Three Effective Field Theory Vignettes

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CERN/EPFL/UOregon

CERN TH Colloquium

October 19, 2022

Effective Field Theory
is everywhere...

Plan

You are here

Intro to EFT

STrEAMlining EFT Matching

Soft de Sitter Effective Theory

Hamiltonian Truncation Effective Theory

Outlook

Intro to EFT

As Scales Become Separated: Lectures on Effective Field Theory

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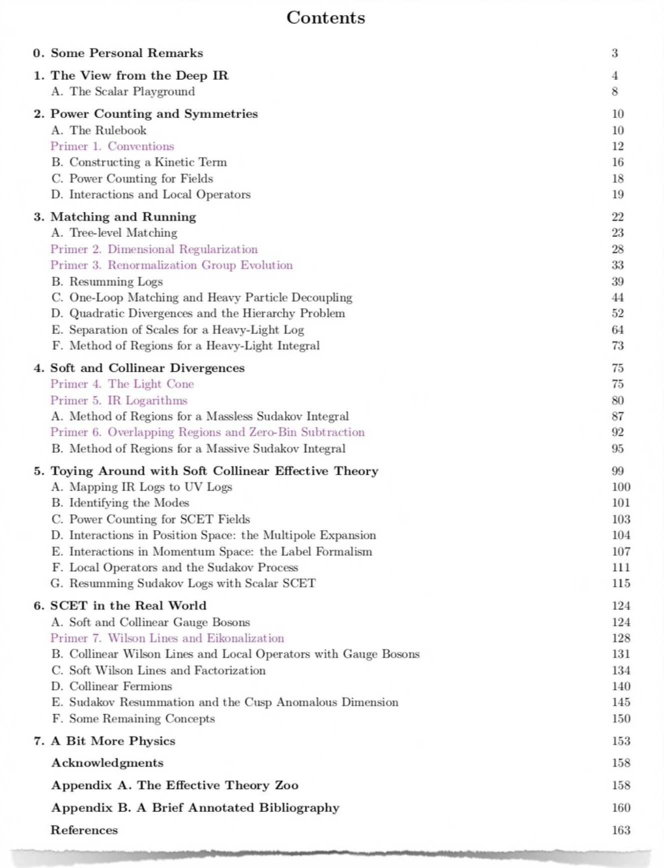
Abstract

These lectures aim to provide a pedagogical introduction to the philosophical underpinnings and technical features of Effective Field Theory (EFT). Improving control of S-matrix elements in the presence of a large hierarchy of physical scales $m \ll M$ is emphasized. Utilizing $\lambda \sim m/M$ as a power counting expansion parameter, we show how matching a UV model onto an EFT makes manifest the notion of separating scales. Renormalization Group (RG) techniques are used to run the EFT couplings from the UV to the IR, thereby resumming large logarithms that would otherwise reduce the efficacy of perturbation theory. A variety of scalar field theory based toy examples are worked out in detail. An approach to consistently evolving a coupling across a heavy particle mass threshold is demonstrated. Applying the same method to the scalar mass term forces us to confront the hierarchy problem. The resummation of a logarithm that lacks explicit dependence on the RG scale is performed. After reviewing the physics of IR divergences, we build a scalar toy version of Soft Collinear Effective Theory (SCET), exposing many subtle aspects of these constructions. We show how SCET can be used to resum the soft and collinear IR Sudakov double logarithms that often appear for processes involving external interacting light-like particles. We conclude with the generalization of SCET to theories of gauge bosons coupled to charged fermions. These lectures were presented at TASI 2018.

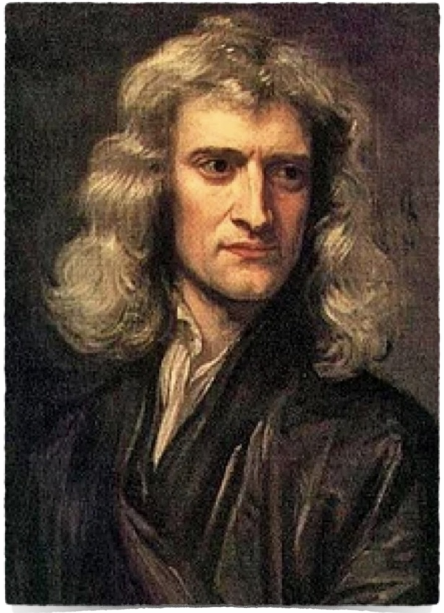


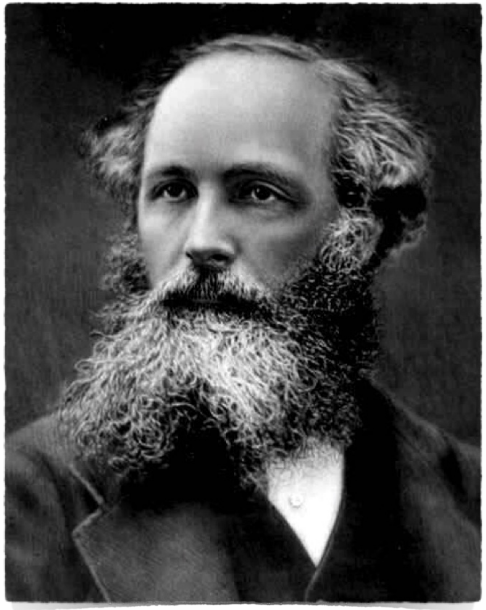
Contents

arXiv: 1903.03622

Reductionism

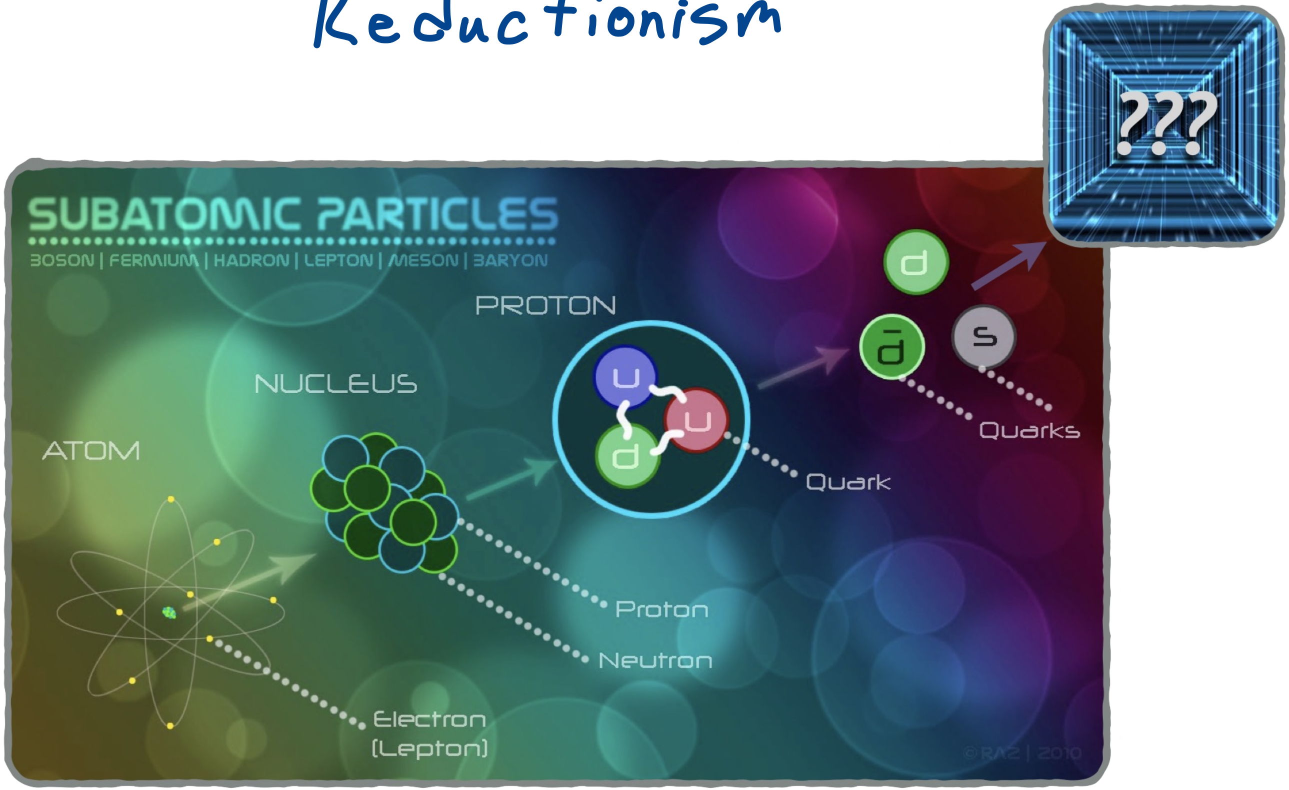
Why didn't Newton and Maxwell need QFT??





"Heavy physics decouples" ⇒ Effective description

Reductionism



Large seperation of scales

How To Build a Theory

1) Degrees of freedom
2) Symmetries
3) Dimensional analysis

Power Counting

"Physics is essentially dimensional analysis and Taylor expansions"

Large separation of scales

$\Rightarrow$ "power counting parameter" λ

Observables can be computed order-by-order in power counting

$\Rightarrow$ Predict theoretical uncertainty

Why EFT?

