QCD and pure SU(3) on the lattice, physical limit

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Nature 443 (2006) 675; QCD transition is analytic
Phys.Rev.D 105 (2022) 7, 074513; quenched: first order

Outline

Lattice formulation

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E} \tag{1}$$

S_E is the Euclidean action

Parameters (the lattice spacing does not appear explicitely):
gauge coupling g
quark masses m_i ($i = 1..N_f$)
(Chemical potentials μ_i)
Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a} \tag{2}$$

Continuum limit: $a \to 0 \Longleftrightarrow N_t \to \infty$; CPU demand scales as N_t^{8-12}

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

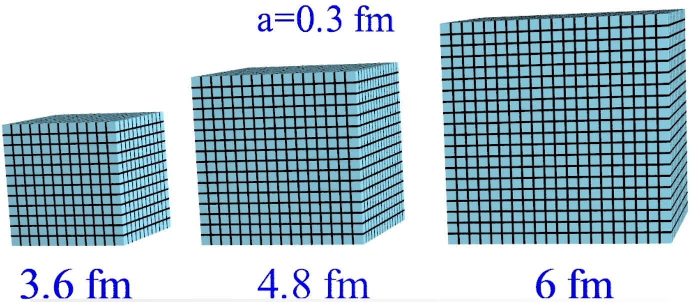
finite size scaling study of the chiral condensate (susceptibility)

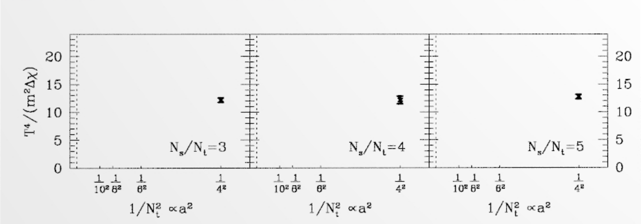
$$\chi=(T/V)\partial^2 \log Z/\partial m^2$$

phase transition: finite V analyticity $V\to\infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

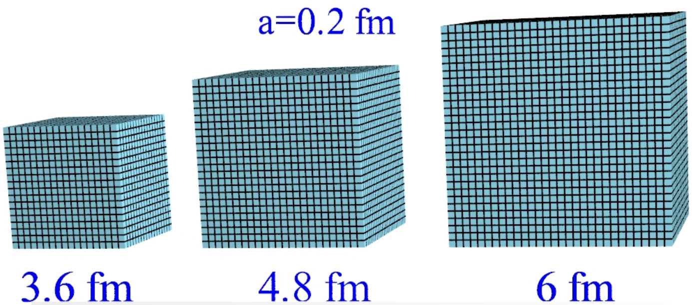
two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.12fm
b. using the continuum extrapolated χ_{max}: finite size scaling

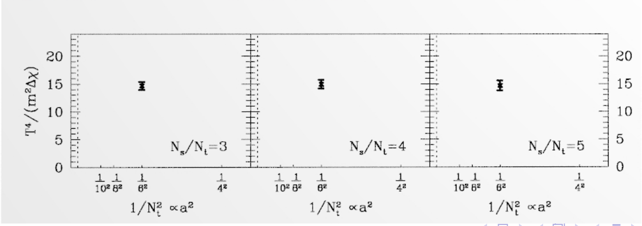
Approaching the continuum limuit





Approaching the continuum limuit





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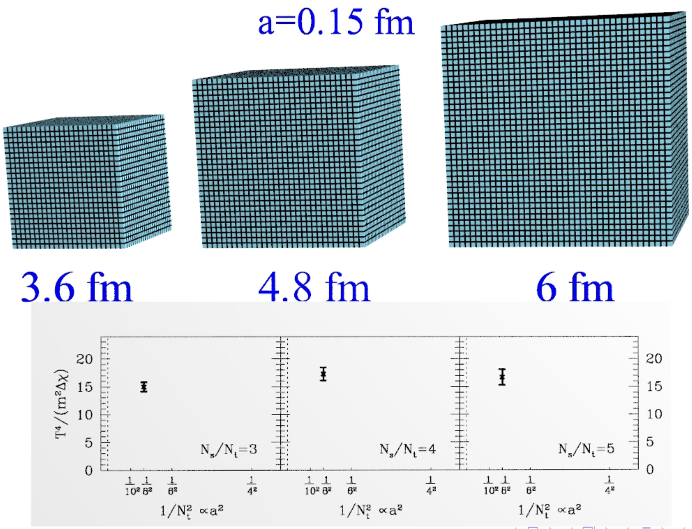

