Neutron stars beyond GR – from theory to astrophysical constraints

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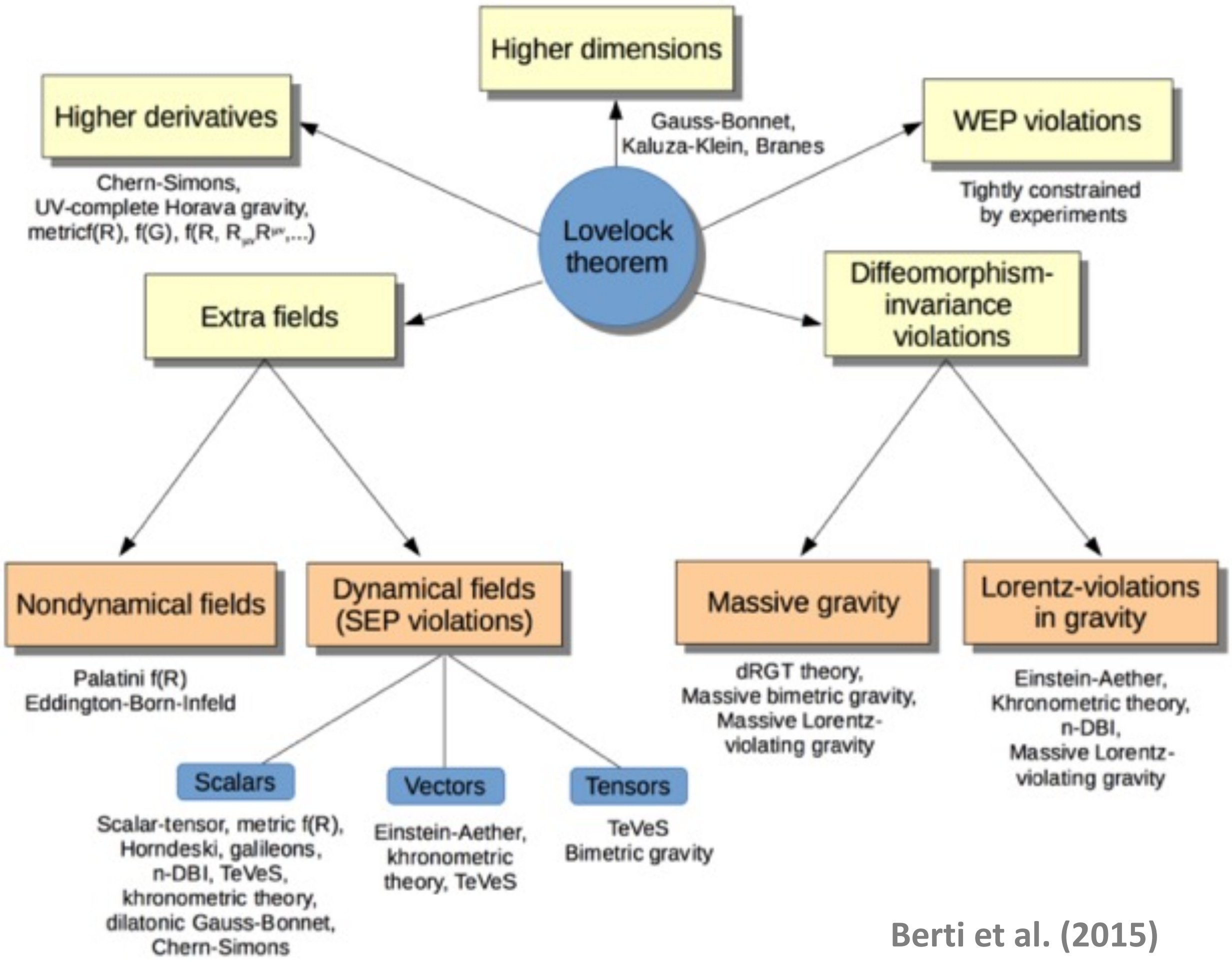
Lovelock's theorem

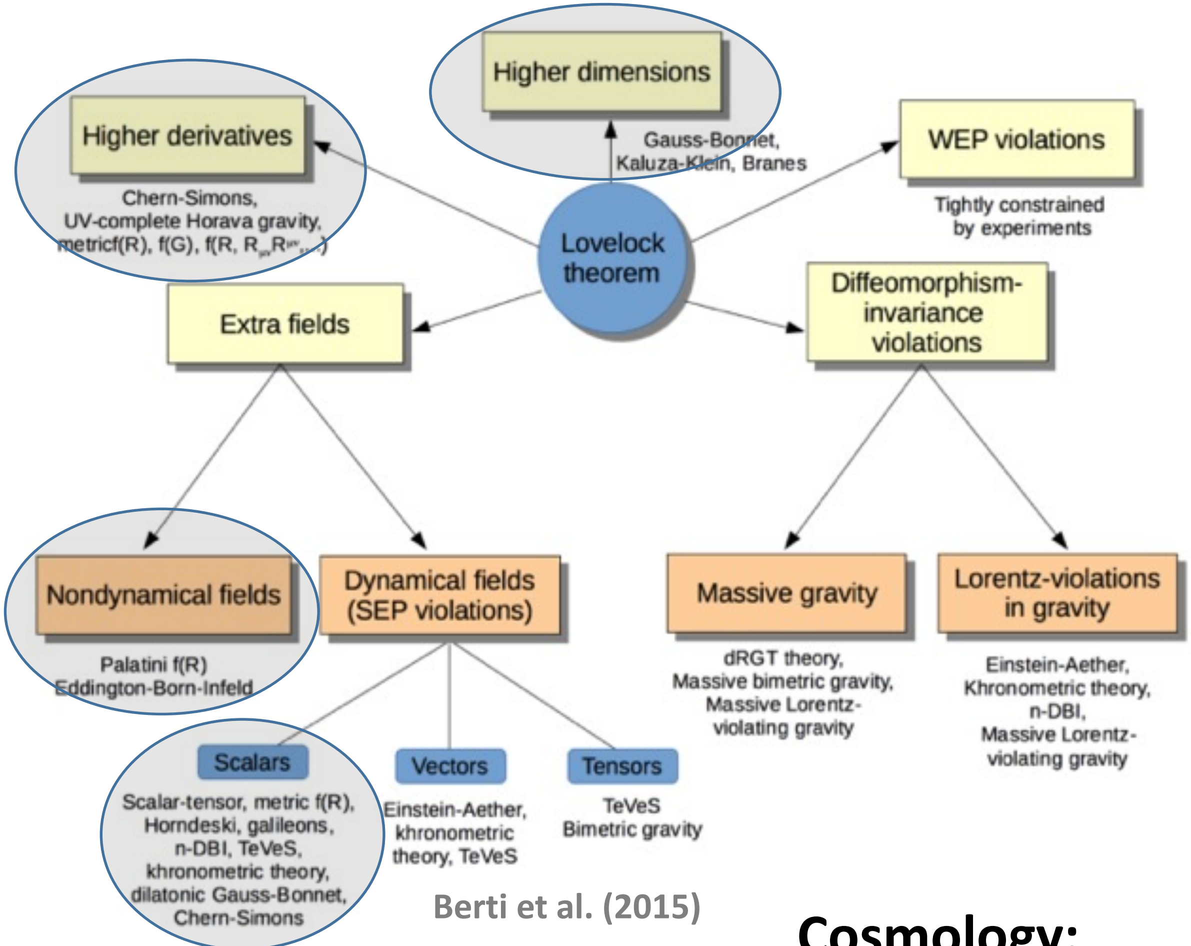
Einstein's field equations are **unique** if:

- ✓ we are working in **four dimensions**
- ✓ **diffeomorphism invariance** is respected
- ✓ the **metric** is the **only field** mediating gravity
- ✓ the equations are **second-order differential equations**.