Conceptual: Exposes relevant physics

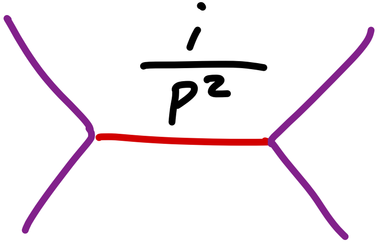
Praticle: "Model Independent" parametrization of low energy physics

Praticle: Facilite hard calculations

Praticle: Improve perturbation theory

From High to Low Energy

1) $E_{cm} \gg M$

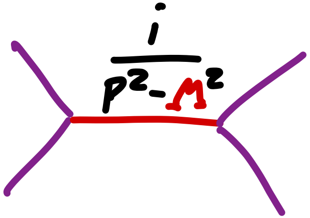


$m \sim 0$

$M \sim 0$

Theory w/ two massless particles ⇒ easy

2) $E_{cm} \sim M$

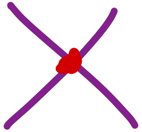


$m \sim 0$

$M \neq 0$

Multiscale Theory ⇒ hard

3) $E_{cm} \ll M$

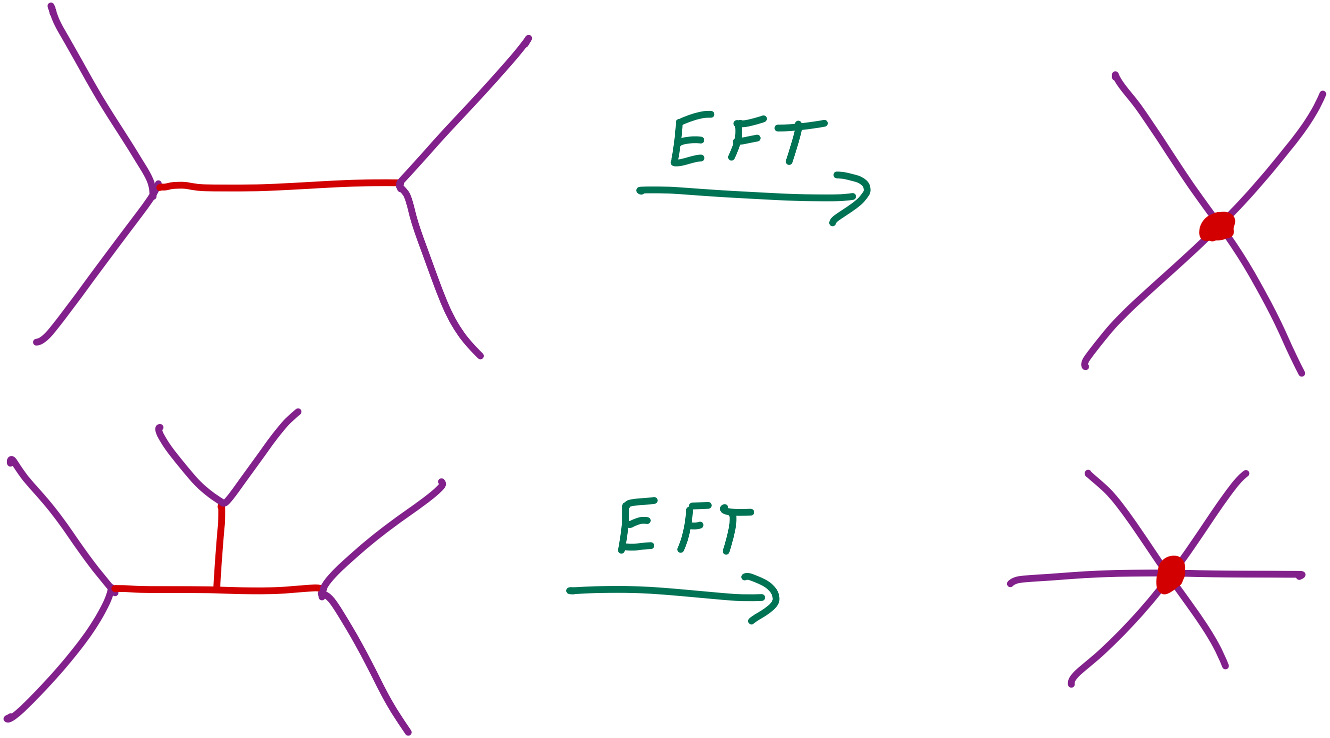


Single particle EFT

⇒ easy

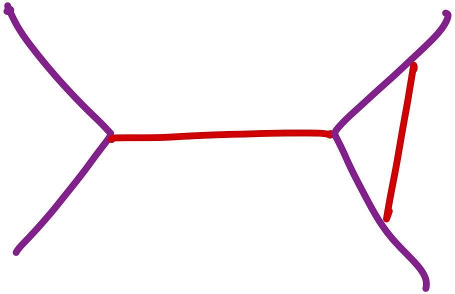
Heavy Physics Decouples

"Integrate out heavy particle"



$m \ll M$

Loops



Generate logs e.g. $\log m/M$

Decoupling more subtle

"Matching" full (UV) theory onto (IR) EFT

EFT and Loops

Loops in QFT $\Rightarrow \log\left(m/M\right) \sim \log \lambda$

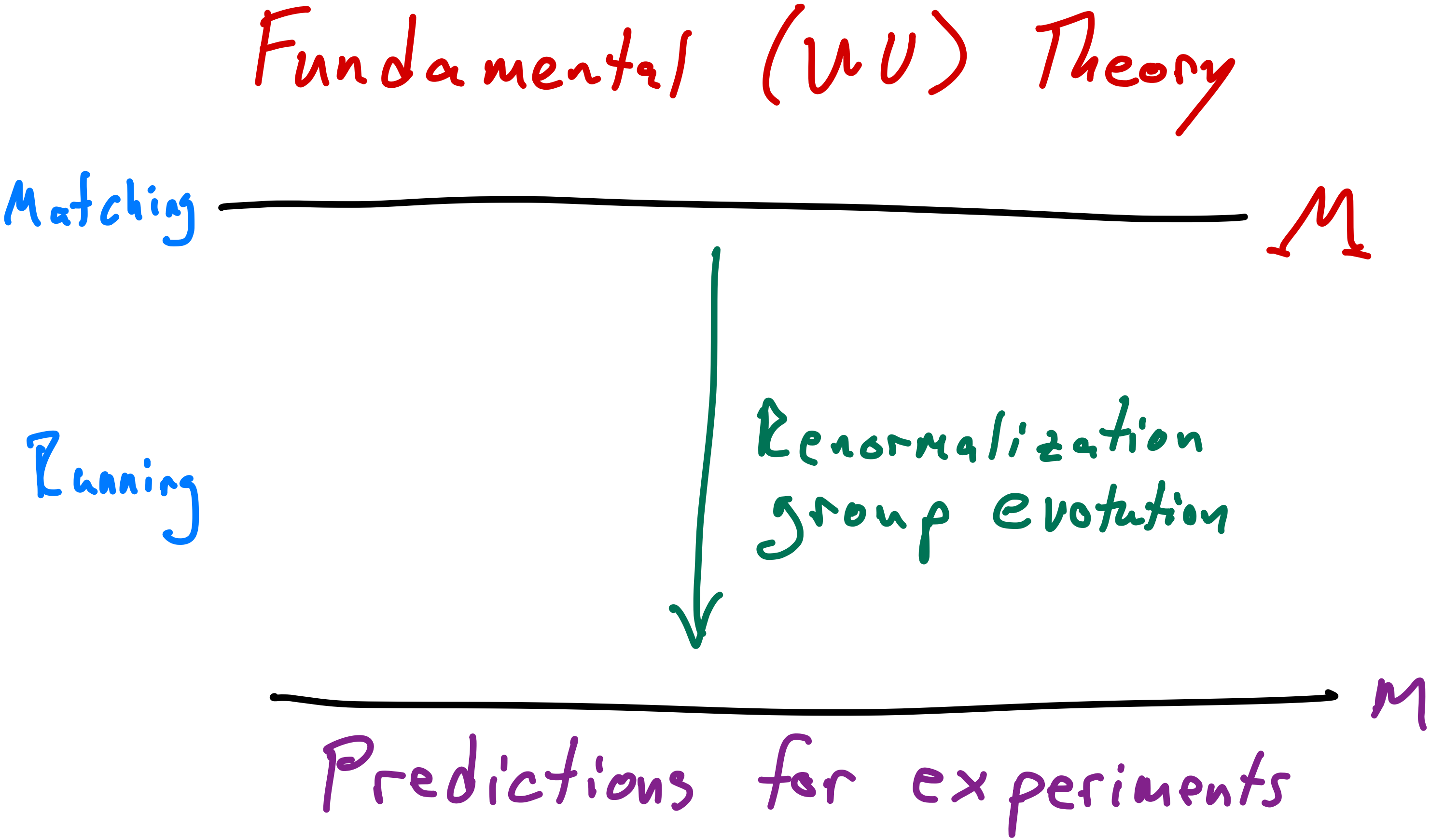
When $m \ll M$, logs can become large

$\Rightarrow$ must resum them

Promote coupling "constants" to running couplings

Renormalization Group Evolution

EFT


Fundamental (UV) Theory
Matching
μ
Running
Renormalization group evolution
μ
Predictions for experiments

Non-relativistic EFT

QFT fields include "particles" and "anti-particles"

Express $\varphi = \varphi_{\text{particle}} + \varphi_{\text{anti-part}}$

Want observables as expansion in $p/m \ll 1$

$\Rightarrow v \ll 1 \Rightarrow$ Power counting

Can "integrate out" $\varphi_{\text{anti-part}} \Rightarrow$ non-rel EFT

Ex: "Heavy Quark Effective Theory"

STrEAMlining EFT Matching

w/ Xiaochuan Lu

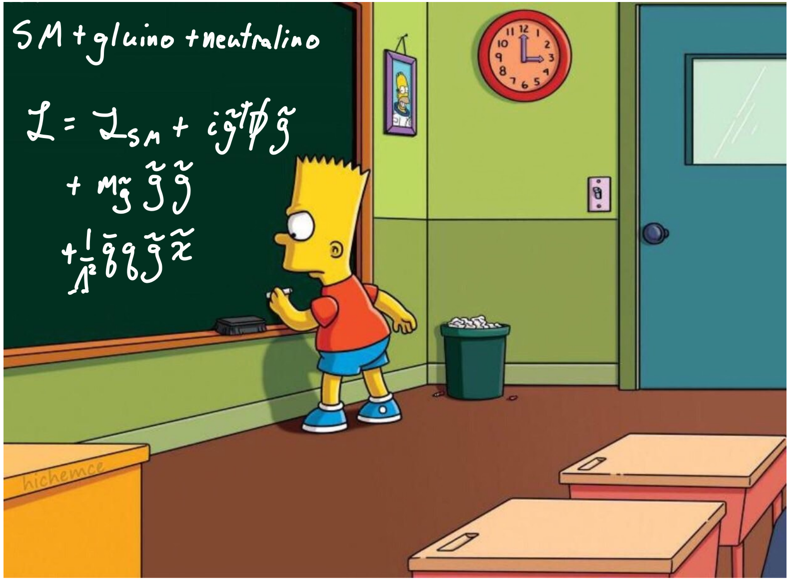
+ Zhengkang (Kevin) Zhang

arXiv: 2011.02484

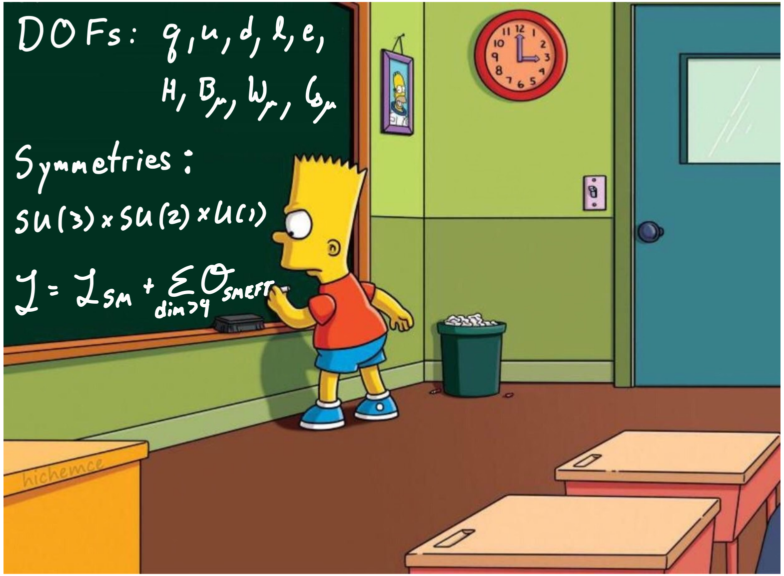
arXiv: 2012.07851

How to organize BSM predictions?

