Three Effective Field Theory Vignettes

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CERN TH Colloquium

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Effective Field Theory is everywhere...
Plan

You are here

Intro to EFT

STREAMlining EFT Matching

Outlook

Soft de Sitter Effective Theory

Hamiltonian Truncation Effective Theory
Intro to EFT
As Scales Become Separated: Lectures on Effective Field Theory

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Abstract
These lectures aim to provide a pedagogical introduction to the philosophical underpinnings and technical features of Effective Field Theory (EFT). Improving control of S-matrix elements in the presence of a large hierarchy of physical scales $m \ll M$ is emphasized. Utilizing $\lambda \sim m/M$ as a power counting expansion parameter, we show how matching a UV model onto an EFT makes manifest the notion of separating scales. Renormalization Group (RG) techniques are used to run the EFT couplings from the UV to the IR, thereby resumming large logarithms that would otherwise reduce the efficacy of perturbation theory. A variety of scalar field theory based toy examples are worked out in detail. An approach to consistently evolving a coupling across a heavy particle mass threshold is demonstrated. Applying the same method to the scalar mass term forces us to confront the hierarchy problem. The resummation of a logarithm that lacks explicit dependence on the RG scale is performed. After reviewing the physics of IR divergences, we build a scalar toy version of Soft Collinear Effective Theory (SCET), exposing many subtle aspects of these constructions. We show how SCET can be used to resume the soft and collinear IR Sudakov double logarithms that often appear for processes involving external interacting light-like particles. We conclude with the generalization of SCET to theories of gauge bosons coupled to charged fermions. These lectures were presented at TASI 2018.
Reductionism

Why didn't Newton and Maxwell need QFT??

"Heavy physics decouples" → Effective description
Reductionism

Large separation of scales
How To Build a Theory

1) Degrees of freedom
2) Symmetries
3) Dimensional analysis
Power Counting

"Physics is essentially dimensional analysis and Taylor expansions"

Large separation of scales
⇒ "power counting parameter" $\lambda$

Observables can be computed order-by-order in power counting
⇒ Predict theoretical uncertainty
Why EFT?

Conceptual: Exposes relevant physics

Praticle: "Model Independent" parametrization of low energy physics

Praticle: Facilitate hard calculations

Praticle: Improve perturbation theory
From High to Low Energy

1) $E_{cm} \gg M$

Theory w/ two massless particles $\Rightarrow$ easy

2) $E_{cm} \sim M$

Multiscale Theory $\Rightarrow$ hard

3) $E_{cm} \ll M$

Single particle EFT $\Rightarrow$ easy
Heavy Physics Decouples

"Integrate out heavy particle"

$\mathbf{EFT}$

$m \ll M$
Loops

Generate logs e.g. $\log \frac{M^m}{m}$

Decoupling more subtle

"Matching" full (UV) theory onto (IR) EFT
EFT and Loops

Loops in QFT \Rightarrow \log \left( \frac{m}{\Lambda} \right) \sim \log \lambda

When \; m \ll M, \; logs \; can \; become \; large

\Rightarrow \text{must resum them}

Promote coupling "constants" to running couplings

Renormalization Group Evolution
EFT
Fundamental (UV) Theory

Matching

Running

Renormalization group evolution

Predictions for experiments
Non-relativistic EFT

QFT fields include "particles" and "anti-particles"

Express \( \Phi = \Phi_{\text{particle}} + \Phi_{\text{anti-part}} \)

Want observables as expansion in \( \frac{P}{m} \ll 1 \)

\( \Rightarrow U \ll 1 \Rightarrow \) Power counting

Can "integrate out" \( \Phi_{\text{anti-part}} \) \( \Rightarrow \) non-rel EFT

Ex: "Heavy Quark Effective Theory"
STREAMlining EFT Matching
w/ Xiaochuan Lu
+ Zhengkang (Kevin) Zhang
arXiv: 2012.07851
How to organize BSM predictions?

Simplified Models

Effective Field Theory

SM + gluino + neutralino

\[ L = L_{SM} + i\bar{\psi}D\psi + \sum m_{ij}\bar{\psi}^i \gamma^5 \psi^j + \frac{1}{4} \bar{\psi}^i \gamma^\mu \psi^j \gamma^\mu \psi^i \]

DOFs: \( q, u, d, l, e, \)

\( H, B_R, W_R, \gamma \)

Symmetries:

\( SU(3)_c \times SU(2)_L \times U(1) \)

\[ L = L_{SM} + \sum_{\text{dim} > 4} E \bar{\psi} \gamma^\mu \psi \]
Constraints on SMEFT

How to interpret?
EFT Matching

Connect UV theories to EFT parameters

Two approaches

Feynman diagrams
(need EFT in advance)