Berti et al. (2015)

Extra scalar field(s)



Berti et al. (2015)

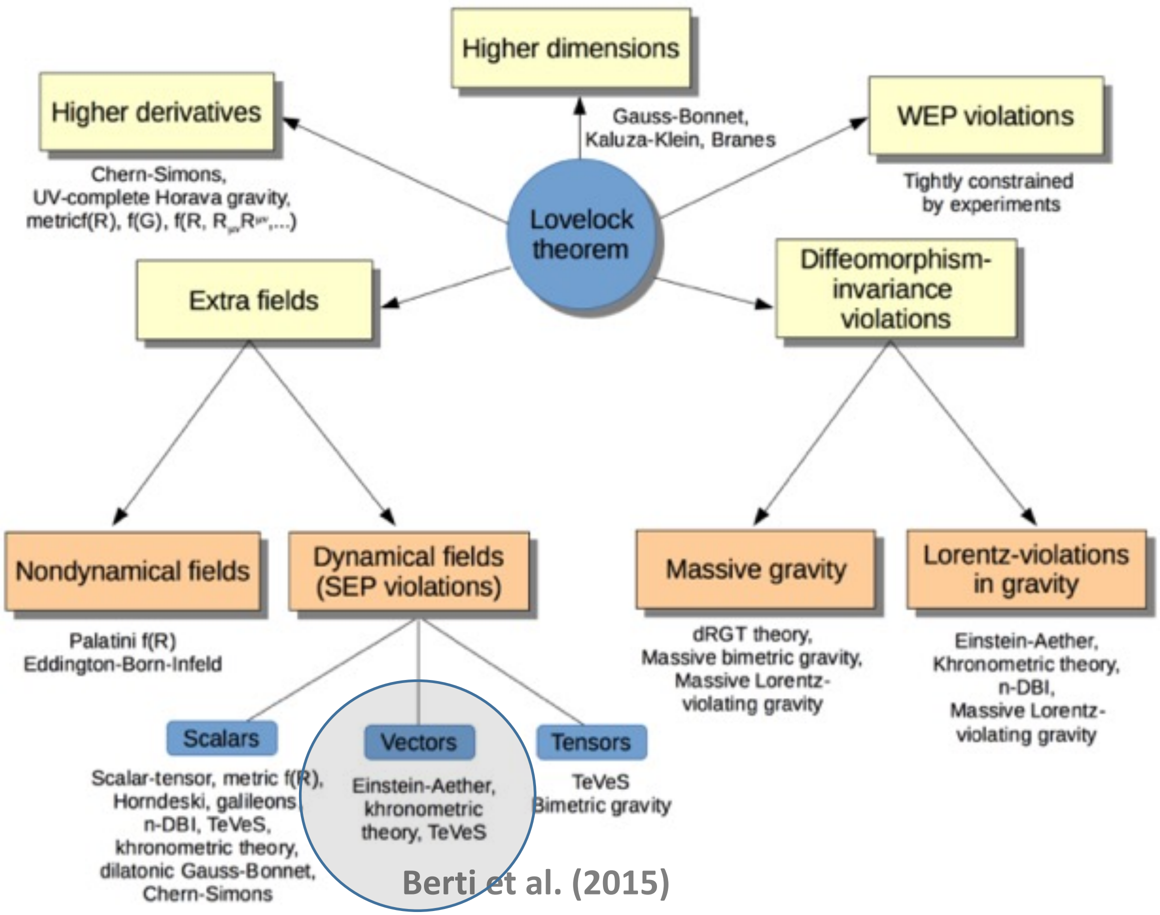
Quantum gravity motivated:

- Gauss-Bonnet gravity
- Chern-Simons gravity

Cosmology:

- Ultralight axion dark matter
- Inflation scalar field
- f(R), Horndeski gravity

Extra scalar field(s)

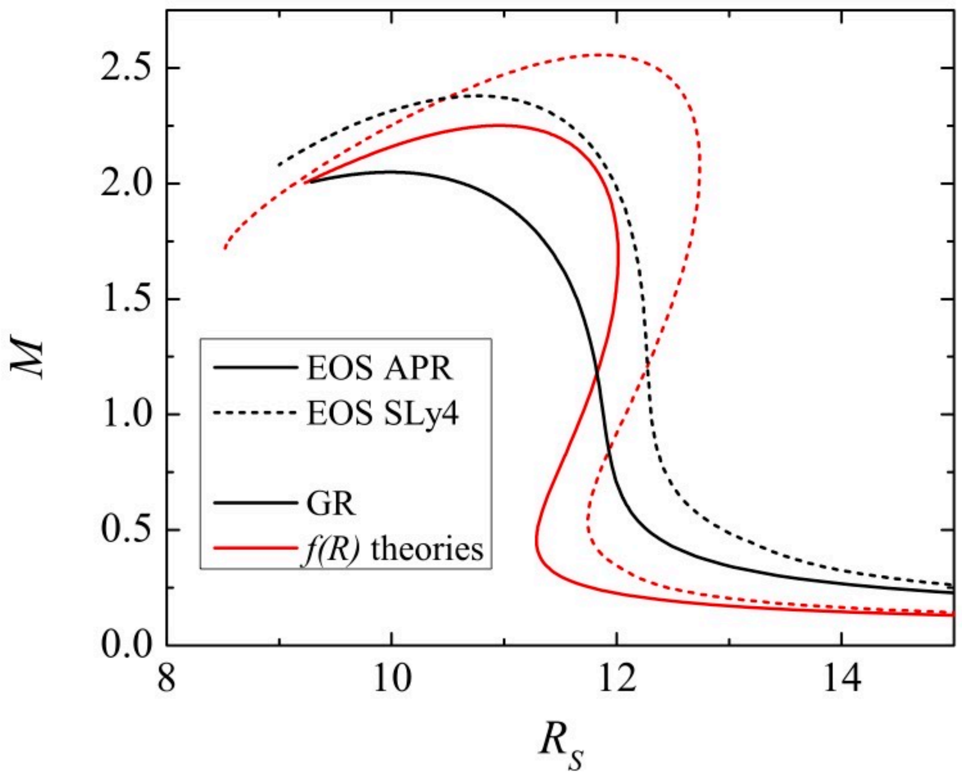


Berti et al. (2015)

Vector fields:

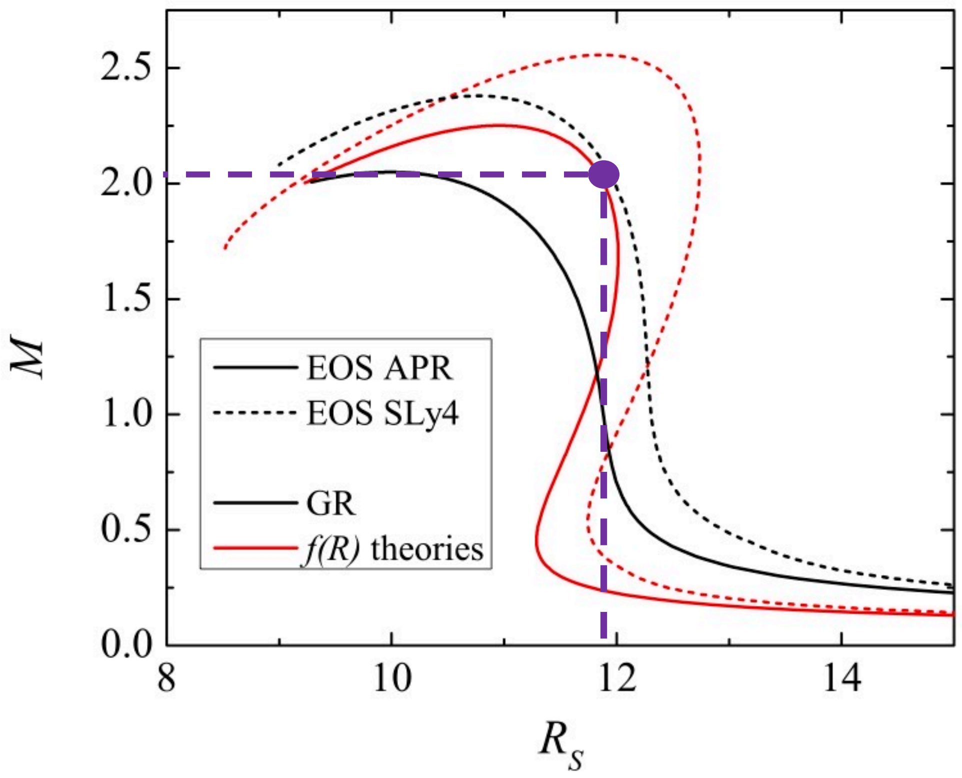
- Instabilities that can not be easily overcome (see Silva, Coates, Ramazanoğlu, Sotiriou (2021))

EOS uncertainty vs Modifying gravity



Modifying the theory of gravity ⟺ EOS uncertainty

EOS uncertainty vs Modifying gravity



Modifying the theory of gravity ⟺ EOS uncertainty

Quantitative vs. Qualitative

Scalarized neutron star models

Scalarization – general idea

- General idea:
  - ✓ **Modified theory** of gravity **possessing** an additional mediator of the gravitational interaction – a **scalar field** $\boldsymbol{\varphi}$
  - ✓ **Perturbative equivalent to GR**
  - ✓ Nonlinear effects for strong fields - **scalarization**

Scalarization – general idea

- **Action in GR**

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{g}\, R + S_m[\psi_m, g_{\mu\nu}]$$

Matter action

Scalarization – general idea

- **Action in GR**

$$S = \frac{1}{16\pi G_*}\int d^4x\,\sqrt{g}\,R + S_m[\psi_m, g_{\mu\nu}]$$

Matter action

- **Action in scalar tensor theories** (scalar field φ): Einstein frame

$$S = \frac{1}{16\pi G_*}\int d^4x\,\sqrt{g}\,[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\varphi\varphi - 4V(\varphi)] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

Kinetic term

Potential term

Coupling term

Scalarization – general idea

- **Scalar field equation**

$$\Box\varphi = -4\pi G_* \alpha(\varphi) T,$$

where $\alpha(\varphi) = \frac{d \ln A(\varphi)}{d\varphi}$

- **Spontaneous scalarization** – the pure **GR** solution is **unstable against scalar perturbations** $\delta\varphi$

$$(\Box - \mu_{\text{eff}}^2)\delta\varphi = 0, \text{ where } \mu_{\text{eff}}^2 = \frac{d\alpha}{d\varphi}|_{\varphi=0} 4\pi G_\star T$$

- If $\boldsymbol{\mu_{\text{eff}}^2 < 0}$ a **tachyonic instability** is present leading to a development of the scalar field

Scalarized neutron stars – DEF model

- **Scalarization of neutron stars** Damour&Esposito-Farese PRL (1993) due to a **nonzero trace** of the energy momentum tensor. **Energetically more favorable** over the GR solutions.