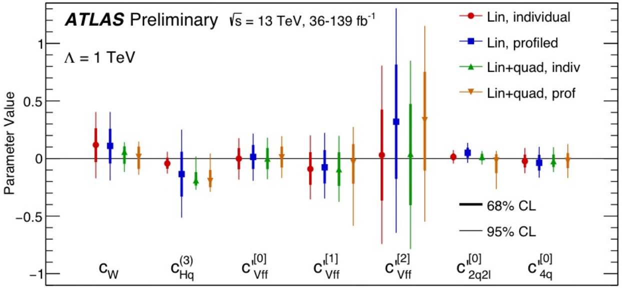
Simplified Models



Effective Field Theory



Constraints on SMEFT



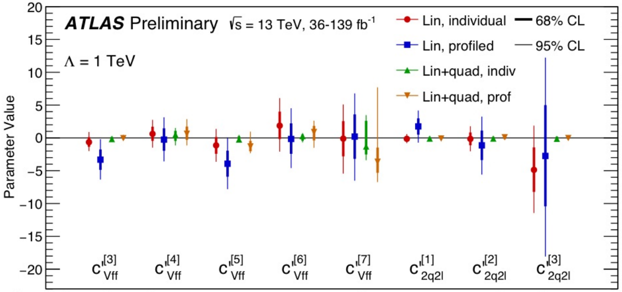
$c'^{[0]}_{Vff} \approx 0.81c_{HWB} + 0.38c_{HD} + 0.13c_{Hl}^{(1)} + 0.37c_{Hl}^{(3)} - 0.14c_{ll}^{(1)} + 0.12c_{Hq}^{(1)}$

$c'^{[1]}_{Vff} \approx 0.73c_{Hl}^{(1)} - 0.28c_{Hl}^{(3)} - 0.48c_{He} + 0.38c_{ll}^{(1)} + 0.13c_{Hq}^{(1)}$

$c'^{[2]}_{Vff} \approx 0.37c_{HWB} + 0.17c_{HD} - 0.31c_{Hl}^{(1)} - 0.53c_{Hl}^{(3)} + 0.25c_{He} + 0.59c_{ll}^{(1)} - 0.21c_{Hq}^{(1)}$

$c'^{[0]}_{2q2l} \approx -0.37c_{lq}^{(1)} + 0.89c_{lq}^{(3)} - 0.11c_{lu} - 0.21c_{eu} - 0.13c_{qe}$

$c'^{[0]}_{4q} \approx 0.11c_{qq}^{(11)} + 0.22c_{qq}^{(18)} + 0.95c_{qq}^{(31)} - 0.2c_{qq}^{(38)}$



$c'^{[3]}_{Vff} \approx -0.19c_{Hl}^{(1)} - 0.14c_{Hl}^{(3)} + 0.86c_{Hq}^{(1)} + 0.41c_{Hu} - 0.17c_{Hd}$

$c'^{[4]}_{Vff} \approx -0.35c_{HWB} + 0.49c_{HD} + 0.26c_{Hl}^{(1)} + 0.35c_{Hl}^{(3)} + 0.51c_{He} + 0.38c_{ll}^{(1)} + 0.18c_{Hq}^{(1)}$

$c'^{[5]}_{Vff} \approx 0.25c_{HD} + 0.33c_{Hl}^{(1)} - 0.22c_{Hl}^{(3)} + 0.18c_{He} - 0.35c_{ll}^{(1)} - 0.3c_{Hq}^{(1)} + 0.71c_{Hu} - 0.16c_{Hd}$

$c'^{[6]}_{Vff} \approx -0.22c_{Hl}^{(1)} + 0.52c_{Hl}^{(3)} - 0.39c_{He} + 0.44c_{ll}^{(1)} - 0.22c_{Hq}^{(1)} + 0.52c_{Hu}$

$c'^{[7]}_{Vff} \approx -0.28c_{HWB} + 0.71c_{HD} - 0.31c_{Hl}^{(1)} - 0.21c_{Hl}^{(3)} - 0.5c_{He} - 0.14c_{ll}^{(1)}$

$c'^{[1]}_{2q2l} \approx 0.56c_{lq}^{(1)} + 0.44c_{lq}^{(3)} + 0.61c_{eu} - 0.1c_{ed} + 0.34c_{qe}$

$c'^{[2]}_{2q2l} \approx 0.68c_{lq}^{(1)} + 0.15c_{lq}^{(3)} + 0.33c_{lu} - 0.51c_{eu} + 0.13c_{ed} - 0.37c_{qe}$

$c'^{[3]}_{2q2l} \approx -0.27c_{lq}^{(1)} + 0.79c_{lu} - 0.39c_{ld} + 0.26c_{eu} - 0.22c_{ed} - 0.16c_{qe}$

SMEFT
= Standard
Model
EFT

How to interpret?

EFT Matching

Connect UV theories to EFT parameters

Two approaches

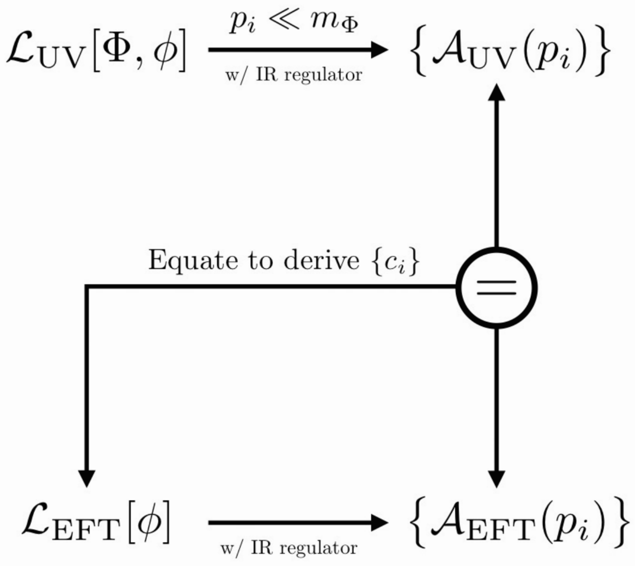
Feynman diagrams (need EFT in advance)

Functional Methods (evaluate path integral directly)

Matching

Amplitude matching

(with Feynman diagrams)



$\mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \xrightarrow[\text{w/ IR regulator}]{p_i \ll m_\Phi} \{\mathcal{A}_{\mathrm{UV}}(p_i)\}$

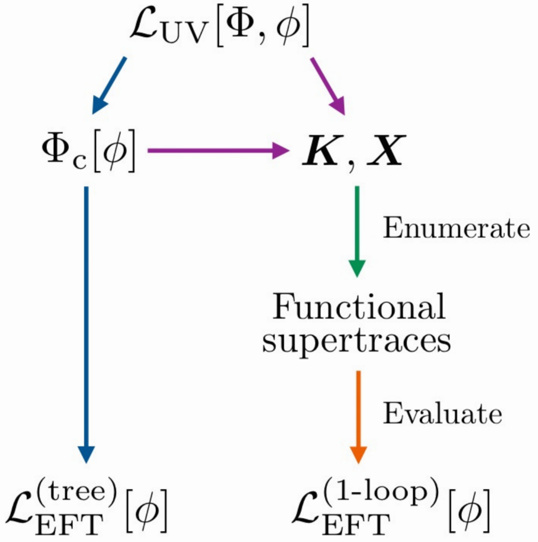
Equate to derive $\{c_i\}$

$=$

$\mathcal{L}_{\mathrm{EFT}}[\phi] \xrightarrow[\text{w/ IR regulator}]{} \{\mathcal{A}_{\mathrm{EFT}}(p_i)\}$

Functional matching

(our prescription)



$\mathcal{L}_{\mathrm{UV}}[\Phi, \phi]$

$\Phi_{\mathrm{c}}[\phi] \longrightarrow \boldsymbol{K}, \boldsymbol{X}$

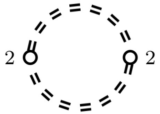
Enumerate

Functional supertraces

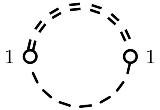
Evaluate

$\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi]$ $\mathcal{L}_{\mathrm{EFT}}^{(1\text{-loop})}[\phi]$

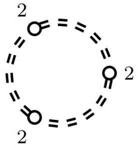
Diagramatic Prescription



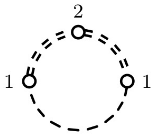
$$= -\frac{i}{2}\,\frac{1}{2}\,\mathrm{STr}\left[\left(\frac{1}{P^2-M^2}\,U^{[2]}_{SS}\right)^2\right]\Big|_{\text{hard}}\,,$$



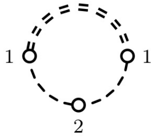
$$= -\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}\,.$$



$$= -\frac{i}{2}\,\frac{1}{3}\,\mathrm{STr}\left[\left(\frac{1}{P^2-M^2}\,U^{[2]}_{SS}\right)^3\right]\Big|_{\text{hard}}\,,$$



$$= -\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[2]}_{SS}\,\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}\,,$$



$$= -\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,U^{[2]}_{HH}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}\,.$$

• • •

Supertraces evaluated using STrEAM package

Example: Singlet Extended SM

Operator	**Coefficient** $\times\, 16\pi^2$
$\lvert H\rvert^2$	$\left[\frac{1}{2}\left(\kappa M^2-\mu_S A\right)+A^2\left(1+\frac{m^2}{M^2}+\frac{m^4}{M^4}\right)\right]\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^4$	$\frac{\kappa^2}{4}\left(-\log\frac{M^2}{\mu^2}\right)+\frac{\mu_S A}{M^2}\left(\frac{\kappa}{2}-\frac{\mu_S A}{4M^2}+\frac{A^2}{M^2}\right)$ $+\frac{A^2}{M^2}\left[\left(\frac{\lambda_S}{4}+3\lambda_H\right)\left(1-\log\frac{M^2}{\mu^2}\right)-2\left(\kappa+\frac{A^2}{M^2}\right)\left(\frac{3}{2}-\log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{m^2}{M^2}\frac{A^2}{M^2}\left[6\lambda_H\left(1-\log\frac{M^2}{\mu^2}\right)-3\left(\kappa+\frac{2A^2}{M^2}\right)\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)+\frac{\mu_S A}{M^2}\left(2-\log\frac{M^2}{\mu^2}\right)\right]$
$\lvert D_\mu H\rvert^2$	$\frac{A^2}{2M^2}+\frac{A^2m^2}{M^4}\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$

