Model-Independent Searches For Anisotropic Stochastic Gravitational Wave Background

Jishnu Suresh

Centre for Cosmology, Particle Physics and Phenomenology - CP3

Université catholique de Louvain

BelGrav UCLouvain

Belgian-Dutch Gravitational Wave Meeting, 2022

IRMP

Stochastic Gravitational-Wave Background

A stochastic gravitational-wave background (SGWB) is created by the superposition of individually undetectable signals.

- individually undetectable (sub-threshold)
- but detectable as a collectivity via their common influence on multiple detectors
- combined signal described statistically — stochastic gravitational-wave background

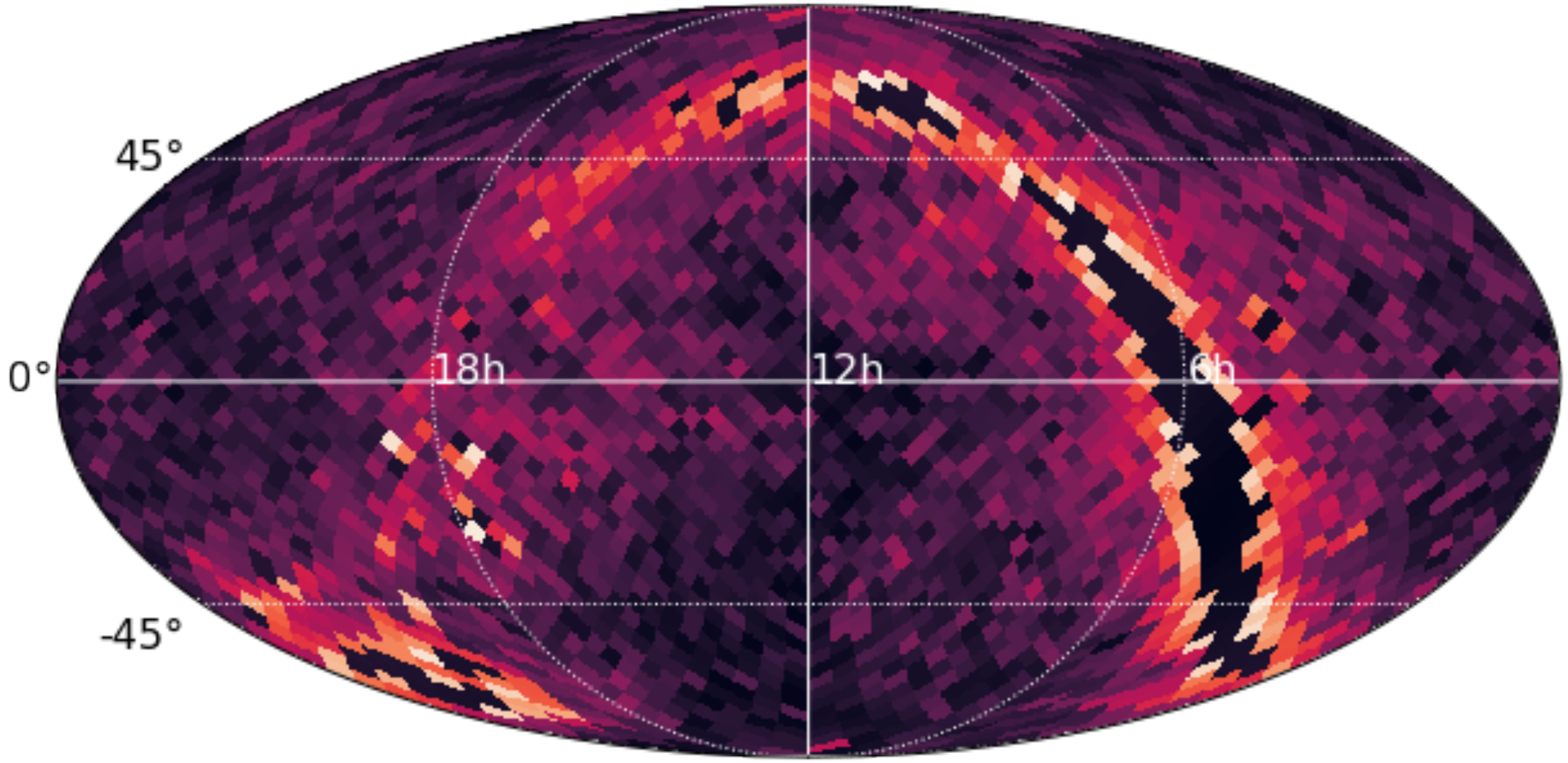
Constraints have been set on the total energy density of SGWB integrated over the sky, as a function of frequency.

