Holographic QCD Equation of State Modeling in the Bayesian Era

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J. Noronha-Hostler, C. Ratti and R. Rougemont

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Illinois Center for Advanced Studies of the Universe

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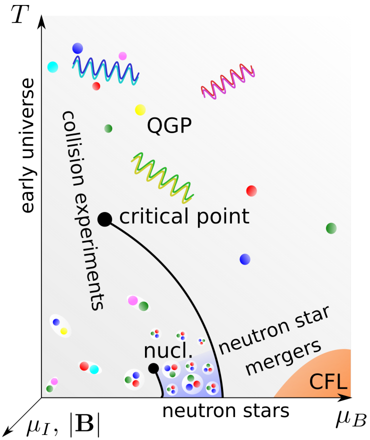
Introduction

- Lattice QCD: First-principles EoS for small μ.
- Strongly coupled, nearly inviscid behavior of the QGP: Holographic model via AdS black-hole dual.

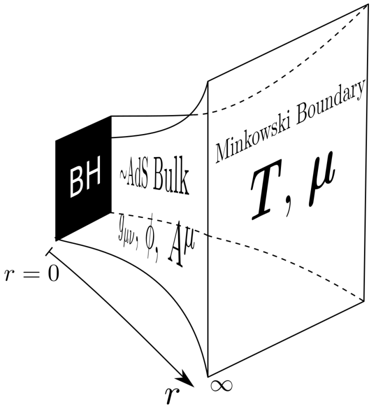
  P. Kovtun, D. T. Son, A. O. Starinets, PRL **94** (2005)
- "Black-hole engineering": Extrapolate from lattice results to make predictions on phase diagram.

  S. S. Gubser and A. Nellore, PRD **78** (2008)
  O. DeWolfe, S. S. Gubser and C. Rosen, PRD **83** (2011)
  R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **96** (2017)
  J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **104** (2021)



Black-Hole Engineering: Theory

- Gauge-gravity duality.
  J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998)
- 5D bulk: Classical gravity with asymptotically Anti-deSitter (AdS_5) geometry.
- 3+1D Boundary: Strongly coupled fluid in Minkowski spacetime.
- AdS radius r: $\sim$ RG energy scale.
- Black-hole horizon at $r = 0$: Infrared. Hawking temperature.

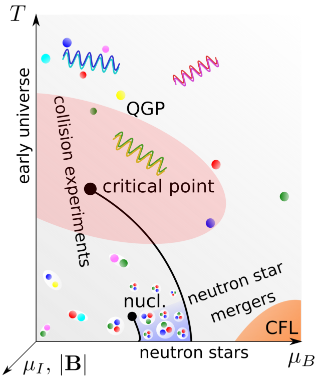


Limitations

- No asymptotic freedom and no quasi-particle excitations.
- Poor description of hadronic phase.

Advantages

- Able to handle out-of-equilibrium physics and predict transport properties.
  S. S. Gubser, A. Nellore, S. S. Pufu and F. D. Rocha, PRL **101**, (2008)
- Bulk viscosity (non-conformal effect) compatible with heavy-ion analyses.
  J. Grefa, M. Hippert, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, PRD **106** (2022)
- Can predict critical point and first-order line.
  O. DeWolfe, S. S. Gubser and C. Rosen, PRD **83** (2011)



Einstein-Maxwell-Dilaton model

- Breaking of conformal symmetry: dilaton field ϕ.
- Dual to baryon chemical potential μ: Abelian gauge field A^μ.
- Action:

$$S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right],$$

- Two potentials, $V(\phi)$ and $f(\phi)$, tweaked to fit lattice QCD results.

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Phenomenological holographic potentials

Polynomial-Hyperbolic Parametrization

- Interpolates between `arXiv:1706.00455` and `arXiv:2201.02004`

$$V(\phi) = -12\cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

$$f(\phi) = \frac{\operatorname{sech}(c_1\phi + c_2\phi^2 + c_3\phi^3)}{1+d_1} + \frac{d_1}{1+d_1}\operatorname{sech}(d_2\phi)$$

Parametric Approach

- Similar shapes, more interpretable parameters

$$V(\phi) = -12\cosh\left[\left(\frac{\gamma_1\,\Delta\phi_V^2 + \gamma_2\,\phi^2}{\Delta\phi_V^2 + \phi^2}\right)\phi\right]$$

$$f(\phi) = 1-(1-A_1)\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi-\phi_1}{\delta\phi_1}\right)\right] - A_1\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi-\phi_2}{\delta\phi_2}\right)\right]$$

New C++ code within MUSES Framework

- Development within the MUSES Framework: Multi-institutional collaboration for a unified solver for the equation of state, bridging models and applications.
- Support and advising by cyberinfrastructure and computer-science experts T. Andrew Manning and Roland Haas.
- Improved method to extract asymptotic UV scalings and thermodynamics.
- Large boost in performance and numerical stability.