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{g} \left[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\varphi\varphi - 4V(\varphi)\right] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

Kinetic term: $2g^{\mu\nu}\partial_\mu\varphi\partial_\varphi\varphi$

Potential term: $4V(\varphi)$

Coupling term: $A^2(\varphi)$

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Kinetic term

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Kinetic term

Coupling term

- **Coupling function** – polynomial expansion in φ

$$\alpha(\varphi) = \frac{d\ln\big(A(\varphi)\big)}{d\varphi} = \alpha_0 + \beta_0\varphi$$

- Reminder: $\Box\varphi = -4\pi G_*\alpha(\varphi)T$

Scalarized neutron stars – DEF model

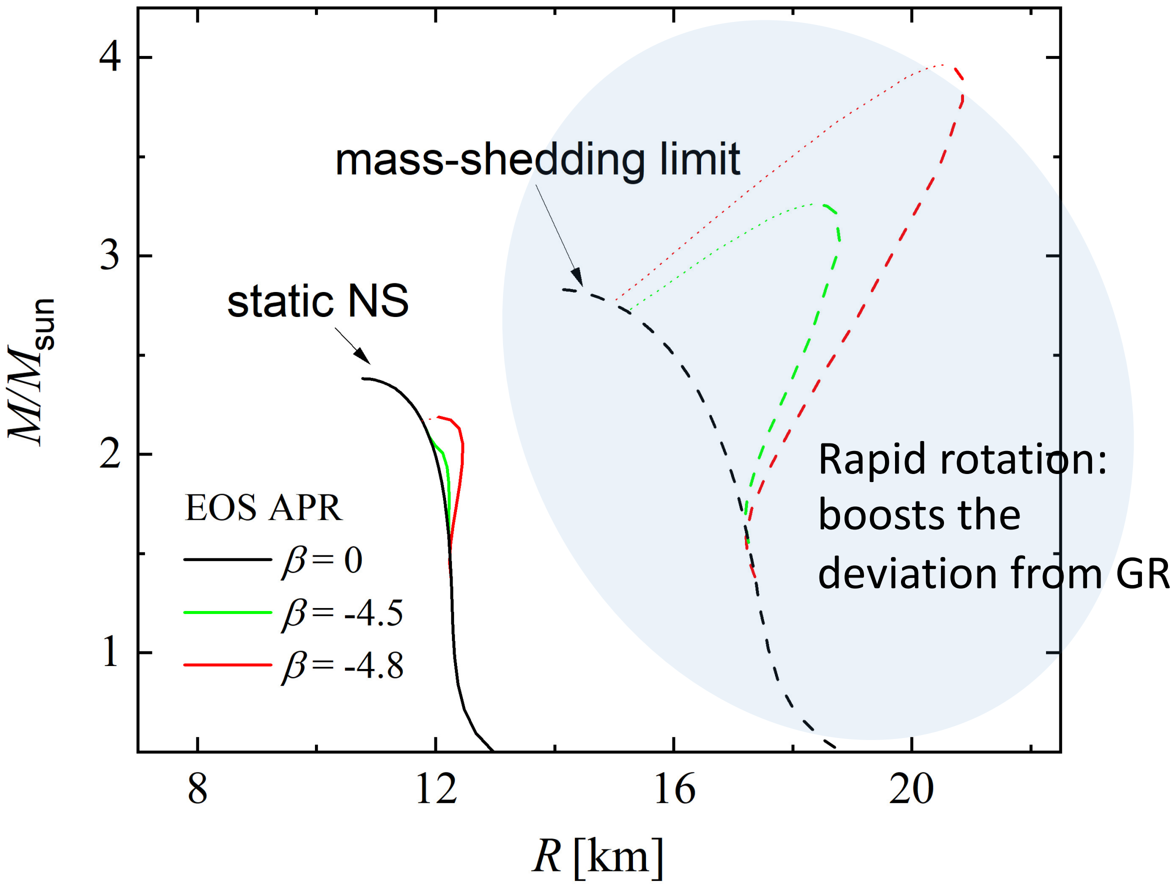
$$\alpha(\varphi) = \alpha_0 + \cancel{\beta_0}\varphi$$

- **Brans-Dicke theory** – $\varphi = 0$ NOT a solutions, **ruled out** by weak field observations

Scalarized neutron stars – DEF model

$\alpha(\varphi) = \cancel{\alpha_0} + \beta_0\varphi$ (reminder $\mu_{\text{eff}}^2 = \frac{d\alpha}{d\varphi}|_{\varphi=0} 4\pi G_\star T$<0)

- Original DEF model Damour&Esposito-Farese (1993)

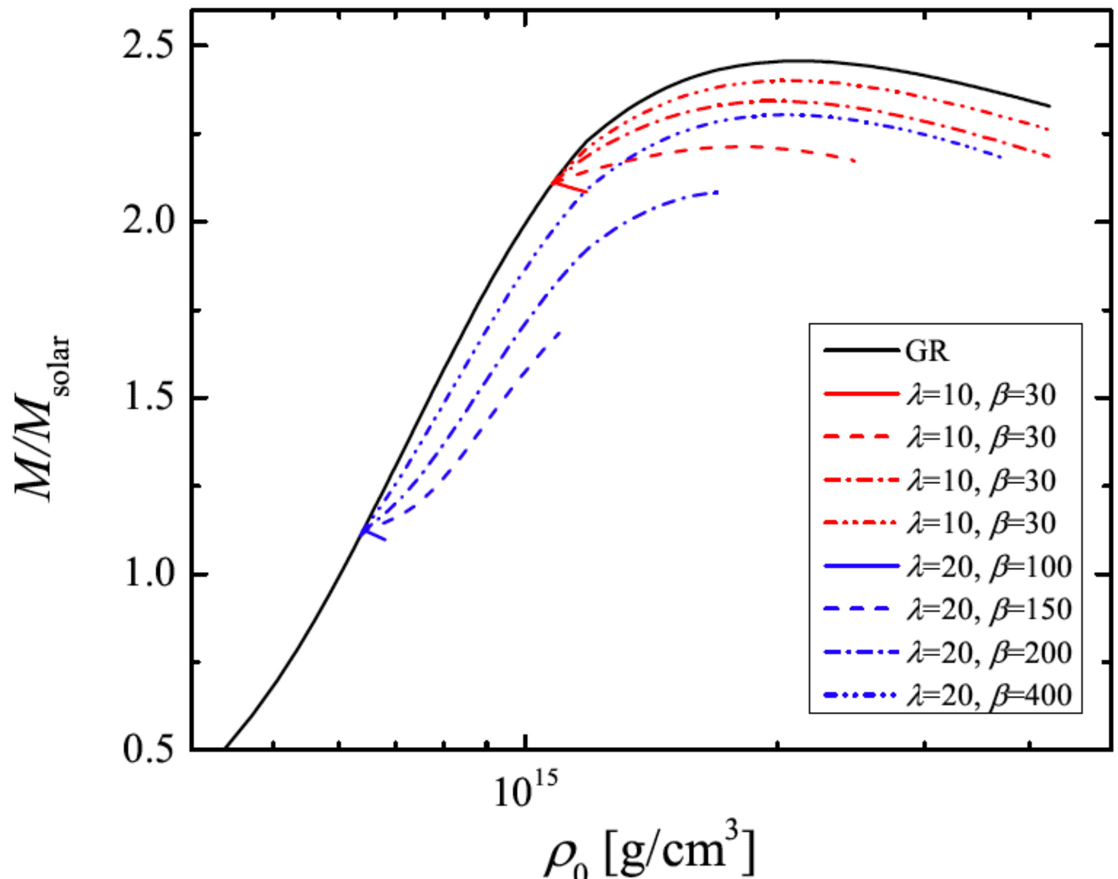


DD, Yazadjiev, Stergioulas, Kokkotas (2013,2014)

