Speed of sound and the effective QCD phase diagram

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Department of Physics and Astronomy

Nuclear seminar at ISU

Organizers:

- Matteo Buzzegoli
- Saúl Hernández
- Semeon Valgushev

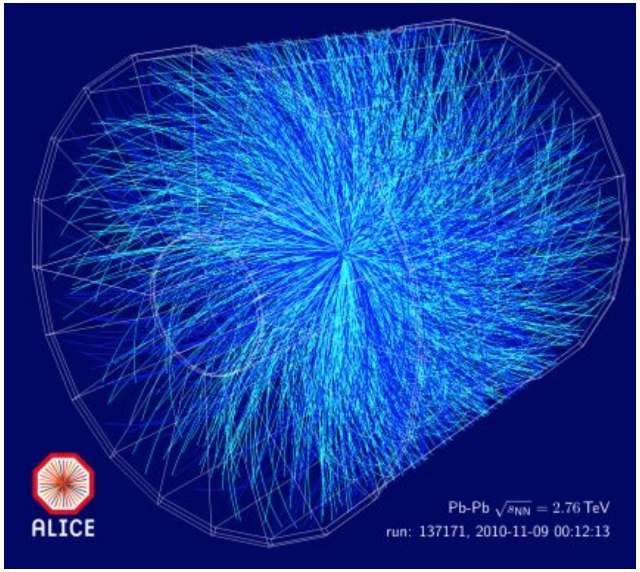
- Thursdays at 4:00 pm via Zoom.
- Follows us in Youtube for previous seminars.

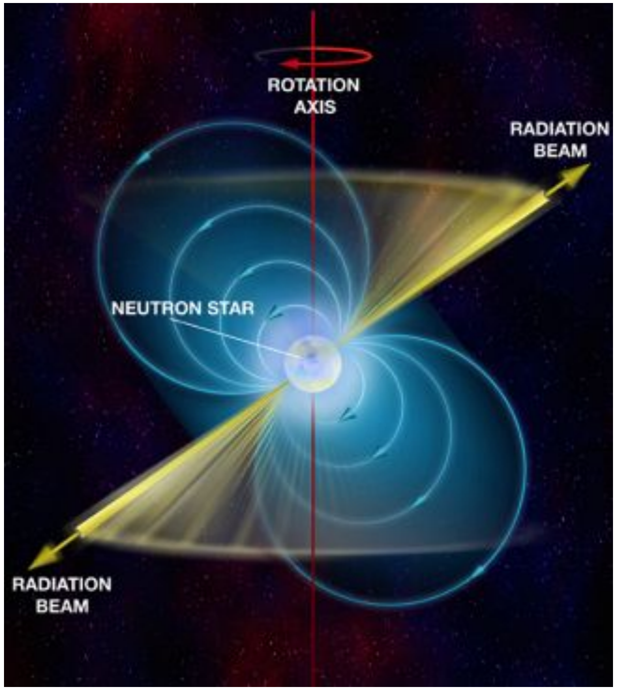
Outline

- Motivation.
- The Linear Sigma model.
- Effective QCD phase diagram.
- The speed of sound.
- Final Comments.

Motivation

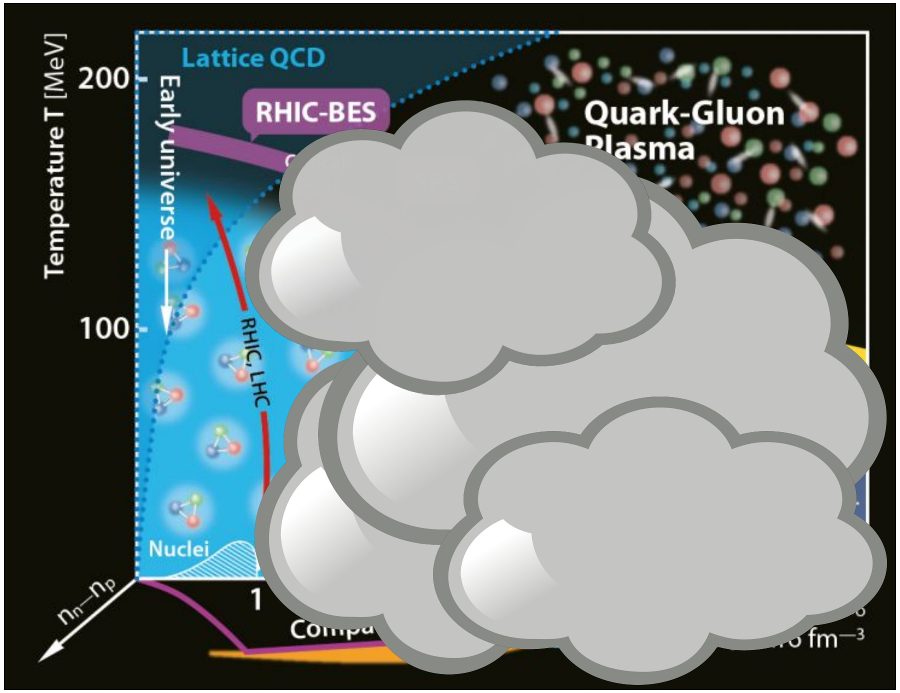
- QCD under extreme conditions (temperature and finite quark density) play an important role in understanding the transitions that took place in the early universe.

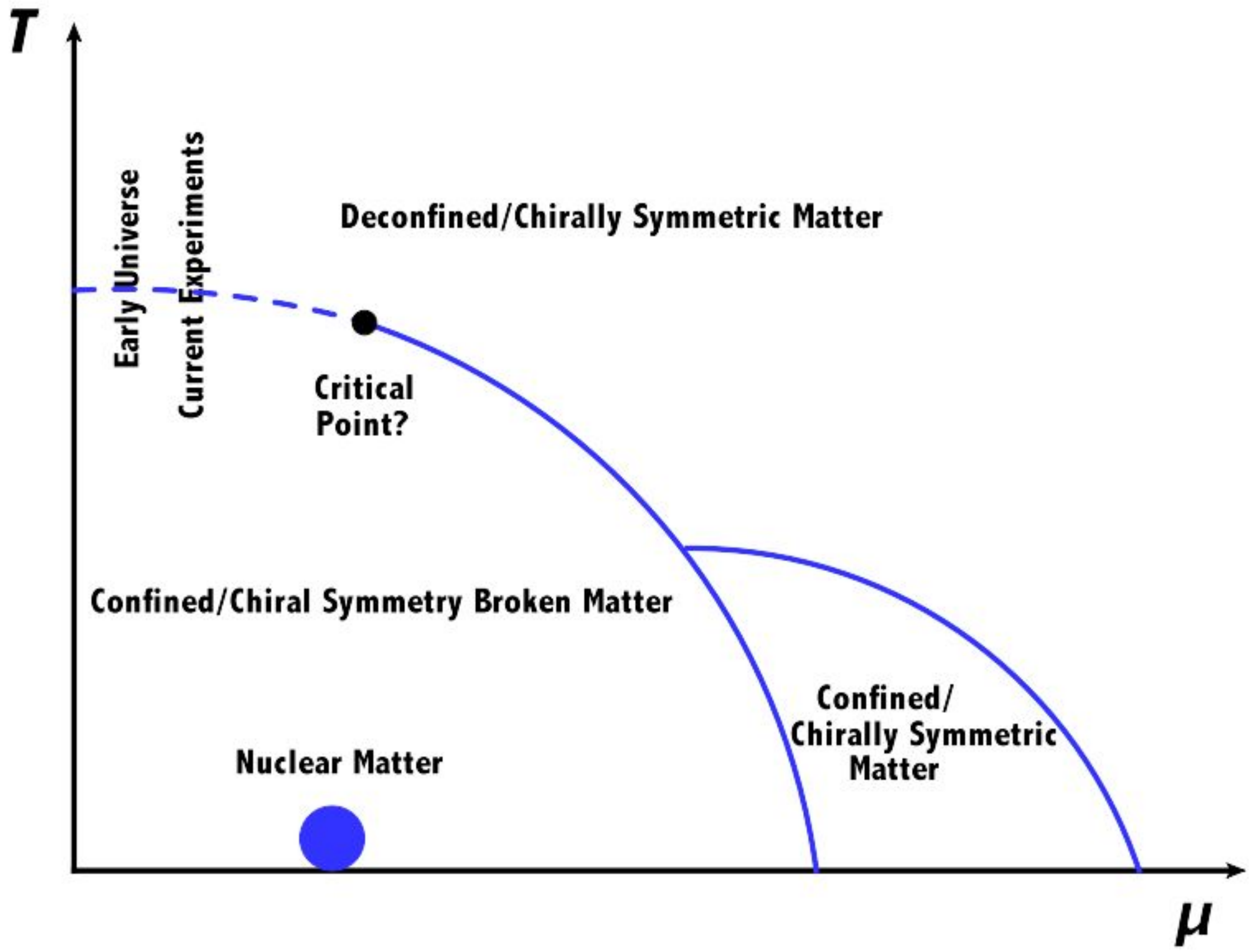




Motivation

- There is only reliable information at low densities.
- There are experimental efforts to dissipate doubts at higher densities.
    - NICA
    - RHIC(BESII)
    - JPARC
    - HADES




T
Early Universe
Current Experiments
Deconfined/Chirally Symmetric Matter
Critical
Point?
Confined/Chiral Symmetry Broken Matter
Nuclear Matter
Confined/
Chirally Symmetric
Matter
μ

Quantum Chromodynamics

- QCD Lagrangian with massless quarks.

$$\mathcal{L}^0_{QCD} = \overline{\psi}(x) i\gamma_\mu \partial^\mu \psi + \mathcal{L}_{quark-gluon} + \mathcal{L}_{glue}$$

- Gauge theory with the local symmetry group $SU(N_C)$.
- The fundamental fields are the quarks and the gluons (gauge field).
- In the limit that each on of the N_f quark fields is massless, QCD shows chiral symmetry.