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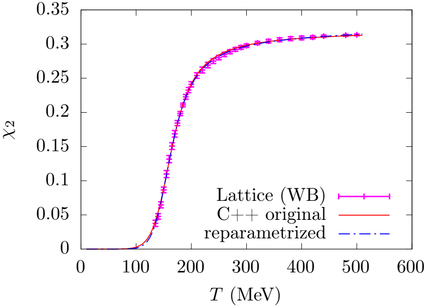
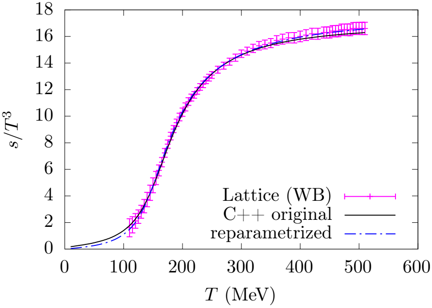
Claudia Ratti — Thursday 11:30

Model calibration: Zero-density EoS

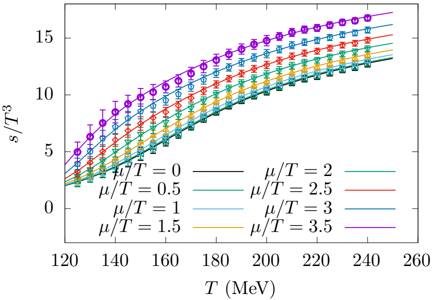


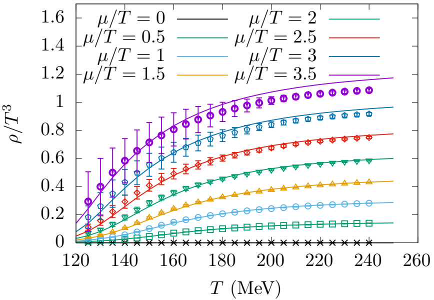


- Baryon susceptibility and entropy density from the lattice used to refit model.
- Also used as inputs in Bayesian analysis further on.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, PRL **730** (2014)
Bellwied, Borsanyi, Fodor et al., PRD **92** (2015)
MH, J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, to appear.

Postdictions: Finite-density EoS





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Phase diagram

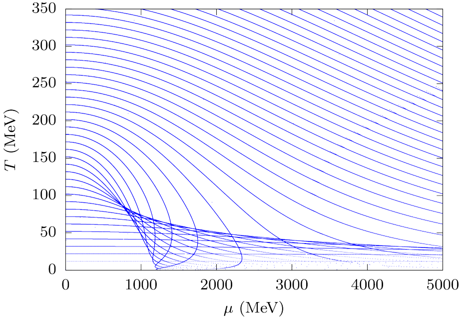
- Metastable solutions → phase transition.

- Initial conditions

$$\begin{cases} \phi_0 \equiv \phi(0) \\ \Phi_1 \equiv \Phi'(0) \end{cases}$$

for dilaton and (bulk) electric field.

- Constant ϕ_0 lines, varying Φ_1.



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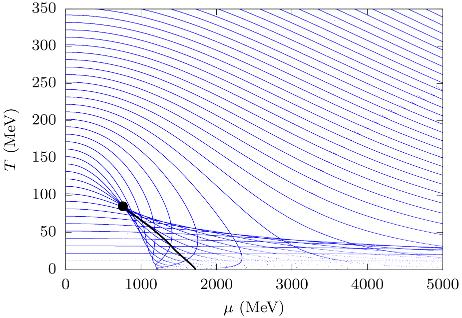

- Constant ϕ_0 lines, varying Φ_1.
- Critical endpoint (CEP) at crossing point.

- Powerful, flexible model capable of describing crossover region and beyond.

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- Accurate prediction of χ_8 supported by lattice results.

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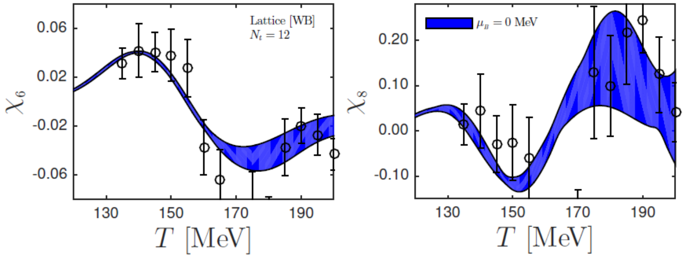
S. Borsanyi, Z. Fodor, J. N. Guenther, S. K. Katz, K. K. Szabo, A. Pasztor, I. Portillo and C. Ratti, JHEP **10** (2018)
R. Rougemont, R. Critelli and J. Noronha, PRD **98** (2018)



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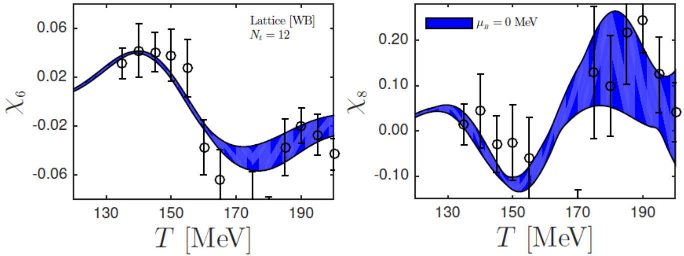


- $\gtrsim 10$ parameters. What is their role in predictions?
- Do lattice results favor a critical point?