NS scalarization in Gauss-Bonnet gravity

- Scalar field trigerred by the curvate itself through R_{GB}^2

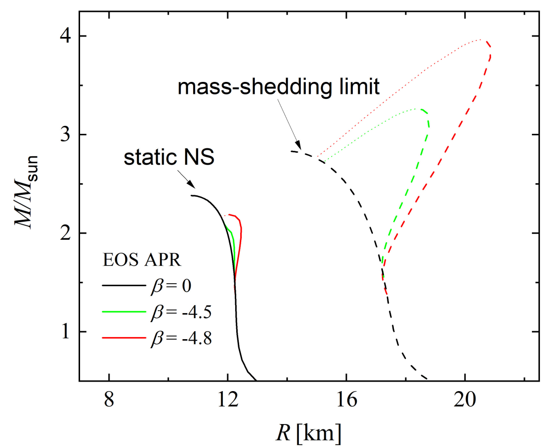
$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right] + S_{\rm matter}(g_{\mu\nu},\chi)$$



DD, Yazadjiev JCAP (2019)

$$f(\varphi) = \frac{1}{2\beta}\left[\exp(-\beta\varphi^2) - 1\right]$$

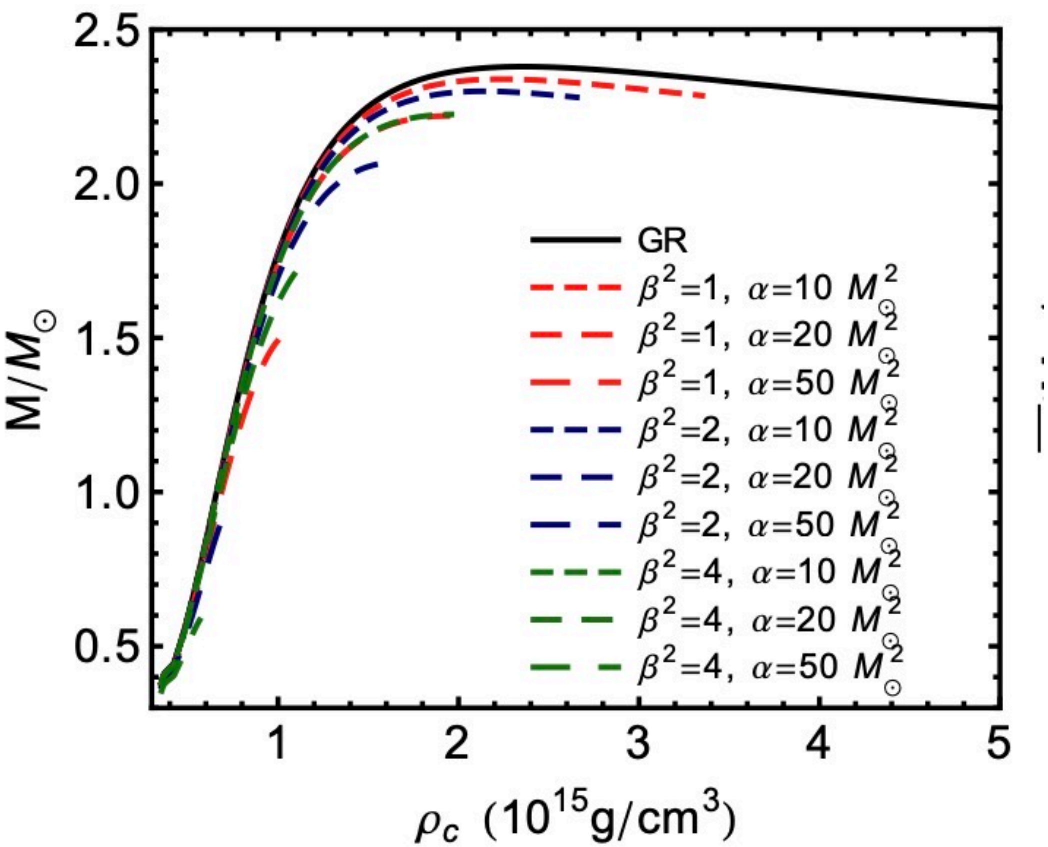
DEF model



Other NS models beyond GR

NS in Gauss-Bonnet gravity WITHOUT scalarization

- **Scalarization not possible** – NS have always scalar field and $\varphi = 0$ not a solution of the field equations
- **Static models** Pani at al (2011) and **rapid rotation** Kleinhaus at al (2016)



Pani at al (2011)

$$f(\varphi) = \frac{\alpha}{16\pi} e^{\beta\varphi}$$

Other NS models beyond GR

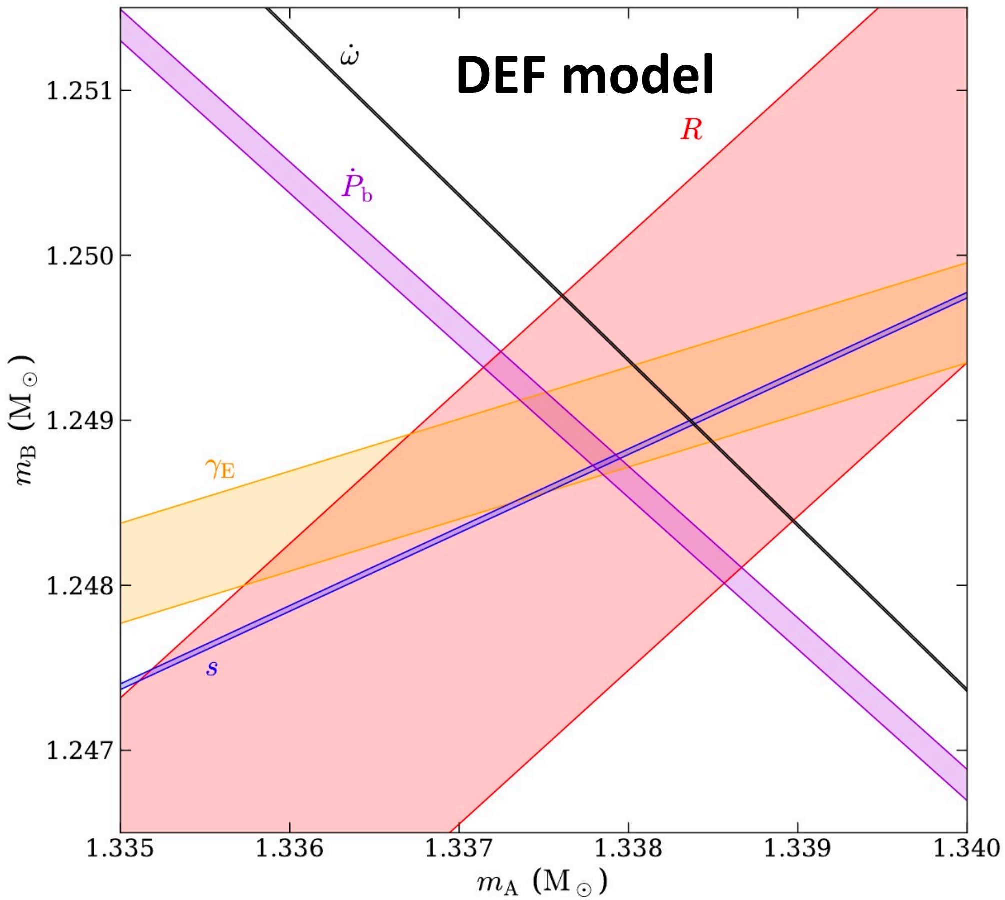
A selected list of theories with NS models:

- dynamcial Chern-Simons gravity **Yunes at al PRD (2009)**
- k-essence theories **Bezares at al PRL (2022)**
- $f(R)$ gravity **DD, Yazadjiev JCAP (2014)**
- Tensor multi-scalar theories **Horbatsch CQG (2015), DD,Yazadjiev PRD (2020)**

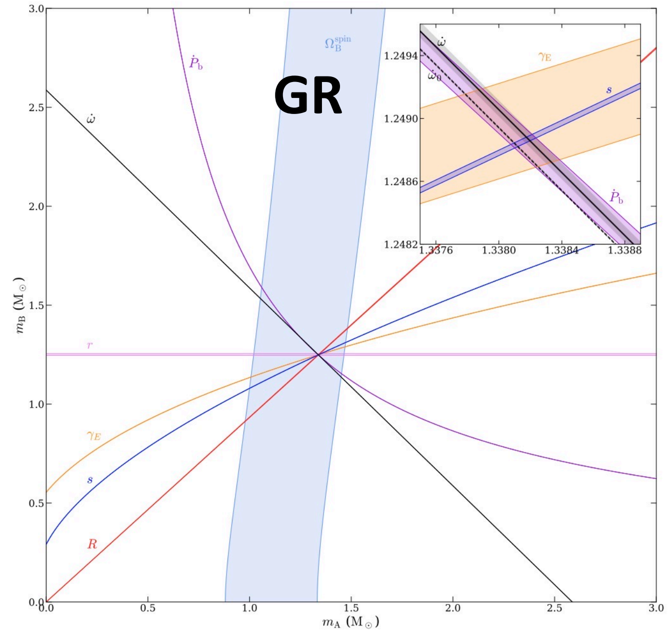
Binary pulsar constraints

Binary pulsar constraints

- Lines fail to intersect in a single region (DEF model, $\alpha_0 = 5\times10^{-2}, \beta = -4$)



vs.