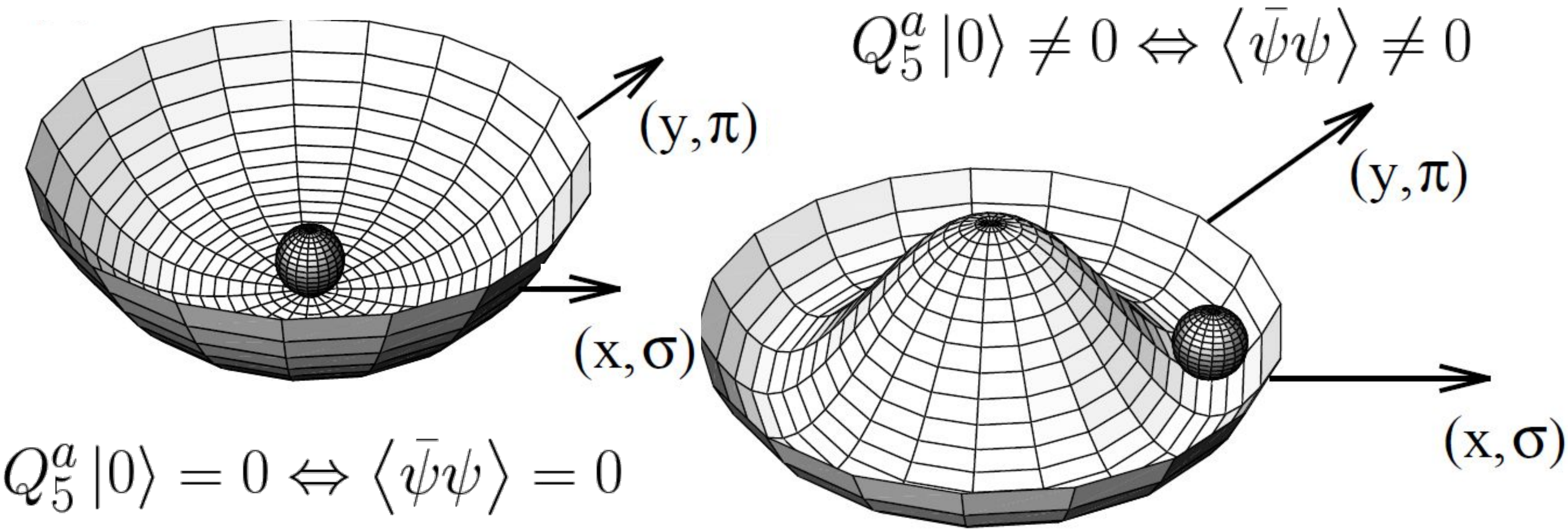
Old Reliable!!!

Linear Sigma Model

- Effective model for low-energy QCD.
- Effects of quarks and mesons on the chiral phase transition.
- Implement ideas of chiral symmetry and spontaneous symmetry breaking

Linear Sigma Model

- Spontaneous breaking of the chiral symmetry.



Linear Sigma Model

- Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma_\mu\partial^\mu\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi,$$

- To allow for spontaneous symmetry breaking

$$\sigma \to \sigma + v$$

$$\langle\sigma\rangle = v; \qquad \langle\pi\rangle = 0.$$

- where v is identified as the order parameter

Linear Sigma Model

- After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] \underline{-\frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2}$$
$$-\frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}] \underline{-\frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2}$$
$$+ i\bar{\psi}\gamma^\mu D_\mu \psi \underline{- gv\bar{\psi}\psi} + \frac{a^2}{2}v^2 - \frac{\lambda}{4}v^4$$
$$-\frac{\lambda}{4}[(\sigma^2 + \pi_0^2)^2 + 4\pi^+\pi^-(\sigma^2 + \pi_0^2 + \pi^+\pi^-)]$$
$$-g\hat{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2$$
$$m_\pi^2 = \lambda v^2 - a^2$$
$$m_f = gv$$

$$a = \sqrt{\frac{m_\sigma^2 - 3m_\pi^2}{2}}$$

Linear Sigma Model

- We calculate the effective potential for fermions and bosons at finite temperature and chemical potential

$$V_b = s_b T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(D^{-1}\right)^{1/2}, \qquad V_f = s_f T \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left(S^{-1}\right)$$

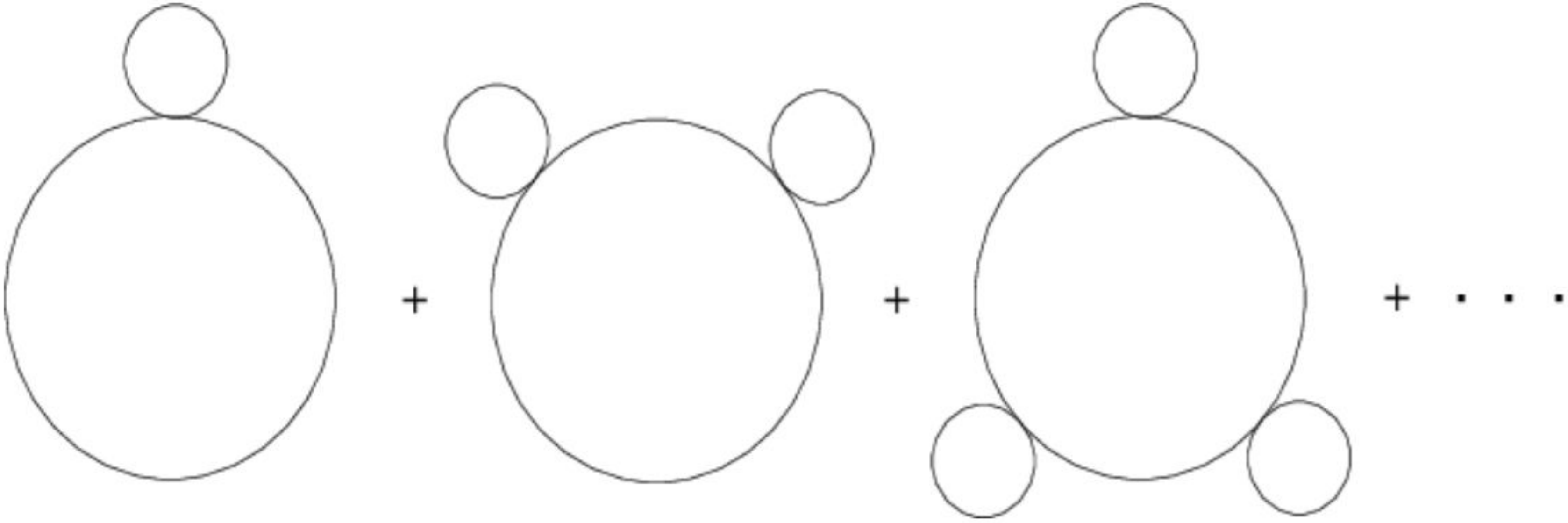
.

where the thermal boson and fermion propagators are given by

$$D = \frac{1}{k^2 + m_b^2 + \omega_n^2}, \qquad S = \frac{\not{k} + m_f}{k^2 + m_f^2 + (\omega_n - i\mu)^2}.$$

Linear Sigma Model

- In order to include medium effects on the mesons we need to go beyond mean field and include the Ring diagrams to the boson contribution

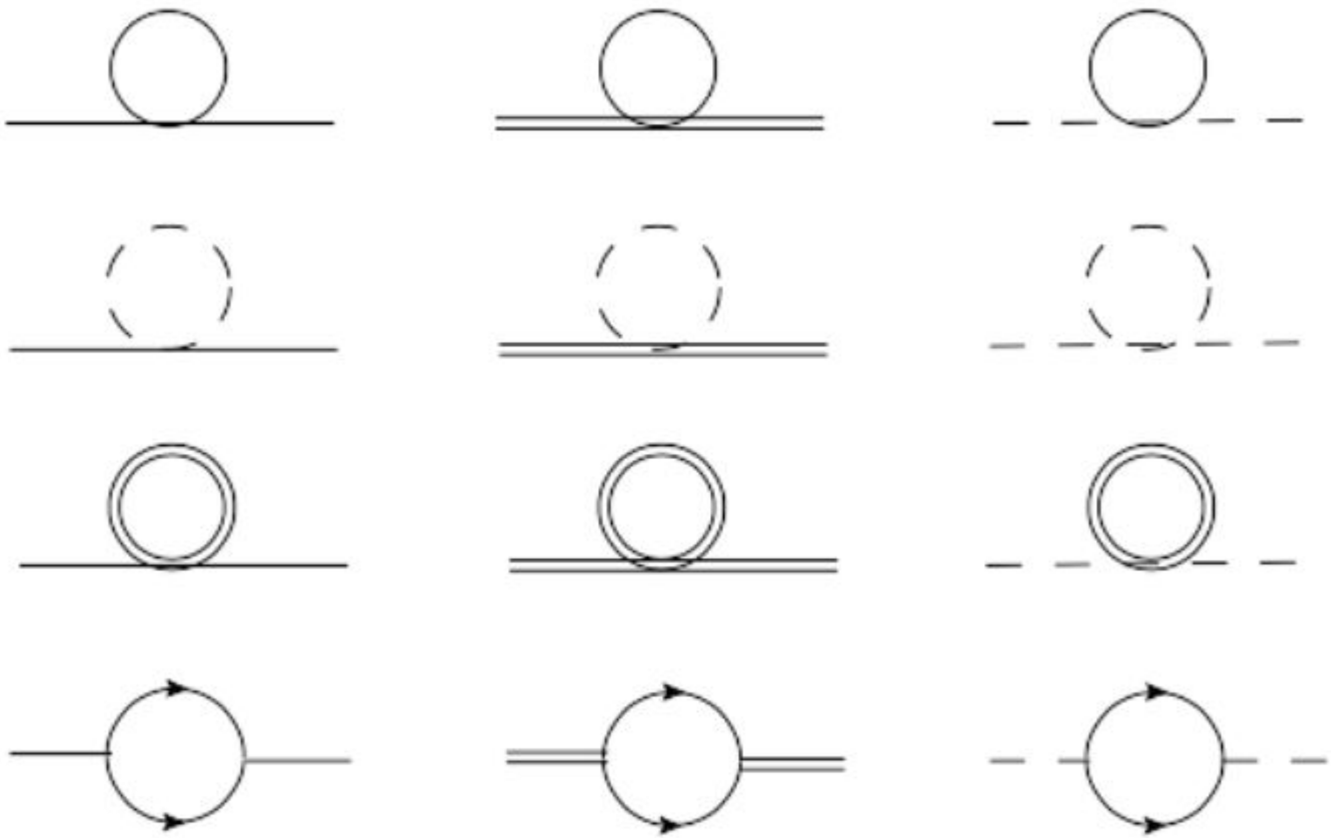


Then the full effective potential is:

$$
\begin{aligned}
V^{\mathrm{eff}} &= V^{\mathrm{tree}} + V^{\mathrm{b}} + V^{\mathrm{f}} + V^{\mathrm{Ring}} \\
V^{\mathrm{tree}}(v) &= -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \\
V^{\mathrm{b}}(v,T) &= T\sum_n \int \frac{d^3k}{(2\pi)^3} \ln D_{\mathrm{b}}(\omega_n,\vec{k})^{1/2} \\
V^{\mathrm{f}}(v,T,\mu) &= -T\sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}[\ln S_{\mathrm{f}}(\tilde{\omega}_n,\vec{k})^{-1}] \\
V^{\mathrm{Ring}}(v,T,\mu) &= \frac{T}{2}\sum_n \int \frac{d^3k}{(2\pi)^3} \ln[1+\Pi_{\mathrm{b}} D(\omega_n,\vec{k})]
\end{aligned}
$$

with Π the self energy

$$\Pi = \lambda\frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2}[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$



High Temperature

$$V^{(eff)} = \boxed{-\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4} + \sum_{i=\sigma,\pi^0} \left\{ \frac{m_i^4}{64\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) - 2\gamma_E + 1 \right] \right.$$

$$\left. - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi} \boxed{(m_i^2 + \Pi)^{3/2}} \right\}$$

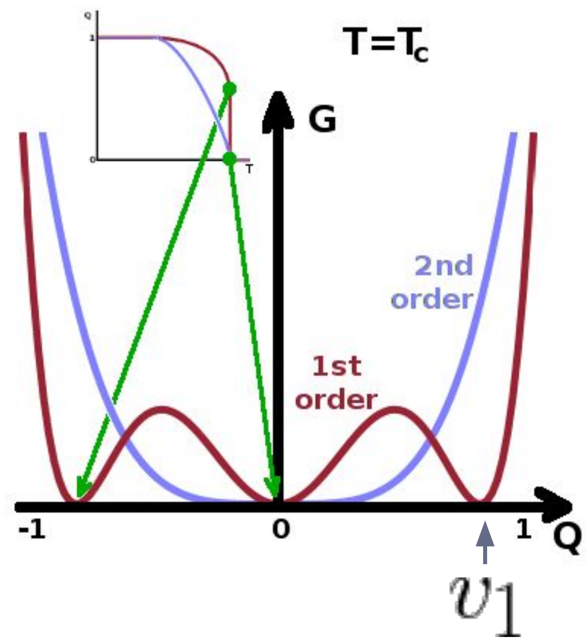
$$- N_c \sum_{f=u,d} \left[\frac{m_f^4}{16\pi^2} \left[\ln\left(\frac{(4\pi T)^2}{2a^2} \right) + \psi^0 \left(\frac{1}{2} + \frac{i\mu}{2\pi T} \right) \right. \right.$$

$$\left. + \psi^0 \left(\frac{1}{2} - \frac{i\mu}{2\pi T} \right) \right] + 8 m_f^2 T^2 [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

$$\left. - 32 T^4 [Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] \right]$$

Criticality

- Now the criterion to find the temperature and the chemical potential where the chiral symmetry is restored, is the following.