Bayesian analysis

Predictions

- Full EoS compatible with lattice results at $\mu = 0$.
- Critical point, first order line and metastable phases.
- Good postdiction of T^{μ}_{μ} and finite density EoS.

Uncertainties

- How strong are constraints from low-density EoS?
- Statistical error from lattice results?
- Systematic fit: optimal set of parameters?

Use tools of Bayesian inference!

Probabilities

Bayes' Theorem

$$\underbrace{P\left(\text{model} \,\middle|\, \text{results}\right)}_{\text{posterior } \mathcal{P}} \times P\left(\text{results}\right) = \underbrace{P\left(\text{results} \,\middle|\, \text{model}\right)}_{\text{likelihood } \mathcal{L}} \times \underbrace{P\left(\text{model}\right)}_{\text{prior knowledge}}$$

Gaussian Likelihood

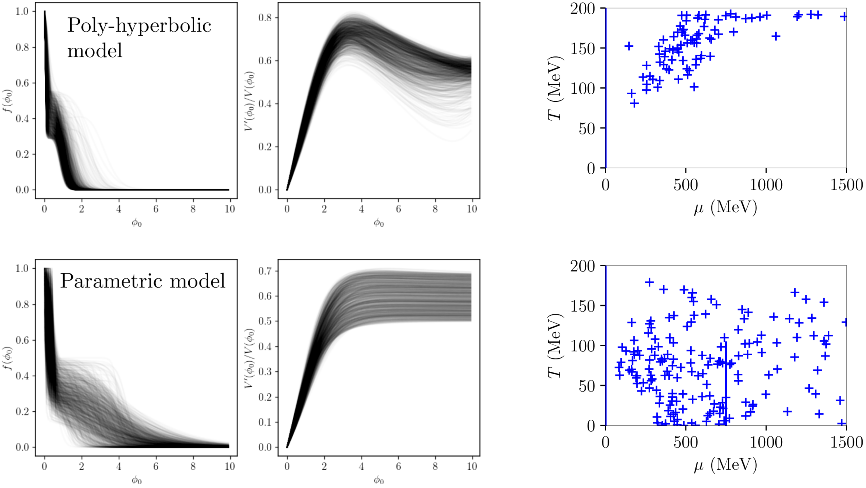
$$\mathcal{L} = \exp\left\{-\frac{1}{2}\boldsymbol{\delta x}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\delta x} - \frac{1}{2}\log\det\boldsymbol{\Sigma} + \text{constant}\right\}$$

- $\boldsymbol{\delta x}$: deviation for $s(T)$ and $\chi_2^{(B)}(T)$ at $\mu = 0$.
- Correlation $\Gamma \equiv \exp\left(-\Delta T/\xi_T\right)$ between neighboring points $\rightarrow$ extra model parameter.

MH, J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, to appear.

Priors

Parametric model leads to more uniform coverage of critical point positons and potential values.



Markov Chain Monte-Carlo (MCMC)

- Start from prior (here, uniform).
- Random evolution to sample from posterior.
- Transition probabilities such that $\mathcal{P}$ is stationary limit.

Metropolis-Hastings algorithm

1. Make small random changes to parameters.
2. Compute $\mathcal{P}$ from model EoS.
   - If $\mathcal{P}/\mathcal{P}_0 > 1$, transition to new parameters.
   - Otherwise, accept transition with probability $\mathcal{P}/\mathcal{P}_0$.
3. Repeat.

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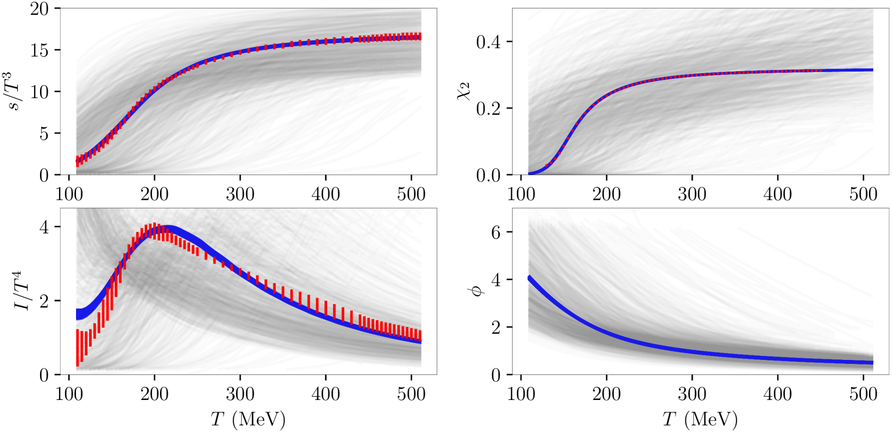
3. Repeat.