Operator	**Coefficient** $\times\, 16\pi^2$
$\lvert H\rvert^6$	$\frac{1}{M^2}\left(-\frac{\kappa^3}{12}-\frac{\kappa^2\mu_S A}{4M^2}+\frac{\kappa\mu_S^2A^2}{2M^4}-\frac{\lambda_S A^4}{2M^4}-\frac{\mu_S^3A^3}{6M^6}+\frac{\mu_S^2A^4}{M^6}\right)$ $+\frac{\kappa A^2}{M^4}\left[3\kappa\left(\frac{11}{6}-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_S}{4}\left(2-\log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{9\lambda_H A^2}{M^4}\left[-\kappa\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)+\lambda_H\left(1-\log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{\mu_S A^3}{M^6}\left[-\kappa\left(5-\log\frac{M^2}{\mu^2}\right)+\frac{\lambda_S}{12}\left(4-\log\frac{M^2}{\mu^2}\right)+3\lambda_H\left(2-\log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{A^4}{M^6}\left[\frac{21\kappa}{2}\left(\frac{37}{21}-\log\frac{M^2}{\mu^2}\right)-18\lambda_H\left(\frac{4}{3}-\log\frac{M^2}{\mu^2}\right)\right]$ $-\frac{7\mu_S A^5}{2M^8}\left(\frac{15}{7}-\log\frac{M^2}{\mu^2}\right)+\frac{9A^6}{M^8}\left(\frac{43}{27}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\partial^2\lvert H\rvert^2)$	$-\frac{\kappa^2}{24M^2}-\frac{5\kappa\mu_S A}{12M^4}$ $+\frac{A^2}{M^4}\left[2\kappa\left(\frac{17}{12}-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_S}{2}\left(1-\log\frac{M^2}{\mu^2}\right)-\frac{\lambda_H}{2}\left(\frac{9}{2}-\log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{11\mu_S^2A^2}{24M^6}-\frac{4\mu_S A^3}{3M^6}+\frac{3A^4}{2M^6}\left(\frac{20}{9}-\log\frac{M^2}{\mu^2}\right)-\frac{3g_2^2A^2}{8M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\lvert D_\mu H\rvert^2$	$\frac{A^2}{M^4}\left[\left(\lambda_H-\frac{A^2}{M^2}\right)\left(\frac{9}{2}-\log\frac{M^2}{\mu^2}\right)-\frac{3\kappa}{2}+\frac{\mu_S A}{2M^2}\right]-\frac{3g_2^2A^2}{2M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\frac{1}{2}\left(H^\dagger\overleftrightarrow{D}^\mu H\right)^2$	$\frac{3g_1^2A^2}{4M^4}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)$
$\lvert D^2H\rvert^2$	$\frac{A^2}{6M^4}$

Operator	**Coefficient** $\times\, 16\pi^2$
$ig_2(D^\mu H)^\dagger\sigma^I(D^\nu H)\,W^I_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$ig_1(D^\mu H)^\dagger(D^\nu H)\,B_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$\frac{ig_2}{2}\left(H^\dagger\sigma^I\overleftrightarrow{D}^\mu H\right)(D^\nu W^I_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$\frac{ig_1}{2}\left(H^\dagger\overleftrightarrow{D}^\mu H\right)(\partial^\nu B_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2W^I_{\mu\nu}W^{I\mu\nu}$	$\frac{g_2^2A^2}{16M^4}$
$\lvert H\rvert^2B_{\mu\nu}B^{\mu\nu}$	$\frac{g_1^2A^2}{16M^4}$
$H^\dagger\sigma^I HW^I_{\mu\nu}B^{\mu\nu}$	$\frac{g_1g_2A^2}{8M^4}$

Operator	**Coefficient** $\times\, 16\pi^2$
$\left(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H\right)\left(\bar{q}\sigma^I\gamma^\mu q\right)$	$\frac{A^2}{8M^4}\left(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger+\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\right)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{q}\gamma^\mu q\right)$	$-\frac{A^2}{8M^4}\left(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger-\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\right)\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{u}\gamma^\mu u\right)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{d}\gamma^\mu d\right)$	$-\frac{A^2}{4M^4}\,\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H\right)\left(\bar{l}\sigma^I\gamma^\mu l\right)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{l}\gamma^\mu l\right)$	$\frac{A^2}{8M^4}\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{e}\gamma^\mu e\right)$	$-\frac{A^2}{4M^4}\,\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$i\left(\widetilde{H}^\dagger(D_\mu H)\right)\left(\bar{u}\gamma^\mu d\right)$ (+h.c.)	$-\frac{A^2}{2M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_d\left(\frac{5}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger\sigma^I H\right)\left(\bar{q}\,\sigma^I i\overleftrightarrow{\not{D}}\,q\right)$	$-\frac{A^2}{8M^4}\left(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger-\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\right)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\left(\bar{q}\,i\overleftrightarrow{\not{D}}\,q\right)$	$\frac{A^2}{8M^4}\left(\boldsymbol{y}_u\boldsymbol{y}_u^\dagger+\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\right)\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\left(\bar{u}\,i\overleftrightarrow{\not{D}}\,u\right)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\left(\bar{d}\,i\overleftrightarrow{\not{D}}\,d\right)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\left(H^\dagger\sigma^I H\right)\left(\bar{l}\,\sigma^I i\overleftrightarrow{\not{D}}\,l\right)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\left(\bar{l}\,i\overleftrightarrow{\not{D}}\,l\right)$	$\frac{A^2}{8M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\left(\bar{e}\,i\overleftrightarrow{\not{D}}\,e\right)$	$\frac{A^2}{4M^4}\,\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\,u\,\widetilde{H}$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_u\boldsymbol{y}_u^\dagger\boldsymbol{y}_u\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\,d\,H$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_d\boldsymbol{y}_d^\dagger\boldsymbol{y}_d\left(1-\log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{l}\,e\,H$ (+h.c.)	$\frac{A^2}{M^4}\,\boldsymbol{y}_e\boldsymbol{y}_e^\dagger\boldsymbol{y}_e\left(1-\log\frac{M^2}{\mu^2}\right)$

One loop matching is "solved"

Stay Tuned

Functional matching relies on dim reg and method of regions.

What about γ_5???

We have developed a novel 4D regulator for the anomaly

Facilitates integrating out Weyl fermions.

Soft de Sitter Effective Theory

w/ Dan Green (UCSD)

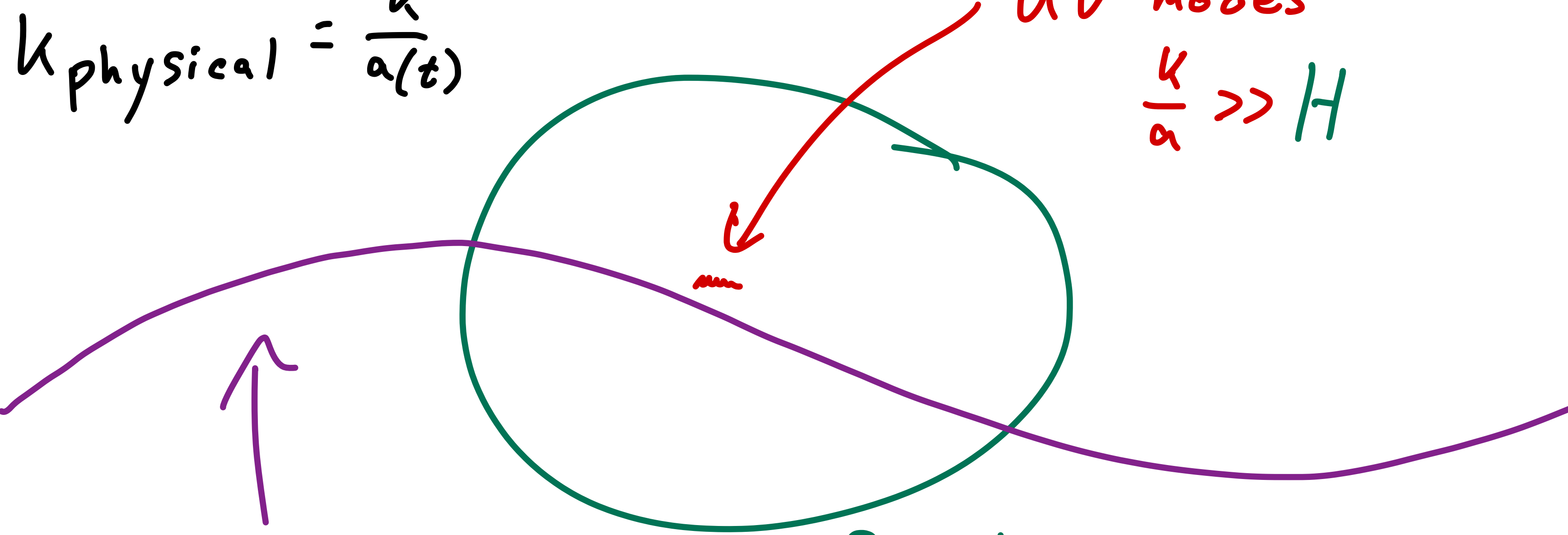
arXiv: 2007.03693

+ Akhil Premkumar + Alec Ridgway

arXiv: 2106.09728, 2111.09332

Soft de Sitter Effective Theory

$$k_{\text{physical}} = \frac{k}{a(t)}$$



UV modes

$$\frac{k}{a} \gg H$$

$$\sim H^{-1}$$

SdSET

$$\frac{k}{a} \ll H$$

Power counting

$$\lambda = \frac{k}{aH}$$

Scalar Fields in dS

EOM $\ddot{\phi} + 3\dot{\phi} + \frac{k^2}{(aH)^2}\phi + \frac{m^2}{H^2}\phi = 0$

Soft limit $\phi_s = (aH)^{-3/2+\nu}\varphi_s$

w/ $\nu = \pm\sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$

or $\alpha = \frac{3}{2} - \nu$ $\beta = \frac{3}{2} + \nu$ s.t. $\alpha + \beta = 3$