[Federico's talk]

Anisotropic Stochastic Gravitational-Wave Background

1 There may be inhomogeneities in the GW source mechanisms, for example a particular distribution of the sources on the sky, which produces an anisotropic signal.

2 As gravitational-wave propagate, they accumulate line-of-sight effects, crossing different matter density fields which are inhomogeneously distributed in the Universe.



The anisotropies can be characterized in terms of a dimensionless GW energy density parameter

$$\Omega_{\mathrm{GW}}(f, \hat{n}) = \frac{2\pi^2}{3H_0^2} f^3 \mathscr{P}(f, \hat{n})$$

We assumes $\mathscr{P}(f, \hat{n}) = H(f)\, \mathscr{P}(\hat{n})$

Spectral Index

In most of our analyses, we model the spectral dependence in terms of power-laws

$$H(f) = \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha-3}$$

$\alpha = 0$ —> consistent with many cosmological models, such as slow roll inflation and cosmic strings;
$\alpha = 2/3$ —> compatible with an astrophysical background dominated by compact binary inspirals ; and
$\alpha = 3$ —> indicating a generic flat strain spectrum.

The anisotropies can be characterized in terms of a dimensionless GW energy density parameter

$$\Omega_{\mathrm{GW}}(f,\hat{n}) = \frac{2\pi^2}{3H_0^2} f^3 \mathscr{P}(f,\hat{n})$$

We assumes

$$\mathscr{P}(f,\hat{n}) = H(f)\,\mathscr{P}(\hat{n})$$

In most of our analyses, we model the spectral dependence in terms of power-laws

$$H(f) = \left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha-3}$$

Spectral Index

Reference frequency (25 Hz in the O3 analysis)

Cross-Correlation estimators $\hat{S}_h = \int_{-T/2}^{T/2} \mathrm{d}t \int_{-T/2}^{T/2} \mathrm{d}t'\, d_1(t)\, d_2(t')\, Q(t,t')$ an arbitrary filter

Choose Q to maximize signal-to-noise ratio for fixed spectral shape

In usual stochastic searches, we assume a fiducial model for the spectral shape and perform the optimal filtering.

$$\tilde{Q}(f) \propto \frac{\gamma^{I*}_{ft,u}\, H(f)}{P_{\mathcal{I}_1}(t;f)\, P_{\mathcal{I}_2}(t;f)}$$

weight the frequencies which agree with the expected signal spectrum.

de-weight the frequencies that correspond to large detector noise.

Stochastic Gravitational-Wave Background search strategies

- ~~Isotropic stochastic gravitational wave background search~~ [Federico's talk]
- Anisotropic stochastic gravitational wave searches are run as three different cases:
  - **Broadband radiometer**: point sources with different power-law spectra (spectral shape α = 0, 2/3, 3)
  - **Narrowband radiometer**: point sources having narrow GW frequency band (SN 1987A, ScoX-1, GC)
  - **Spherical harmonics search**: Extended or diffuse sources - measure angular power spectra

			Upper limit ranges (10^{-8})		Upper limit range (10^{-9})	
α	$\Omega_{\rm GW}$	$H(f)$	O1+O2+O3 (HLV)	O1 + O2 (HL)	O1+O2+O3 (HLV)	O1 + O2 (HL)
0	constant	$\propto f^{-3}$	1.7 – 7.6	4.4 – 21	3.2–9.3	7.8–29
2/3	$\propto f^{2/3}$	$\propto f^{-7/3}$	0.85 – 4.1	2.3 – 12	2.4–9.3	6.4–25
3	$\propto f^{3}$	constant	0.013 – 0.11	0.046 – 0.32	0.57–3.4	1.9–11

Direction	Best upper limit (10^{-25})	Frequency band (Hz)
Scorpius X-1	2.1	189.31 – 190.31
SN 1987A	1.7	185.13 – 186.13
Galactic Center	2.1	202.56 – 203.56

