

Exclusive central production of $\pi^+\pi^-$ and pp: distributions and questions

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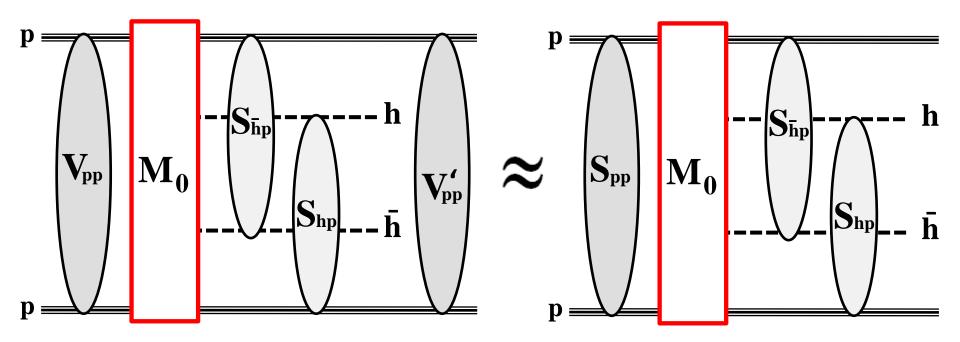
Plan

- 1. Motivations
- 2. Details of processes
- 3. Model formulae
- 4. Distributions and questions
- 5. Summary

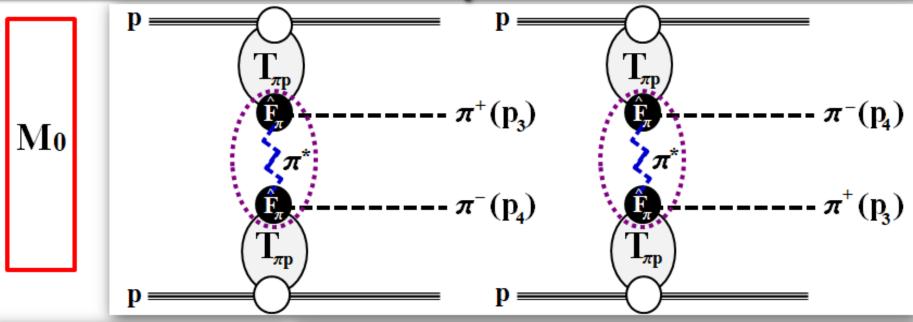
Motivations

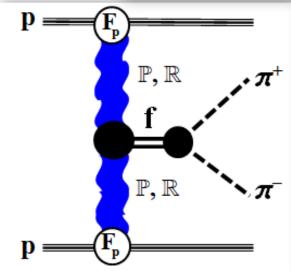
- **CEDP** (Central Exclusive Diffractive Production) of hadron pairs is one of the basic "**standard candles**"
- investigation of hadron resonances ($p+p \Longrightarrow p+\{R \longrightarrow \pi^{\dagger}\pi^{-},h\bar{h}\}+p$), extracting their couplings to hadrons and reggeons
- fine tuning of diffractive models (elastic amplitudes, rescattering corrections, form-factors, couplings)
- clarification of the off-shell hadron form-factors
- large cross sections (crucial factor for low luminocity)
- clear kinematics (missing mass method)
- CEDP distributions are very sensitive to changes in the model (sub amplitudes, form factors, unitarization, reggeization procedure, distributions in central missing mass, t, rapidity and azimuthal angle)
- possible extraction of reggeon-hadron cross-sections
- investigate spin effects, baryon trajectories, search for Odderon