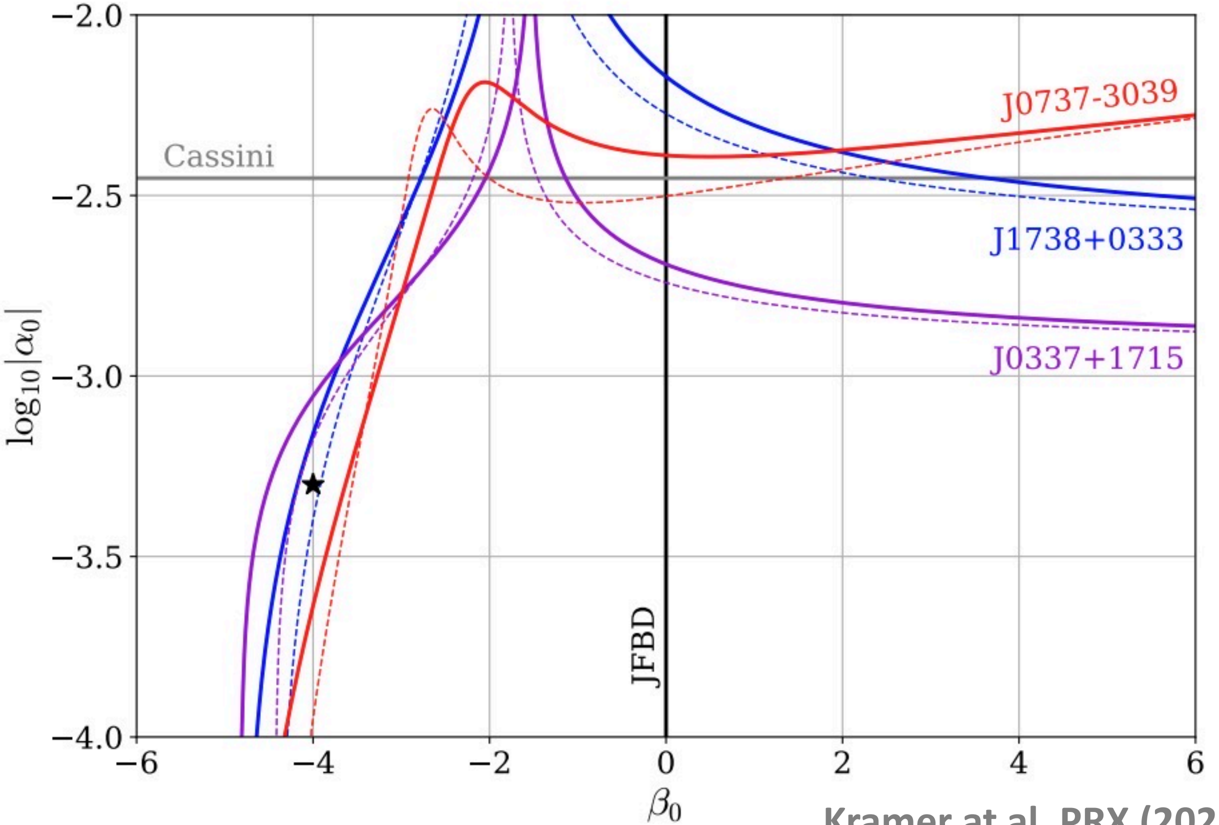
Kramer at al. PRX (2021)

Binary pulsar constraints – DEF model

- Scalarization window closed for the original DEF model Zhao et al. CCQ (2022)

$$S = \frac{1}{16\pi G_*}\int d^4x\,\sqrt{g}\,[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\varphi\varphi] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

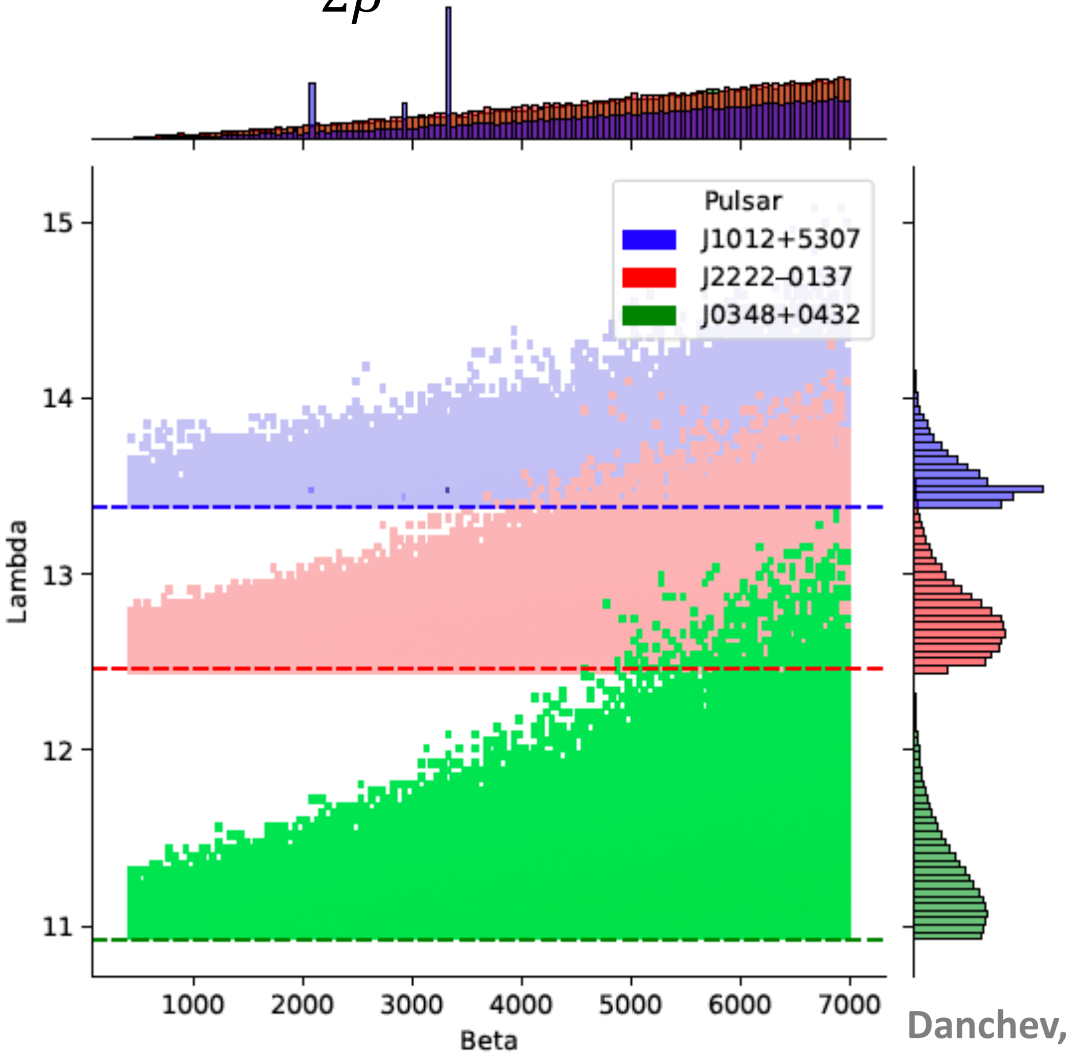
$$\alpha(\varphi) = d\ln A(\varphi)\,/d\varphi = \alpha_0 + \beta_0\varphi$$



Kramer at al. PRX (2021)

Binary pulsar constraints – Gauss-Bonnet gravity

$$f(\varphi) = \frac{\lambda^2}{2\beta}\left(1 - e^{-\beta\varphi^2}\right)$$



Danchev, DD, Yazadjiev PRD (2022)