  - Second Order

$$\left.\frac{\partial^2}{\partial v^2}V^{eff}\right|_{v=0} = 0$$

  - First Order

$$V^{eff}(0) = V^{eff}(v_1); \quad \left.\frac{\partial}{v\partial v}V^{eff}\right|_{v=0} = \left.\frac{\partial}{v\partial v}V^{eff}\right|_{v=v_1} = 0$$

Model Parameters

- The parameter space consists of the λ and g coupling constants and the mass parameter a, which can be fix by LQCD data (PRL 125, 052001 (2020)).

Fixing a with:

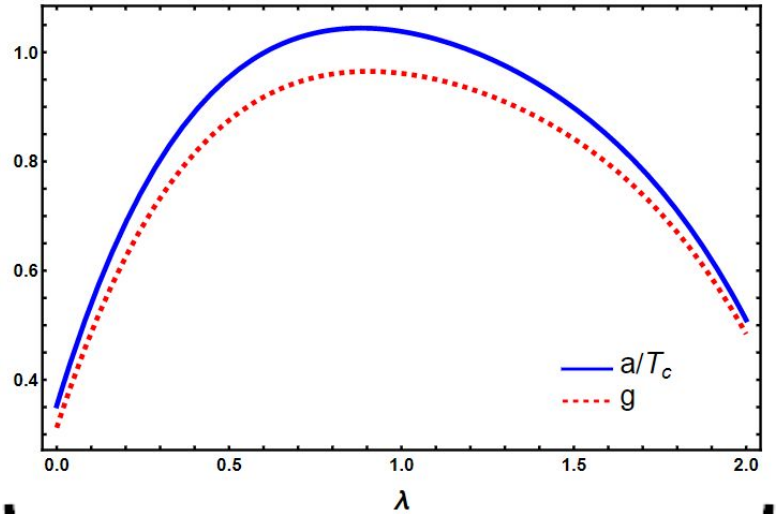
$$6\lambda\left(\frac{T_c^2}{12}-\frac{T_c}{4\pi}\left(\Pi_b(T_c,\mu_B=0)-a^2\right)^{1/2}+\frac{a^2}{16\pi^2}\left[\ln\left(\frac{\tilde{\mu}^2}{T_c^2}\right)\right]\right)+g^2T_c^2-a^2=0.$$

Fixing λ and g with the collection of curves that obey this relation:

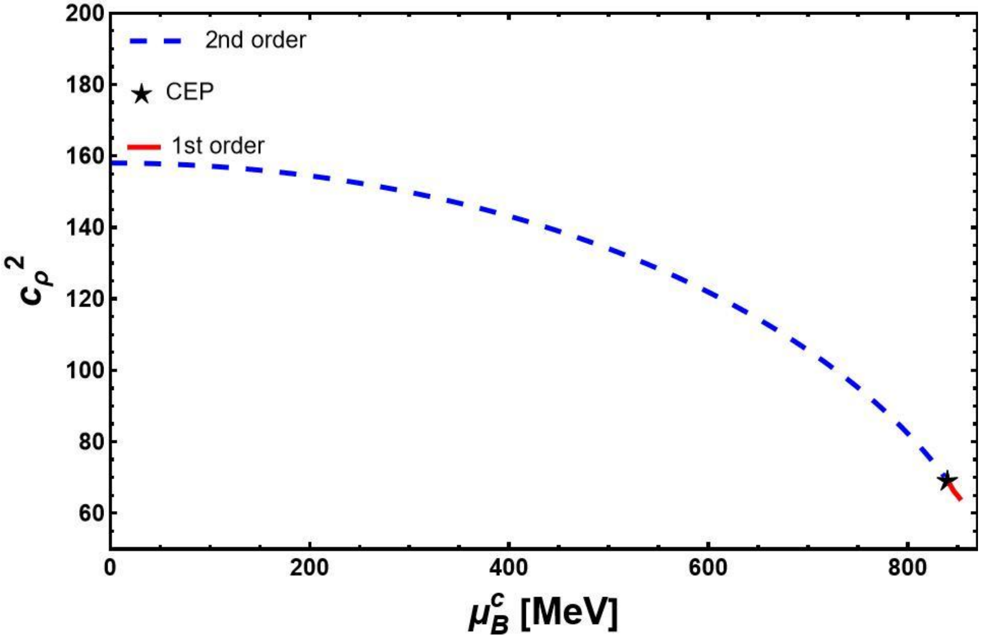
$$\frac{T_c(\mu_B)}{T_c^0}=1-\kappa_2\left(\frac{\mu_B}{T_c^0}\right)^2+\kappa_4\left(\frac{\mu_B}{T_c^0}\right)^4$$

$\kappa_2 = 0.0153$ and $\kappa_4 = 0.00032$

Results



$$\lambda = 0.4, \;\; g = 0.88 \;\; \text{and} \;\; a = 141.38 \text{ MeV}$$



$$768 \text{ MeV} < \mu_B^{CEP} < 849 \text{ MeV}$$

$$69 \text{ MeV} < T^{CEP} < 70.3 \text{ MeV}$$

Ayala, A, et al, Eur. Phys. J. A 58, 87 (2022).

Comments so far..

- The linear sigma model is an effective tool that allows the analytical analysis of properties of great interest in QCD.
- Up to this point we have learned that the parameter space with physical relevance is neither large nor arbitrary.
- There is still room for improvement and plenty of things to learn from this model and its possible extensions.

What is next?

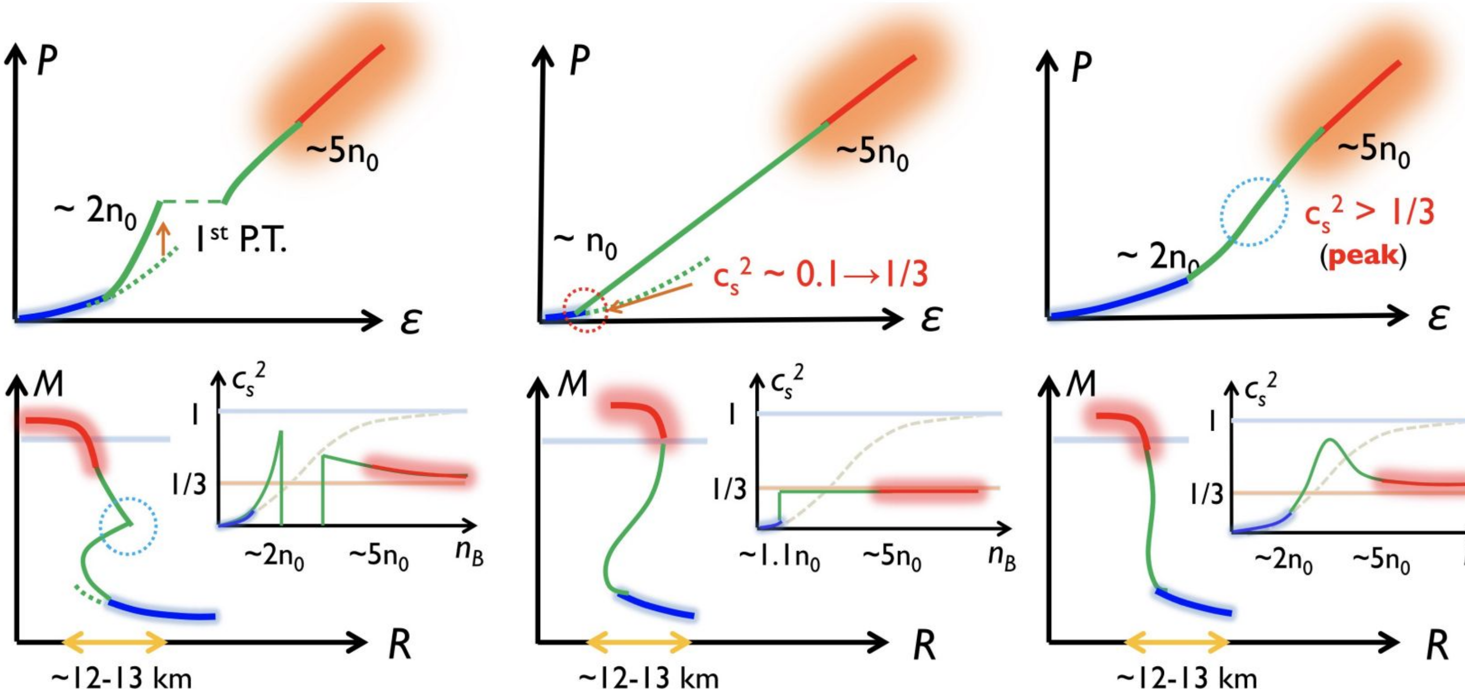
- Speed of sound is closely related with the thermodynamics properties of any system, including the EoS.

- For example, in neutron start researches, the c_s behavior as a function of baryon number density influences the mass-radius relationship.

- In HIC, c_s also conveys relevant information; for example, it displays a local minimum at a crossover transition.

What is next?

- TOV and EoS can give some insight about the transition quark-nucleon matter. Directly related with the speed of sound behavior.



T. Kojo arXiv:2011.10940v2 (2020)

Speed of sound

- The square of the speed of sound is usually defined as

$$c_\chi^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_\chi$$

where χ denotes the parameter fixed in the calculation of the speed of sound.

- According to the properties on the propagation medium, it may be more useful to keep one quantity fixed rather than another.