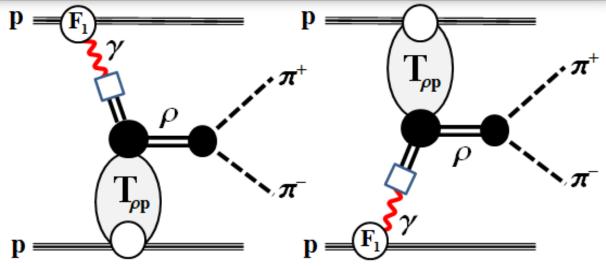
Details of processes



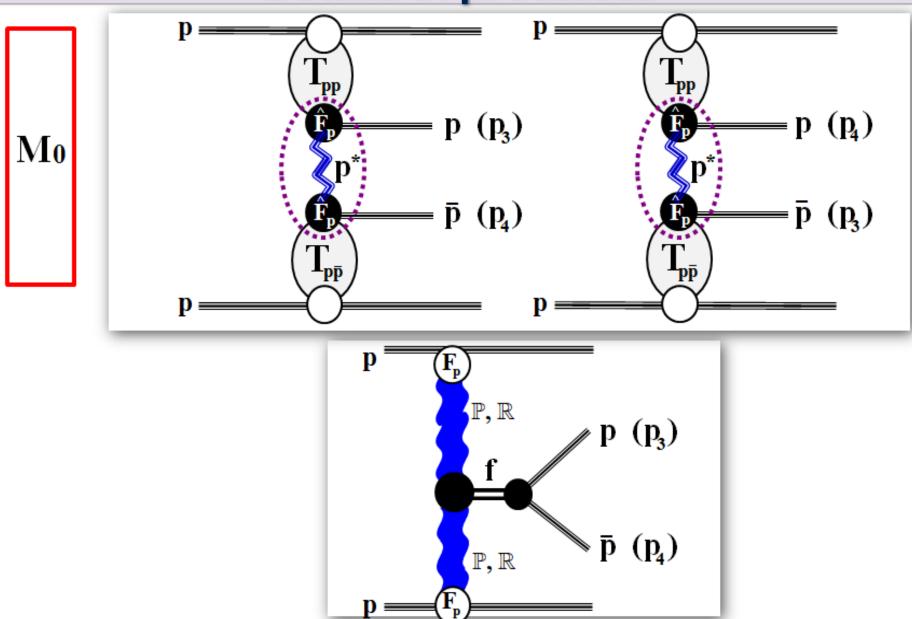
Details of processes



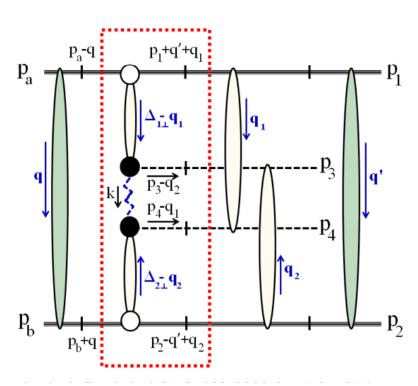




Details of processes



Model formulae (kinematics)



$$\frac{d\sigma_{2\to 4}}{\prod_{i=1}^{3} dp_{i\perp} d\phi_{i} \cdot dy_{3} dy_{4}} = \frac{1}{2\beta s} \cdot \frac{\prod_{i=1}^{3} p_{i\perp}}{2^{4} (2\pi)^{8} \cdot \frac{1}{2} \lambda_{0}^{1/2}} |T|^{2} =$$

$$= \frac{\prod_{i=1}^{3} p_{i\perp}}{2^{12} \pi^{8} \beta s \lambda_{0}^{1/2}} |T|^{2}.$$

$$d\Phi_{2\to 4} = (2\pi)^4 \,\delta^4 \left(p_a + p_b - \sum_{i=1}^4 p_i \right) \prod_{i=1}^4 \frac{d^3 p_i}{(2\pi)^3 2E_i} =$$

$$= \frac{1}{2^4 (2\pi)^8} \prod_{i=1}^3 p_{i\perp} dp_{i\perp} d\phi_i \cdot dy_3 dy_4 \cdot \mathcal{J};$$

$$\mathcal{J} = \frac{dp_{1z}}{E_1} \frac{dp_{2z}}{E_2} \delta \left(\sqrt{s} - \sum_{i=1}^4 E_i \right) \delta \left(\sum_{i=1}^4 p_{iz} \right) =$$

$$= \frac{1}{\left| \tilde{E}_2 \tilde{p}_{1z} - \tilde{E}_1 \tilde{p}_{2z} \right|},$$

where $p_{i\perp} = |\vec{p}_i|$, $\tilde{p}_{1,2z}$ are appropriate roots of the system

$$\begin{cases} A = \sqrt{s} - E_3 - E_4 = \sqrt{m_{1\perp}^2 + p_{1z}^2} + \sqrt{m_{2\perp}^2 + p_{2z}^2}, \\ B = -p_{3z} - p_{4z} = p_{1z} + p_{2z}, \end{cases}$$

$$\tilde{p}_{1z} = \frac{B}{2} + \frac{1}{2(A^2 - B^2)} \left[B \left(m_{1\perp}^2 - m_{2\perp}^2 \right) + A \cdot \lambda_0^{1/2} \right],$$

$$\lambda_0 = \lambda \left(A^2 - B^2, m_{1\perp}^2, m_{2\perp}^2 \right).$$

Here
$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$
, and then $\mathcal{J}^{-1} = \lambda_0^{1/2}/2$.