Functional Methods
(evaluate path integral directly)
Matching

Amplitude matching (with Feynman diagrams)

\[ \mathcal{L}_{\text{UV}}[\Phi, \phi] \xrightarrow{p_i \ll m_\Phi} \{ \mathcal{A}_{\text{UV}}(p_i) \} \]

Equate to derive \( \{ c_i \} \)

\[ \mathcal{L}_{\text{EFT}}[\phi] \xrightarrow{\text{w/ IR regulator}} \{ \mathcal{A}_{\text{EFT}}(p_i) \} \]

Functional matching (our prescription)

\[ \mathcal{L}_{\text{UV}}[\Phi, \phi] \]

\[ \Phi_c[\phi] \rightarrow K, X \]

Enumerate

Functional supertraces

\[ \mathcal{L}_{\text{EFT}}^{(\text{tree})}[\phi] \]

\[ \mathcal{L}_{\text{EFT}}^{(1\text{-loop})}[\phi] \]

Evaluate
Diagrammatic Prescription

\[ -\frac{i}{2} \frac{1}{2} \text{Str} \left( \left( \frac{1}{P^2 - M^2} U_{SS}^{[2]} \right)^2 \right) \bigg|_{\text{hard}}, \]

\[ -\frac{i}{2} \text{Str} \left( \frac{1}{P^2 - M^2} U_{SH}^{[1]} \frac{1}{P^2 - m^2} U_{HS}^{[1]} \right) \bigg|_{\text{hard}}, \]

\[ -\frac{i}{2} \frac{1}{3} \text{Str} \left( \left( \frac{1}{P^2 - M^2} U_{SS}^{[2]} \right)^3 \right) \bigg|_{\text{hard}}, \]

\[ -\frac{i}{2} \text{Str} \left( \frac{1}{P^2 - M^2} U_{SS}^{[2]} \frac{1}{P^2 - m^2} U_{SH}^{[1]} \frac{1}{P^2 - M^2} U_{HS}^{[1]} \right) \bigg|_{\text{hard}}, \]

\[ -\frac{i}{2} \text{Str} \left( \frac{1}{P^2 - M^2} U_{SH}^{[1]} \frac{1}{P^2 - m^2} U_{HH}^{[2]} \frac{1}{P^2 - M^2} U_{HS}^{[1]} \right) \bigg|_{\text{hard}}. \]
Example: Singlet Extended SM

One loop matching is "solved"
Stay Tuned

Functional matching relies on dim reg and method of regions. What about $Y_5$???

We have developed a novel 4D regulator for the anomaly. Facilitates integrating out Weyl fermions.
Soft de Sitter Effective Theory

w/ Dan Green (UCSD)


+ Akhil Premkumar + Alec Ridgway

arXiv: 2106.09728, 2111.09332
Soft de Sitter Effective Theory

\[ \kappa_{\text{physical}} = \frac{\kappa}{a(t)} \]

\[ \frac{\kappa}{a} \ll H \]

\[ \tau \sim H^{-1} \]

UV modes

\[ \frac{\kappa}{a} \gg H \]

Power counting

\[ \lambda = \frac{\kappa}{aH} \]
Scalar Fields in dS

EOM \[ \ddot{\phi} + 3 \dot{\phi} + \frac{k^2}{(aH)^2} \phi + \frac{m^2}{H^2} \phi = 0 \]

Soft limit \( \phi_s = (aH)^{-3/2 + \nu} \phi_s \)

with \( \nu = \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \)

or \( \alpha = \frac{3}{2} - \nu \quad \beta = \frac{3}{2} + \nu \) s.t. \( \alpha + \beta = 3 \)

WLOG \( \alpha < \beta \)
One-to-many Mode Expansion

Factorize into soft and hard modes
\[ \phi(\vec{x}, t) = \phi_S(\vec{x}, t) + \phi_H(\vec{x}, t) \]

Integrate out hard modes
\[ \Rightarrow \text{Local operator expansion} \]

Observables order-by-order in power counting
SdSET Fields

Two IR degrees of freedom

- "Growing" mode $\varphi_+$
- "Decaying" mode $\varphi_-$

$\phi_s = H((aH)^{-\alpha} \varphi_+ + (aH)^{-\beta} \varphi_-)$

Time dependence factorizes
Starobinsky's Stochastic Inflation

Massless scalar field in dS (1986)

\[ \frac{3}{2} P(\phi, t) = \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} P(\phi, t) + \frac{1}{3H} \frac{\partial}{\partial \phi} [V'(\phi) P(\phi, t)] \]
Starobinsky's Stochastic Inflation