O3 directional paper, R. Abbott et. al - Phys. Rev. D 104, 022005 (2021)

The anisotropies can be characterized in terms of a dimensionless GW energy density parameter

$$\Omega_{\mathrm{GW}}(f,\hat{n}) = \frac{2\pi^2}{3H_0^2} f^3 \mathscr{P}(f,\hat{n})$$

We assumes $\mathscr{P}(f,\hat{n}) = H(f)\,\mathscr{P}(\hat{n})$

If we free this assumption, then we are interested in estimating the SGWB anisotropies at every frequency!

$$\mathscr{P}(f,\hat{n}) = \sum_p \mathscr{P}_p(f)\, e_p(\hat{n})$$

where the choice of the basis function $e_p(\hat{n})$ is done based on the source properties.

Following the standard Maximum Likelihood estimators as the statistics, one can write

In pixel-basis $\hat{\mathscr{P}}(f, \hat{n}) = \Gamma^{-1}_{\hat{n},\hat{n}'}(f)\,.\,X_{\hat{n}'}(f)$

In spherical harmonic-basis $\hat{\mathscr{P}}_{lm}(f) = \Gamma^{-1}_{lm,lm}(f)\,.\,X_{lm}(f)$

Dirtymap
$$X_p(f) \;=\; \sum_{\mathscr{I}t} \frac{\gamma^{\mathscr{I}*}_{ft,p}\, C^{\mathscr{I}}(t;f)}{P_{\mathscr{I}_1}(t;f)\, P_{\mathscr{I}_2}(t;f)}$$

$$\gamma^{\mathscr{I}}_{ft,p} :\equiv \sum_A F^A_{\mathscr{I}_1}(\hat{\mathbf{n}}_p, t)\, F^A_{\mathscr{I}_2}(\hat{\mathbf{n}}_p, t)\, e^{2\pi i f\, \hat{\mathbf{n}}_p\cdot\Delta\mathbf{x}_{\mathscr{I}}(t)/c}$$

Overlap Reduction Function

Fisher matrix
$$\Gamma_{pp'}(f) \;=\; \sum_{\mathscr{I}t} \frac{\gamma^{\mathscr{I}*}_{ft,p}\, \gamma^{\mathscr{I}}_{ft,p'}}{P_{\mathscr{I}_1}(t;f)\, P_{\mathscr{I}_2}(t;f)}$$