Model formulae (general amplitude)

$$\begin{array}{c|c} \mathbf{p} & & \\ \hline & \mathbf{V_{pp}} & \mathbf{M_0} & \\ \mathbf{M_0} & & \\ \mathbf{S_{hp}} & -\mathbf{h} & \mathbf{V_{pp}'} \\ \mathbf{p} & & \\ \end{array}$$

$$pprox S_{pp}$$

$$M_0 = S_{\bar{h}p} - h$$

$$S_{hp} - \bar{h}$$

$$\left\{ \begin{array}{l} \mathbf{S}_{\text{pp}} \\ \mathbf{M}_{\mathbf{0}} \end{array} \right\} = \left\{ \begin{array}{l} \mathbf{N}_{\mathbf{0}} \\ \mathbf{N}_{\text{pp}} \end{array} \right\} = \left\{ \begin{array}{l} \mathbf{N}_{\mathbf{0}} \\ \mathbf{N}_{\text{pp}} \end{array} \right\} = \left\{ \begin{array}{l} \mathbf{h} \\ \mathbf{N}_{\text{pp}}$$

$$\{p\} \equiv \{p_a, p_b, p_1, p_2, p_3, p_4\}$$

$$\{\tilde{p}\} \equiv \{p_a - q, p_b + q; p_1 + q' + q_1, p_2 - q' + q_2, p_3 - q_2, p_4 - q_1\},$$

$$\tilde{s}_{14} = (p_1 + p_4 + q')^2, \ \tilde{s}_{23} = (p_2 + p_3 - q')^2,$$

 $s_{ij} = (p_i + p_j)^2, \ t_{1,2} = (p_{a,b} - p_{1,2})^2,$
 $\hat{s} = (p_3 + p_4)^2, \ \hat{t} = (p_a - p_1 - p_3)^2$

Model formulae (continuum and resonance)

$$M_0^C(\{p\}) = T_{hp}^{el}(s_{13}, t_1) \mathcal{P}_h(\hat{s}, \hat{t}) \left[\hat{F}_h(\hat{t})\right]^2 T_{\bar{h}p}^{el}(s_{24}, t_2)$$

$$\hat{F}_h = e^{(\hat{t} - m_h^2)/\Lambda_h^2}$$

(Reggeized) hadron propagator

$$S_{h_1h_2}(s,q^2) = \int d^2\vec{b} \, e^{i\vec{q}\vec{b}} \left(1 + 2iT_{h_1h_2}^{el}(s,b) \right) =$$

$$= \int d^2\vec{b} \, e^{i\vec{q}\vec{b}} e^{-2\Omega_{h_1h_2}^{el}(s,b)} = (2\pi)^2 \delta^2 \left(\vec{q} \right) + 2\pi \bar{T}_{h_1h_2}(s,q^2),$$

$$\bar{T}_{h_1h_2}(s,q^2) = \int_0^\infty b \, db \, J_0 \left(b\sqrt{-q^2} \right) \left[e^{-2\Omega_{h_1h_2}^{el}(s,b)} - 1 \right]$$

$$V_{h_1h_2}(s,q^2) = \int d^2\vec{b} \, e^{i\vec{q}\vec{b}} \sqrt{1 + 2iT_{h_1h_2}^{el}(s,b)} =$$

$$= \int d^2\vec{b} \, e^{i\vec{q}\vec{b}} e^{-\Omega_{h_1h_2}^{el}(s,b)} = (2\pi)^2 \delta^2 \left(\vec{q} \right) + 2\pi \tilde{T}_{h_1h_2}$$

$$\tilde{T}_{h_1h_2} = \int_0^\infty b \, db \, J_0 \left(b\sqrt{-q^2} \right) \left[e^{-\Omega_{h_1h_2}^{el}(s,b)} - 1 \right]$$

Off-shell hadron form-factor

$$T_{h_1h_2}^{el}(s,b) = \frac{e^{-2\Omega_{h_1h_2}^{el}(s,b)} - 1}{2i},$$

$$\Omega_{h_1h_2}^{el}(s,b) = -i \,\delta_{h_1h_2}^{el}(s,b),$$

$$\delta_{h_1h_2}^{el}(s,b) = \frac{1}{16\pi s} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \delta_{h_1h_2}^{el}(s,t)$$

Corrections

Elastic amplitudes

[A.A. Godizov, Effective transverse radius of nucleon in high-energy elastic diffractive scattering, Eur. Phys. J. C 75, 224 (2015)]
[A.A. Godizov, Asymptotic properties of Regge trajectories and elastic pseudoscalar-meson scattering on nucleons at high energies, Yad. Fiz. 71, 1822 (2008)]

 $M_0^R\left(\{\tilde{p}\}\right)$ Amplitude of resonance di-hadron production

Model formulae (di-pion continuum)