Speed of sound

- For this work, we will focus on

$$c_{\rho_B}^2 = \frac{\partial(p,\rho_B)}{\partial(\epsilon,\rho_B)} = \frac{s\chi_{\mu\mu} - \rho_B\chi_{\mu T}}{T(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},$$

$$c_s^2 = \frac{\partial(p,s)}{\partial(\epsilon,s)} = \frac{\rho_B\chi_{TT} - s\chi_{\mu T}}{\mu_B(\chi_{TT}\chi_{\mu\mu} - \chi_{\mu T}^2)},$$

$$c_{s/\rho_B}^2 = \frac{\partial(p,s/\rho_B)}{\partial(\epsilon,s/\rho_B)} = \frac{c_{\rho_B}^2 Ts + c_s^2\mu_B\rho_B}{Ts + \mu_B\rho_B}.$$

Speed of sound

- The pressure, entropy and baryon number densities can be derived using the thermodynamics relations in the grand canonical ensemble as

$$p = -\Omega, \qquad \epsilon = -p + Ts + \mu_B \rho_B$$

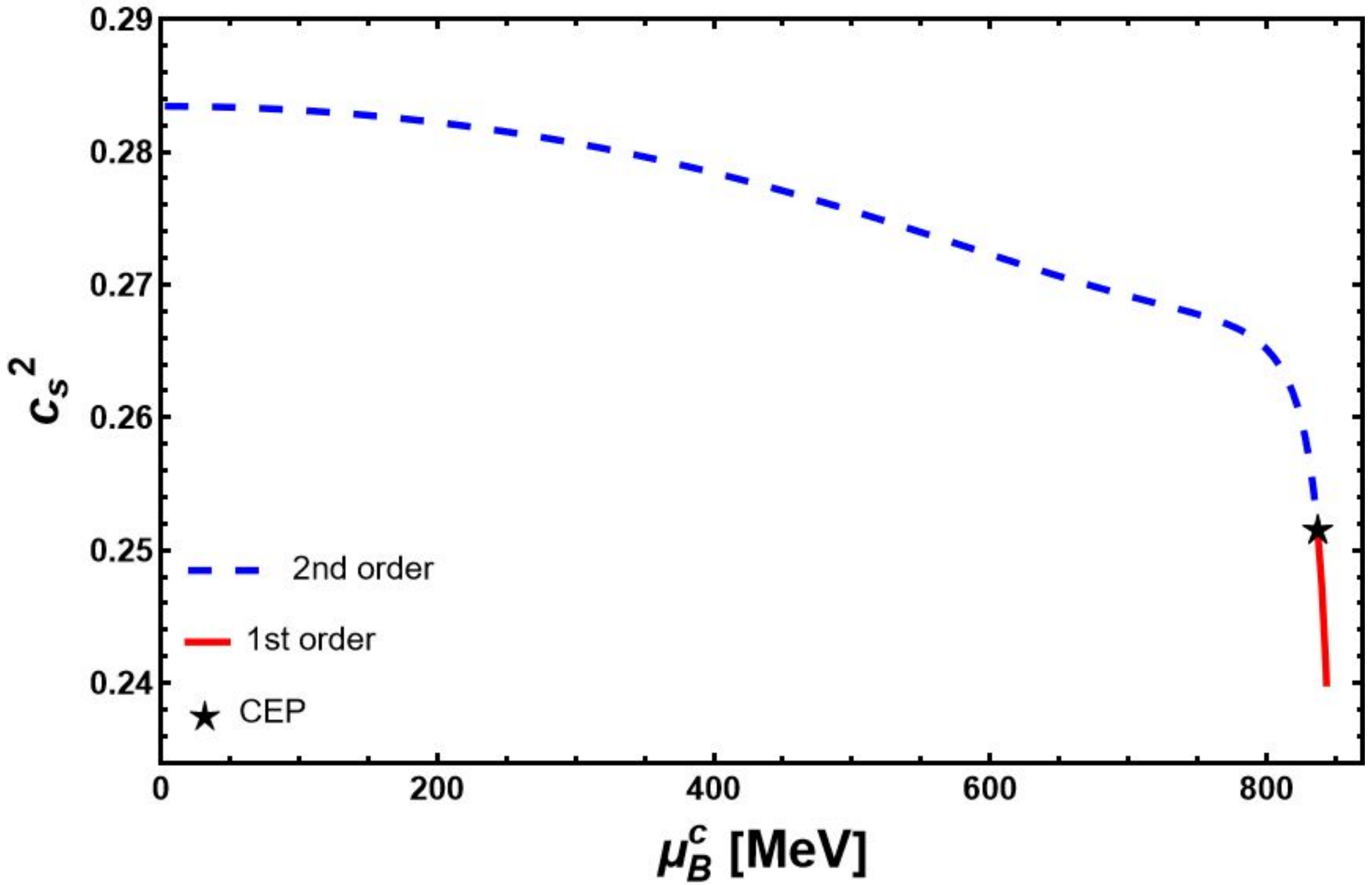
$$s = \left(\frac{\partial p}{\partial T}\right)_{\mu_B} \quad \text{and} \quad \rho_B = \left(\frac{\partial p}{\partial \mu_B}\right)_T .$$

where

$$\Omega(T, \mu) = V^{(eff)}(v = 0, T, \mu)$$

Results

$$\lambda = 0.4,\ \ g = 0.88 \ \ \text{and} \ \ a = 141.38\ MeV$$



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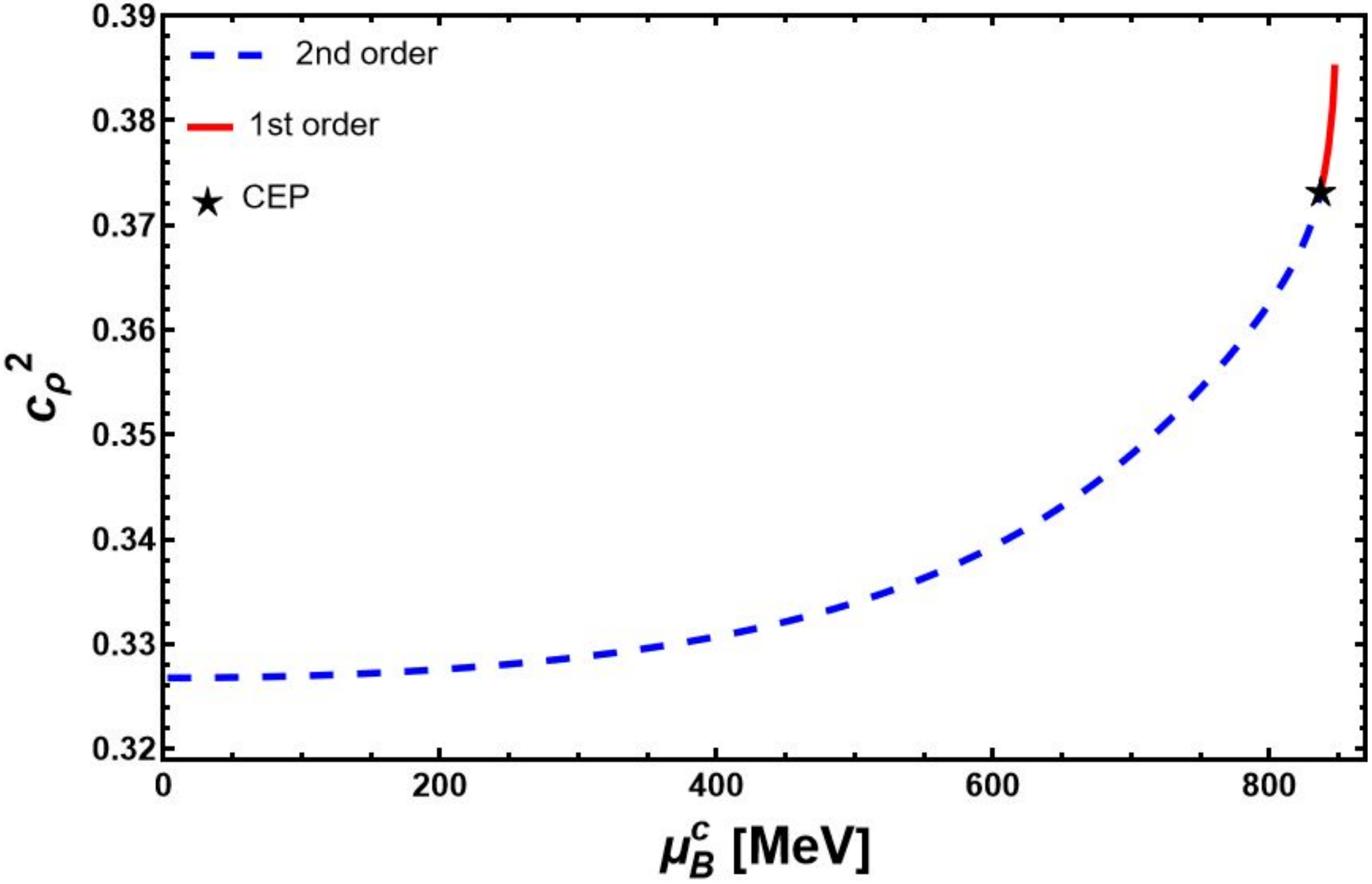