WLOG $\alpha < \beta$

One-to-many Mode Expansion

Factorize into soft and hard modes

$$\phi(\vec{x},t) = \phi_S(\vec{x},t) + \Phi_H(\vec{x},t)$$

Integrate out hard modes

$\Rightarrow$ Local operator expansion

Observables order-by-order in power counting

SdSET Fields

Two IR degrees of freedom

- "Growing" mode φ_+ ← Correlators of interest
- "Decaying" mode φ_-

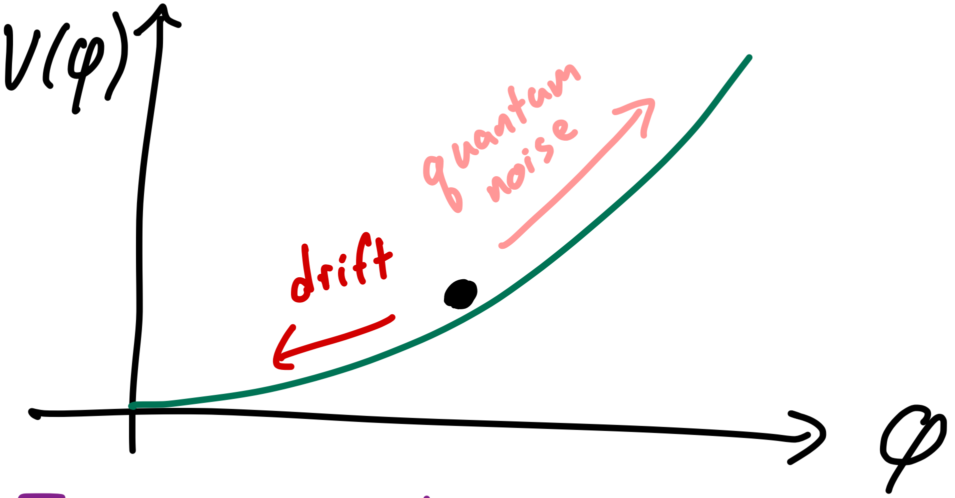
w/ $\phi_s = H\left((aH)^{-\alpha}\varphi_+ + (aH)^{-\beta}\varphi_-\right)$

Time dependence factorizes :)

Starobinsky's Stochastic Inflation

(1986)

Massless scalar field in dS

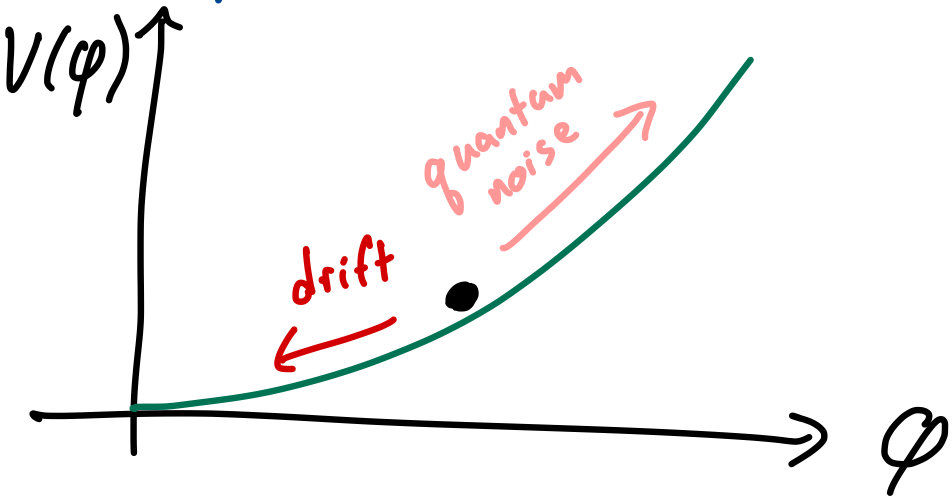


(φ is not the inflaton)

$\Rightarrow$ Fokker-Planck equation:

$$\frac{\partial}{\partial t}P(\varphi,t) = \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\varphi^2}P(\varphi,t) + \frac{1}{3H}\frac{\partial}{\partial\varphi}\left[V'(\varphi)P(\varphi,t)\right]$$

Starobinsky's Stochastic Inflation

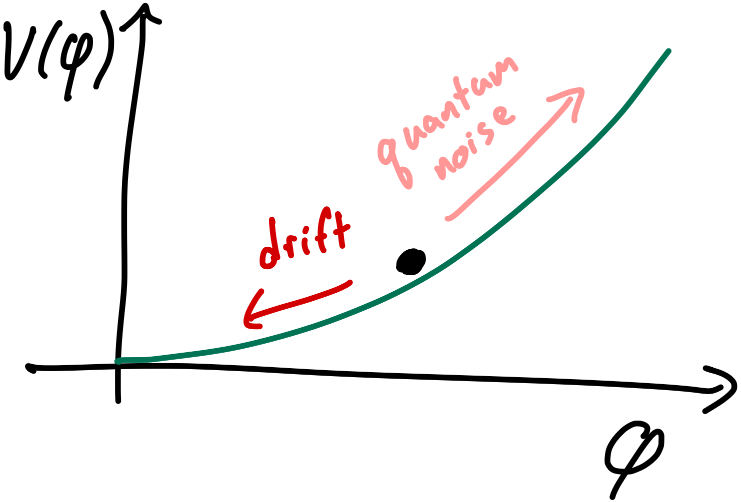


Gaussian noise

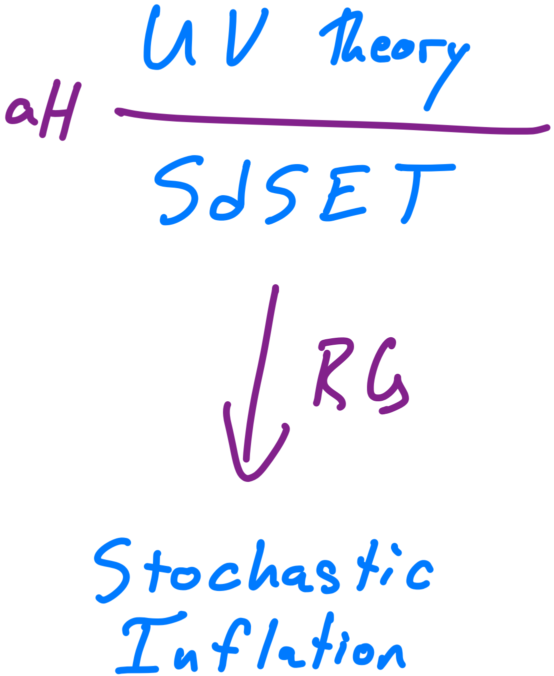
Tree-level potential

Systematic corrections?

Stochastic Inflation ⟺ RG flow



⟺


UV Theory

aH

SdSET

RG

Stochastic Inflation

Light Scalars in dS

Composite operators

$$\mathcal{O}_n = \varphi_+^n \sim \left(k/aH\right)^{n\alpha} \to \mathcal{O}(1)$$

RG mixing expected

Contract any two legs

$$\langle \mathcal{O}_n \ldots \rangle \supset \langle \mathcal{O}_{n-2} \cdots \rangle \binom{n}{2} \frac{C_\alpha^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$

Light Scalars in dS

$$\int \frac{d^3p}{(2\pi)^3}\,\frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$ is scaleless and diverges as $\alpha \to 0$

Isolate UV divergence

$$p^2 \to p^2 + K_{IR}^2$$

$$\langle \mathcal{O}_n \ldots \rangle \supset \langle \mathcal{O}_{n-2} \cdots \rangle \binom{n}{2} \frac{c_\alpha^2}{4\pi^2}\left(-\frac{1}{2\alpha} - \gamma_E - \log\frac{aH}{K_{IR}}\right)$$

Dynamical RG ⟺ Stocastic Inflation

Resum time dependent logs:

$$\frac{\partial}{\partial t}\langle O_n \dots \rangle = -\frac{n}{3}\sum_{m>1}\frac{c_m}{m!}\langle O_{n-1}\, O_m \dots \rangle + \frac{n(n-1)}{8\pi^2}\langle O_{n-2} \dots \rangle$$

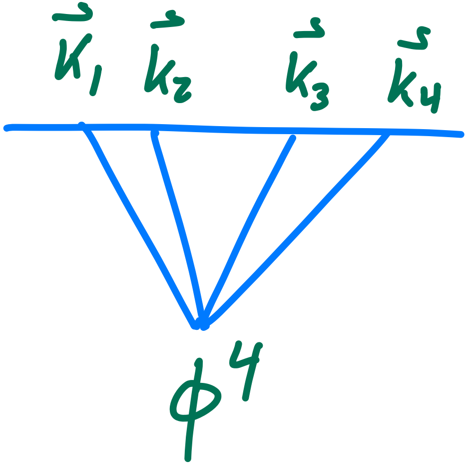
insertion of potential $V \sim c_m \varphi_+^m \varphi_-$

(Starobinsky; Starobinsky, Yokoyama)

Is equivalent to a Fokker-Planck eq for $P(\varphi,t)$ w/ $\langle \varphi^n \rangle = \int d\varphi\, P(\varphi,t)\, \varphi^n$ (Baumgart + Sundrum)