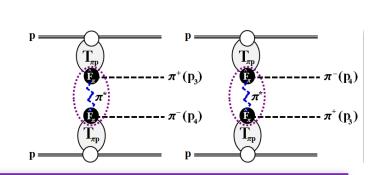
$$M_0^{C,\pi}(\{p\}) = T_{\pi+p}^{el}(s_{13}, t_1) \mathcal{P}_{\pi}(\hat{s}, \hat{t}) \left[\hat{F}_{\pi}(\hat{t}) \right]^2 T_{\pi-p}^{el}(s_{24}, t_2)$$

$$\mathcal{P}_{\pi}(\hat{s}, \hat{t}) = \left(\operatorname{ctg} \frac{\pi \alpha_{\pi}(\hat{t})}{2} - i\right) \cdot \frac{\pi \alpha_{\pi}'}{2\Gamma(1 + \alpha_{\pi}(\hat{t}))} \left(\frac{\hat{s}}{s_0}\right)^{\alpha_{\pi}(\hat{t})}$$

$$\alpha_{\pi}(\hat{t}) = 0.7(\hat{t} - m_{\pi}^2)$$

$$\hat{F}_{\pi} = e^{(\hat{t} - m_{\pi}^2)/\Lambda_{\pi}^2},$$
where $\Lambda_{\pi} \sim 1.2 \text{ GeV}$

from STAR data



Model formulae (di-pion f resonances)

$$\begin{split} M_0^{pp \to p\{f_0 \to \pi^+ \pi^-\}p} &= \\ &= -F_{\mathbb{P}}(t_1, \xi_1) F_{\mathbb{P}}(t_2, \xi_2) \, g_{\mathbb{PP}f_0}(t_1, t_2, M_c^2) \times \\ &\times \frac{g_{f_0 \pi \pi} \left(\mathcal{F}(M_c^2, m_{f_0}^2) \right)^2 F_M(t_1) F_M(t_2)}{\left(M_c^2 - m_{f_0}^2 \right) + B_{f_0}(M_c^2, m_{f_0}^2)}, \\ B_{f_0}(M_c^2, m_{f_0}^2) &= \\ &= \mathrm{i} \, \Gamma_{f_0} \left(\mathcal{F}(M_c^2) \right)^2 \left[\frac{1 - 4 m_{\pi}^2 / M_c^2}{1 - 4 m_{\pi}^2 / m_{f_0}^2} \right]^{1/2} \\ \mathcal{F}(M_c^2, m_f^2) &= F^{\mathbb{PP}f}(M_c^2, m_f^2) = F^{f \pi \pi}(M_c^2, m_f^2) = \\ &= \exp \left(\frac{-(M_c^2 - m_f^2)^2}{\Lambda_f^4} \right), \, \Lambda_f \sim 1 \, \mathrm{GeV}, \\ F_M(t) &= 1/(1 - t / m_0^2), \, m_0^2 = 0.5 \, \mathrm{GeV}^2 \end{split}$$

$$\begin{split} M_0^{pp \to p\{f_2 \to \pi^+ \pi^-\}p} &= \\ &= F_{\mathbb{P}}(t_1, \xi_1) F_{\mathbb{P}}(t_2, \xi_2) \, g_{\mathbb{P}Pf_2} \times \\ &\times \frac{\left(g_{f_2\pi\pi}/2\right) \left(\mathcal{F}(M_c^2, m_{f_2}^2)\right)^2 F_M(t_1) F_M(t_2)}{\left(M_c^2 - m_{f_2}^2\right) + B_{f_2}(M_c^2, m_{f_2}^2)} \mathcal{P}_2 \\ \text{where} \\ \mathcal{P}_2 &= \left(\Delta_1 \Delta_{34}\right)^2 - \frac{\left(M_c^2 - 4m_\pi^2\right) \lambda \left(M_c^2, t_1, t_2\right)}{12 M_c^2} \\ B_{f_2}(M_c^2, m_{f_2}^2) &= \\ &= \mathrm{i} \, \mathcal{F}_{f_2} \left(\mathcal{F}(M_c^2, m_{f_2}^2)\right)^2 \left[\frac{1 - 4m_\pi^2/M_c^2}{1 - 4m_\pi^2/m_{f_2}^2}\right]^{5/2} \frac{M_c^4}{m_{f_2}^4} \end{split}$$

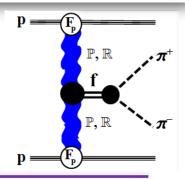
$$F_{\mathbb{P}}(t,\xi) = g_{pp\mathbb{P}}(t)^2 \left(i + \tan \frac{\pi(\alpha_{\mathbb{P}}(t) - 1)}{2} \right) \frac{\pi \alpha'_{\mathbb{P}}(t)}{\xi^{\alpha_{\mathbb{P}}}(t)}$$

 $g_{\mathbb{PP}f_0(500)} = 0.88,$ $g_{\mathbb{PP}f_0(980)} = 0.43,$

 $g_{\mathbb{PP}f_2(1270)} = 1.72.$

from STAR data 0.64 for the "glueball"