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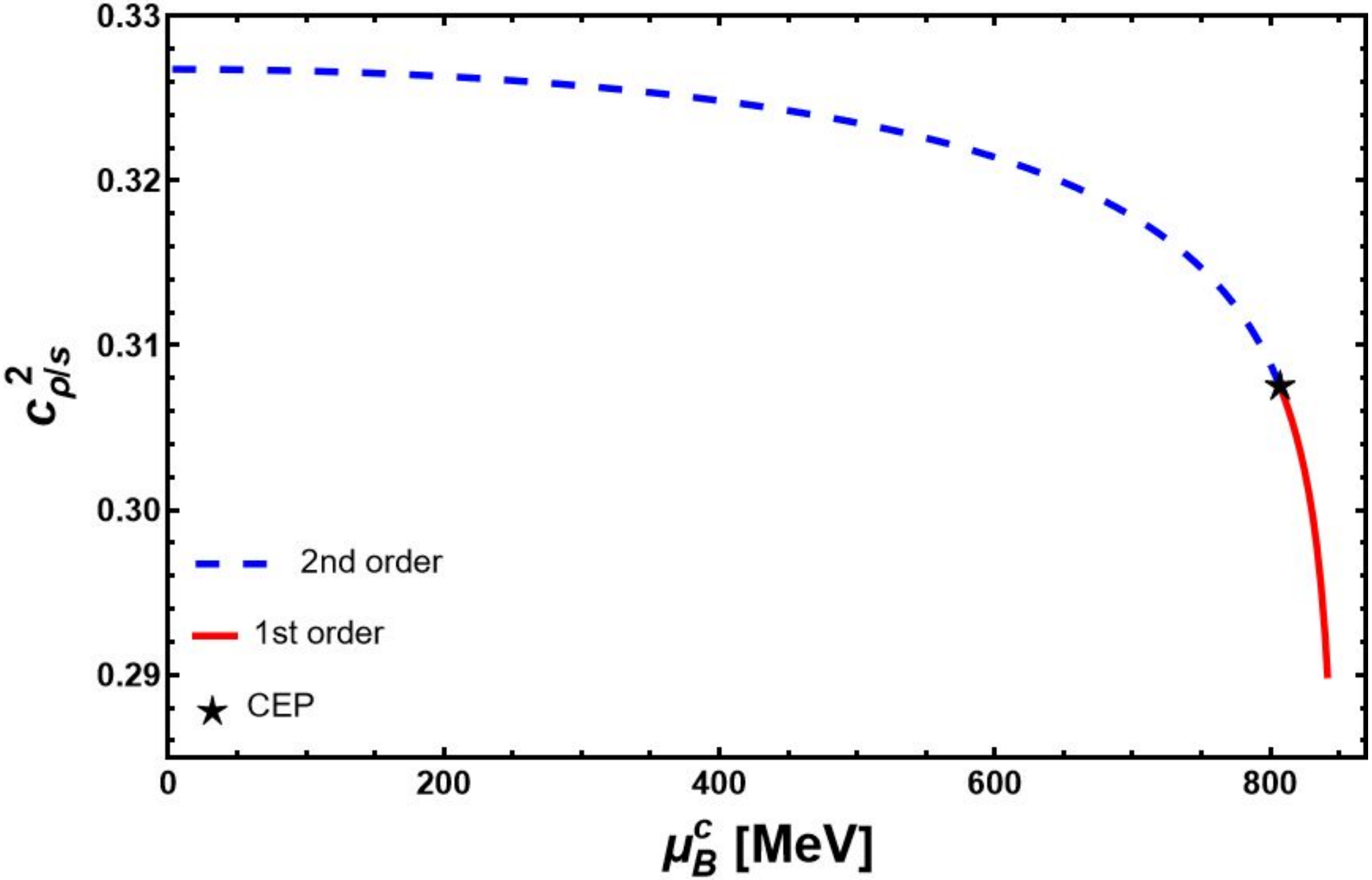
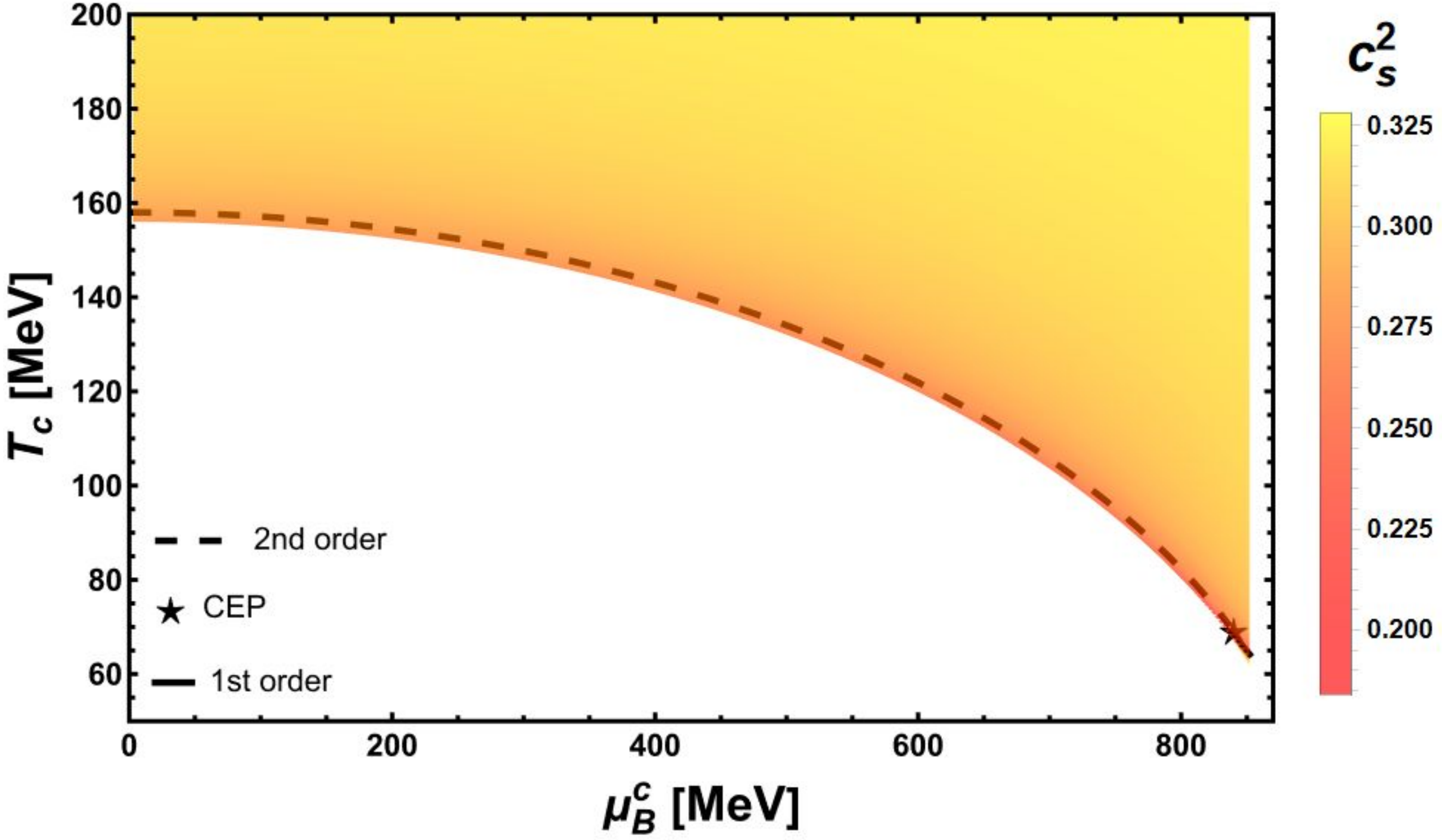
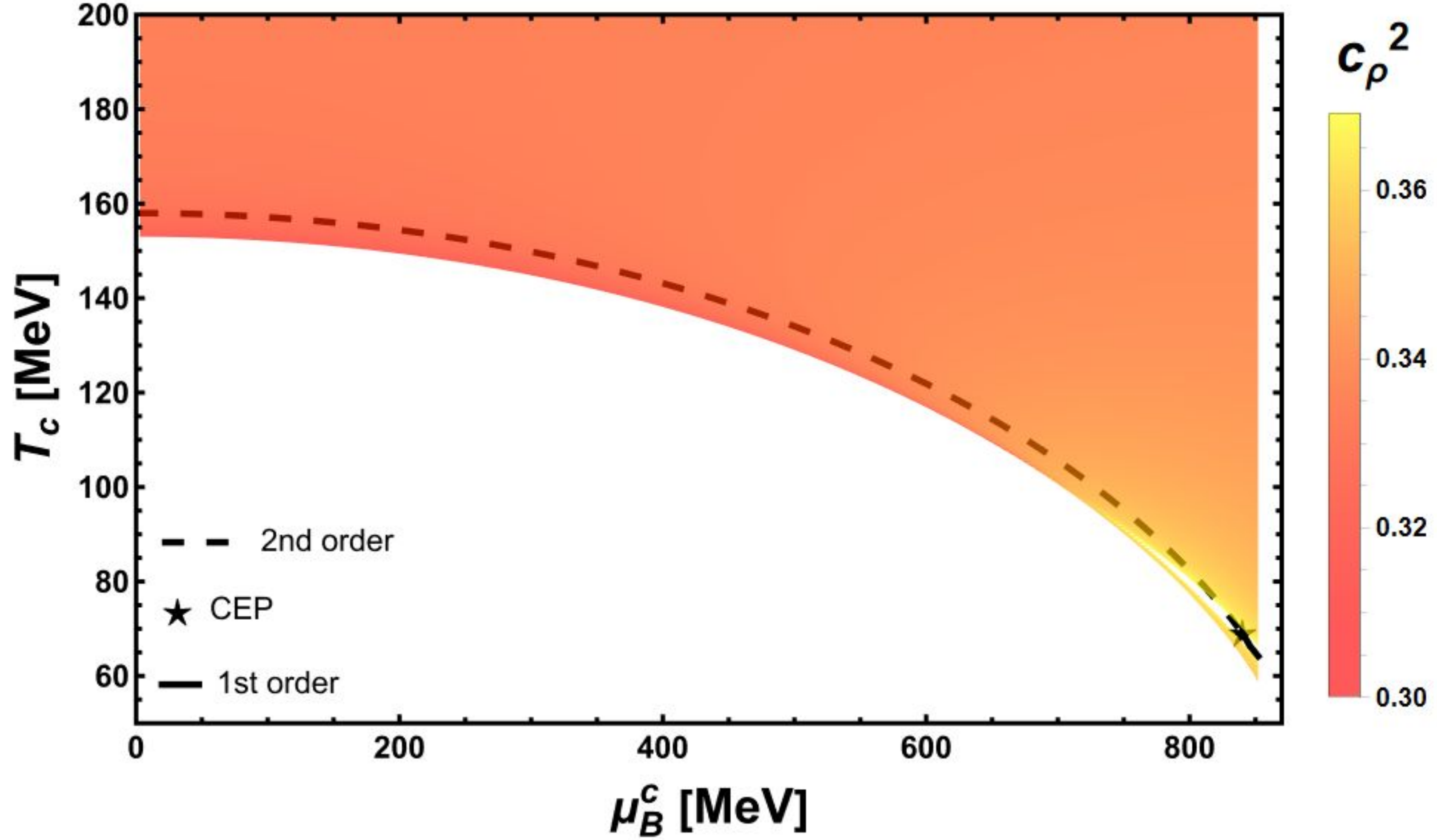


Results



200
180
160
140
120
100
80
60
0
200
400
600
800
T_c [MeV]
μ_B^c [MeV]
2nd order
CEP
1st order
c_s^2
0.325
0.300
0.275
0.250
0.225
0.200

Results



200
180
160
140
120
100
80
60
T_c [MeV]
0
200
400
600
800
μ_B^c [MeV]
2nd order
CEP
1st order
c_ρ^2
0.36
0.34
0.32
0.30

Comments about the CEP

- The main idea is to establish that the speed of sound exhibits different behaviors around the critical point that can be directly related to drastic changes in the medium.

