Modelling Neutron Star Matter

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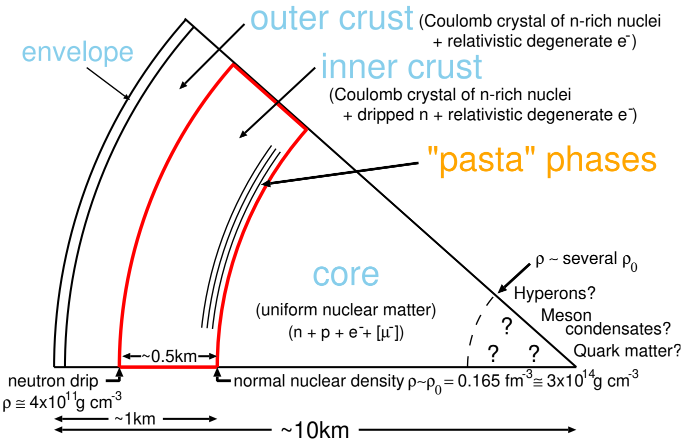
Outline

- A very brief introduction to neutron stars (NSs)
- Description of nuclear matter
- Models specific to this work and the constraints used
- Results
- Summary

Structure of a Neutron Star



envelope

outer crust (Coulomb crystal of n-rich nuclei + relativistic degenerate e^-)

inner crust (Coulomb crystal of n-rich nuclei + dripped n + relativistic degenerate e^-)

"pasta" phases

core

(uniform nuclear matter)

($n + p + e^- + [\mu^-]$)

$\rho \sim$ several ρ_0

Hyperons?

Meson condensates?

Quark matter?

? ? ?

~0.5km

neutron drip

$\rho \cong 4x10^{11}$ g cm^{-3}

normal nuclear density $\rho \sim \rho_0 = 0.165$ fm$^{-3} \cong 3x10^{14}$ g cm^{-3}

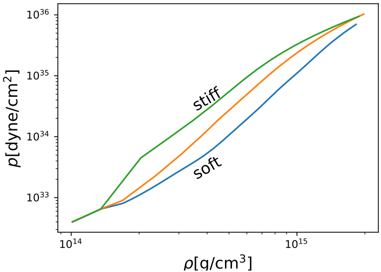
~1km

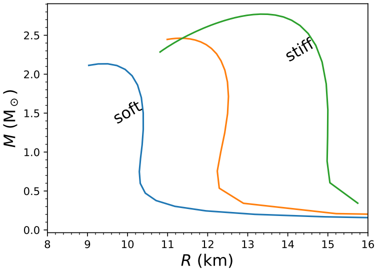
~10km

Figure: **Schematic picture of a NS Interior**

Equation of State and Mass-Radius

$$\boxed{P = P(\rho)} \longleftrightarrow \boxed{M = M(R)}$$





One to one correspondence between Equation of State (EOS) and mass-radius

Tidally Deformed Stars

- If a static spherically symmetric star of mass M and radius R is placed in a time-independent external tidal field $\mathcal{E}_{ij}$, a quadrupole moment Q_{ij} is induced onto the star and to linear order

$$Q_{ij} = -\lambda \mathcal{E}_{ij},$$

- Tidal deformation parameter λ related to the $l = 2$ dimensionless Love number k_2

$$\lambda = \frac{2}{3} k_2 R^5.$$

- Observational parameter in LIGO-Virgo, ET: $\Lambda = \lambda / M^5$

NS Observations that an EOS must satisfy

- Precise mass-measurement of massive NSs:

  $(1.908 \pm 0.016) M_\odot$ Arzoumanian et al, ApJS 235, 37 (2018).

  $(2.01 \pm 0.04) M_\odot$ Antoniadis et al, Science 340, 448 (2013).

  $(2.08 \pm 0.07) M_\odot$ E. Fonseca et al, ApJL 915 L12 (2021).

- BNS merger event GW170817 provides bounds on tidal deformability (Λ), and pressure at $2\rho_0$; Abbott et al, PRL 121, 161101 (2018):

  $\Lambda_{1.4} = 190^{+390}_{-120} \Rightarrow \Lambda_{1.4} \leq 580,\ P(2\rho_0) = 3.5^{+2.7}_{-1.7} \times 10^{34}$ dyn/cm^2

- NICER collaboration provided:

  1) Simultaneous mass-radius measurements of PSR J0030+0451

  $M = 1.34^{+0.15}_{-0.16} M_\odot,\ R = 12.71^{+1.14}_{-1.19}$km Riley et al, ApJL, 887, L21 (2019).