$V(\phi) \uparrow$

Drift

Gaussian noise

Tree-level potential

Systematic corrections?
Stochastic Inflation $\Rightarrow$ RG flow

$V(\varphi)$

- Drift
- Quantum noise

$\varphi$

$U V$ Theory

$SdSET$

$\downarrow$ RG

Stochastic Inflation
Light Scalars in dS

Composite operators

\[ O_n = \phi^n \sim \left( \frac{\kappa}{a_T} \right)^n \rightarrow O(1) \]

RG mixing expected

Contract any two legs

\[
\langle O_n \ldots \rangle \rightarrow \langle O_{n-2} \ldots \rangle \left( \frac{n}{2} \right) \frac{C_2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}
\]
Light Scalars in dS

\[ \int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}} \] is scaleless and diverges as \( \alpha \to 0 \)

Isolate UV divergence

\[ p^2 \to p^2 + \frac{K_{\text{IR}}}{p} \]

\[ \langle \Theta_n \ldots \rangle \to \langle \Theta_{n-2} \ldots \rangle \left( \frac{n}{2} \right) \frac{C_{\alpha}^2}{4\pi^2} \left( \frac{-1}{2\alpha} - \gamma_E - \log \frac{aH}{K_{\text{IR}}} \right) \]
Dynamical RG \(\Rightarrow\) Stochastic Inflation

Resum time dependent logs:

\[
\frac{\partial}{\partial t} \langle \Omega_n \cdots \rangle = -\frac{n}{3} \leq \frac{C_m}{m!} \langle \Omega_{n-1} \Omega_m \cdots \rangle
\]

Insertion of potential

\[
V \sim C_m \phi_+ \phi_-
\]

\[
+ \frac{n(n-1)}{8 \pi^2} \langle \Omega_{n-2} \cdots \rangle
\]

(Starobinsky; Starobinsky, Yokoyama)

Is equivalent to a Fokker-Planck eq

for \( p(\phi, t) \) \( \Rightarrow \)

\[
\langle \phi^n \rangle = \int d\phi \ p(\phi, t) \phi^n
\]

(Baumgart + Sundrum)
Tree Matching

Assume UV theory is $d\phi^4$

\[ \phi^4 \]

\[ \phi^2 \]

\[ \phi_+^3 \phi_1 \]

Initial Conditions

\[ \langle \phi_+(k_1) \phi_+(k_2) \rangle \]
One Loop Matching

\[ \delta \alpha = \frac{\lambda}{8\pi^2} \frac{1}{3}(\gamma_E - \frac{2}{3}) \]

Other terms removed by \( \phi_- \rightarrow \phi_- + \frac{\lambda}{9} (2\lambda H)^3 \phi_+^2 \)
One Loop Matching

\[ C_{3,1} = \lambda - \frac{\lambda^2}{4\pi^2} \left( \frac{1}{9} \gamma_E (2 + 3 \gamma_E) + \frac{5}{12} \pi^2 \right) \]

+ \( \mathcal{O}(\lambda^2) \) impact on initial conditions (contributes to NNNLO RG)
Initial Conditions to $\chi^2$

\[ \vec{k}_1 \quad \vec{k}_2 \quad \vec{k}_3 \quad \vec{k}_4 \quad \vec{u}_5 \quad \vec{u}_6 \]
\[ T_1 \quad T_2 \]

$= \Theta(\lambda^2)$

$+ \langle \varphi_+(\vec{u}_1) \ldots \varphi_+(\vec{u}_6) \rangle_{\mathcal{I}_C}$
\[ NLO \ \Phi_+(0) \ \text{RG} \]

\[ \langle \Phi_+^2 [x=0] \Phi_+(\vec{k}_1) \Phi_+(\vec{k}_2) \rangle = \lambda P(k_1) P(k_2) \left( \frac{1}{48 \pi^2 \alpha^2} + \frac{(4 - 3 \gamma_e - 3 \log \frac{M}{\alpha})}{72 \pi^2 \alpha} + \text{finite} \right) \]
\[ \langle \varphi^2 [\vec{x} = 0] \varphi_+(\vec{u}_1) \ldots \varphi_+(\vec{u}_4) \rangle \sim \]
\[ \frac{\lambda^2}{8 \pi^2 \alpha} \frac{1}{27} \left[ 16 + 4 \delta_\varepsilon \left( -11 + 3 \delta_\varepsilon \right) + 3 \pi^2 + 12 (\log 2)^2 \right] + \ldots \]
$\text{NNLO } \Phi^3(0) \text{ RG}$

\[
\left< \Phi_+^3 \left[ \vec{x} = 0 \right] \Phi_+ (\vec{k}_1) \right> = \frac{\lambda}{16 \pi^2} \frac{1}{12} P(k_1) \left[ \frac{1}{\alpha} + \ldots \right]
\]
Put it all together