Inputs: Baryon susceptibility and entropy density from the lattice.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, PRL **730** (2014)
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

Posterior distribution: Equation of State



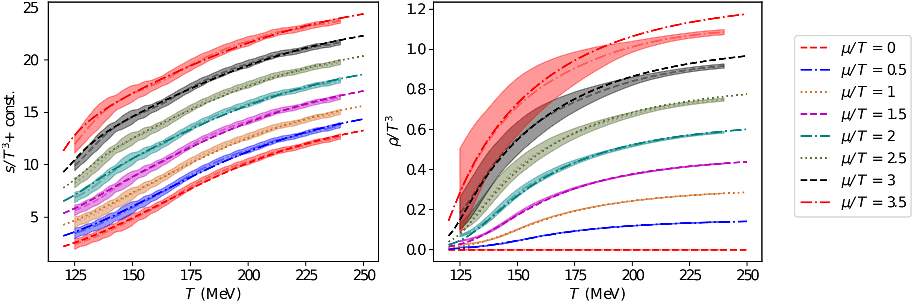
- Very tight constraints on entropy density and baryon susceptibility.

Bellwied, Borsanyi, Fodor et al., PRD **92** (2015)
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

Validation: MAP postdictions

- Excellent postdictions of lattice results at zero and *finite density*.
- Parametric model yields best MAP fit.

(model selection to be performed)



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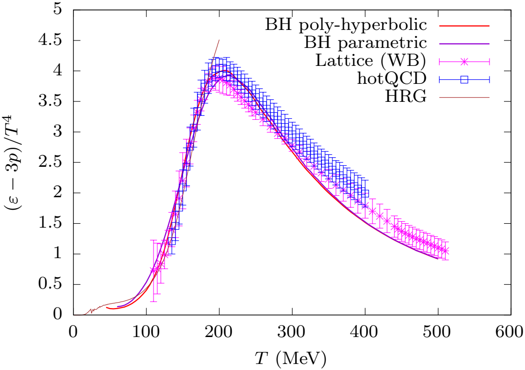
Validation: MAP postdictions

- Slightly worse agreement with the trace anomaly.



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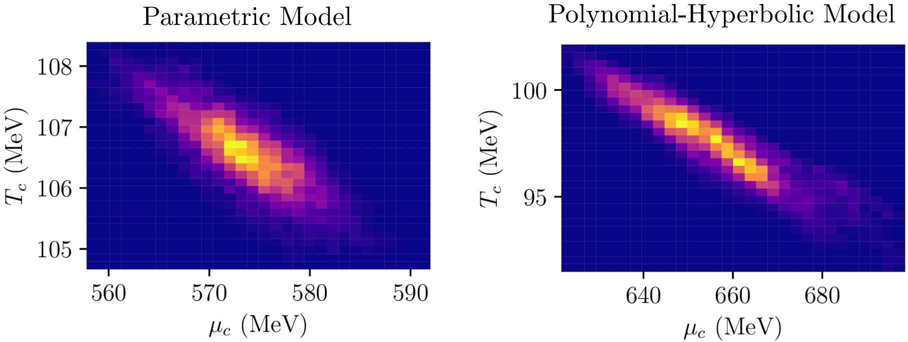
Finding the critical point



- Critical point successfully found in 88.8% – 99.9% of samples.

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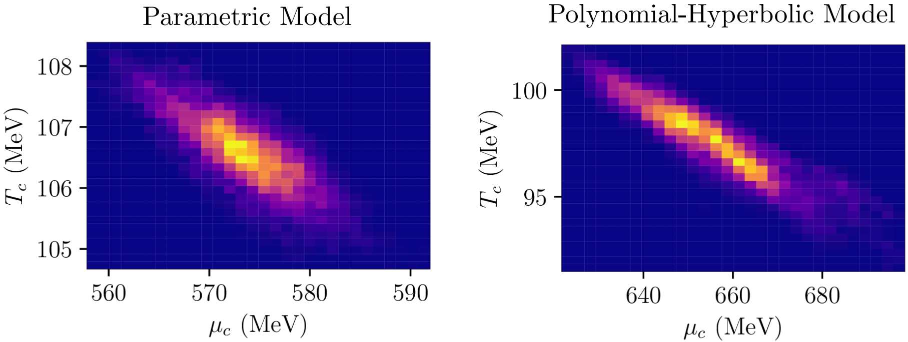

- Critical point successfully found in 88.8% – 99.9% of samples.
- In prior, only found in $\sim 10 - 13\%$ of samples.