[A.A. Godizov, The ground state of the Pomeron and its decays to light mesons and photons, Eur. Phys. J. C 76, 361 (2016)]



Model formulae (di-pion rho)

$$M_{0}^{pp \to p\{\rho_{0} \to \pi^{+}\pi^{-}\}p} = i T_{\rho_{0}p}^{el}(s_{2}, t_{2}) \frac{C_{T}^{\rho_{0}}(t_{1})\mathcal{P}_{\rho}}{|t_{1}|} \times \frac{(g_{\rho_{0}\pi\pi}/2)\mathcal{F}_{\rho}(M_{c}^{2})\mathcal{F}_{\rho}(t_{1})}{(M_{c}^{2} - m_{\rho_{0}}^{2}) + B_{\rho_{0}}(M_{c}^{2}, m_{\rho_{0}}^{2})} + (1 \leftrightarrow 2),$$

$$s_{2} = (p_{3} + p_{4} + p_{2})^{2} = (p_{a} - p_{1} + p_{b})^{2},$$

$$s_{1} = (p_{3} + p_{4} + p_{1})^{2} = (p_{b} - p_{2} + p_{a})^{2}.$$

$$\delta_{\rho_{0}p}^{el}(s, t) \simeq g_{pp\mathbb{P}}(t)g_{\rho\rho\mathbb{P}}(t)\left(i + \tan\frac{\pi(\alpha_{\mathbb{P}}(t) - 1)}{2}\right) \times \pi\alpha_{\mathbb{P}}'(t)\left(\frac{s}{2s_{0}}\right)^{\alpha_{\mathbb{P}}(t)},$$

$$g_{\rho\rho\mathbb{P}}(t) = g_{\rho\rho\mathbb{P}}(0) = 7.07 \,\mathrm{GeV}$$
 [A.A. Godizov]

$$\begin{split} \left(\vec{\Delta}_{34}^* \vec{p}_a^*\right) &= -\Delta_{34} p_a - \Delta_{34,z}^* p_{a,z}^*, \\ \Delta_{34,z}^* &= -\Delta_1 \Delta_{34} \frac{2M_c}{\lambda^{1/2}}, \\ p_{a,z}^* &= \left(\frac{p_a p_c (M_c^2 + t_1 - t_2)}{2M_c^2} - \frac{t_1}{2}\right) \frac{2M_c}{\lambda^{1/2}}, \\ \lambda &\equiv \lambda (M_c^2, t_1, t_2), \ p_c = p_3 + p_4. \end{split}$$

$$C_T^{\rho_0}(t) = \sqrt{\frac{3\Gamma_{\rho \to e^+ e^-}}{\alpha_e m_\rho}} \frac{m_\rho^2}{m_\rho^2 - t},$$

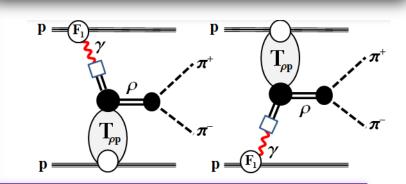
$$\mathcal{F}_\rho(p^2) = (1 + p^2(p^2 - m_\rho^2)/\Lambda_\rho^4)^{-n_\rho},$$

$$F_1(t) = \frac{1 - \kappa t/(4m_\rho^2)}{1 - t/(4m_\rho^2)} (1 - t/m_D^2)^{-2},$$

$$B_{\rho_0}(M_c^2, m_{\rho_0}^2) =$$

$$= i \Gamma_{\rho_0} \left(\mathcal{F}_{\rho}(M_c^2) \right)^2 \left[\frac{1 - 4m_{\pi}^2 / M_c^2}{1 - 4m_{\pi}^2 / m_{\rho_0}^2} \right]^{3/2} \frac{M_c^2}{m_{\rho_0}^2}$$

Constants can be found in [P. Lebiedowicz, O. Nachtmann, A. Szczurek]



Model formulae (p pbar continuum)

$$M_0^{C,p}(\{p\}) = T_{pp}^{el}(s_{13}, t_1) \mathcal{P}_p(\hat{s}, \hat{t}) \left[\hat{F}_p(\hat{t}) \right]^2 T_{\bar{p}p}^{el}(s_{24}, t_2),$$

$$\mathcal{P}_p(\hat{t}) = 1/(\hat{t} - m_p^2)$$
 "scalar" proton approximation

General case with reggeization

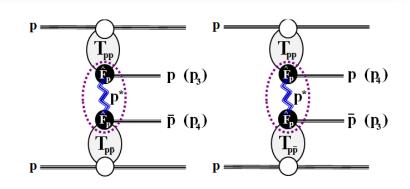
$$\alpha_p(\hat{t}) = -0.4 + 0.9\hat{t} + 0.125\hat{t}^2.$$