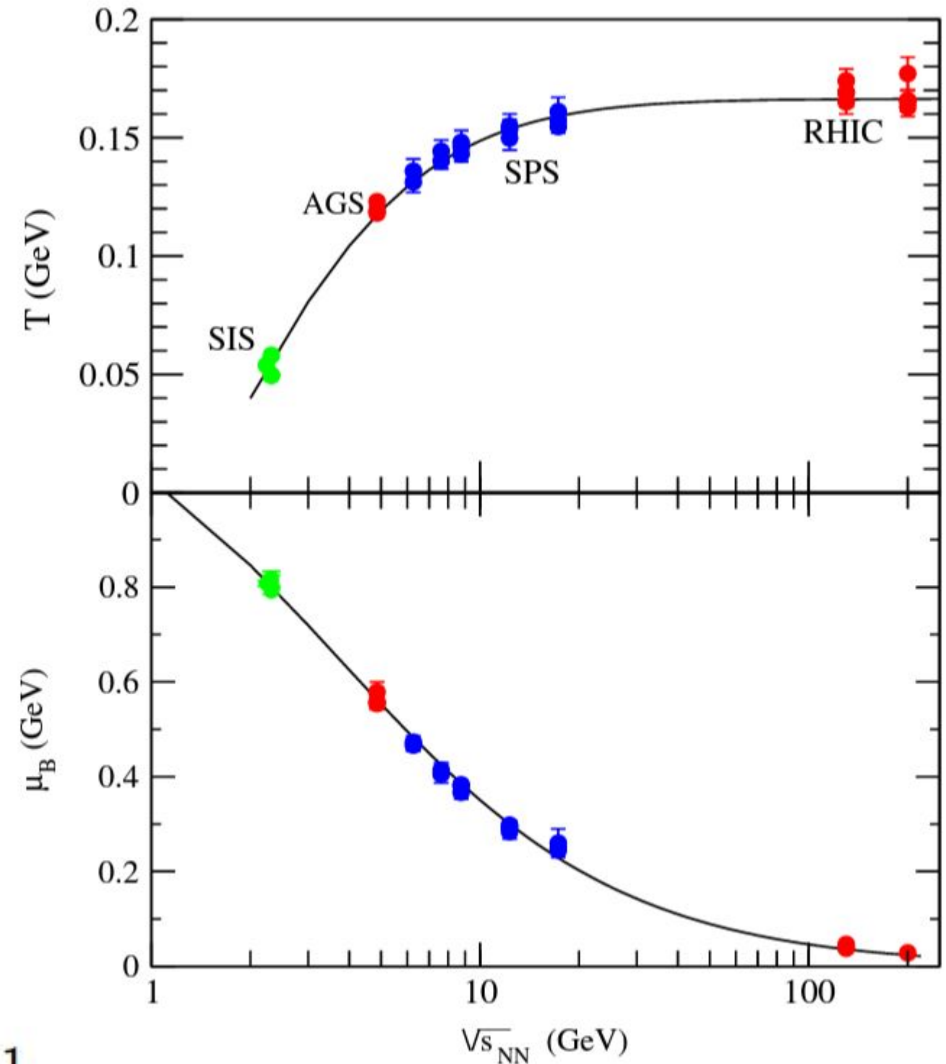
- For example, a drastic change in the behavior of the speed of sound at fix entropy, it's a clear sign that we are approaching the CEP.

- This criterion can be complemented with the other criteria that have been shown in previous works of the collaboration.

- For example, we can parameterize the baryon chemical potential in terms of the collision energy to compare the fluctuations of the cumulants and their dependence on it.

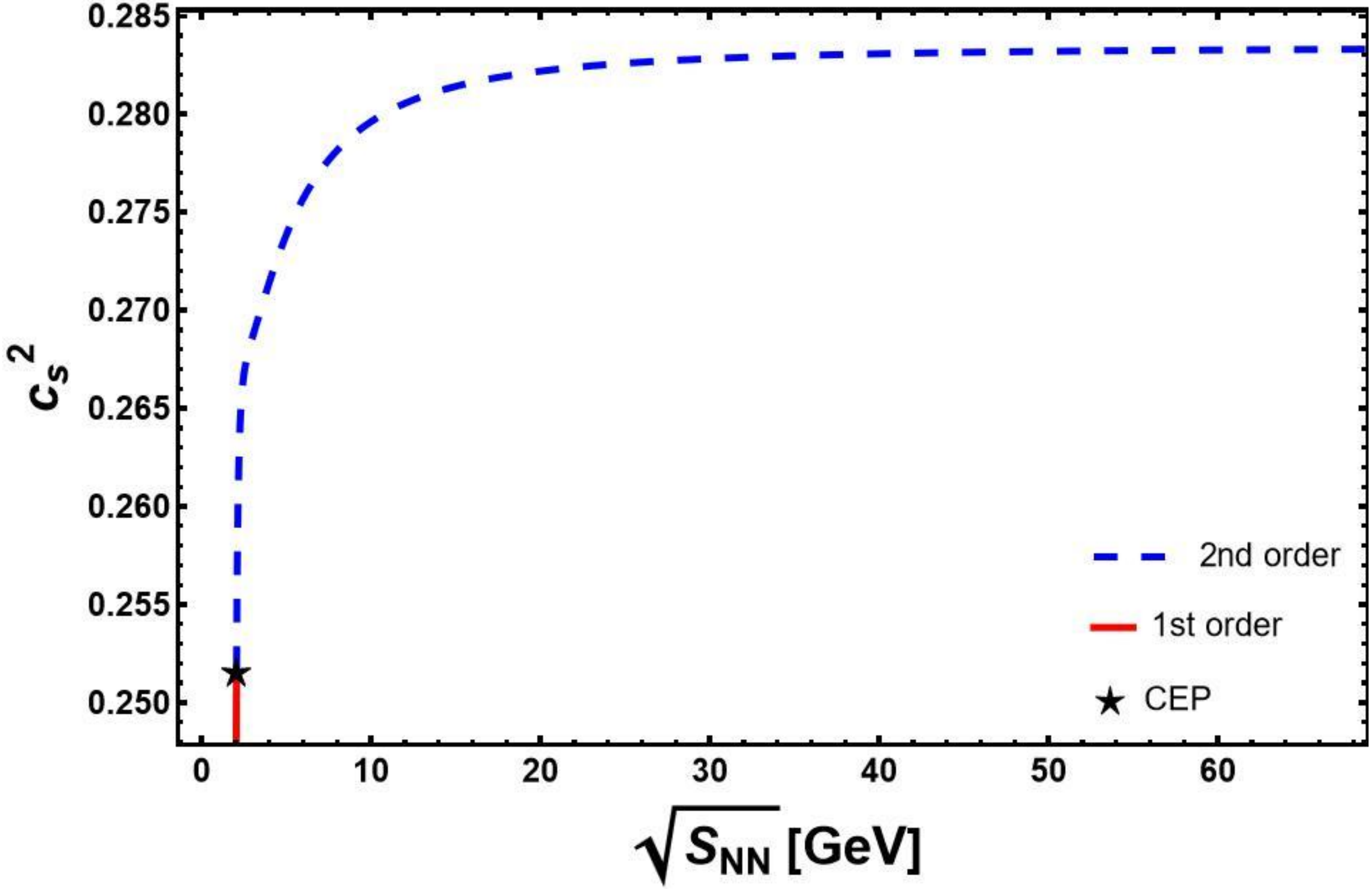
$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

$$d = 1.308\text{GeV}, \;\; e = 0.273\text{GeV}^{-1}$$

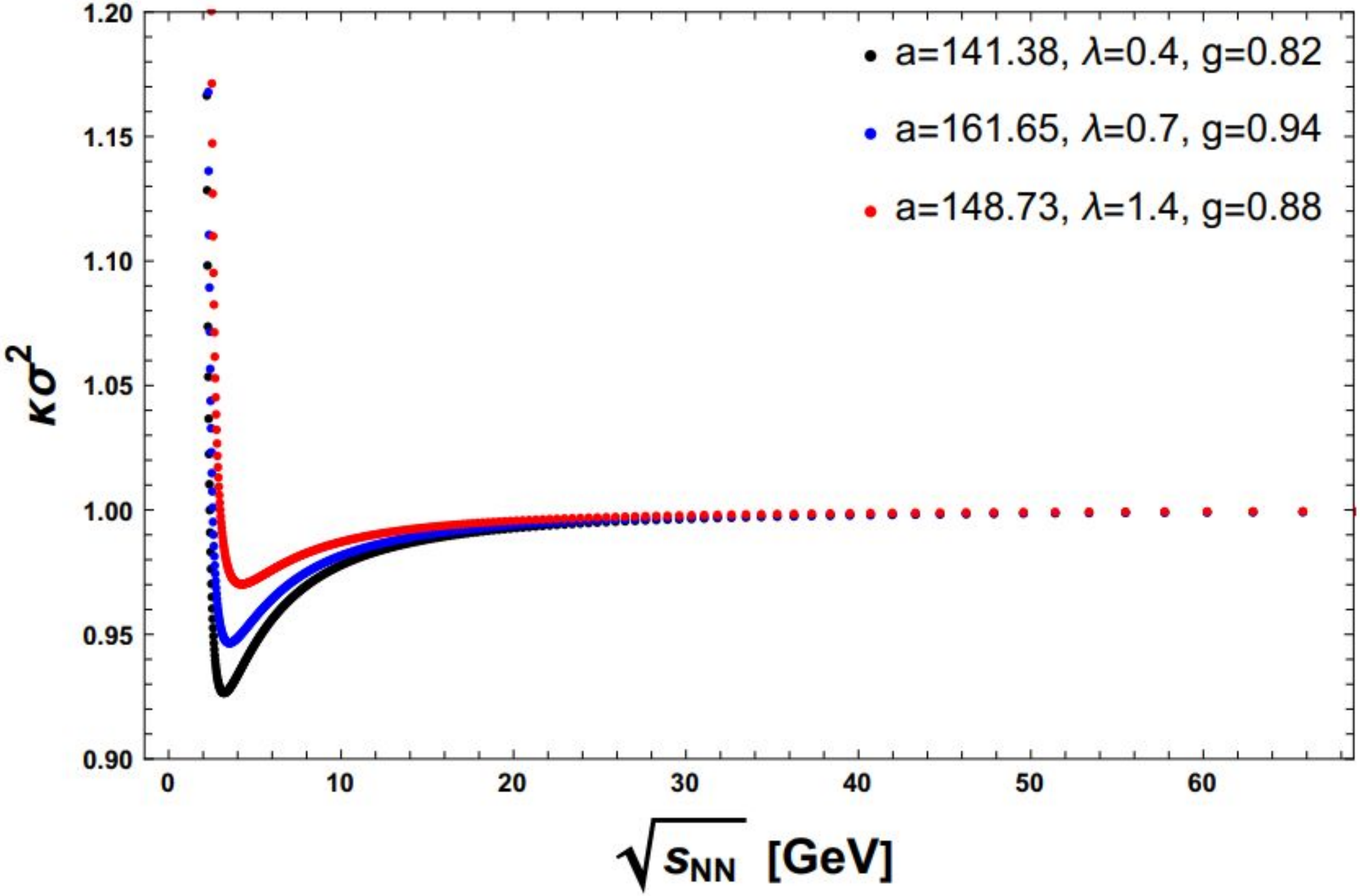


Phys. Rev. C **73**, 034905

Results



0.285
0.280
0.275
0.270
0.265
0.260
0.255
0.250
c_s^2
0
10
20
30
40
50
60
$\sqrt{S_{NN}}$ [GeV]
2nd order
1st order
CEP


a=141.38, λ=0.4, g=0.82
a=161.65, λ=0.7, g=0.94
a=148.73, λ=1.4, g=0.88
$\kappa\sigma^2$
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0
10
20
30
40
50
60
$\sqrt{s_{NN}}$ [GeV]

Final Comments

- Ring diagrams inclusion is equivalent to introducing screening effects at finite T and μB .