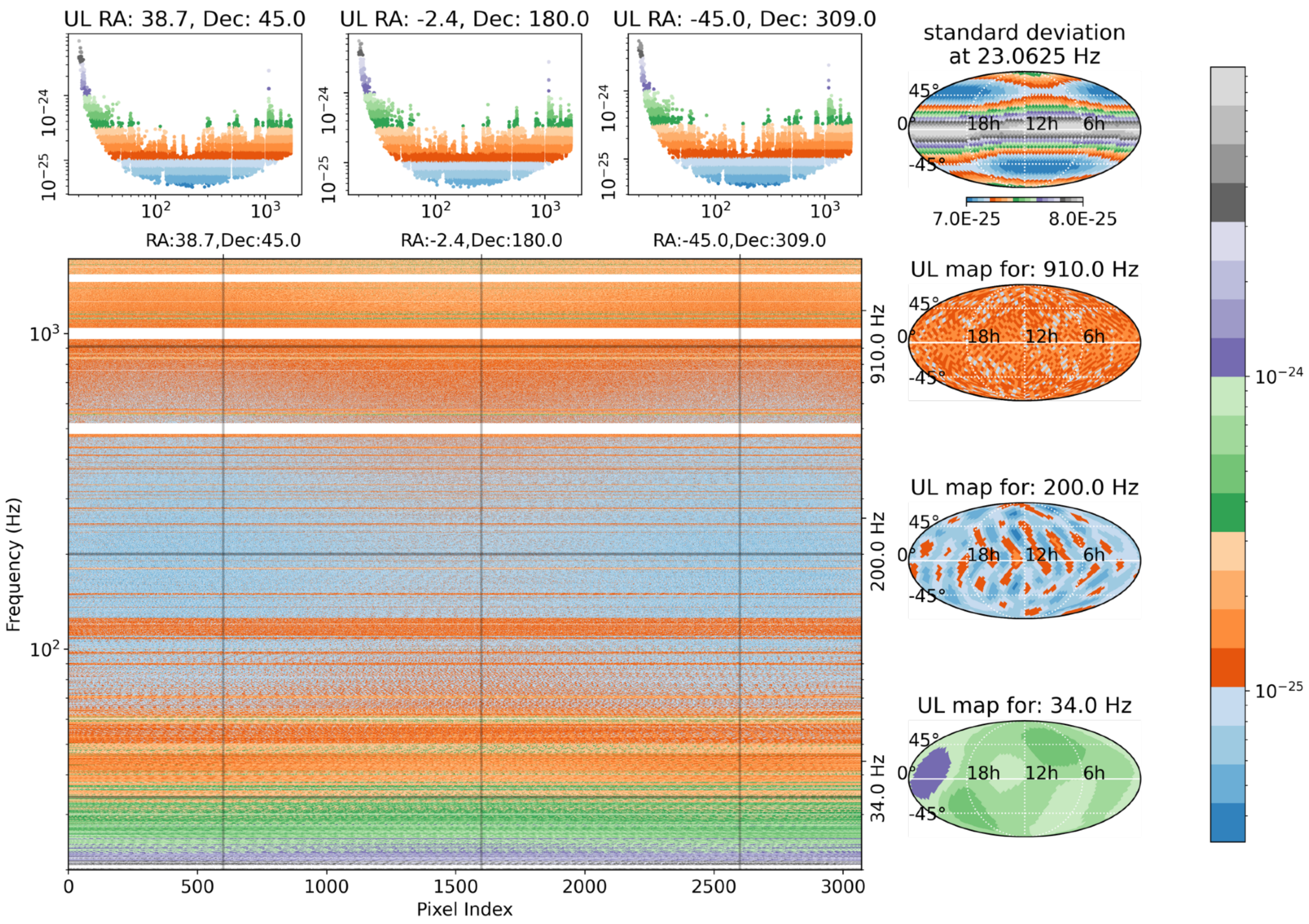
Following the standard Maximum Likelihood estimators as the statistics, one can write

In pixel-basis $\hat{\mathscr{P}}(f,\hat{n}) = \Gamma^{-1}_{\hat{n},\hat{n}'}(f)\,.\,X_{\hat{n}'}(f)$

