Black hole multipoles in higher-derivative gravity

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based on arXiv:2208.01044

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Belgian-Dutch GW meeting
Gent, October 14th 2022

Multipole moments in GR

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Theory-dependent test: higher-derivative corrections. Still no hair, but multipoles are different

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$$\mathcal{L}_{(6)} = \lambda_{\rm ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu} + \lambda_{\rm odd}\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} \tilde{R}_{\delta\gamma}{}^{\mu\nu}$$
$$\mathcal{L}_{(8)} = \epsilon_1 \ell^6 \mathcal{C}^2 + \epsilon_2 \ell^6 \tilde{\mathcal{C}}^2 + \epsilon_3 \ell^6 \mathcal{C}\tilde{\mathcal{C}}$$

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Observe: the size of the corrections is determined by

$$\hat{\lambda}_i = \frac{\ell^4}{M^4}\lambda_i\,, \qquad \hat{\epsilon}_i = \frac{\ell^6}{M^6}\epsilon_i$$

Rotating BH solutions can be obtained as a series in the spin $\chi = a/M$ Cano, Ruipérez '19

6-derivative corrections

$$Z_n = M_n + iS_n$$

We find

$$Z_n = Z_n^{\text{Kerr}} \times \left[1 + \left(\hat{\lambda}_{\text{ev}} + i\hat{\lambda}_{\text{odd}}\right) f_n(\chi)\right]$$

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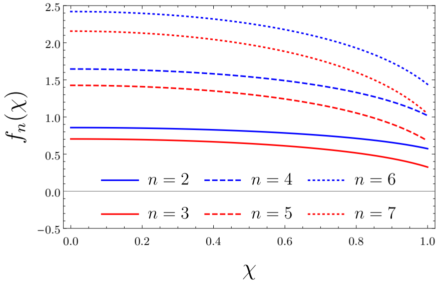

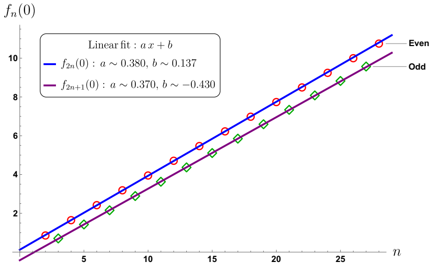

8-derivative corrections

$$Z_n = Z_n^{\text{Kerr}} \times \left[1 + (\hat{\epsilon}_1 + \hat{\epsilon}_2)\, g_n(\chi) + (\hat{\epsilon}_1 - \hat{\epsilon}_2 + i\,\hat{\epsilon}_3)\, h_n(\chi)\right]$$

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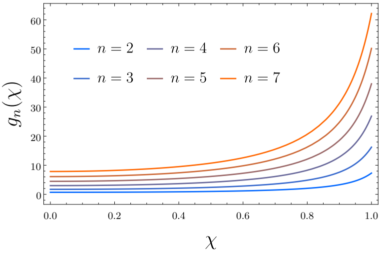
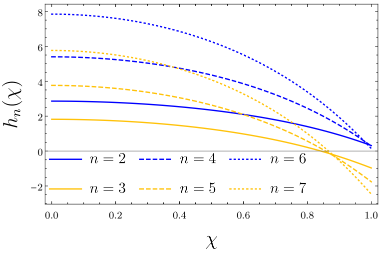
$$g_2(\chi) = \frac{7}{10} + \frac{39\chi^2}{44} + \frac{179\chi^4}{208} + \frac{49\chi^6}{64} + \frac{818629\chi^8}{1244672} + \mathcal{O}(\chi^{10})$$

$$h_2(\chi) = -\frac{8}{25\,\chi^{10}}\Big(64 - 80\chi^{10} + 660\chi^8 - 1545\chi^6 + 1500\chi^4 - 600\chi^2 - 8(1-\chi^2)^{5/2}(8 + 35\chi^4 - 55\chi^2)\Big)$$





Constraining higher-derivative corrections

BH binary with masses $M_{(i)}$, spins $\chi_{(i)}$ and quadrupole $M_{2,(i)}$

Consider

$$\delta\kappa^{(s)} = -\frac{1}{2}\left(\frac{M_{2,(1)}}{M_{(1)}^3\chi_{(1)}^2} + \frac{M_{2,(2)}}{M_{(2)}^3\chi_{(2)}^2}\right) - 1$$

[1]From LIGO Scientific, VIRGO, KAGRA collaboration 2112.06861

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Einstein Telescope[2]: $\delta\kappa^{(s)} \lesssim 10^{-2} \Rightarrow \ \ell|\lambda_{\rm ev}|^{1/4} \lesssim 0.1 - 1\,\text{km}$

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Conclusions

- Higher-derivative gravity induces no hair but modified multiple structure of black holes
- First detailed analysis of BH multipole moments beyond GR. Interesting structure, parity-breaking corrections, non-polynomial dependence on χ.
- Impact of modified multipoles on the inspiral?
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Thank you for your attention

Bonus

Can we really see higher-derivative corrections? Depends on the scale ℓ

The first corrections are $\sim \ell^4 Riem^3$. The relative deviation Δ with respect to GR is of order

$$\Delta \sim \frac{\ell^4 (GM)^2}{r^6}$$

$$\Delta_{Surf.Sun} \sim \left(\frac{\ell}{5\times 10^8 \text{km}}\right)^4 , \quad \Delta_{Surf.Earth} \sim \left(\frac{\ell}{2\times 10^8 \text{km}}\right)^4 , \quad \Delta_{BH}(10 M_\odot) \sim \left(\frac{\ell}{40 \text{km}}\right)^4$$

We may measure this if $\ell \sim$ km.

Constraints from cosmology? NO

BHs$\rightarrow$ Weyl curvature. Cosmology $\rightarrow$ Ricci curvature