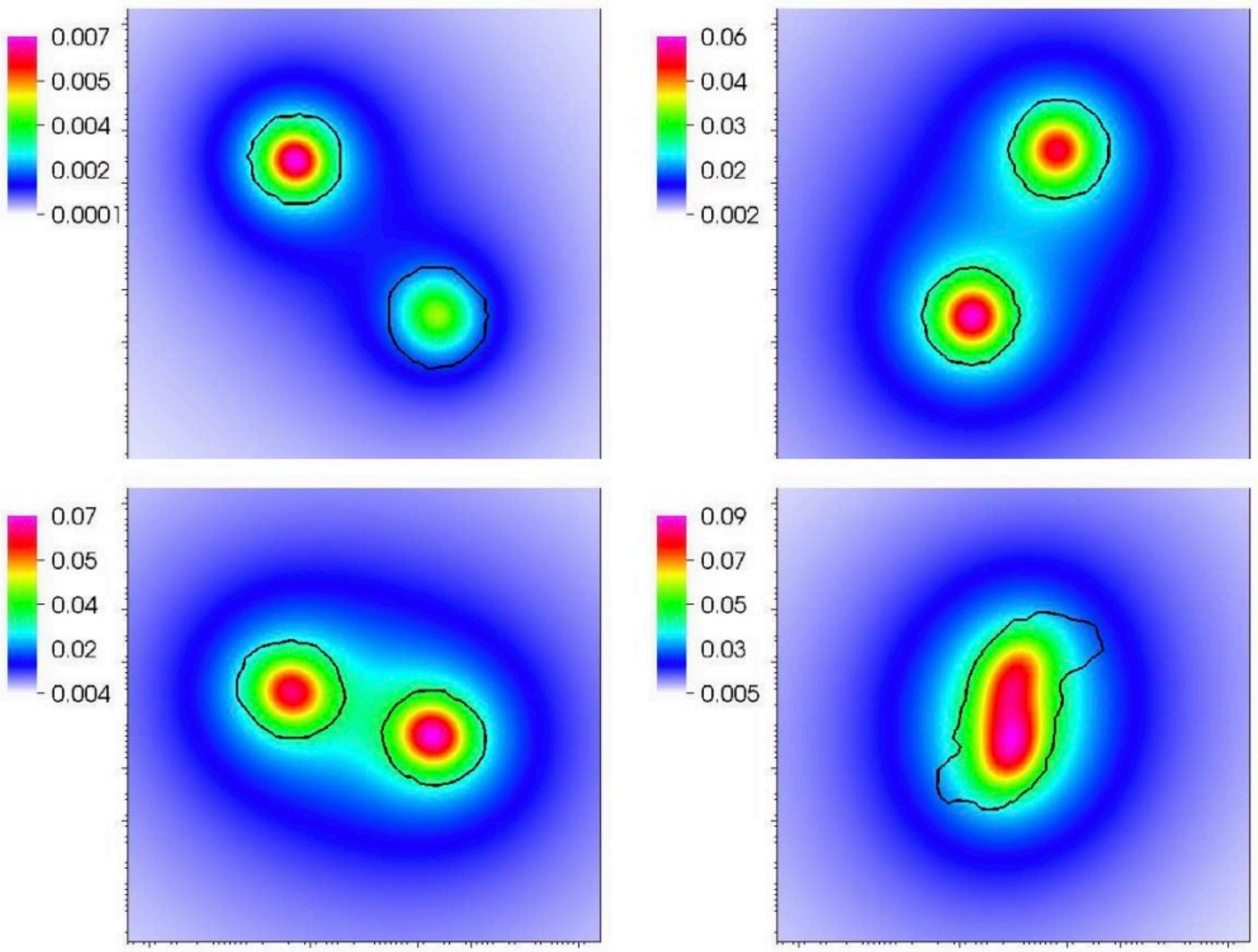
Evading binary pulsar constraints

- A way out:
  - ✓ **massive scalar field, e.g.** $V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \lambda \varphi^4$

    Ramazanoglu,Pretorius(2016), Yazadjiev,DD(2016), Rosca-Mead et al. (2020)
  - ✓ $\varphi|_{r\to\infty} \sim 1/r^2$ (e.g. tensor-multi scalar theories)
  - ✓ **rapid rotation**
  - ✓ etc.

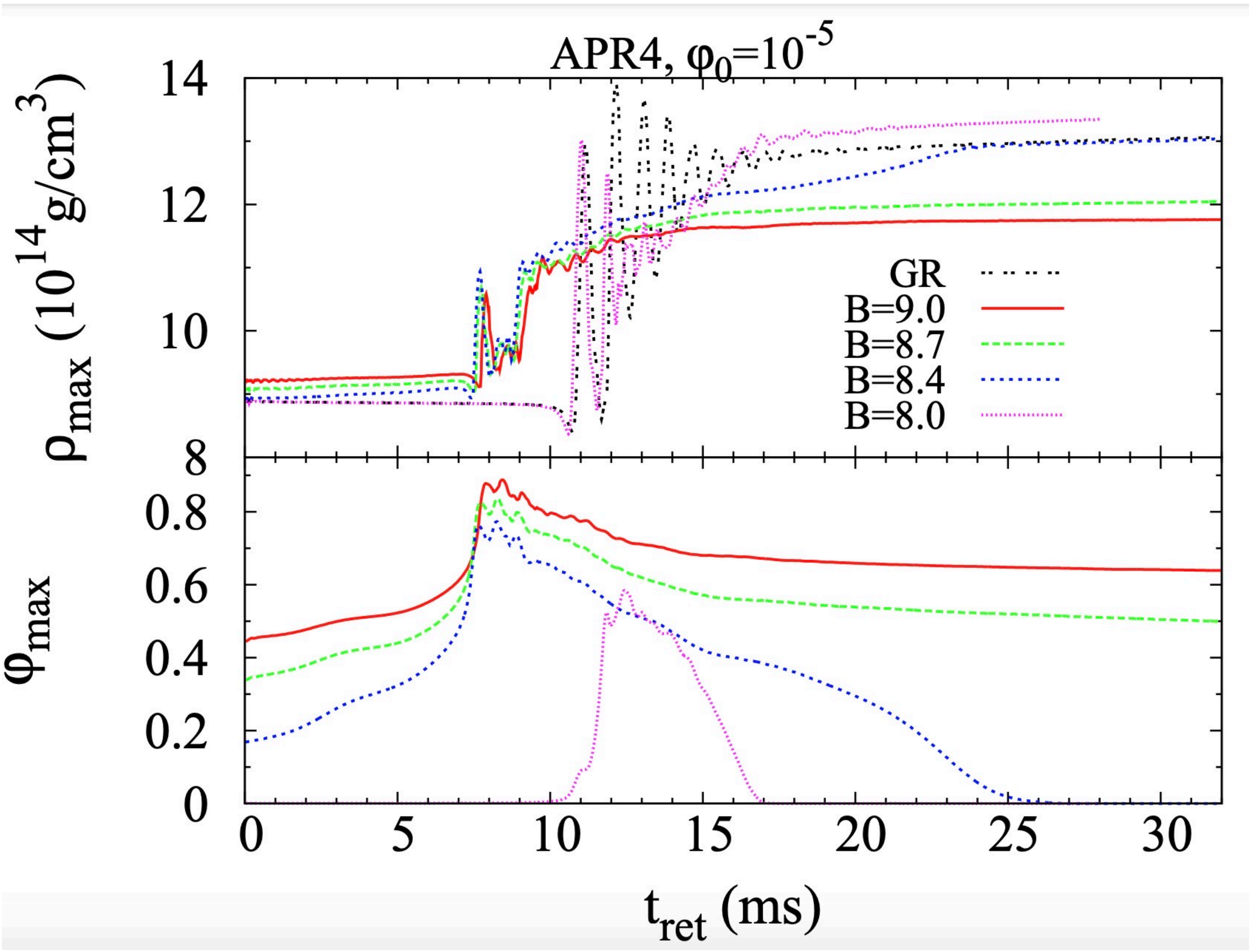
Binary mergers

Dynamical scalarization – DEF model



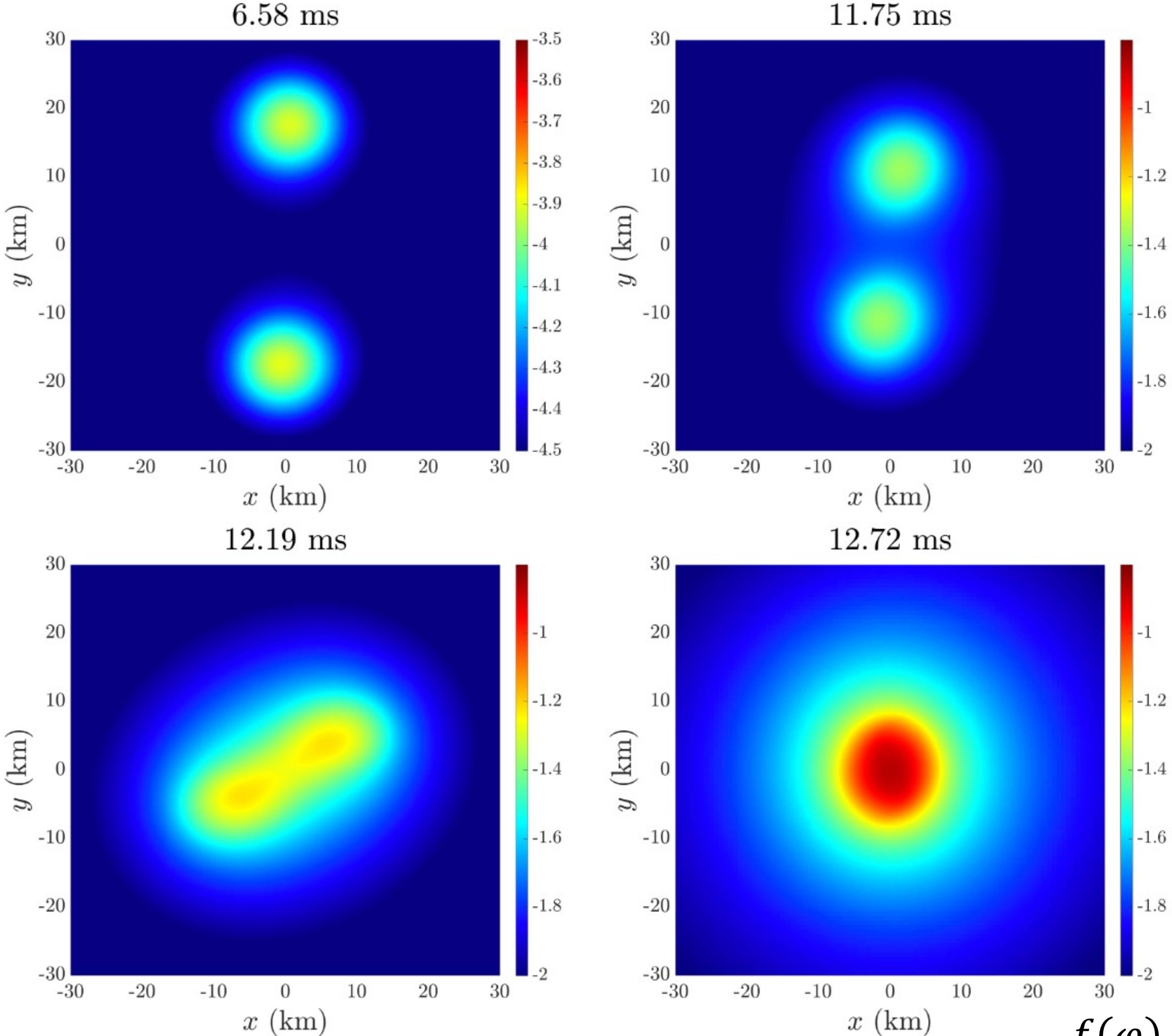
Barausse at al. PRD (2012)