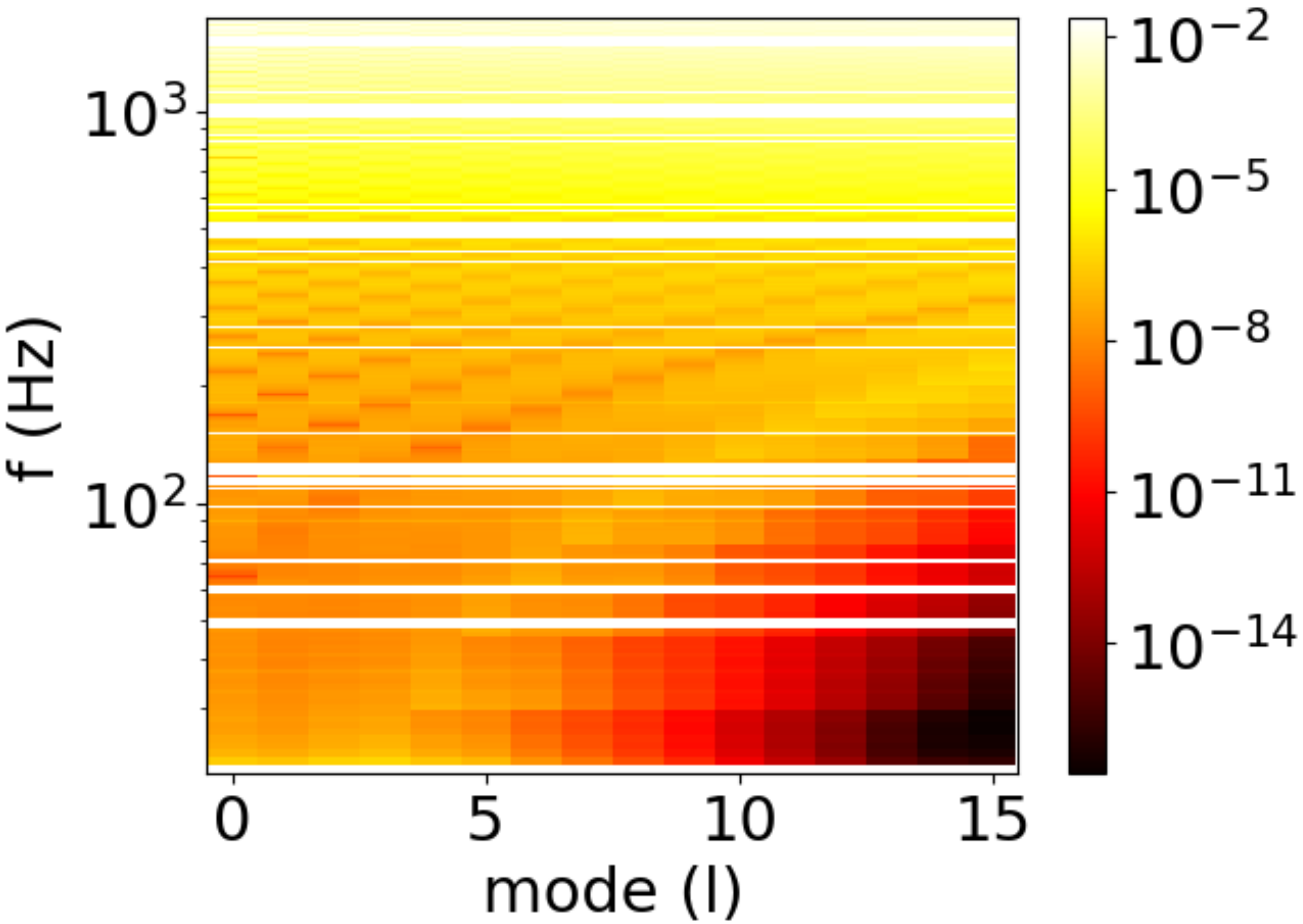
In spherical harmonic-basis $\hat{\mathscr{P}}_{lm}(f) = \Gamma^{-1}_{lm,lm}(f)\,.\,X_{lm}(f)$

Model-independant angular power spectra

$$\hat{C}_{\ell}(f) = \frac{1}{2l+1}\sum_{m}\left[\,|\,\hat{\mathscr{P}}_{lm}(f)\,|^2 - \left[\Gamma'^{-1}\,\Gamma\,\Gamma'^{-1}\right]_{lm,lm}(f)\right]$$



(Phys.Rev.D 105 12, 122001 2022)



(In-preparation)