  $M = 1.44^{+0.15}_{-0.14} M_\odot,\ R = 13.02^{+1.24}_{-1.06}$km Miller et al, ApJL, 887, L24 (2019).

  2) Radius measurements of J0740+6620

  $R = 12.39^{+1.30}_{-0.98}$km Riley et al, ApJL, 918, L27 (2021).

  $R = 13.7^{+2.6}_{-1.5}$km Miller et al, ApJL, 918, L28 (2021).

Description of Nuclear Matter:

pressure as a function of energy

energy per particle of nuclear matter

$$P(\rho,\delta) = \rho^2 \frac{d}{d\rho}(E_0(\rho,\delta))$$

symmetric nuclear matter

symmetry energy

$$E_0(\rho,\delta) \approx E_0(\rho) + E_{sym}\delta^2$$

$$E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2}\chi^2 + \frac{Q_0}{6}\chi^3$$

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_0) + L_{\mathrm{sym}}\chi + \frac{K_{\mathrm{sym}}}{2}\chi^2 + \frac{Q_{\mathrm{sym}}}{6}\chi^3$$

$$\delta = \text{“Isospin asymmetry”} = (\rho_{\mathrm{n}} - \rho_{\mathrm{p}})/\rho, \qquad \chi = (\rho - \rho_0)/3\rho_0$$

J. Margueron, R. Hoffmann Casali, and F. Gulminelli, Phys. Rev. C 97, 025805 (2018)

EOS of Dense Matter from Nuclear Physics

Difficulties

- Constituents are not known.
- Interaction between constituents are not fully known.
- Uncertainties in the many-body description.

⇒ EOS is model dependent.

Phenomenological approaches are most widely used.

- Based on effective Interaction.
  1. Non-relativistic Skyrme-Interaction (~ 240)
  2. Relativistic Mean Field (RMF) models (~ 270)

Dutra et al. PRC 85, 035201 (2012); Dutra et al. PRC 90, 055203 (2014);

Oertel et al. RMP 89, 015007 (2017)

Our main objective: Exploring the parameter space to quantify the uncertainties.

RMF model

- Interaction between baryons is described via exchange of mesons.
- The most general form of the interaction Lagrangian density:

$$
\begin{aligned}
\mathcal{L}_{\rm DD} &= \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \Gamma_\sigma(\rho)\sigma\bar{\psi}\psi - \Gamma_\omega(\rho)\bar{\psi}\gamma^\mu\omega_\mu\psi - \frac{\Gamma_\rho(\rho)}{2}\bar{\psi}\gamma^\mu\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau}\psi \\
&+ \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu,
\end{aligned}
$$

σ, ω_μ,and $\boldsymbol{\rho_\mu}$ are meson fields.

- For the density dependent (DD) models, the coupling parameters Γ_σ, Γ_ω, and Γ_ρ are density dependent and do not have nonlinear terms.

$$\Gamma_i(\rho) = a_i + (b_i + d_i x^3)e^{-c_i x},$$

for $i = \sigma, \omega, \rho$, and $x = n/n_0$.

P. Gogelein, E. N. E. van Dalen, C. Fuchs, and H. Muther, Phys. Rev. C 77, 025802 (2008)

Saturation properties of nuclear matter:

Paremeter sets are obtained by exploring the uncertainties of the saturation properties of nuclear matter:

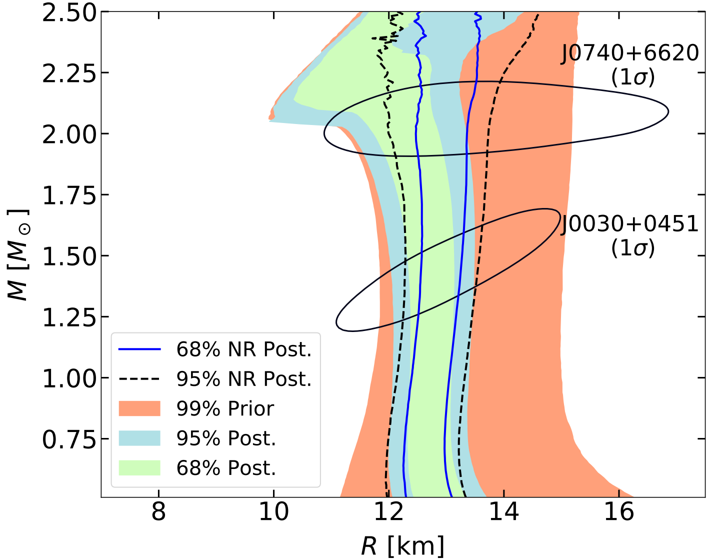
- Saturation density: $\rho_{sat} = (0.135, 0.195)$ fm^{-3}
- Binding energy per nucleon: $E_{sat} = (-14, -17)$ MeV.
- Incompressibility: $K_{sat} = (150, 350)$ MeV.
- Symmetry energy: $E_{sym} = (20, 45)$ MeV.
- Symmetry energy slope : $L_{sym} = (20, 180)$ MeV.

