QCD equation of state at finite density with a critical point from an alternative expansion scheme

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muses

NP3M

NuSTEAM
Nuclear Science in Texas to Enhance and Advance Minorities

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DEPARTMENT OF ENERGY UNITED STATES OF AMERICA

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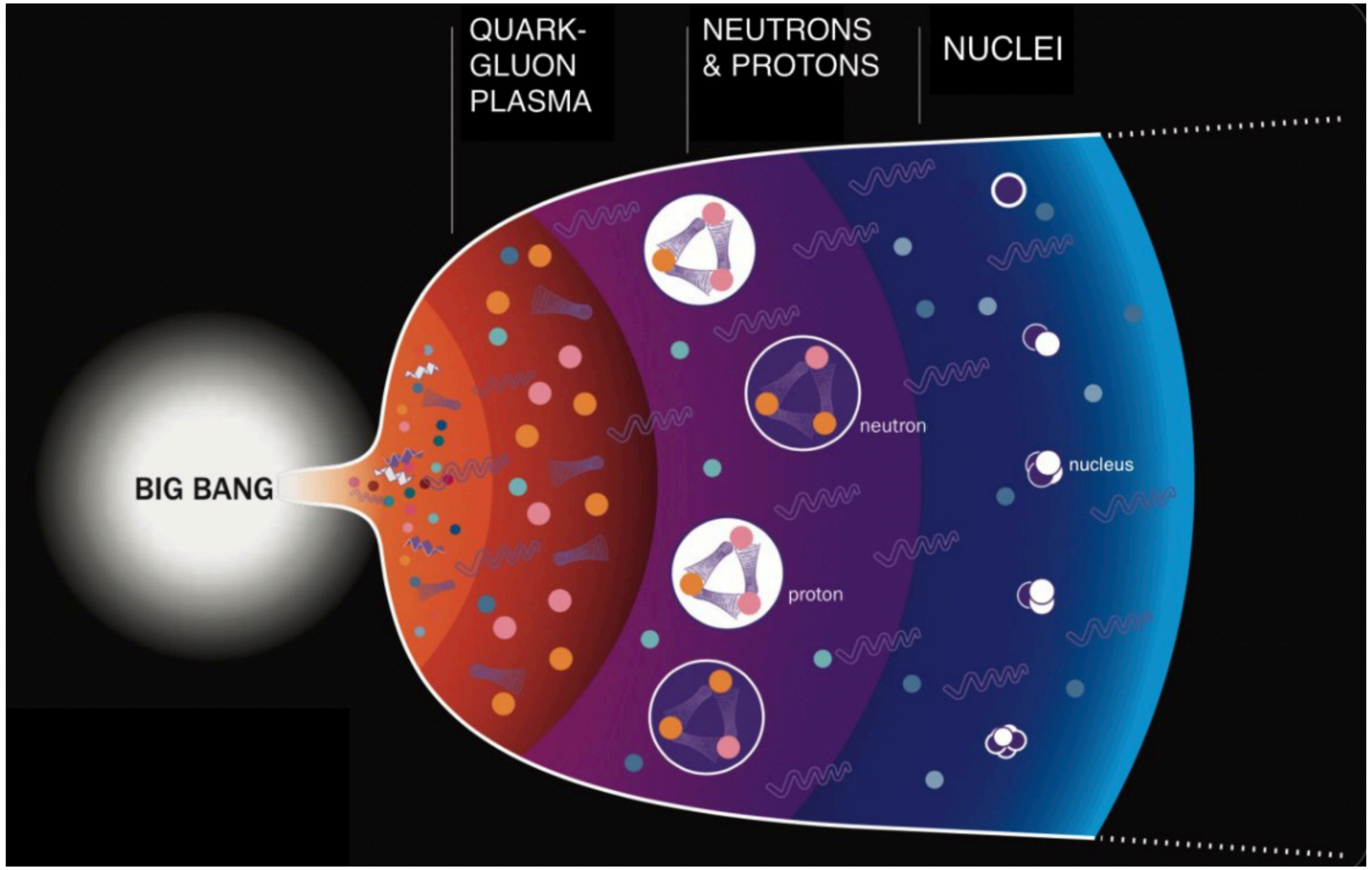
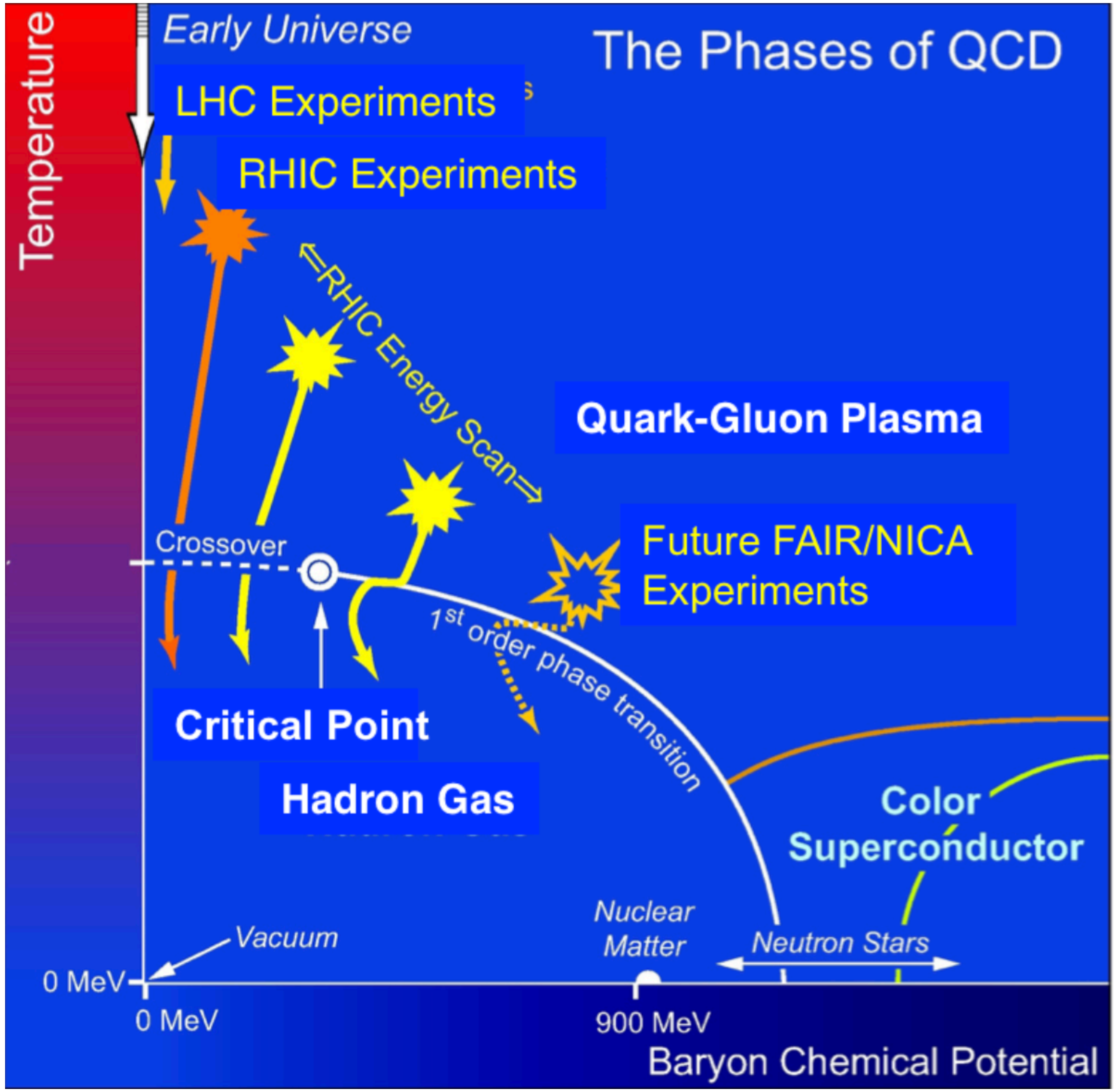
Introduction