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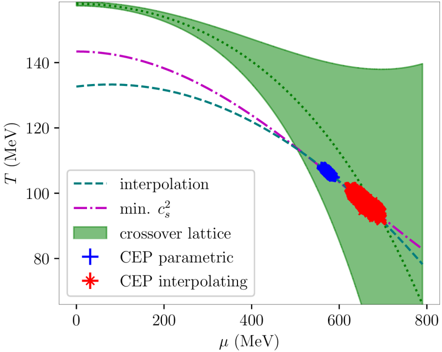
Predictions for the CEP



Parametric model
$(573 \pm 5,\ 106.6 \pm 0.6)$ MeV

Poly-hyperbolic model
$(655 \pm 13,\ 97.6 \pm 1.7)$ MeV

- Tight constraints. Compatible with lattice pseudo-critical line.
- Most of the uncertainty along same curve.

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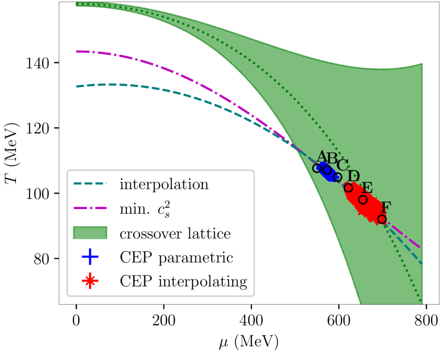
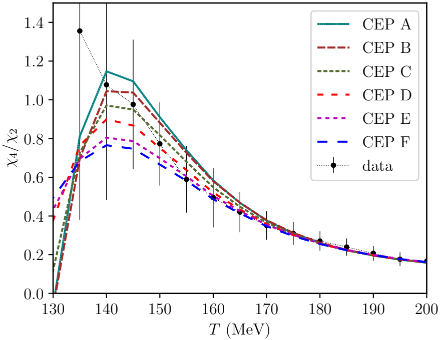
J. Grefa, MH, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti and R. Rougemont, PRD **106** (2022)
S. Borsanyi et al., PRL **125** (2020)

Predictions for the CEP





- Tight constraints. Compatible with lattice pseudo-critical line.
- Most of the uncertainty along same curve.
- Degeneracy may be broken by results on χ_4 near the crossover.

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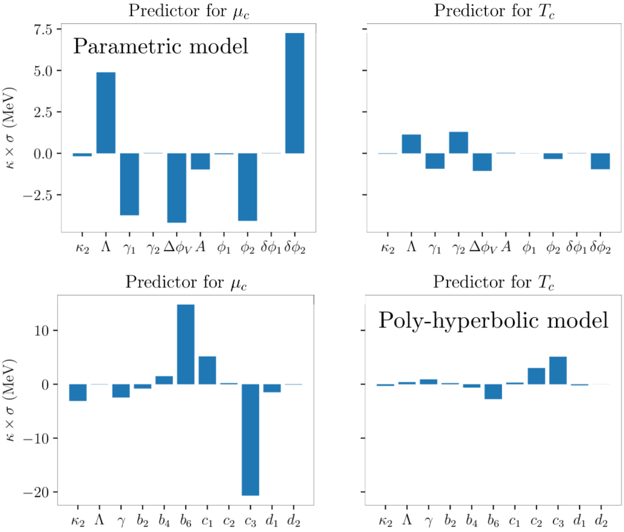
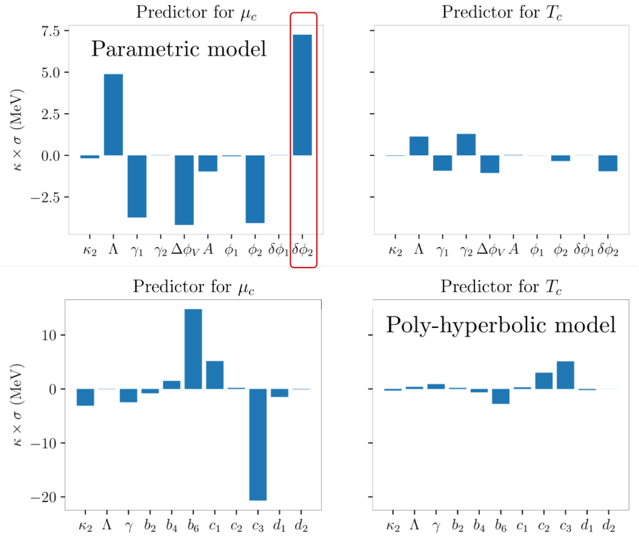
Parameter sensitivity — Linear predictors