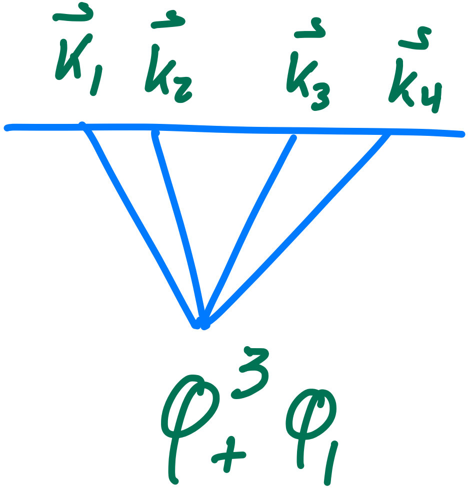
Tree Matching

Assume UV theory is $\lambda\phi^4$

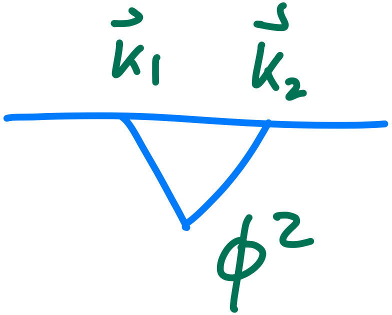


$$=$$

$$\lambda = C_{3,1}$$

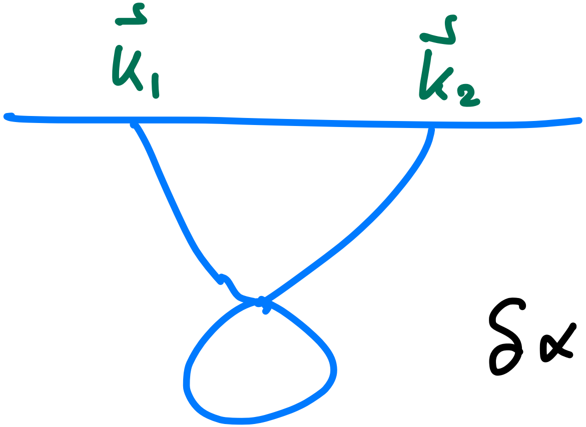


Initial Conditions

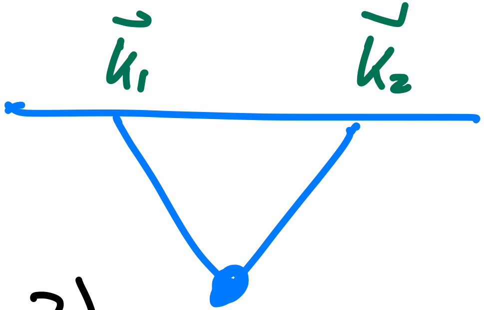


$$= \langle \varphi_+(k_1)\, \varphi_+(k_2) \rangle_{IC}^{Gauss}$$

One Loop Matching



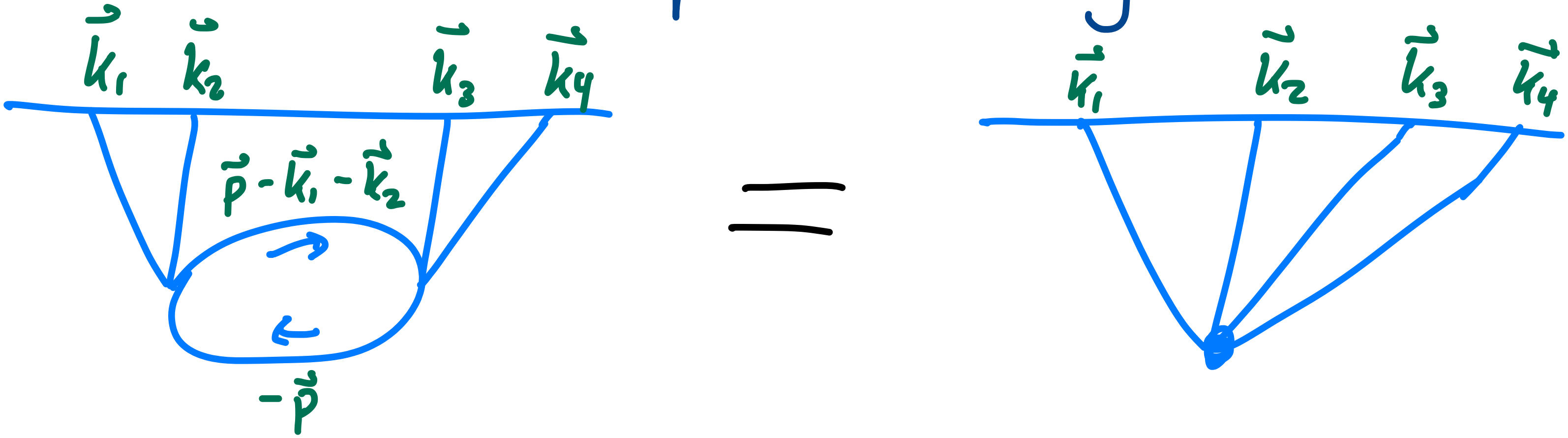
$$=$$



$$\delta\alpha = \frac{\lambda}{8\pi^2}\frac{1}{3}\left(\gamma_E - \frac{7}{3}\right)$$

Other terms removed by $\varphi_- \rightarrow \varphi_- + \frac{\lambda}{9}(aH)^3\varphi_+^2$

One Loop Matching

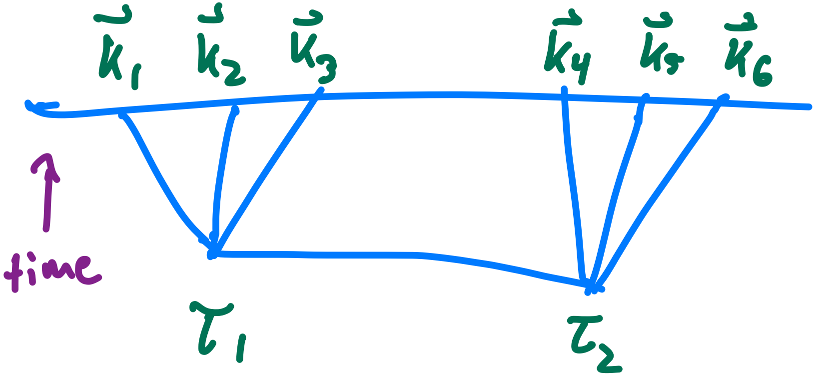


$$C_{3,1} = \lambda - \frac{\lambda^2}{4\pi^2}\left(\frac{1}{9}\gamma_E(2+3\gamma_E)+\frac{5}{12}\pi^2\right)$$

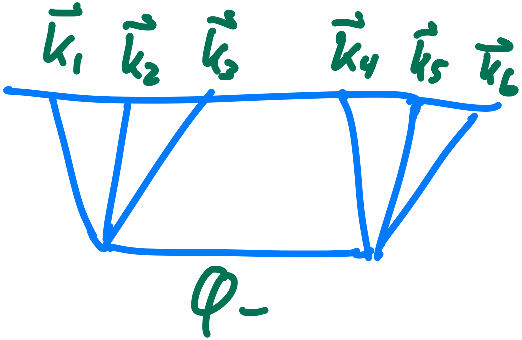
+ $\mathcal{O}(\lambda^2)$ impact on initial conditions

(contributes to NNLO RG)

Initial Conditions to λ^2

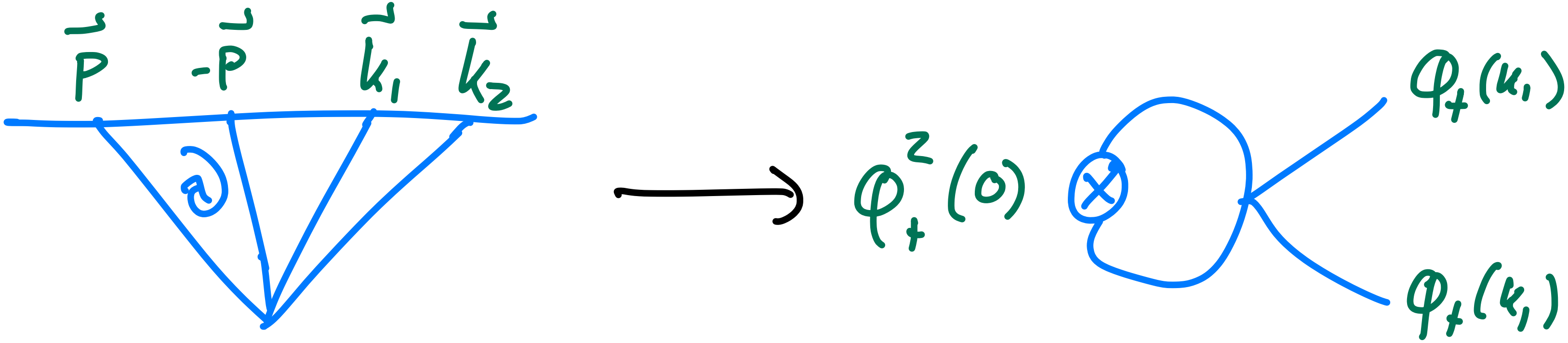


$$\overset{\mathcal{O}(\lambda^2)}{=}$$



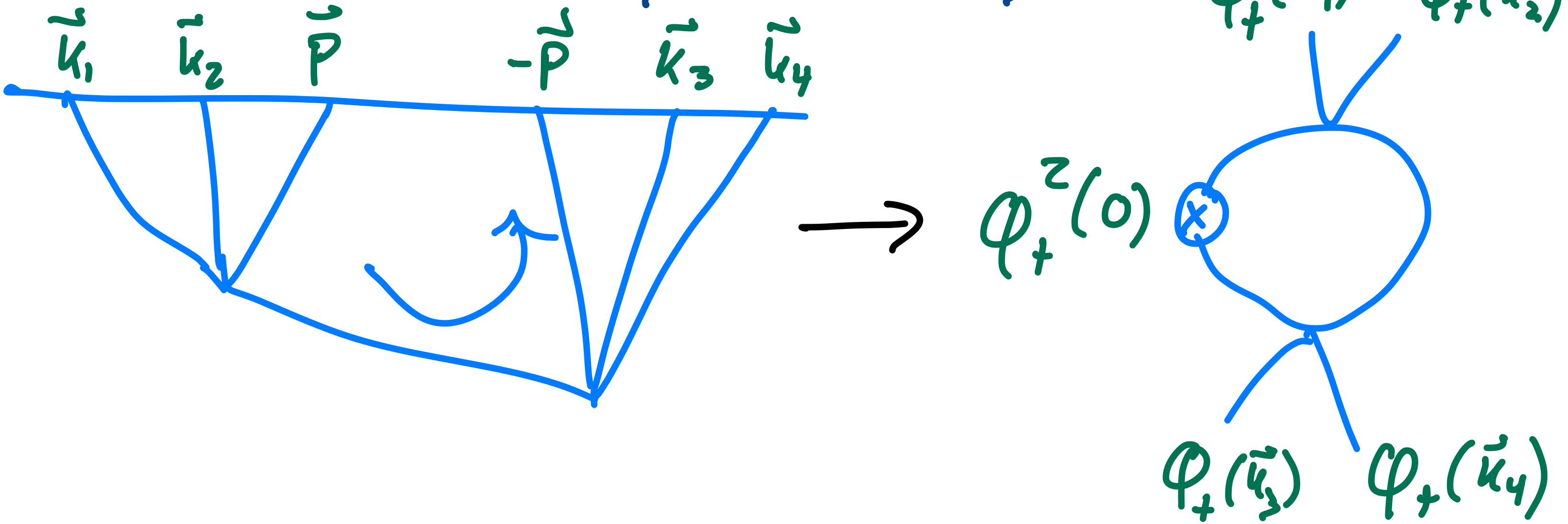
$$+ \left\langle \varphi_+(\vec{k}_1) \ldots \varphi_+(\vec{k}_6) \right\rangle_{IC}$$