- CEP signaled by marked change in the c_s

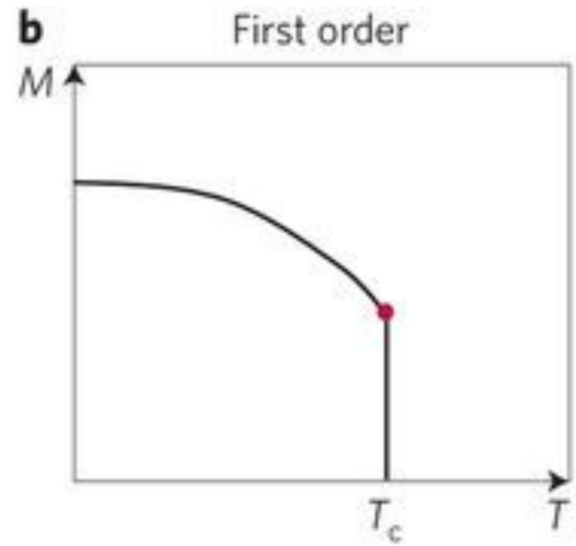
- CEP found at low T and high μB

- the CEP can be located for collision energies , $\sqrt{s_{NN}} \sim 2$ GeV, namely, in the lowest NICA or within the HADES energy domain.

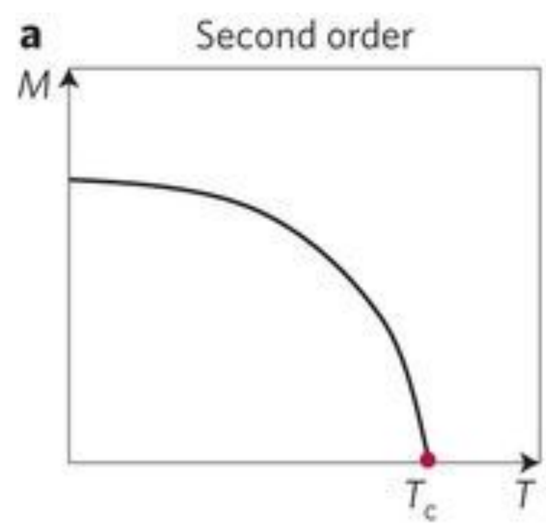
THANKS FOR WATCHING!

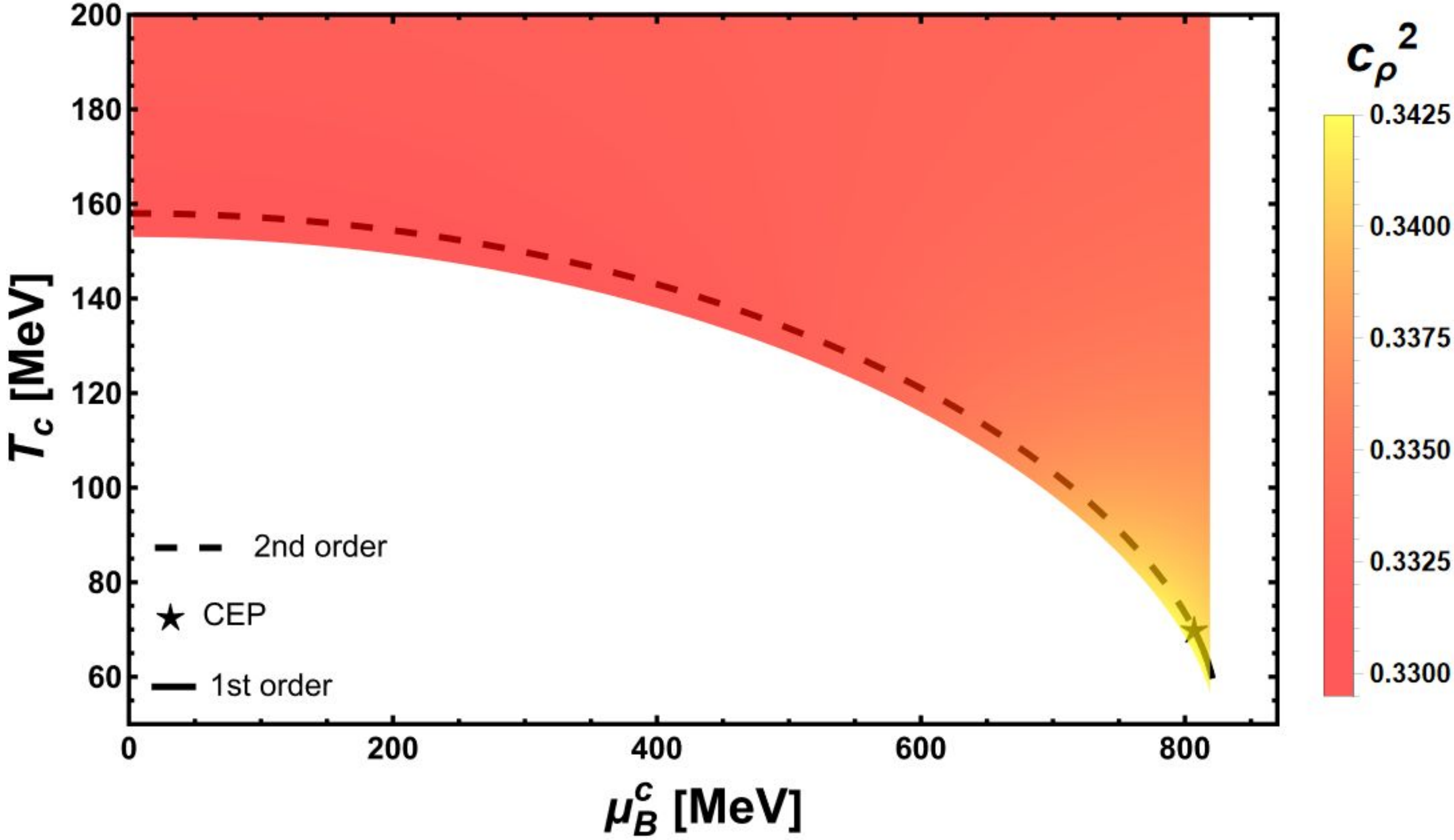
Classifications

- The system is in a "mixed-phase regime" in which some parts of the system have completed the transition and others have not.

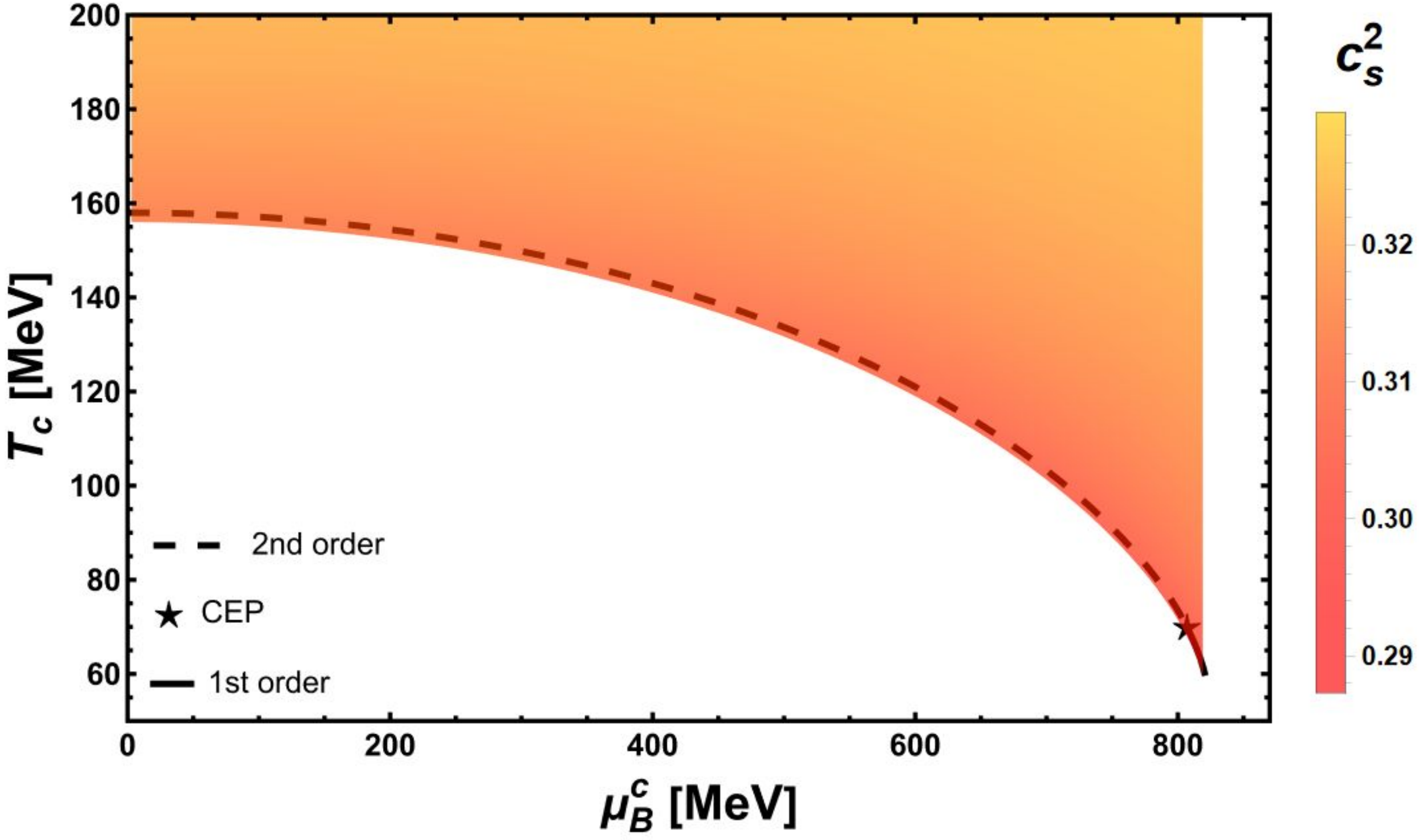


- They are characterized by a divergent susceptibility, an infinite correlation length, and a power law decay of correlations near criticality.