and spin effects will be considered in further papers

$$\hat{F}_p = e^{(\hat{t} - m_p^2)/\Lambda_p^2},$$

where $\Lambda_p \sim 1 \text{ GeV}$

from STAR data



Model formulae (p pbar f resonance)

$$\begin{split} &M_0^{pp \to p\{f_0 \to p\bar{p}\}p} = \\ &= -F_{\mathbb{P}}(t_1, \xi_1) F_{\mathbb{P}}(t_2, \xi_2) \, g_{\mathbb{P}\mathbb{P}f_0}(t_1, t_2, M_c^2) \times \\ &\times \frac{g_{f_0 p\bar{p}} \left(\mathcal{F}(M_c^2, m_{f_0}^2)\right)^2 F_M(t_1) F_M(t_2)}{(M_c^2 - m_{f_0}^2) + B_{f_0}(M_c^2, m_{f_0}^2)}, \end{split}$$

$$B_{f_0}(M_c^2, m_{f_0}^2) =$$

$$= i \Gamma_{f_0} \left(\mathcal{F}(M_c^2) \right)^2 \left[\frac{1 - 4m_p^2 / M_c^2}{1 - 4m_p^2 / m_{f_0}^2} \right]^{1/2}$$

$$\mathcal{F}(M_c^2, m_f^2) = F^{\mathbb{PP}f}(M_c^2, m_f^2) = F^{fp\bar{p}}(M_c^2, m_f^2) =$$

$$= \exp\left(\frac{-(M_c^2 - m_f^2)^2}{\Lambda_f^4} \right), \Lambda_f \sim 1 \text{ GeV},$$

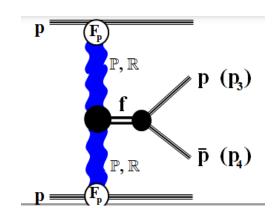
$$F_M(t) = 1/(1 - t/m_0^2), m_0^2 = 0.5 \text{ GeV}^2$$

$$F_{\mathbb{P}}(t,\xi) = g_{pp\mathbb{P}}(t)^2 \left(i + \tan \frac{\pi(\alpha_{\mathbb{P}}(t) - 1)}{2} \right) \frac{\pi \alpha_{\mathbb{P}}'(t)}{\xi^{\alpha_{\mathbb{P}}}(t)}$$

$$g_{\mathbb{PP}f_0(2100)} = 0.64,$$
 [A.A. Godizov] for the "glueball" $g_{p\bar{p}f_0(2100)} = 3.1,$

testing STAR data

Other constants can be found in [P. Lebiedowicz, O. Nachtmann, A. Szczurek]

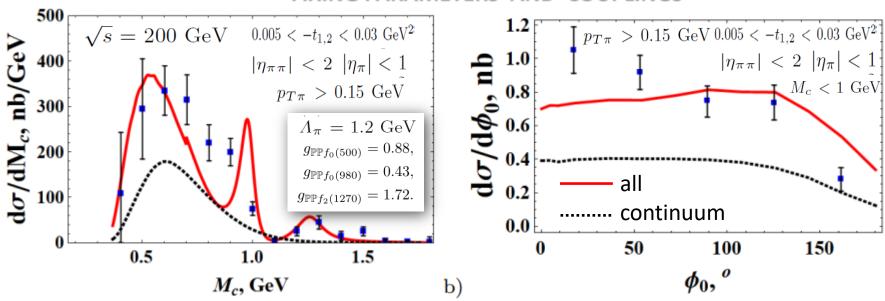


Distributions and questions (di-pion STAR)

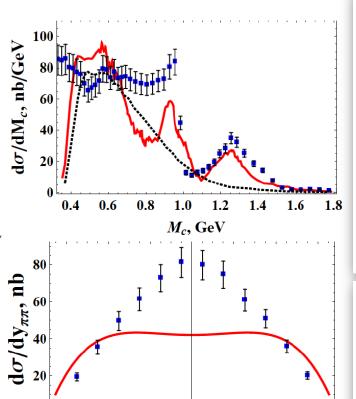
Our basic task is to extract the fundamental information on the interaction of hadrons from different cross-sections ("diffractive patterns"):

- from t-distributions we can obtain size and shape of the interaction region;
- the distribution on the azimuthal angle between final protons gives quantum numbers of the produced system;
- from the **dependence on missing mass** and its influence on t-dependence we can make some conclusions about the **interaction at different space-time scales and interrelation between them**. Also we can extract **couplings of reggeons** to different resonances.