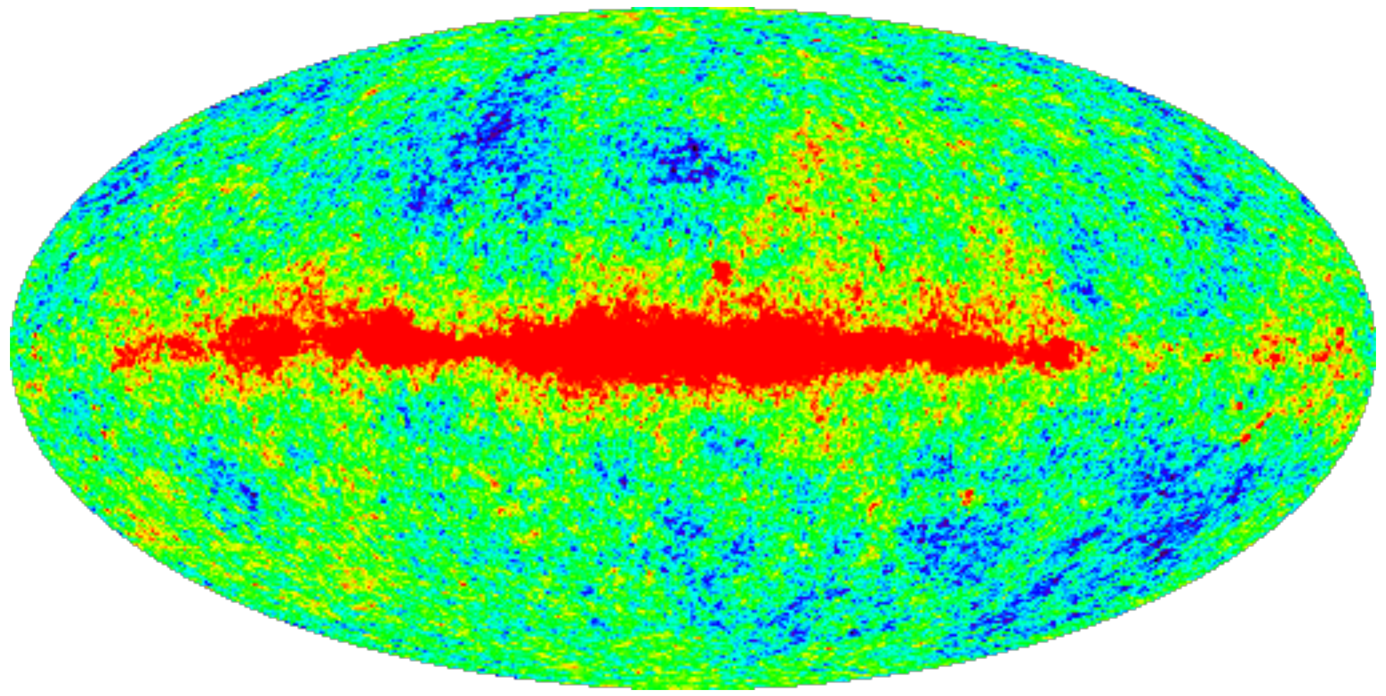
The 95% confidence Bayesian upper limit on the strain amplitude for all-sky directions and all-frequency bins. The color bar denotes the range of UL variations.

The 95% confidence Bayesian upper limit on model-independent angular power spectra at all-frequency bins. The color bar denotes the range of UL variations.

What can you infer from these estimators?

- SGWB anisotropies can be used to explore the formation and evolution of structures in the universe using map-correlation techniques
    - SGWB with Cosmic Microwave Background
    - SGWB with weak lensing
    - SGWB with Galaxy Count …….
- All these survey provides information on the angular power spectrum at different frequency range. So it is essential to compute the same for the SGWB.
- There are also different theoretical predictions for these anisotropies. One can set constrains on these models.