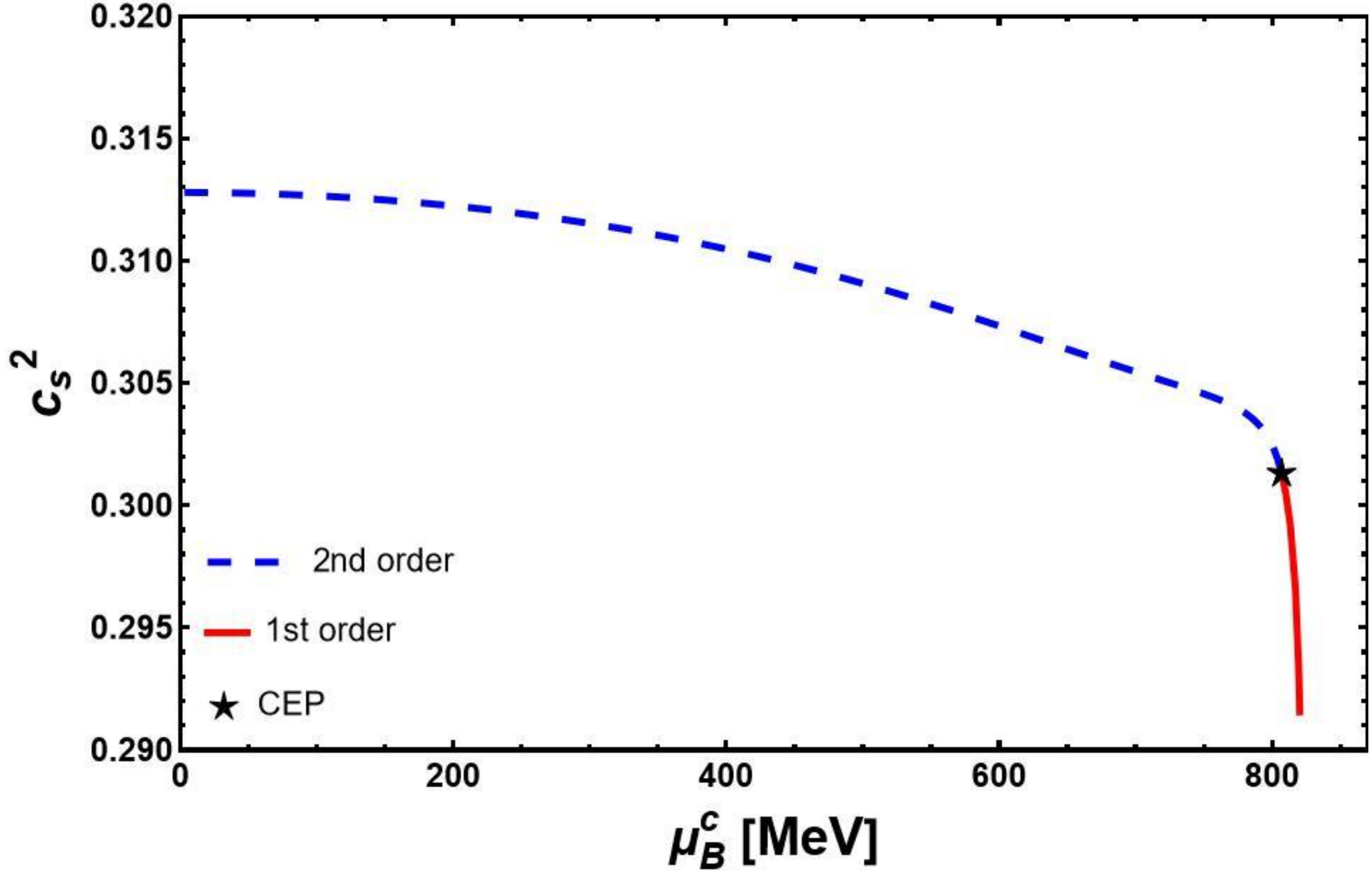




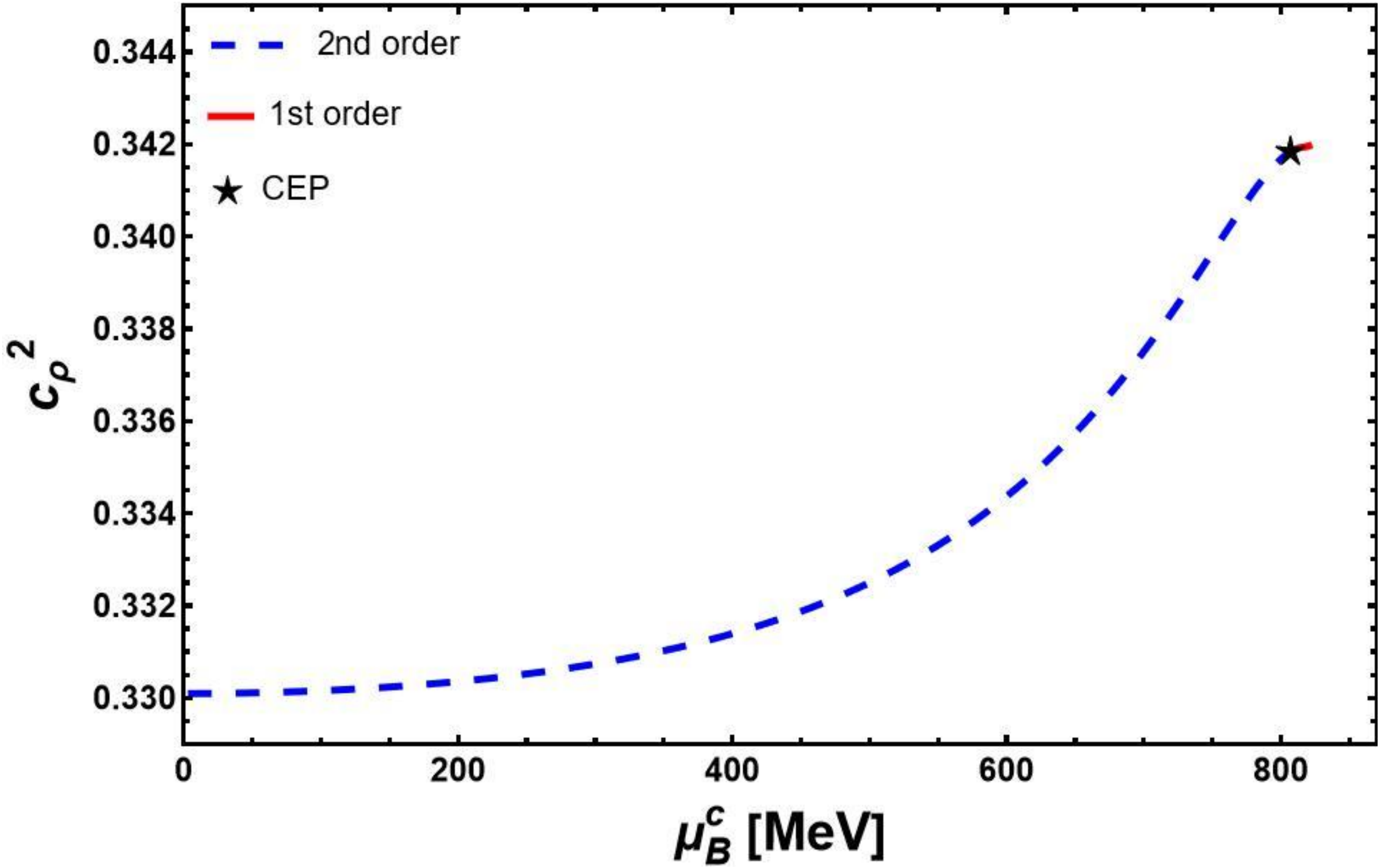
c_ρ^2
0.3425
0.3400
0.3375
0.3350
0.3325
0.3300
200
180
160
140
120
100
80
60
T_c [MeV]
0
200
400
600
800
μ_B^c [MeV]
2nd order
CEP
1st order

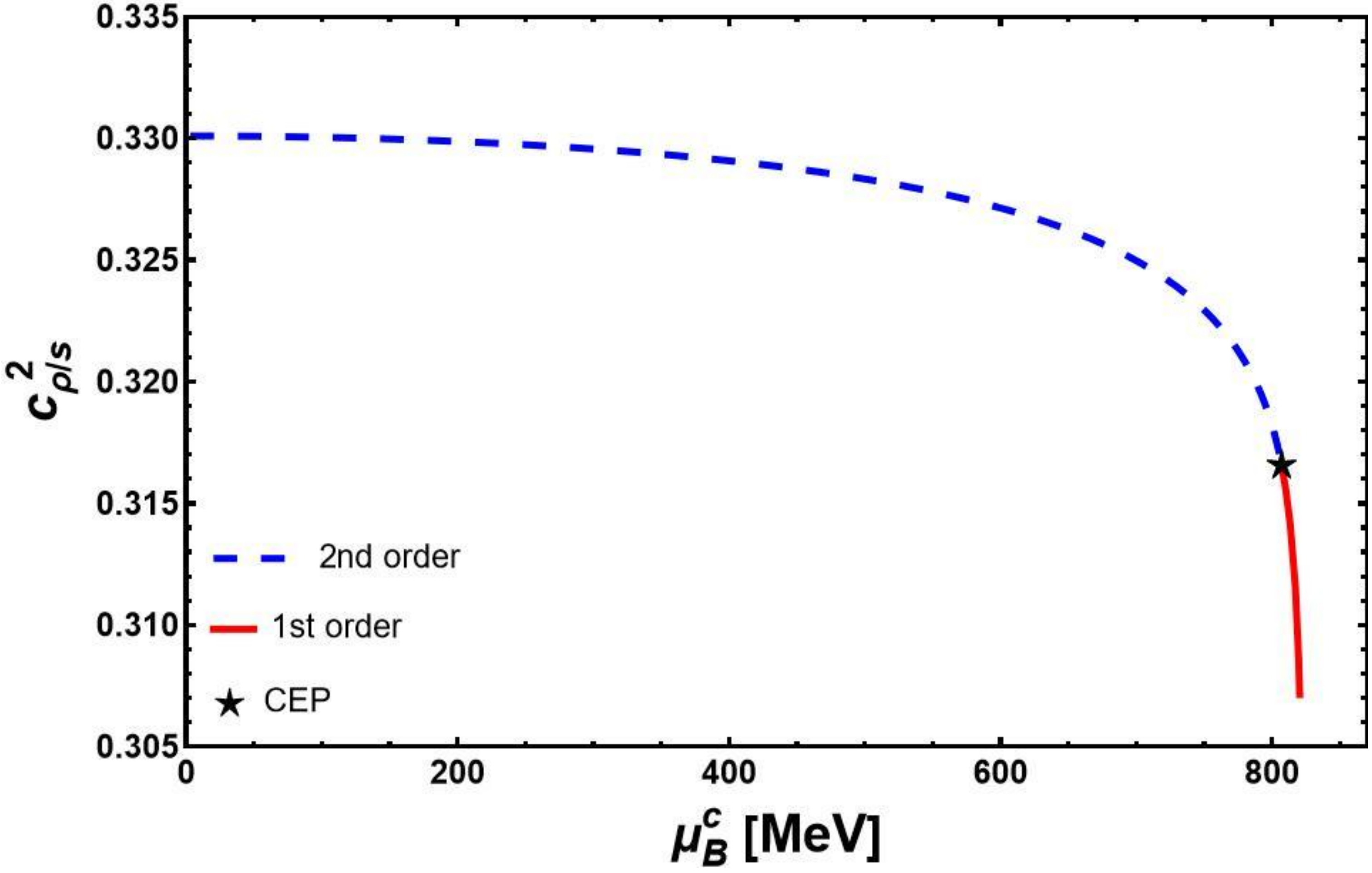


200
180
160
140
120
100
80
60
T_c [MeV]
0
200
400
600
800
μ_B^c [MeV]
2nd order
CEP
1st order
c_s^2
0.32
0.31
0.30
0.29



0.320
0.315
0.310
0.305
0.300
0.295
0.290
c_s^2
0
200
400
600
800
μ_B^c [MeV]
2nd order
1st order
CEP


2nd order
1st order
CEP
0.344
0.342
0.340
0.338
0.336
0.334
0.332
0.330
c_ρ^2
0
200
400
600
800
μ_B^c [MeV]



0.335
0.330
0.325
0.320
0.315
0.310
0.305
0
200
400
600
800
$c^2_{\rho/s}$
μ_B^c [MeV]
2nd order
1st order
CEP