NLO $\varphi_+^2(0)$ RG



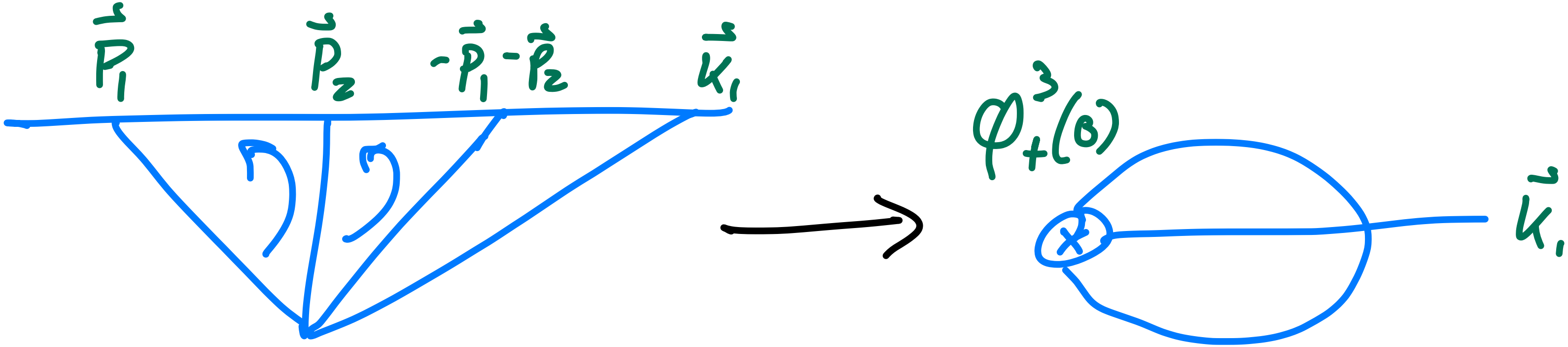
$$\left\langle \varphi_+^2[\vec{x}=0]\,\varphi_+(\vec{k}_1)\,\varphi_+(\vec{k}_2)\right\rangle = \lambda P(k_1)P(k_2)\left(\frac{1}{48\pi^2\alpha^2} + \frac{(4-3\gamma_E-3\log\overset{\text{IR}}{\bar{k}})}{72\pi^2\alpha} + \text{finite}\right)$$

NNLO $\varphi_+^2(0)$ RG



$$\langle \varphi_+^2[\vec{x}=0]\,\varphi_+(k_1)\dots\varphi_+(k_4)\rangle \sim \frac{\lambda^2}{8\pi^2\alpha}\,\frac{1}{27}\left[16+4\gamma_E(-11+3\gamma_E)+3\pi^2+12(\log 2)^2 + \dots\right]$$

NNLO $\varphi^3(0)$ RG



$$\left\langle \varphi_+^3[\vec{x}=0]\,\varphi_+(\vec{k}_1)\right\rangle = \frac{\lambda}{16\pi^2}\,\frac{1}{12}\,P(k_1)\left[\frac{1}{\alpha}+\cdots\right]$$

Put it all together

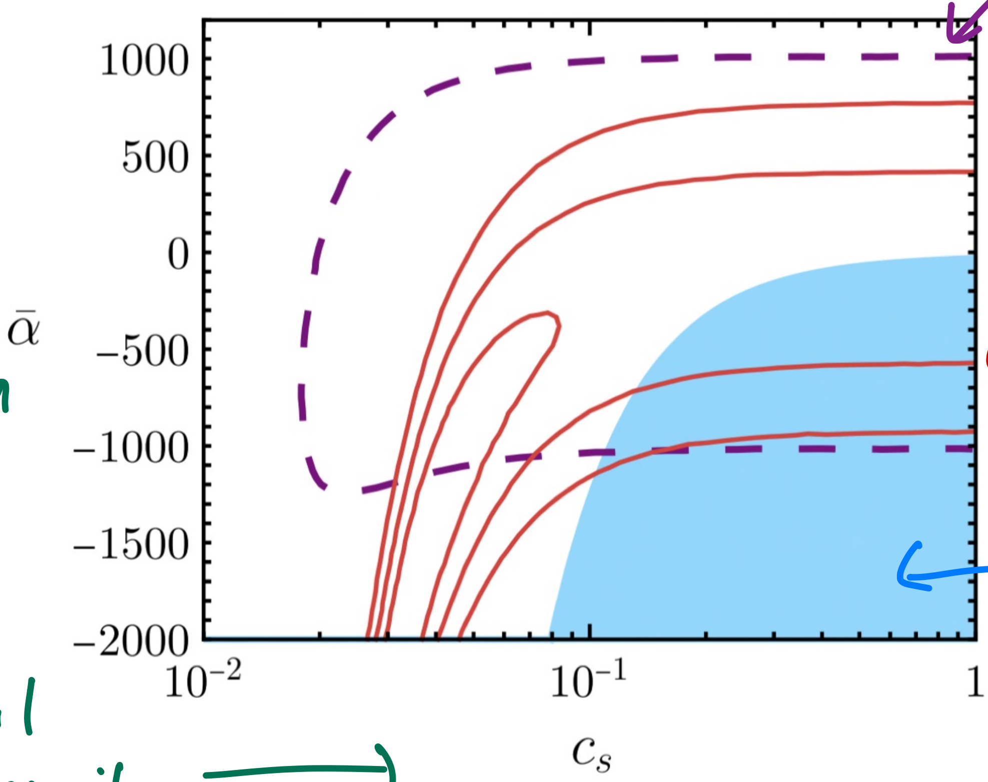
$$\frac{\partial}{\partial t}P = \frac{1}{3}\frac{\partial}{\partial \varphi_+}\left[V_{eff}' P\right] + \frac{1}{8\pi^2}\frac{\partial^2}{\partial \varphi_+^2}P + \frac{\lambda_{eff}}{192\pi^2}\frac{\partial^3}{\partial \varphi_+^3}\left(\varphi_+ P\right)$$

$$V_{eff} = \frac{\lambda_{eff}}{3!}\left(\varphi_+^3 + \frac{\lambda_{eff}}{18}\varphi_+^5 + \frac{\lambda_{eff}}{162}\varphi_+^7 + \dots\right)$$

$$\lambda_{eff} = \lambda - 12b_2 - \frac{\lambda^2}{2\pi^2}\left(\frac{1}{3}\gamma_E(2+3\gamma_E) + \frac{5\pi^2}{4}\right)$$

Compute equilibrium distribution, relaxation eigenvalues, etc.

Apply Same Techniques to Inflaton



perturbative unitarity

Planck Constraints

Large fluctuations out of perturbative control?

Primordial Non-Gaussianity

Stay Tuned

We have developed a comprehensive understanding for the origin of this breakdown.

The tails of these distributions are dominated by a different saddle point $\Rightarrow$ operator scalings change and UV sensitivity emerges.

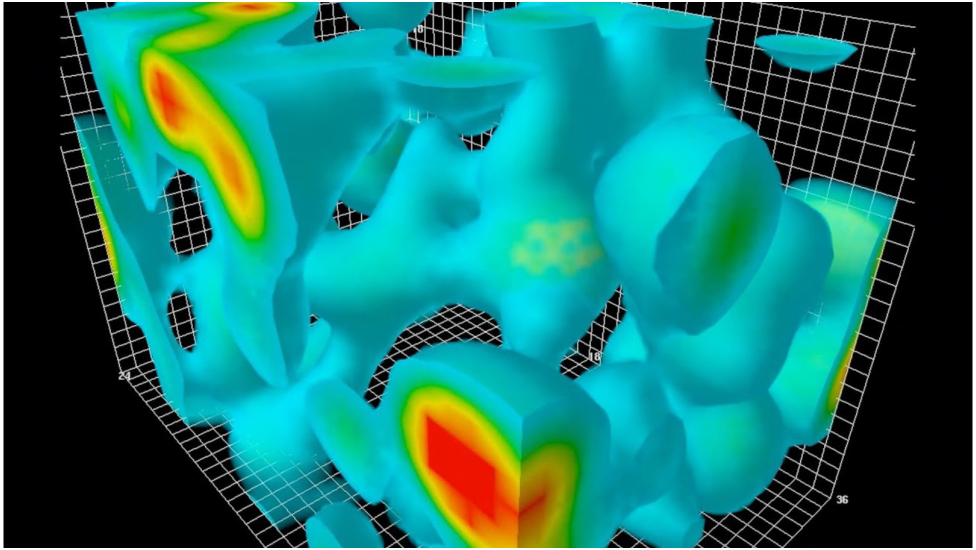
Hamiltionan Truncation Effective Theory

w/ Kara Farnsworth
Rachel Houtz
Markus Luty

arXiv: 2110.08273

Explore Strongly Coupled QFT

Lattice QCD



(from Adelade group)

Hamiltonian Truncation

1) Write $\hat{H} = \hat{H}_0 + \hat{V}$ where H_0 can be solved exactly

$$\hat{H}_0 |E_i\rangle = E_i |E_i\rangle$$

2) Introduce "energy cutoff" E_{max}

3) Compute matrix elements of $\hat{H}$ using truncated basis

4) Diagonalize $\Rightarrow$ approx energy spectrum

Want to Study IR Strong Theories

Simple case study

$\lambda \phi^4$ Theory in 2D

Dimensional analysis

$[\phi] = 0 \Rightarrow [\lambda] = 2$

Relevant operator

$\Rightarrow$ weak in UV and strong in IR

Improve Numerical Predictions

Interpret E_{max} as EFT cutoff scale

Power counting $\lambda \sim E_{IR}/E_{max}$

Write $H_{eff} = H_0 + H_1 + H_2 + \dots$

w/ $H_n = O(V^n)$

finite volume   non-locality

Expect $$H_2 \sim \frac{\lambda^2}{E_{max}^2} \int dx \left[\phi^2 + \phi^4 + \frac{\overset{\swarrow}{R^{-1}} + \overset{\swarrow}{H_0}}{E_{max}} \left(1 + \phi^2 + \phi^4\right) + \dots \right]$$

EFT Expectations

$$H_2 \sim \frac{\lambda^2}{E_{max}^2} \int dx \left[\underbrace{\phi^2 + \phi^4}_{\text{"local approx"}} + \frac{R^{-1} + H_0}{E_{max}} \left(1 + \phi^2 + \phi^4\right) + \dots \right]$$

Compute energy eigenvalues as a function of E_{max}

"Raw truncation" converges as $1/E_{max}^2$

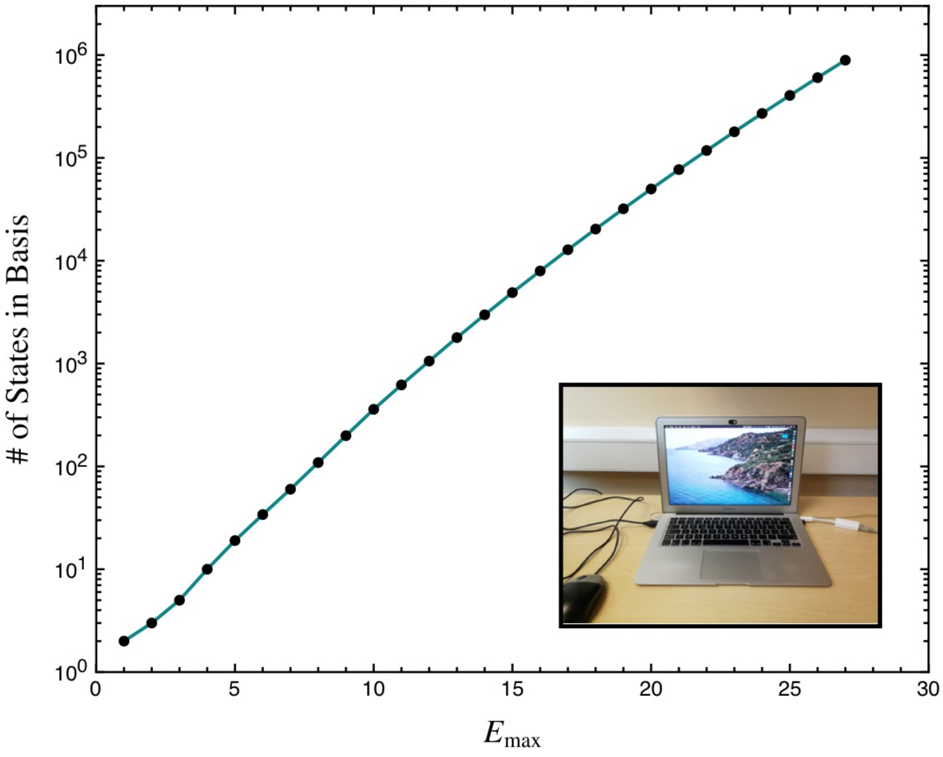
"Improved truncation" converges as $1/E_{max}^3$

Technical Details

- Need IR cutoff ⇒ work in finite volume
- Observable for matching is "transition matrix" (old fashioned perturbation thy)
- Derived set of diagramatic rules
- E_{max} breaks Lorentz invariance

  ⇒ sensitive to states that do not participate in interaction.