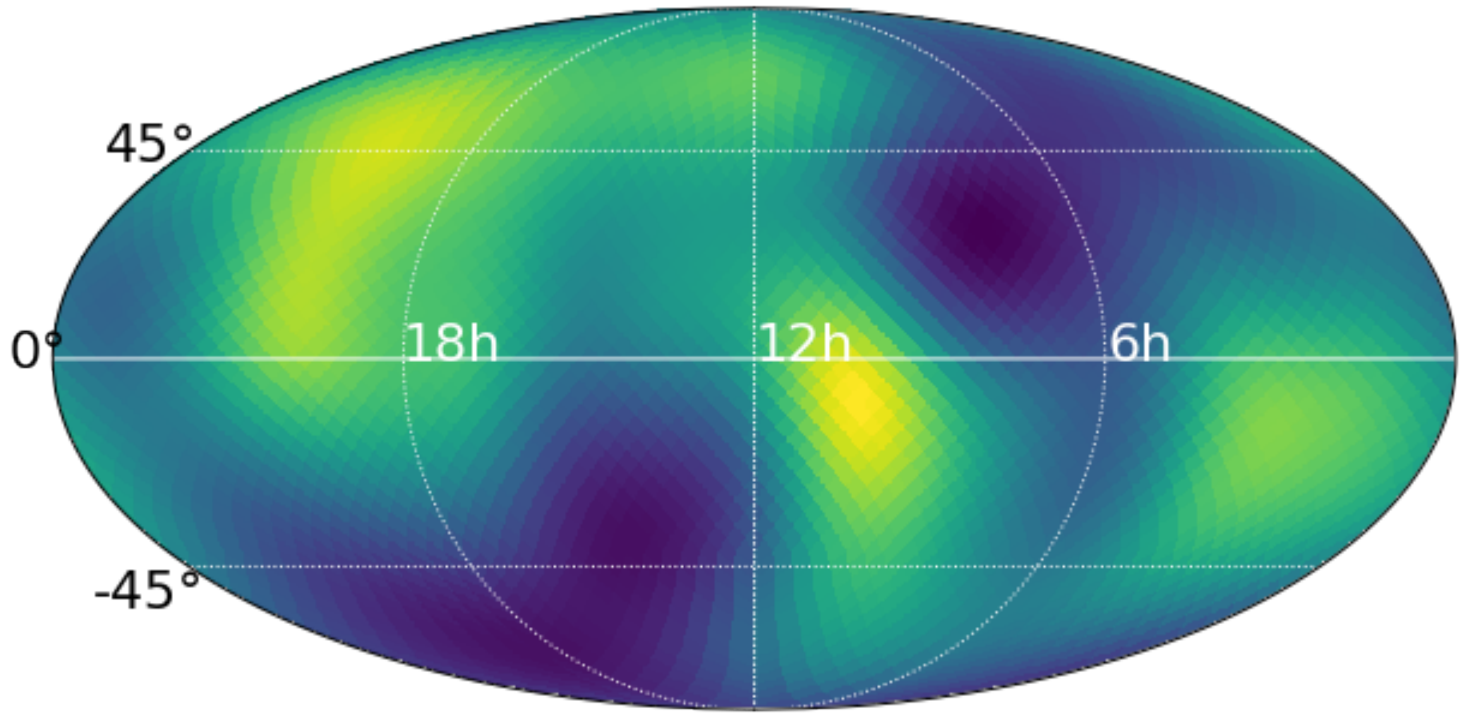
From EM observation



(f)

×

From SGWB estimation

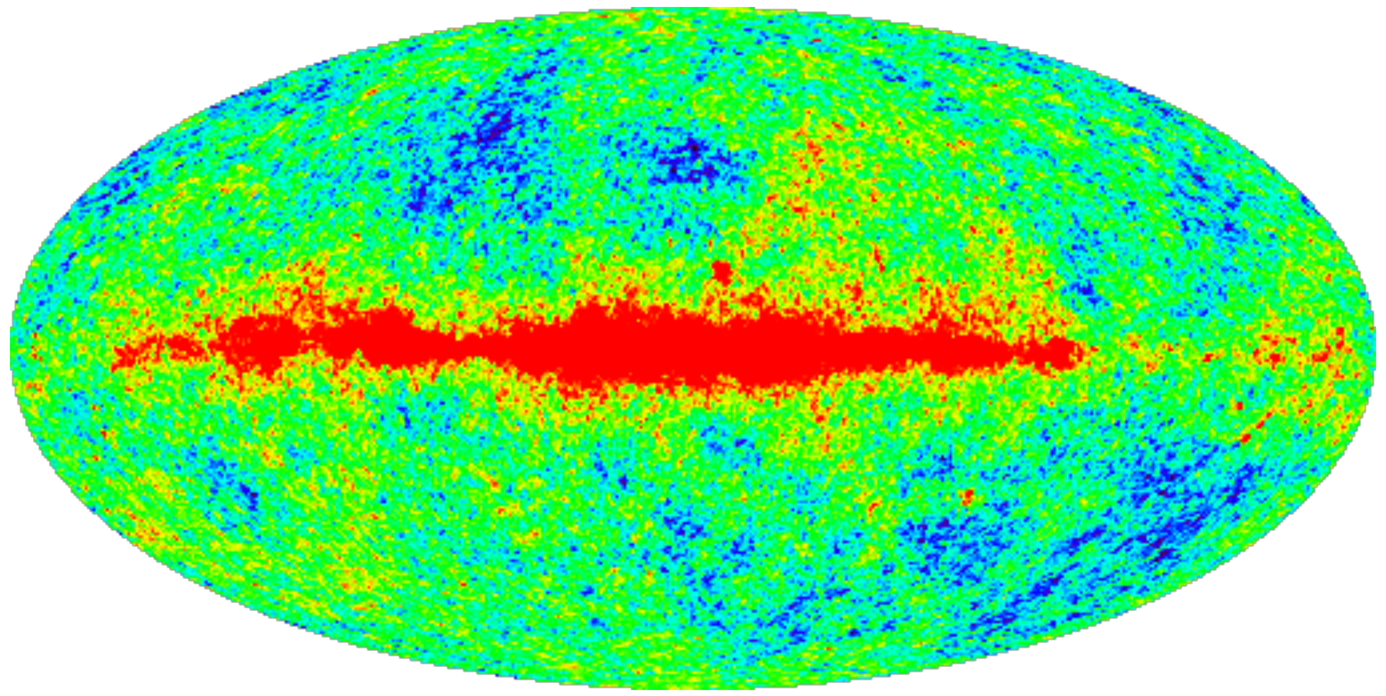


(f)

A model independent estimate of these anisotropies will give us the flexibility to look for different surveys/predictions.