FIXING PARAMETERS AND COUPLINGS



Distributions and questions (di-pion STAR)

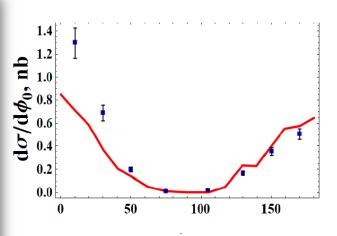


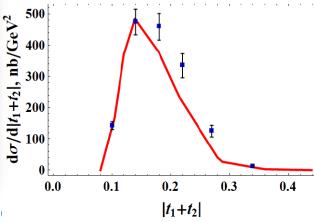
0.0

0.2

$$\sqrt{s} = 200 \; {
m GeV}$$
 $(p_x + 0.3 \, {
m GeV})^2 + p_y^2 < 0.25 \, {
m GeV}^2$
 $0.2 \; {
m GeV} < |p_y| < 0.4 \; {
m GeV}$
 $p_x > -0.2 \; {
m GeV}$
 $p_{T\pi} > 0.2 \; {
m GeV}$
 $|\eta_{\pi}| < 0.7$
— all
...... continuum

difference between the data and predictions at the same energy





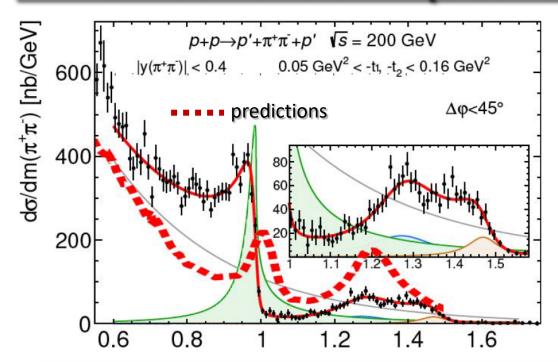
- Pomeron-Pomeron-meson couplings are constants in this approach, but in reality they depends on the azimuthal angle
- predictions slightly overestimate the data in the region of mass \sim 0.6 GeV (it depends on the Pomeron coupling to fo(500) and can be corrected) and underestimate the data in the region of mass \sim 0.8 GeV
- more flat in pseudorapidity (features of the Pomeron flux)

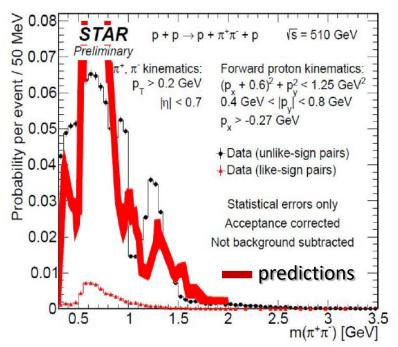
0.4

0.6

-0.4 -0.2

Distributions and questions (di-pion STAR)

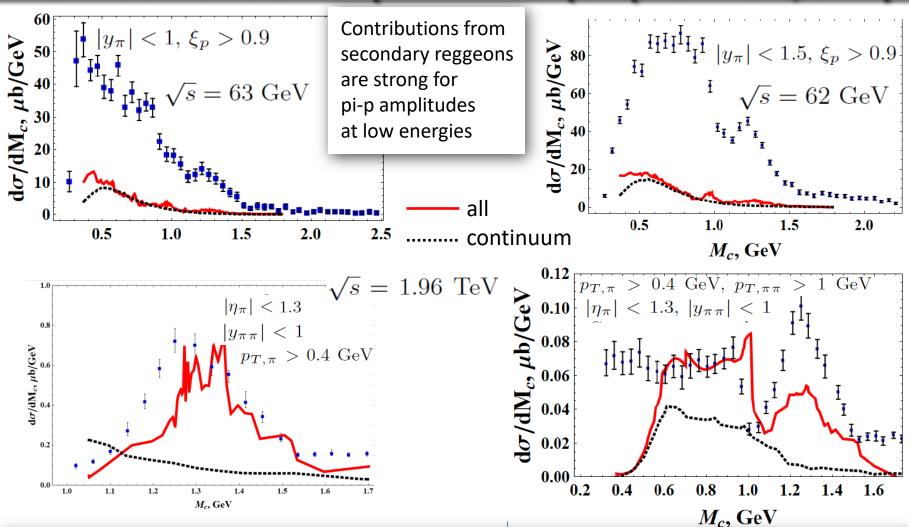




Underestimate the data at low masses and overestimate the data at $f_2(1270)$. Since these data were obtained by some extrapolation, it can be considered only qualitatively.

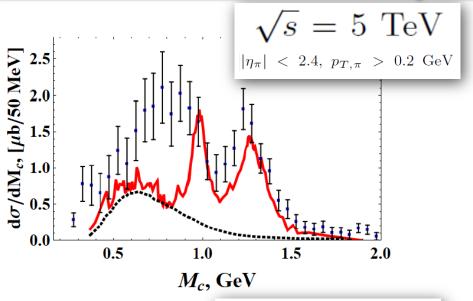
Quality of the data?