\[ \frac{\partial}{\partial t} P = \frac{1}{3} \frac{\partial}{\partial \phi_+} \left[ V_{\text{eff}}' P \right] + \frac{1}{8 \pi^2} \frac{\partial^2}{\partial \phi_+^2} P + \frac{\lambda_{\text{eff}}}{192 \pi^2} \frac{\partial^3}{\partial \phi_+^3} (P_0 P) \]

\[ V_{\text{eff}} = \frac{\lambda_{\text{eff}}}{3!} \left( \phi_+^3 + \frac{\lambda_{\text{eff}}}{18} \phi_+^5 + \frac{\lambda_{\text{eff}}}{162} \phi_+^7 + \ldots \right) \]

\[ \lambda_{\text{eff}} = \lambda - 12 b_2 - \frac{\lambda^2}{2 \pi^2} \left( \frac{1}{3} \delta_E (2 + 3 \delta_E) + \frac{3 \pi^2}{4} \right) \]

Compute equilibrium distribution, relaxation eigenvalues, etc.
Apply Same Techniques to Inflaton

Primordial Non-Gaussianity

Large fluctuations out of perturbative control?

Perturbative unitarity

Planck Constraints

$\bar{\alpha}$

$C_s$
Stay Tuned

We have developed a comprehensive understanding for the origin of this breakdown. The tails of these distributions are dominated by a different saddle point $\Rightarrow$ operator scalings change and UV sensitivity emerges.
Hamiltonian Truncation Effective Theory

w/ Kara Farnsworth
Rachel Houtz
Markus Luty

arXiv: 2110.08273
Explore Strongly Coupled QFT

Lattice QCD

(from Adelaide group)
Hamiltonian Truncation

1) Write $\hat{H} = \hat{H}_0 + \hat{V}$ where $\hat{H}_0$ can be solved exactly
   $\hat{H}_0 |E_i\rangle = E_i |E_i\rangle$

2) Introduce "energy cutoff" $E_{\text{max}}$

3) Compute matrix elements of $\hat{H}$ using truncated basis

4) Diagonalize $\Rightarrow$ approx energy spectrum
Want to Study IR Strong Theories

Simple case study

$\lambda \phi^4$ theory in 2D

Dimensional analysis

$[\phi] = 0 \Rightarrow [\lambda] = 2$

Relevant operator

$\Rightarrow$ weak in UV and strong in IR
Improve Numerical Predictions

Interpret $E_{\text{max}}$ as EFT cutoff scale

Power counting $\lambda \sim E_{\text{IR}} / E_{\text{max}}$

Write $H_{\text{eff}} = H_0 + H_1 + H_2 + \ldots$

with $H_n = O(U^n)$

Expect $H_2 \sim \frac{\lambda^2}{E_{\text{max}}} \int dx \left[ \phi^2 + \phi^4 + \frac{R^{-1} + H_0}{E_{\text{max}}} (1 + \phi^2 + \phi^4) \right] + \ldots$
EFT Expectations

\[ H_2 \sim \frac{1}{E_{\text{max}}} \int dx \left[ \phi^2 + \phi^4 + \frac{R^{-1} + H_0}{E_{\text{max}}^2} (1 + \phi^2 + \phi^4) \right] \]

“Local approx”

Compute energy eigenvalues as a function of \( E_{\text{max}} \)

“Raw truncation” converges as \( \frac{1}{E_{\text{max}}^2} \)

“Improved truncation” converges as \( \frac{1}{E_{\text{max}}^3} \)
Technical Details

• Need IR cutoff \(\Rightarrow\) work in finite volume

• Observable for matching is "transition matrix" (old fashioned perturbation thy)

• Derived set of diagramatic rules

• \(E_{\text{max}}\) breaks Lorentz invariance
  \(\Rightarrow\) sensitive to states that do not participate in interaction.
Numerical Results

\[ m_{NO} = 1, \ 2\pi R = 10 \]
Numerical Results

\[ \lambda/4\pi = 1, \quad m_{NO} = 1, \quad 2\pi R = 10 \]

\[ \Delta E_1 \]

\[ E_{\text{max}} \]

- **Raw**
- **Improved**
Numerical Results

\[ \lambda/4\pi = 1, \ m_{NO} = 1, \ 2\pi R = 10 \]

![Graph 1](image1)

![Graph 2](image2)
Numerical Results

$E_{\text{max}} = 27$, $m_{\text{NO}} = 1$, $2\pi R = 10$
Stay Tuned

Extension to next order which requires incorporating state dependence into matching coefficients.

Extension to 3D $\lambda^3 \phi^4$ requires incorporating UV divergences.
Outlook
Effective Field Theory is everywhere...
• Heavy physics decouples
• EFT is dimensional analysis and Taylor expansions
• One loop matching is "solved" using functional methods
• Stochastic Inflation is EFT
• Hamiltonian Truncation is EFT with a finite energy cutoff