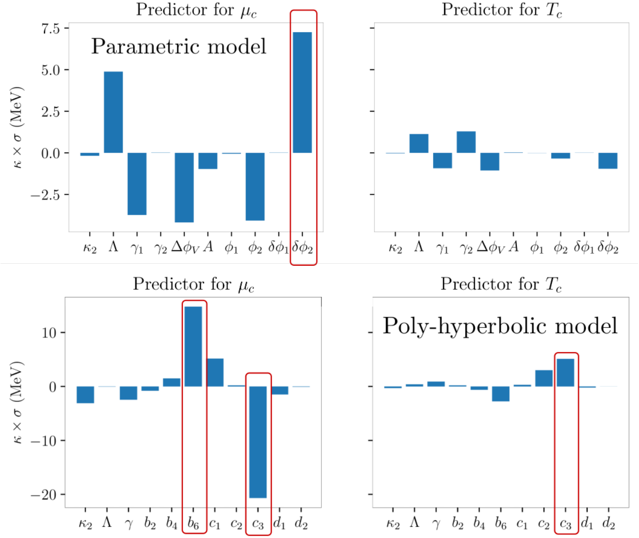
Parameter sensitivity — Linear predictors

- Role of $f(\phi)$ IR falloff.



Parameter sensitivity — Linear predictors

- Role of $f(\phi)$ IR falloff.
- Highest-order polynomial terms.



Outlook

- Signatures of the critical point in and outside of equilibrium.
- How do lattice results determine location of the critical point? Can predictions be made more precise?
- Ensemble of models available to elucidate physical meaning of potentials.

More...

- Integration to MUSES framework and merging with other models.
- Public EoS code expected this year!
- More conserved charges: isospin and strangeness.

Conclusions

1. Powerful description of the strongly coupled QGP, matching *finite-density* lattice results.

Borsányi, Fodor, Guenther et al., PRL **126** (2021)
J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **104** (2021)
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2. Bayesian inference: *ensemble* of equations of state favored by lattice results.

3. Large statistical preference for a CEP, to be more precisely quantified.

4. CEP in range $\mu_c \approx 550 - 700$ MeV and $T_c \approx 90 - 110$ MeV, within ~ 1 MeV-wide band.

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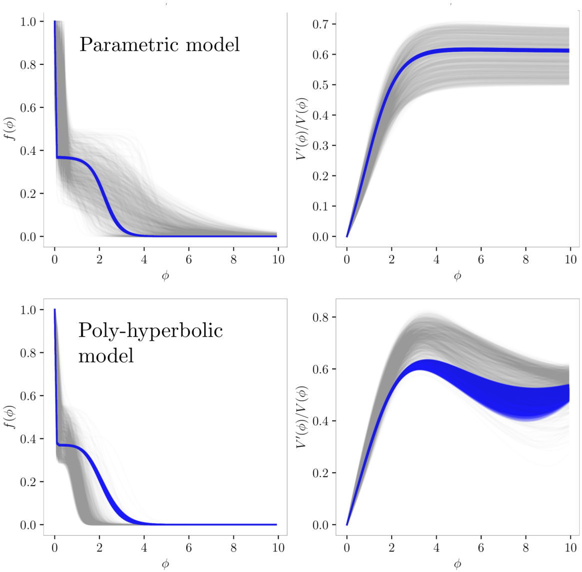
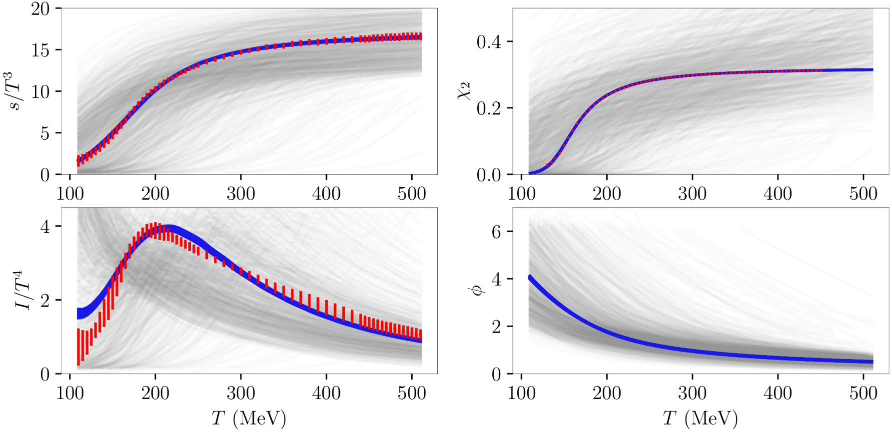
Backup slides...