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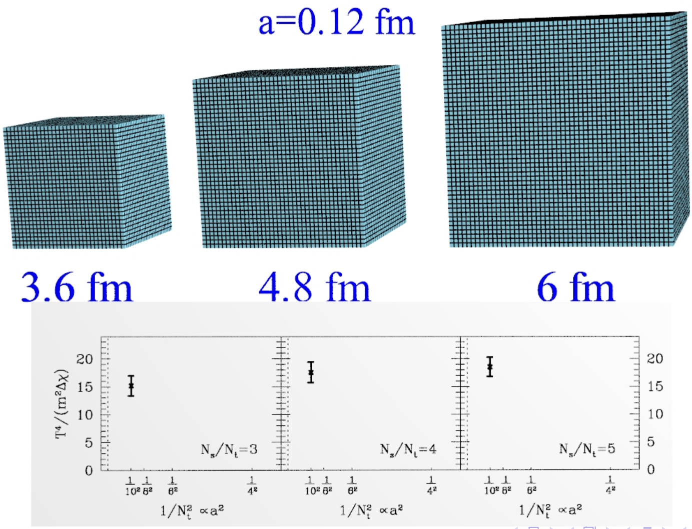

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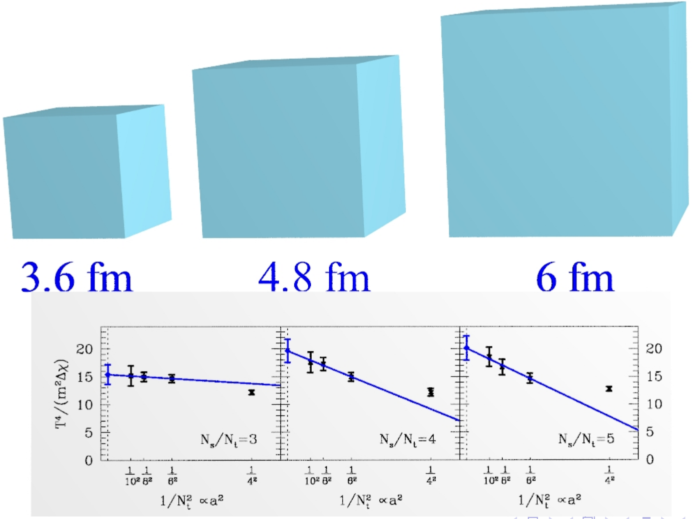


Old (but important) result

The most cited lattice paper ever written (thank to the community)

Important for cosmology: no first order QCD phase transition
No relics to search for (no bubbles, no quark nuggets, etc.)

Important for heavy ion physics: for small μ smooth transition
It manisfests itself in the equation of state, too

questions: a) is "a" small enough b) is V large enough c) staggered

since 2006 Moore's law resulted in 3 orders of magnitude more CPU

(a) CPU costs increase by a^{-8}, thus taking a/2 would eat up 200
(b) AND at the same time we want to double or triple the volume

(c) remnant of the chiral transition:
staggered is chiral but (i) pion tower (ii) rooting
$\Rightarrow$ go to (more) chiral action: 4HEX or overlap fermions
overlap: at least 100 times more CPU demanding, but probably doable

Test case: pure SU(3) theory; is it easier?

usually lattice people take the pure SU(3) theory for testing
it is $\mathcal{O}(100)$ times faster to generate new configurations

the reason for the a^{-8} law is critical slowing down
correlation lengths are larger in lattice units: moving together
we get almost the same configurations: autocorrelation grows

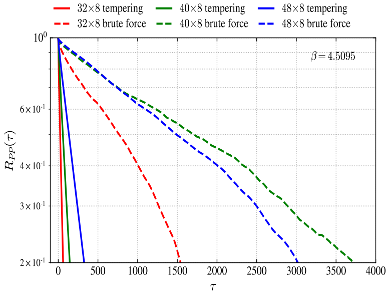
pure SU(3) is a first order finite T transition (we will see)
first order transitions: supercritical slowing down
barrier between the two phases: exponentially difficult
does not move from one phase to the other (bubbles, walls needed)

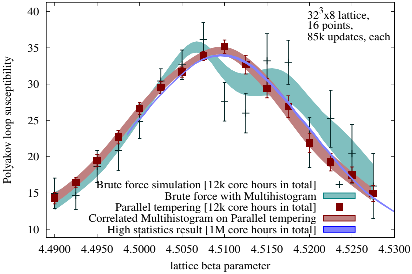
new technique to avoid it: tempering algorithm to generate configs
measure: Polyakov loop (product of the gauge fields in the t-direction)