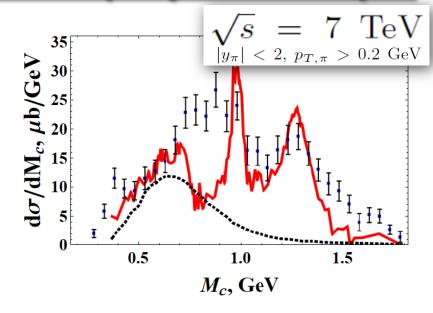
Distributions and questions (di-pion ISR,CDF)

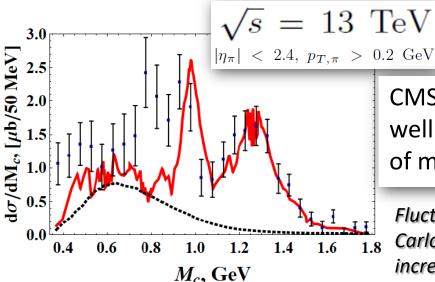


It looks strange and has no explanation at the moment, since this is the data from the same experiment. Interference effects with gamma-gamma or gamma-Odderon fusion? Corrections to pion-pion final interaction?

Distributions and questions (di-pion CMS)







CMS data at all energies are described rather well. Discrepancy is observed only in the region of masses around 0.8 GeV

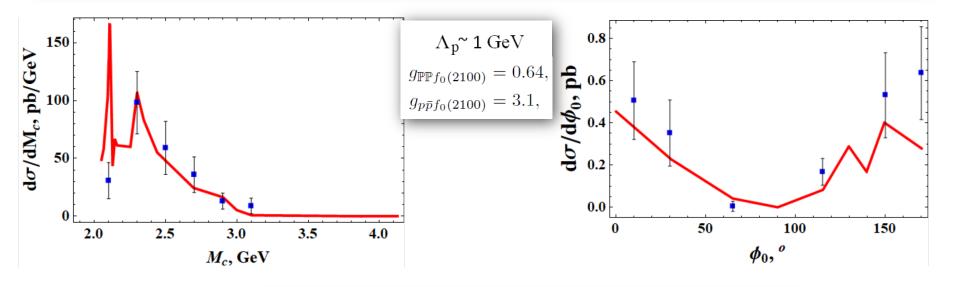
Fluctuations in solid curves are due to complex Monte-Carlo integration process, and they can be smoothed by increasing the integration accuracy

Distributions and questions (p pbar STAR)

$$\sqrt{s} = 200 \text{ GeV}$$

$$|\eta_{p,\bar{p}}| < 0.7 \quad p_T^p(p_T^{\bar{p}}) > 0.4 \text{ GeV} \quad \min(p_T^p, p_T^{\bar{p}}) < 1.1 \text{ GeV} \quad (p_x + 0.3 \text{ GeV})^2 + p_y^2 < 0.25 \text{ GeV}^2$$

$$p_x > -0.2 \text{ GeV} \quad 0.2 \text{ GeV} < |p_y| < 0.4 \text{ GeV}$$



Preliminary calculations for p pbar production and qualitative comparison with STAR data
We do not see f0(2100) at STAR ...

Summary

- The result is crucially dependent on the choice of the off-shell pion form factor
- The description of the old STAR data with (theory: 0.64 for the "glueball")
- Discrepancy in the rho region (small rho contribution in this kinematics)

$$\Lambda_{\pi} = 1.2 \text{ GeV}$$
 $g_{\mathbb{PP}f_0(500)} = 0.88,$
 $g_{\mathbb{PP}f_0(980)} = 0.43,$
 $g_{\mathbb{PP}f_2(1270)} = 1.72.$

$$\hat{F}_{\pi} = e^{(\hat{t} - m_{\pi}^2)/\Lambda_{\pi}^2}$$

- **Rescattering corrections** in p-p and pi-p channels make a **significant contribution** to the values and shape of distributions
- When parameters are fixed from the old STAR data, we have a difference between the data from STAR (new data at 200 & 510 GeV), ISR, CDF and theoretical predictions, which should be explained somehow (complex dependance of Pomeron couplings to mesons on t and azimuthal angle, wrong normalization of the data, missed contributions from some other processes like low mass difractive dissociation, interference with gamma gamma and gamma Odderon processes, effects related to the irrelevance and possible modifications of the Regge approach (for the virtual pion exchange)
- To fix the model parameters correctly we need the comparison with precise (exclusive) experimental data (STAR, ATLAS+ALFA, CMS+TOTEM) simultaneously in several differential observables
- This model will be implemented to the Monte-Carlo event generator ExDiff

THANK YOU!