac{dr}{d\rho} \right) \frac{dT}{dt}$$

[14] S. Kalita, L. Sarmah, AW, PRD 105 (2022) 2, 024028; Universe 8 (2022) 647; PRD 107 (2023) 4, 044072

# White dwarfs' cooling[15]

Cooling equation: $$L = \frac{3k_B\mathcal{M}}{Am_H}\left(-\frac{\bar{c}_v}{3k_B} + \rho_s q \frac{1}{\mathcal{M}}\frac{dm}{dr}\frac{dr}{d\rho}\right)\frac{dT}{dt}$$




Left: $\bar{c}_v$ as a function of $T$ for different carbon WDs; $\rho_c \in [10^5 - 10^{11}]$ g cm$^{-3}$ for $\alpha = 0$ (solid lines), $\alpha = -2 \times 10^{13}\,\mathrm{cm}^2$, and $\alpha = -3 \times 10^{13}\,\mathrm{cm}^2$.

Right: Luminosity as a function of time for carbon WDs with $\rho_c = 10^{11}\,\mathrm{g\,cm^{-3}}$. The initial luminosity corresponds to a surface temperature of $\approx 10^7$ K.

15 S. Kalita, L. Sarmah, AW, PRD 107 (2023) 4, 044072

# How equations of state do depend on (modified) gravity[16]

System of self-gravitating fermions restricted to a box of radius $R$, such that evaporation is prevented, is described by the distribution function $f(\mathbf{r}, \mathbf{p})$.

**Local variables**

$$n = \int f d\mathbf{p}$$

$$\epsilon_{\mathbf{kin}} = \int f \frac{p^2}{2m} d\mathbf{p},$$

$$P = \frac{1}{3} \int f \frac{p^2}{m} d\mathbf{p}$$

$$s = -k_B f_{\mathbf{max}} \int \tilde{f} \; \mathbf{ln} \; \tilde{f} + (1-\tilde{f}) \; \mathbf{ln} \; (1-\tilde{f}) d\mathbf{p}$$

$$f_{\mathbf{max}} =: \frac{g}{h^3}, \quad \tilde{f} =: \frac{f}{f_{\mathbf{max}}}$$

**Global variables** (valid at equilibrium)

$$N = \int n \, 4\pi r^2 dr,$$

$$M = Nm = \int \rho \, 4\pi r^2 dr,$$

$$E = E^t_{\text{kin}} + W,$$

$$S = \int s \, 4\pi r^2 dr,$$

$$W = -\int \rho \mathbf{r} \cdot \nabla \Phi d\mathbf{r}$$

$n$ - local particle number density, $\epsilon_{\text{kin}}$ - kinetic energy density, $P$ - local pressure, $s$ - Fermi-Dirac entropy density, $E$ - energy, $g$ - spin multiplicity of quantum states, $S$ - entropy of the fermion gas, $N$ - particle number, $E^t_{\text{kin}} = \int \epsilon_{\text{kin}} 4\pi r^2 dr$

This is a non-relativistic case (see the relativistic one in[16])

[16] AW, PRD 107 (2023) 4, 044025

# Statistical equilibrium state

The most probable state of an isolated system is obtained by the maximization of the entropy $S$ at fixed mass energy $\mathcal{E}$ and particle number $N$

$$\max\{S \,|\, \mathcal{E}, N \,\text{fixed}\}$$

- **Step 1**: maximizing $s$ at fixed energy density $\epsilon$ and particle number density $n$ wrt variations on $f$ $\rightarrow$ distribution function $f$ and the local variables corresponding to the condition of the **local thermodynamic equilibrium**

$$\frac{\delta s}{k_B} - \beta(r)\delta\epsilon + \alpha(r)\delta n = 0,$$

- **Step 2**: $\rightarrow$ equations describing a system in the **statistical equilibrium state**

$$\frac{\delta S}{k_B} - \beta_0 c^2 \delta M + \alpha_0 \delta N = 0.$$

**A stable thermodynamical equilibrium state is always dynamically stable.**[17]

[17] wrt the Vlasov-Einstein equations in the microcanonical ensemble, J.R. Ipster, Astrophys. J. 238, 1101 (1980).

# Statistical equilibrium state equations

$$\frac{d\Phi}{dr} = -\rho^{-1}\frac{dP}{dr}, \qquad \Phi(r) = -\frac{GM}{r} - 4\pi G\int_r^R \Big(\rho(r)r + \text{modification}\Big)dr.$$

$$f(\mathbf{r},\mathbf{p}) = \frac{g}{h^3}\frac{1}{1+e^{-\alpha_0}e^{\beta(p^2/2m+m\Phi(r))}},$$

$$n(r) = \frac{g}{h^3}\int\frac{d\mathbf{p}}{1+e^{-\alpha_0}e^{\beta(p^2/2m+m\Phi(r))}},$$

$$\epsilon_{kin}(r) = \frac{g}{h^3}\int\frac{p^2/2m\,d\mathbf{p}}{1+e^{-\alpha_0}e^{\beta(p^2/2m+m\Phi(r))}},$$

$$P(r) = \frac{g}{3h^3}\int\frac{p^2/2m\,d\mathbf{p}}{1+e^{-\alpha_0}e^{\beta(p^2/2m+m\Phi(r))}},$$

where $\beta = 1/k_BT$ and $\alpha_0 = \mu_0/k_BT$.
The equation of state for Fermi gas at finite temperature

$$\rho(r) = \frac{4\pi g\sqrt{2}m^{5/2}}{h^3\beta^{3/2}}I_{1/2}[e^{-\alpha_0+\beta m\Phi(r)}],$$

$$P(r) = \frac{8\pi g\sqrt{2}m^{3/2}}{3h^3\beta^{5/2}}I_{3/2}[e^{-\alpha_0+\beta m\Phi(r)}],$$

# Summary and conclusions

- We must be consistent in describing physical systems in different scales
- More research on matter properties in the MG framework is necessary
- Tests of gravity with the use of stars and substellar objects (BD, (exo)-planets, seismology)
- We should consider more realistic models on both sides: gravity and matter - rotating bodies, magnetic fields, ..., opacities (atmosphere)

# Thanks!

awojnar@ucm.es