One can now create a single statistical formalism with these estimators that would do all correlations at once (SGWB, CMB, galaxy counts, & lensing) and constrain model parameters

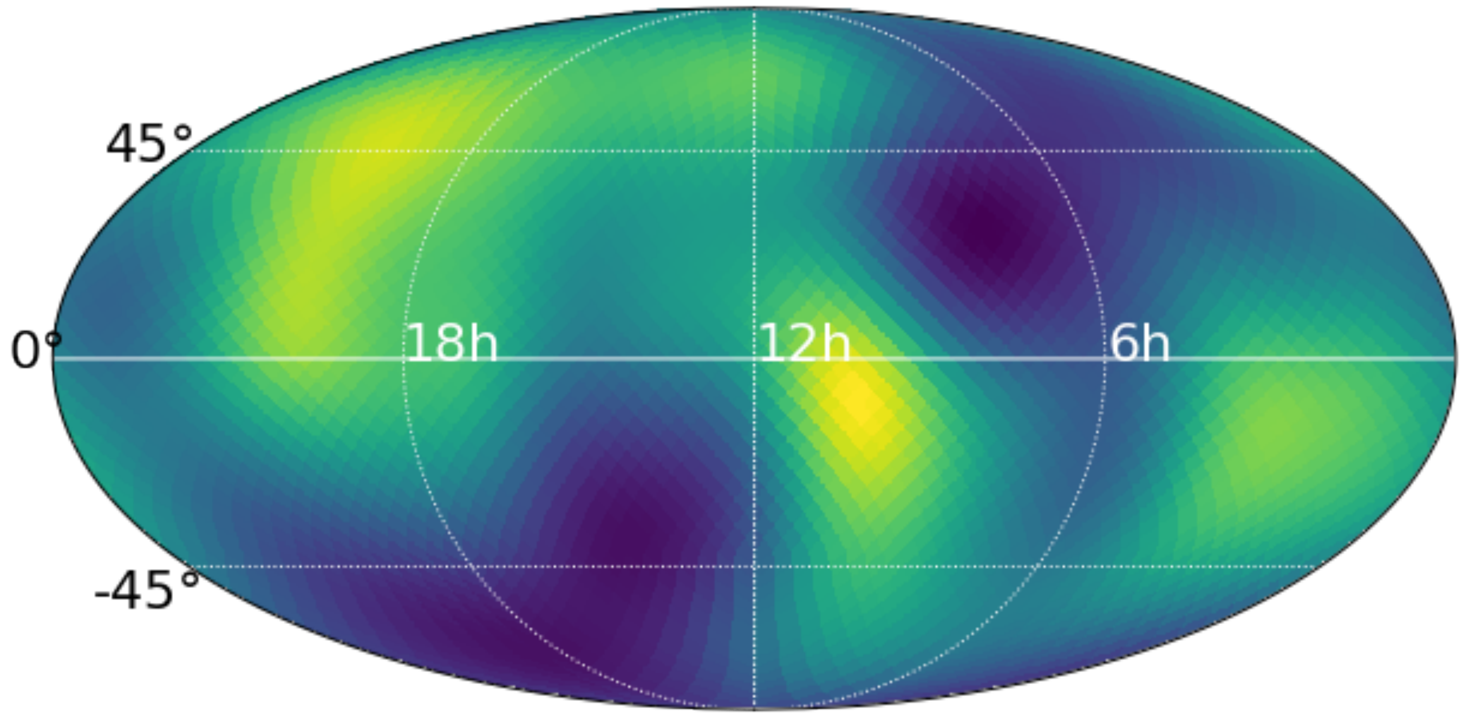
From EM observation



(f)

×

From SGWB estimation



(f)

thank you!