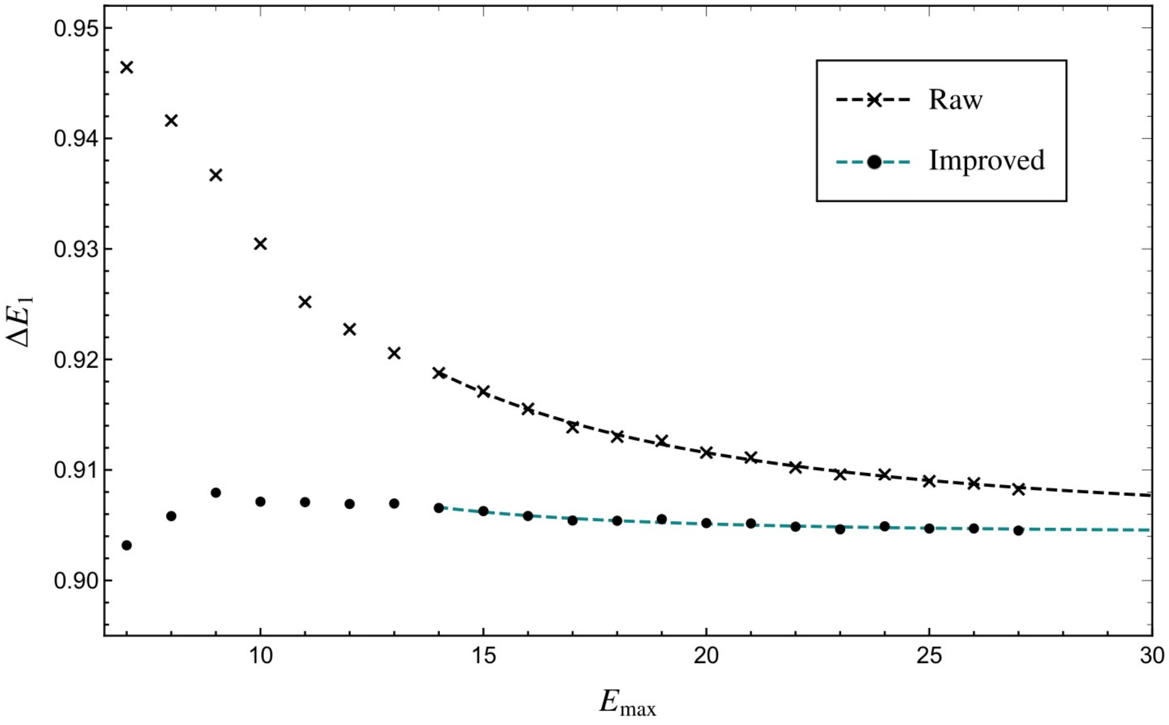
Numerical Results



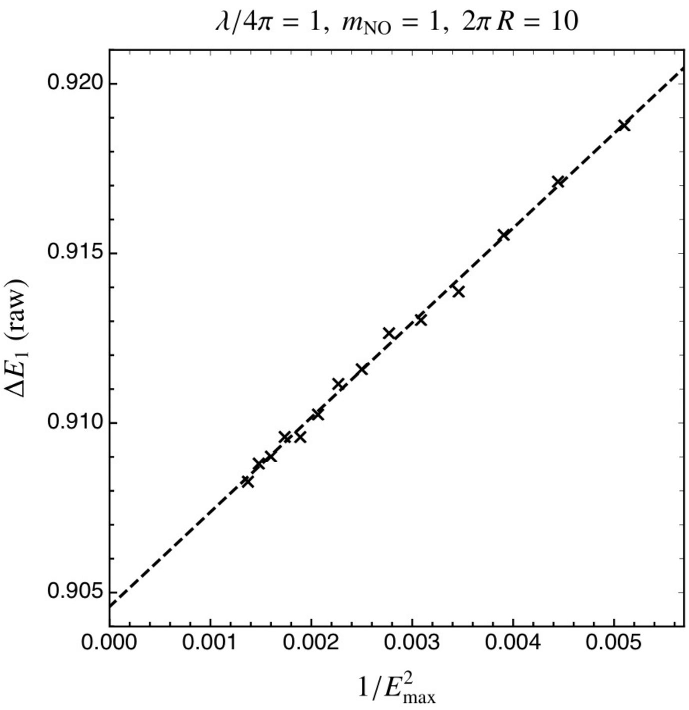
$m_{\mathrm{NO}} = 1,\ 2\pi R = 10$

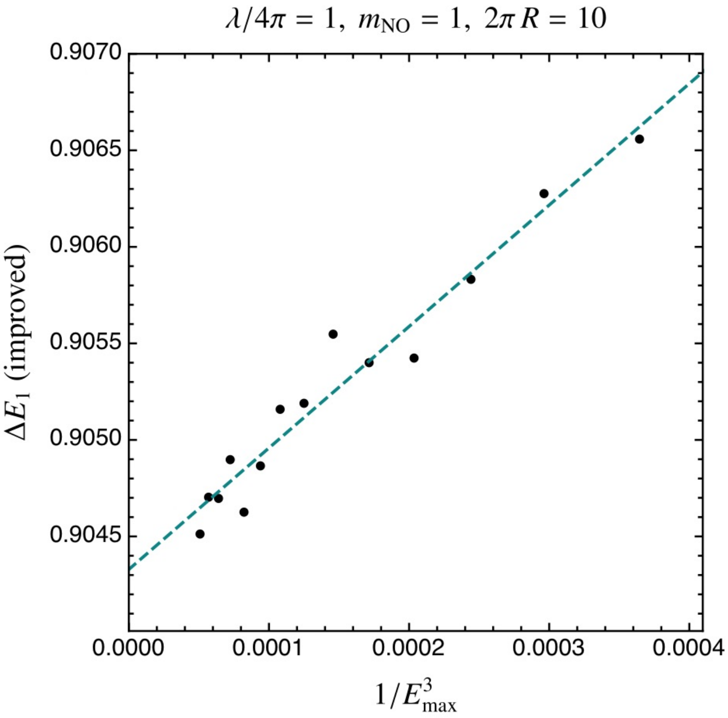
Numerical Results

$\lambda/4\pi = 1,\ m_{\text{NO}} = 1,\ 2\pi R = 10$

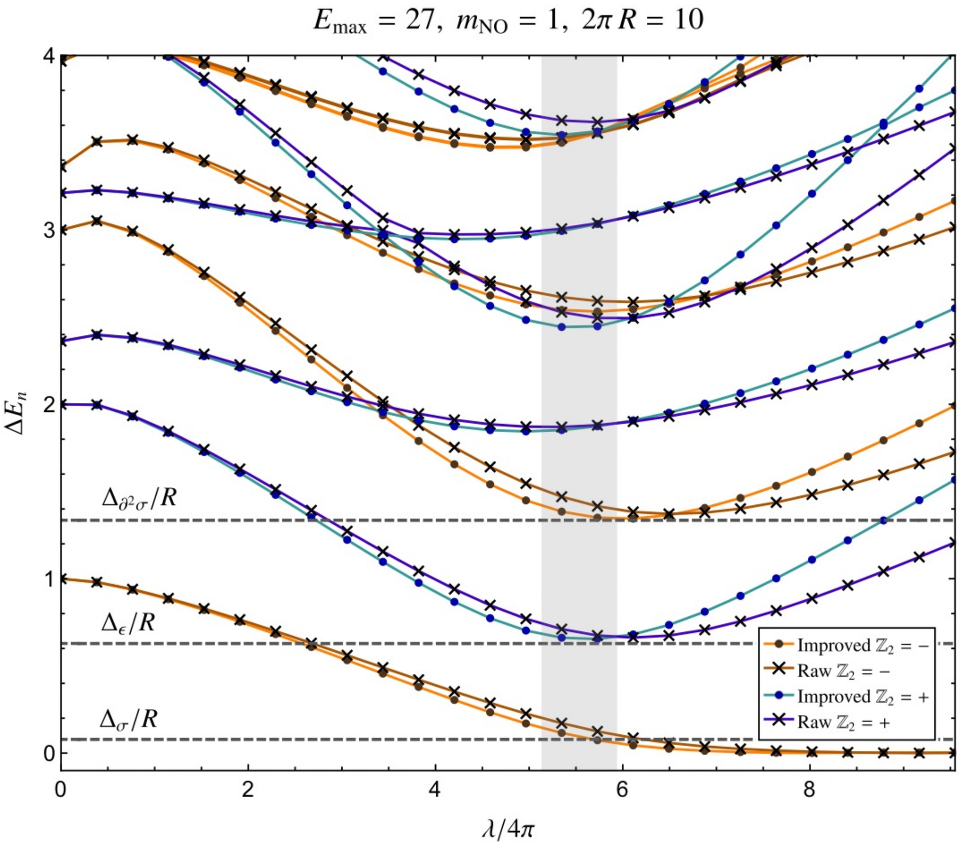


Numerical Results


$\lambda/4\pi = 1,\ m_{\mathrm{NO}} = 1,\ 2\pi R = 10$


$\lambda/4\pi = 1,\ m_{\mathrm{NO}} = 1,\ 2\pi R = 10$

Numerical Results



Stay Tuned

Extension to next order which requires incorporating state dependence into matching coeffecients.

Extension to 3D $\lambda\phi^4$ requires incorporating UV divergences

Outlook

Effective Field Theory is everywhere...

- Heavy physics decouples
- EFT is dimensional analysis and Taylor expansions
- One loop matching is "solved" using functional methods
- Stochastic Inflation is EFT Renormilization Group evolution
- Hamiltionan Truncation is EFT with a finite energy cutoff