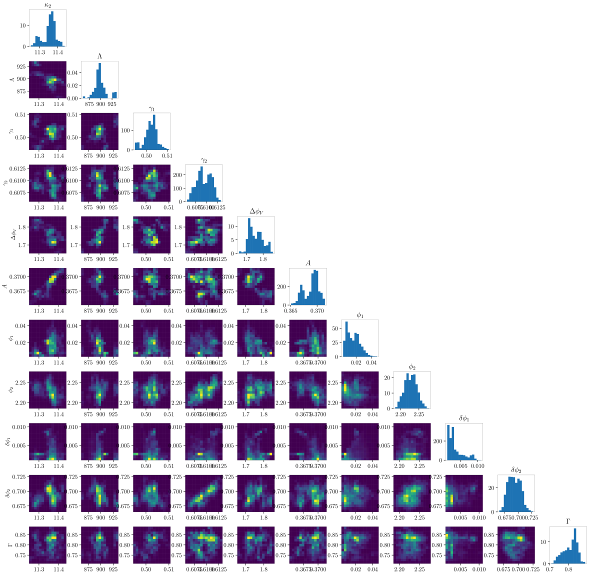
Posterior distribution



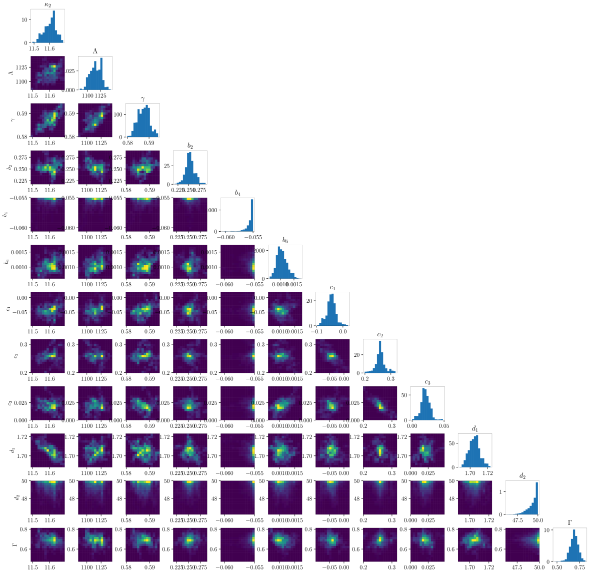
Posterior distribution: Equation of State

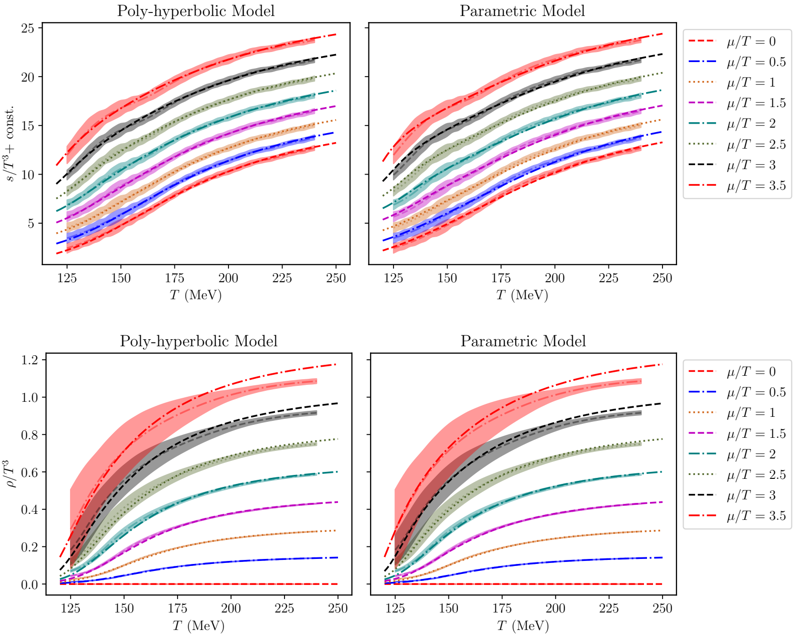


Posterior: Parametric Model



Posterior: Polynomial-Hyperbolic Model




Poly-hyperbolic Model
Parametric Model
s/T^3+ const.
T (MeV)
$\mu/T = 0$
$\mu/T = 0.5$
$\mu/T = 1$
$\mu/T = 1.5$
$\mu/T = 2$
$\mu/T = 2.5$
$\mu/T = 3$
$\mu/T = 3.5$
Poly-hyperbolic Model
Parametric Model
ρ/T^3
T (MeV)

Equations of motion

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r) - B'(r)\right]\phi'(r) - \frac{e^{2B(r)}}{h(r)}\left[\frac{\partial V(\phi)}{\partial \phi} + \right.$$
$$\left. - \frac{e^{-2[A(r)+B(r)]}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial \phi}\right] = 0,$$

$$\Phi''(r) + \left[2A'(r) - B'(r) + \frac{d[\ln f(\phi)]}{d\phi}\phi'(r)\right]\Phi'(r) = 0,$$

$$A''(r) - A'(r)B'(r) + \frac{\phi'(r)^2}{6} = 0,$$

$$h''(r) + [4A'(r) - B'(r)]h'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) +$$
$$+2e^{2B(r)}V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

Extraction of Thermodynamics

- Thermodynamics extracted from scalings after conversion to physical units.
- Requires near-boundary scalings,

$$\phi \sim \phi_A \, e^{-\nu A(r)}, \quad \Phi \sim \Phi_0^{\text{far}} + \Phi_2^{\text{far}} \, e^{-2A(r)}, \quad A \sim A_{-1}^{\text{far}} \, r + A_0^{\text{far}}$$

- Inversion to find ϕ_A and Φ_2^{far}:
  large coefficient $\times$ tiny number $=$ pure noise.