Dynamical scalarization – Post merger evolutions



Shibata et al. PRD (2013)

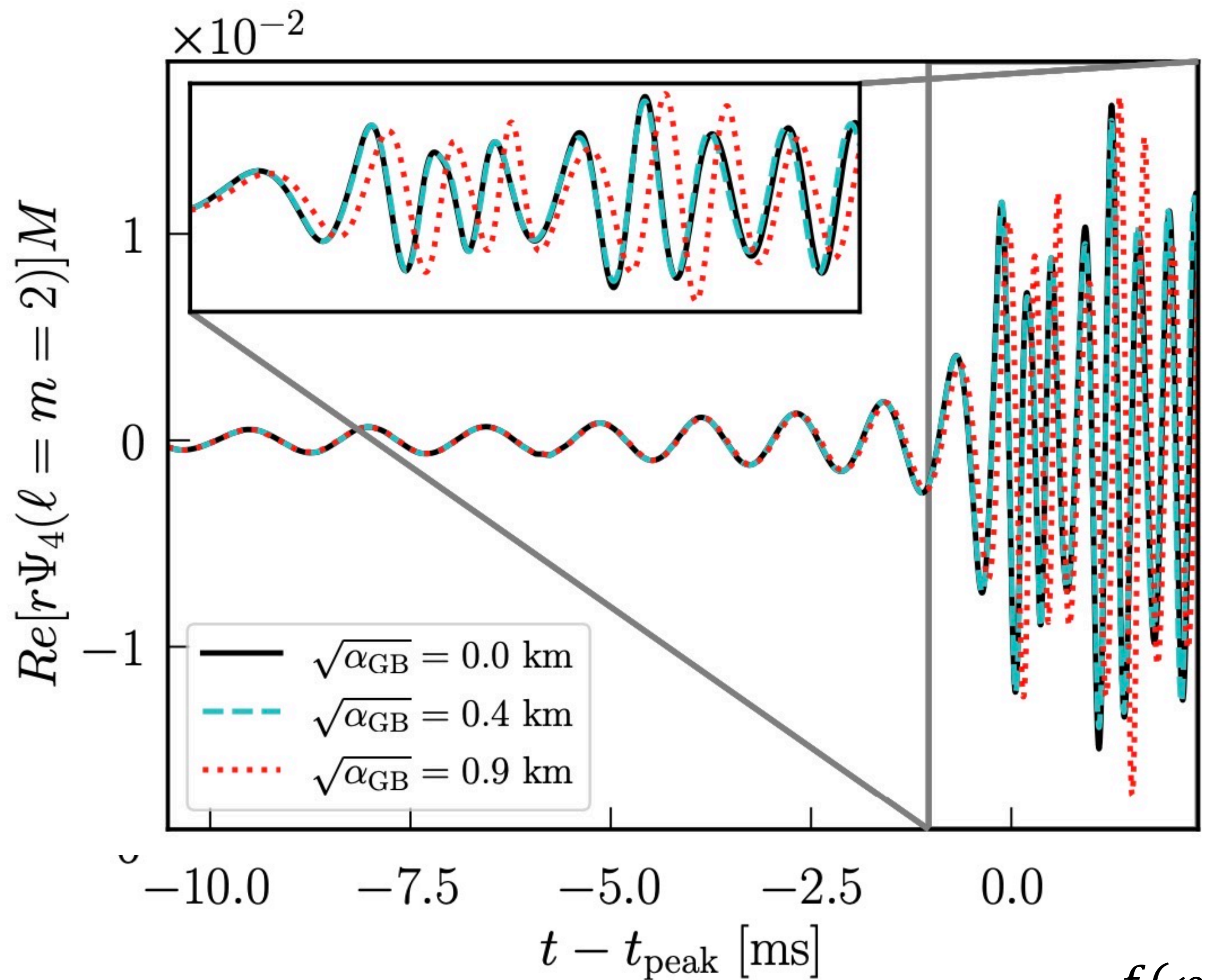
Dynamical scalarization – Gauss-Bonnet theory



$$f(\varphi) = \frac{\lambda^2}{2\beta}\left(1 - e^{-\beta\varphi^2}\right)$$

Kuan, Lam, DD, Yazadjiev, Shibata, Kiuchi (2023)

Shift symmetric Gauss-Bonnet theory – full theory evolution



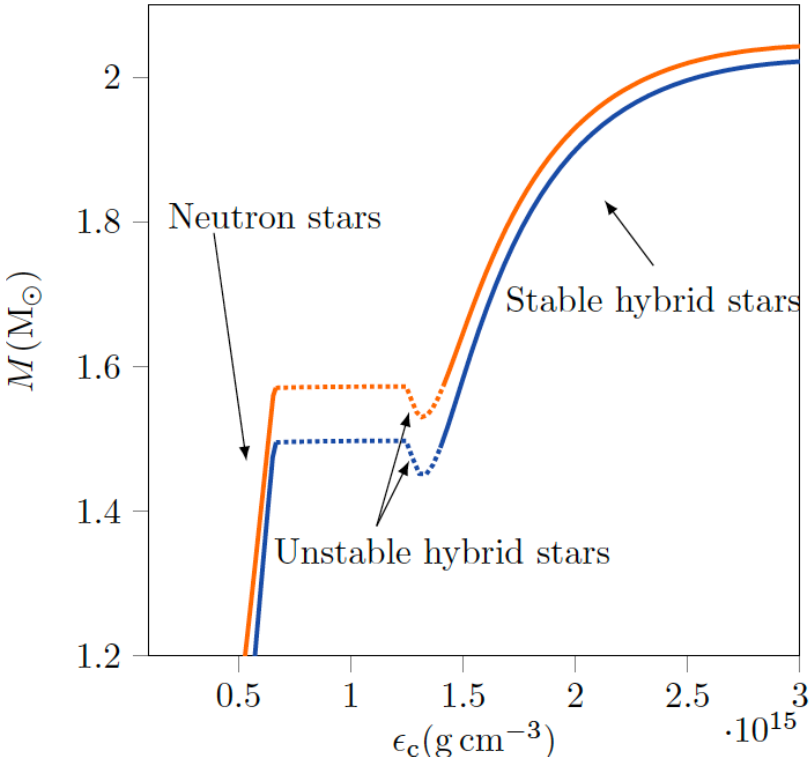
$f(\varphi) = \lambda^2 \varphi$

East, Pretorius PRD (2022)