Additionally, we use the constraints coming from chiral EFT calculations from Drischler et al., Phys. Rev. C 93, 054314 (2016)

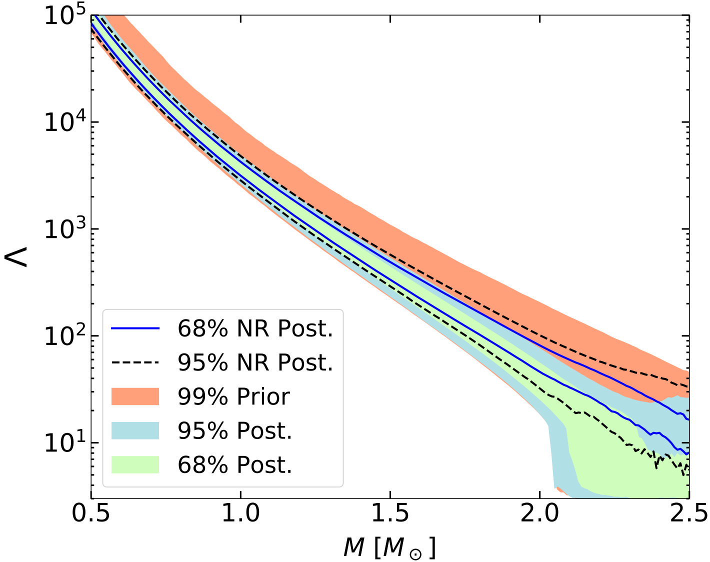
Results: Unified EOS

- High density EOS is constructed for a set of model parameters correspondening to a unique set of nuclear matter parameters
- Low density EOS is calculated within the compressible liquid drop model (CLDM) model for the aformentioned set of nuclear matter parameters.
- β-equlibrium is applied over the whole range.
- The crust and the core are matched with the continuity of pressure and chemical potential.
- We compared our results with the non-relativistic metamodels from H. Dinh Thi, C. Mondal, and F. Gulminelli, Universe 7, 373 (2021)

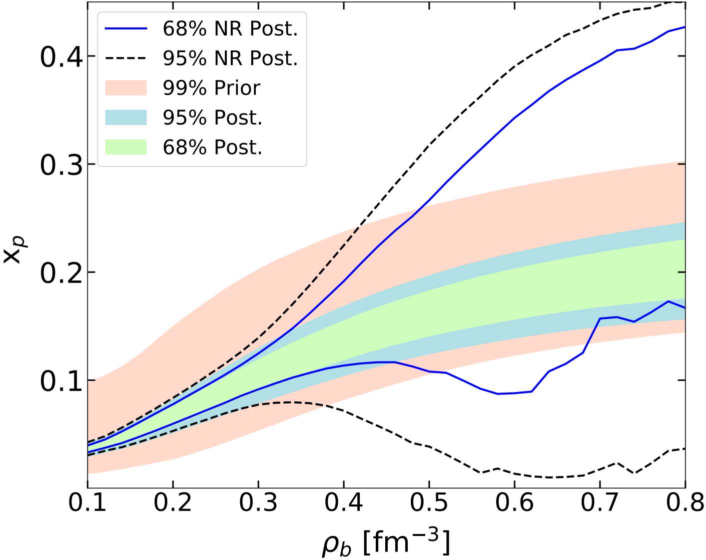
Results: Mass - Radius



Results: Tidal deformability



Results: Proton fraction



Summary:

- Any study of dense matter EOS is heavily model dependent. Therefore, a metamodelling approach to dense matter is very helpful to refine our knowledge.
- Within the GDFM type density-dependent RMF model, a wide range of EOSs can be generated with diverse nuclear matter properties that will be able to satisfy present observational constraints.
- Our future objective is to apply this model to study finite nuclei properties
- Manuscript in preparation.

Thank You