Thermodynamics

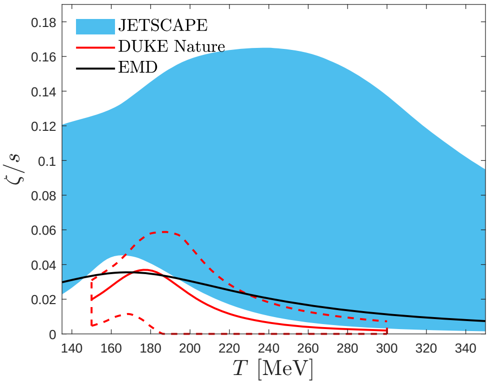
$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda,$$

$$\mu_B = \frac{\Phi_0^{\text{far}}}{\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda,$$

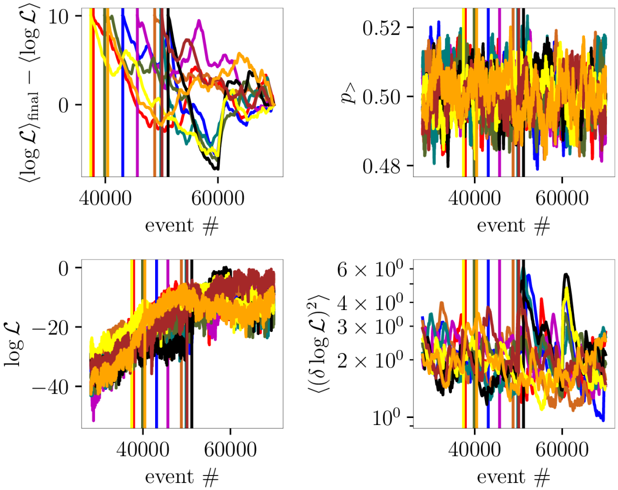
$$s = \frac{2\pi}{\kappa_5^2\phi_A^{3/\nu}}\Lambda^3,$$

$$\rho_B = -\frac{\Phi_2^{\text{far}}}{\kappa_5^2\phi_A^{3/\nu}\sqrt{h_0^{\text{far}}}}\Lambda^3.$$

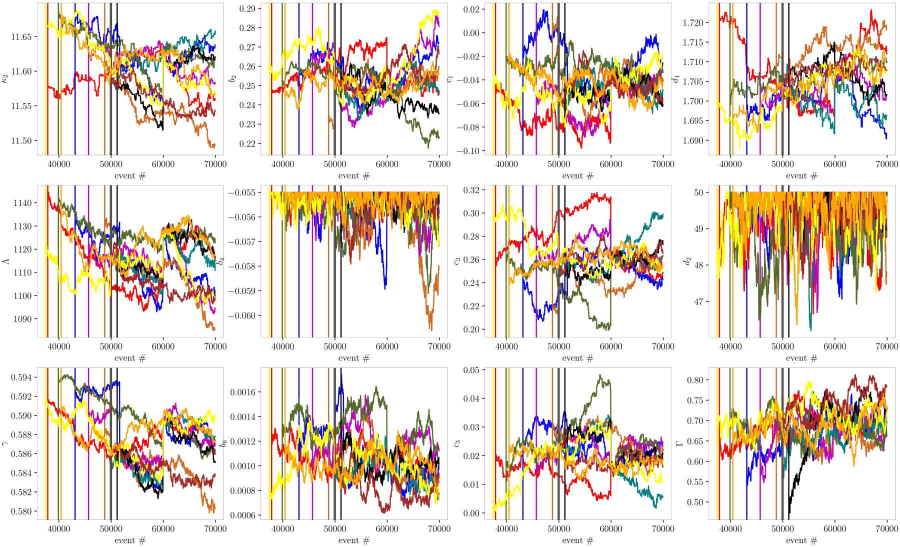
Bulk Viscosity



MCMC


$\langle \log \mathcal{L} \rangle_{\text{final}} - \langle \log \mathcal{L} \rangle$
10
0
40000
60000
event #
$p_>$
0.52
0.50
0.48
40000
60000
event #
$\log \mathcal{L}$
0
−20
−40
40000
60000
event #
$\langle (\delta \log \mathcal{L})^2 \rangle$
6×10^0
4×10^0
3×10^0
2×10^0
10^0
40000
60000
event #

MCMC: Polynomial-Hyperbolic Model



MCMC: Parametric Model