Qualitative changes:
Gravitational phase transitions

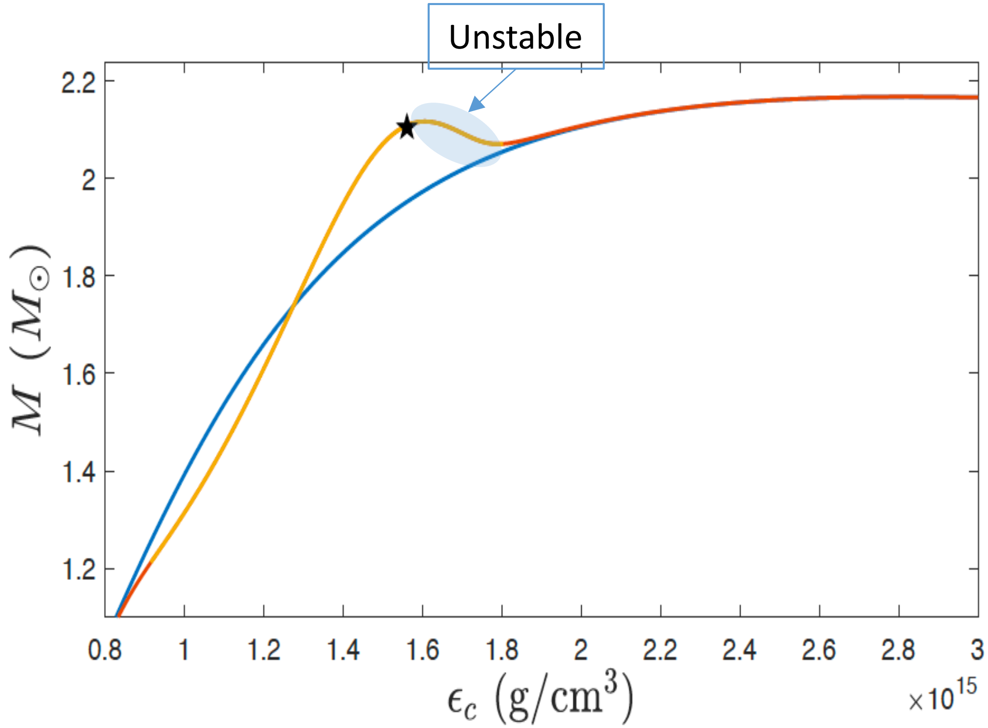
Twin Stars vs. Scalarization

Twin stars



Espino, Paschalidis (2021)

Gravitationally induced phase transition



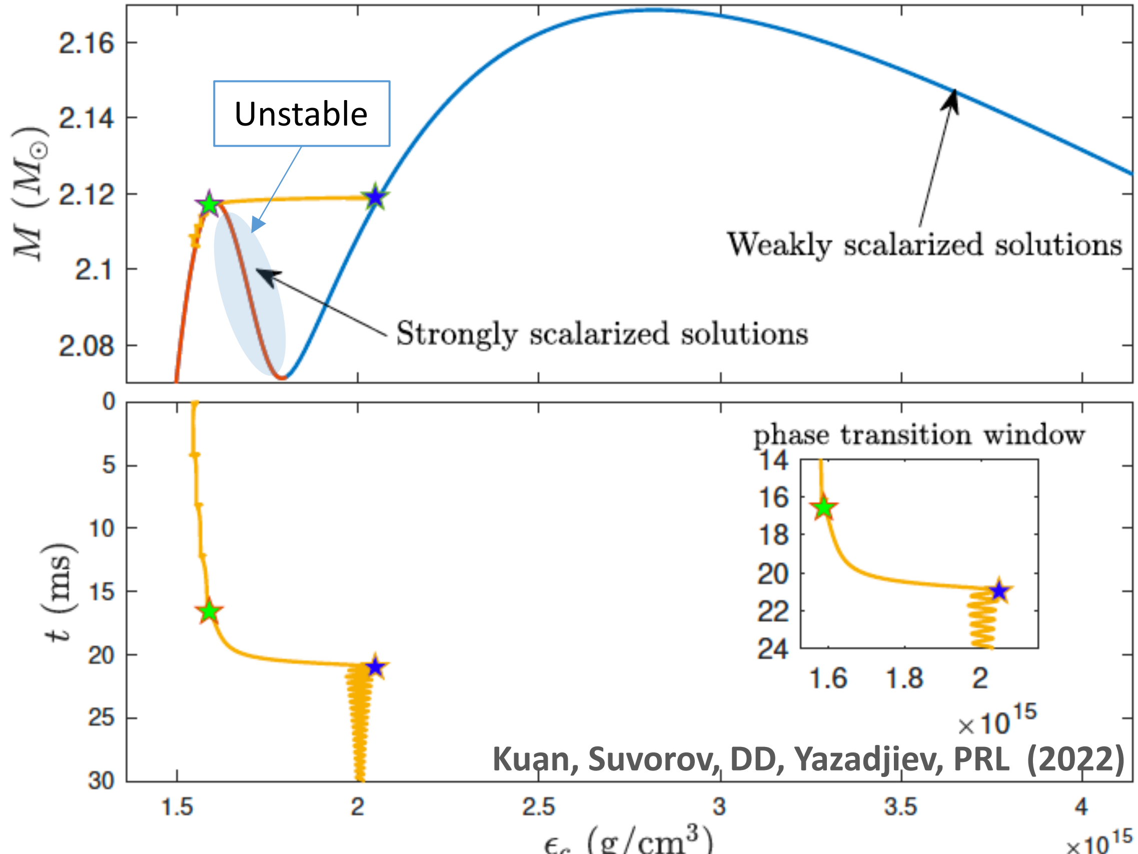
Kuan, Suvorov, DD, Yazadjiev, PRL (2022)

Gravitational phase transition

- **DEF model with a massive scalar field**

$$S = \frac{1}{16\pi G_*}\int d^4x\,\sqrt{g}\,[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\varphi\varphi - 4V(\varphi)] + S_m[\psi_m, A^2(\varphi)g_{\mu\nu}]$$

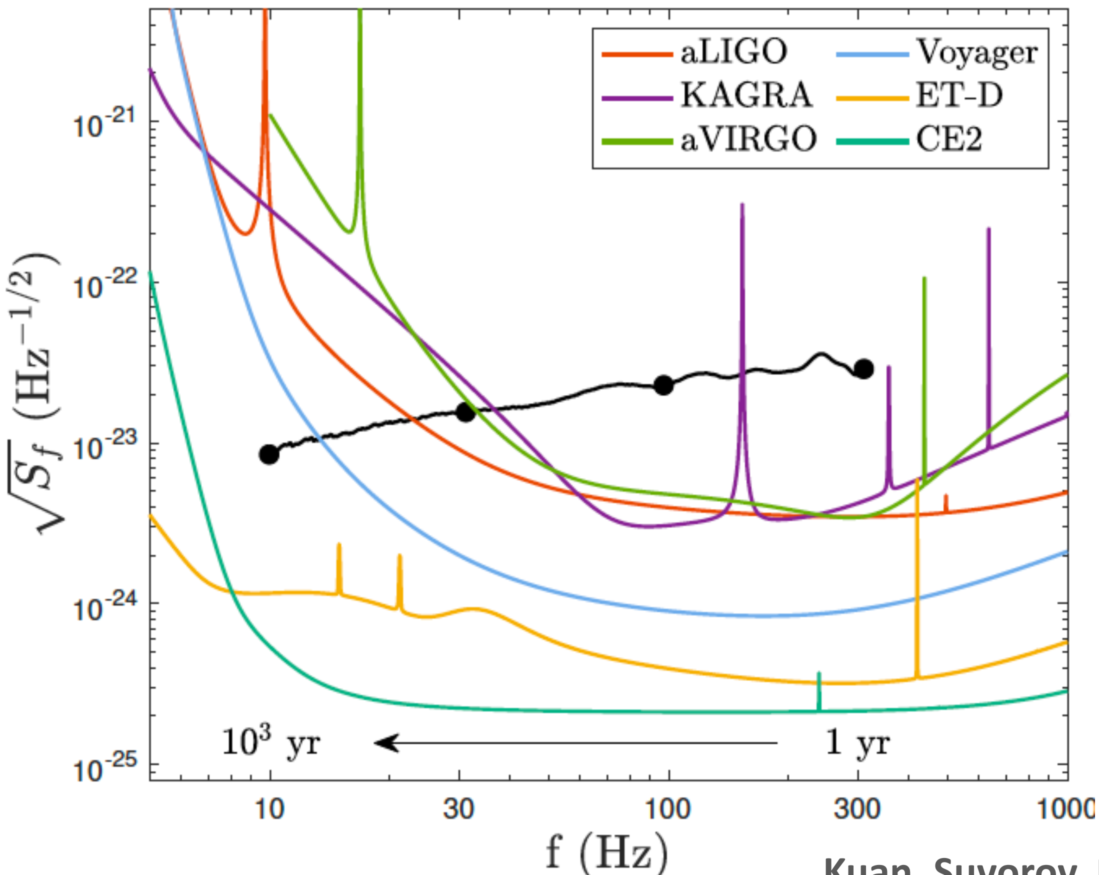
- $V(\varphi) \neq 0$ in order to **avoid binary pulsar constraints** Zhao et al. (2022)



Kuan, Suvorov, DD, Yazadjiev, PRL (2022)

Effective power-spectral density

- Spherically symmetric perturbations ⇒ emission of **breathing modes**
- Observational period 2 months, distance 10kpc



Kuan, Suvorov, DD, Yazadjiev, PRL (2022)

Conclusions

- Neutron stars are **ideal laboratories** to test modified gravity with a plethora of observations
- **Matter uncertainties** hinder our abilities BUT it is **easier** to deviate from GR **compared to black holes**
- Some of the **best GR constraints** come from **neutron star** observations
- Even though there is a plethora of equilibrium models there are **very few rotating ones** and **dynamics is highly unexplored.**

THANK YOU!