Tempering algorithm

Several sub-ensemble with slightly different parameters (temperature)

Two kinds of transition:
(a) Transitions within a sub-ensemble (traditional step)
(b) Swapping update of two sub-ensembles (takes from "different T")





Order parameters: susceptibility and third cumulant

Use two quantities sensitive to a first order phase transition

(a) susceptibility of the Polyakov loop:

$$\chi = N_s^3 \left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)$$

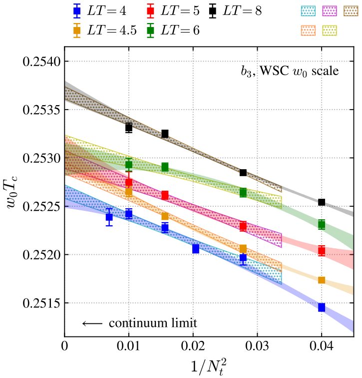
peak around T_c: getting narrower and higher with V
this behavior is linear if the transition is first order

(b) third-order Binder cumulant of the Polyakov loop

$$b_3 = \frac{\langle |P|^3 \rangle - 3\langle |P| \rangle \langle |P|^2 \rangle + 2\langle |P| \rangle^3}{\left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)^{3/2}}$$

b_3 has a zero-crossing at a particular coupling which we call T_c
the slope of b_3 at the zero-crossing increases linearly with V

Continuum limit for T_c



continuum limit goes as a^2 or $1/N_t^2$

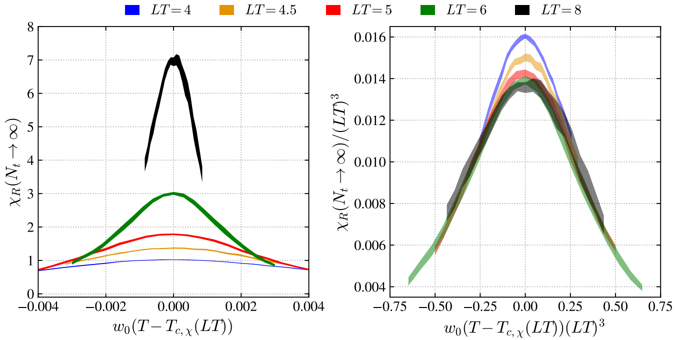
w_0: practical unit easy to measure on the lattice 0.17236[70] fm

we have five volumes and 4-6 lattice spacings

largest aspect ratio is 8 (for QCD it was 5), thus four times larger V

Volume dependence of the susceptibility

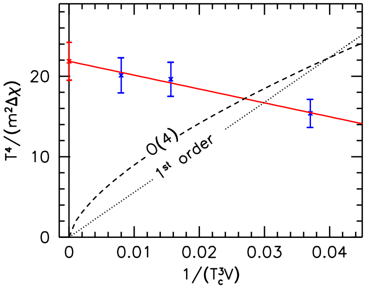
continuum extrapolated renormalized Polyakov loop susceptibilities narrower and higher: rescale it with the volume:

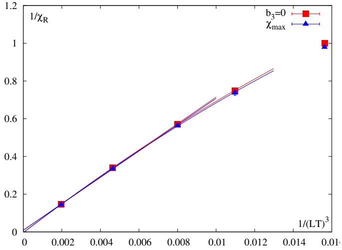


certainly not consistent with a constant behavior

Infinite volume extrapolation

infinite V extrapolation of the inverse Polyakov loop susceptibility
results are renormalized and continuum extrapolated





pure SU(3): linear behavior with small subleading term(s)

the errors are hardly visible, V→0 result is consistent with zero

$\chi_R^{-1}(V=\infty) = 0.0023(58)_{\text{stat}}(65)_{\text{sys}}$

Conclusions/Plans

Old (2006) and important qualitative result:
The finte T QCD transition: analytic (certain level of rigor)

New (2022) result for the pure SU(3) theory:
High precision (more rigorous) proof that it is of first order
(tempering algorithm to overcome supercritical slowing down)

Other high precision results (didn't have time to discuss them):
Various discontinuities:
energy density, Polyakov loop, topological susceptibility

Revisit the old and important result for full QCD with Moore's law
smaller lattice spacings, larger volumes and chiral action

the calculation needs leading edge CPU