QCD Phase Diagram

- **Different phases of strongly interacting matter in equilibrium are captured in the QCD phase diagram**





Lattice QCD results

Fermi Sign Problem!

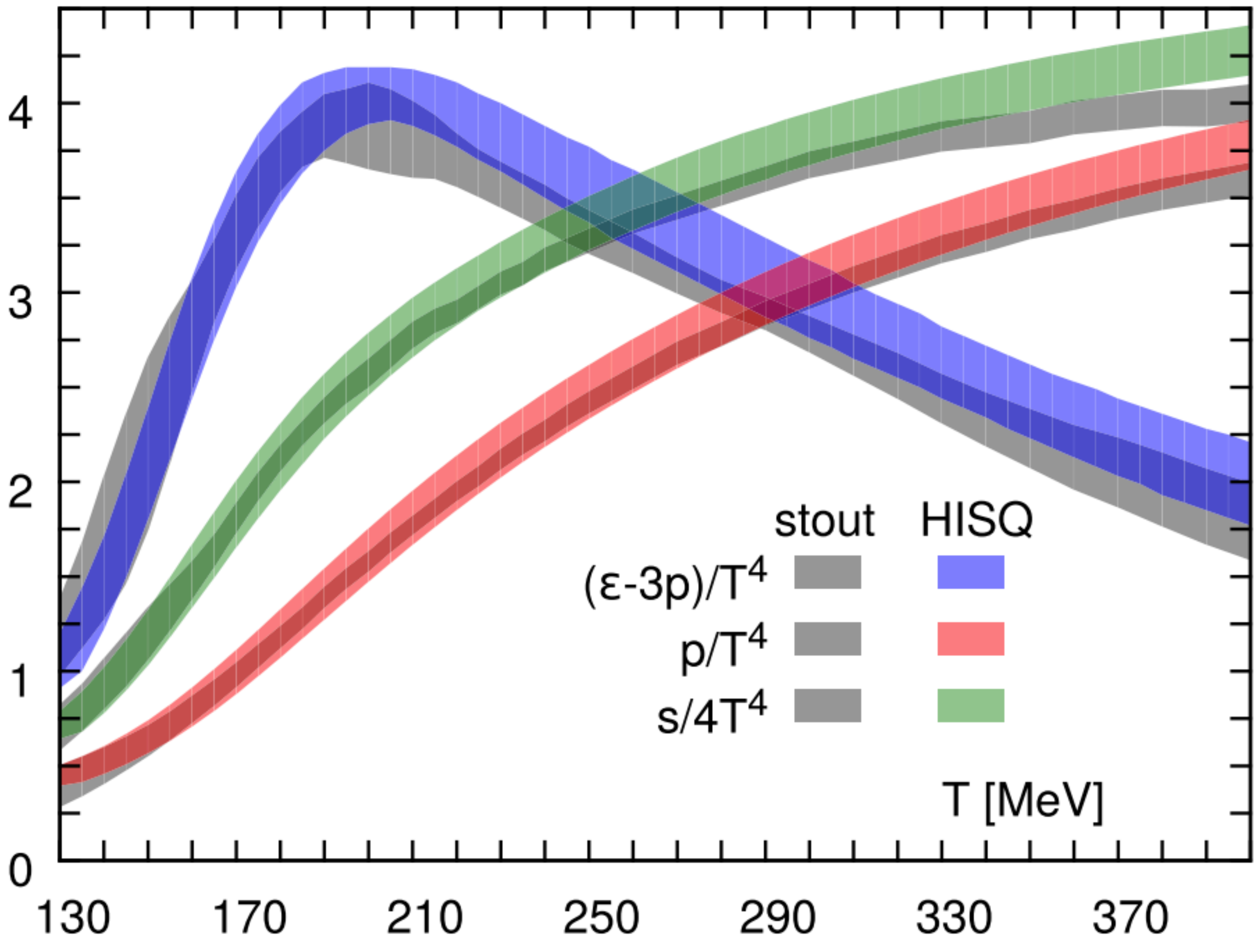
- **Simulation at finite μ_B is Impossible**

$\mu_B = 0$

Way out!

- **Simulating Imaginary μ_B $\rightarrow$ Analytical Continuation**
- **Expansion around $\mu_B = 0$**

[Wuppertal-Budapest (stout) and HotQCD (HISQ) collaborations]



[S. Borsanyi et al Phys. Lett. B 730 (2014) 99]

[A. Bazavov et al.,PhysRevD.90.094503(2014)]

Part 1: Lattice EoS: Taylor Expansion

Lattice EoS at Finite Density

Taylor Expansion around $\mu_B = 0$

Most Straightforward

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!}\chi_n(T,\mu_B=0)\left(\frac{\mu_B}{T}\right)^n$$

$$c_n(T) = \frac{\chi_n^B(T,\mu_B=0)}{n!} = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\Big|_{\mu_B=0}$$

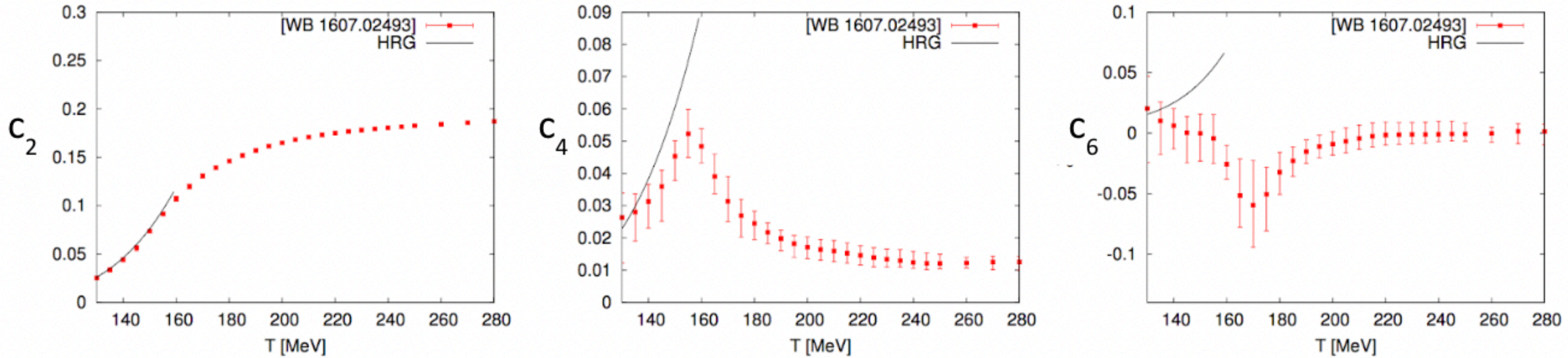
Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 2.5$ despite great Computational Power**
- **Adding one more Higher-Order term does not help in convergence**
- **Taylor expansion is carried out at T= constant and doesn't cope well with μ_B-dependent transition temperature**

[Borsányi, S. et al. PRL (2021)]

- **Re-summation with Pade' Approximation can reach $\frac{\mu_B}{T} = 3.0$**

D. Bollweg et al., arXiv:2212.09043v1 [hep-lat] (2022)



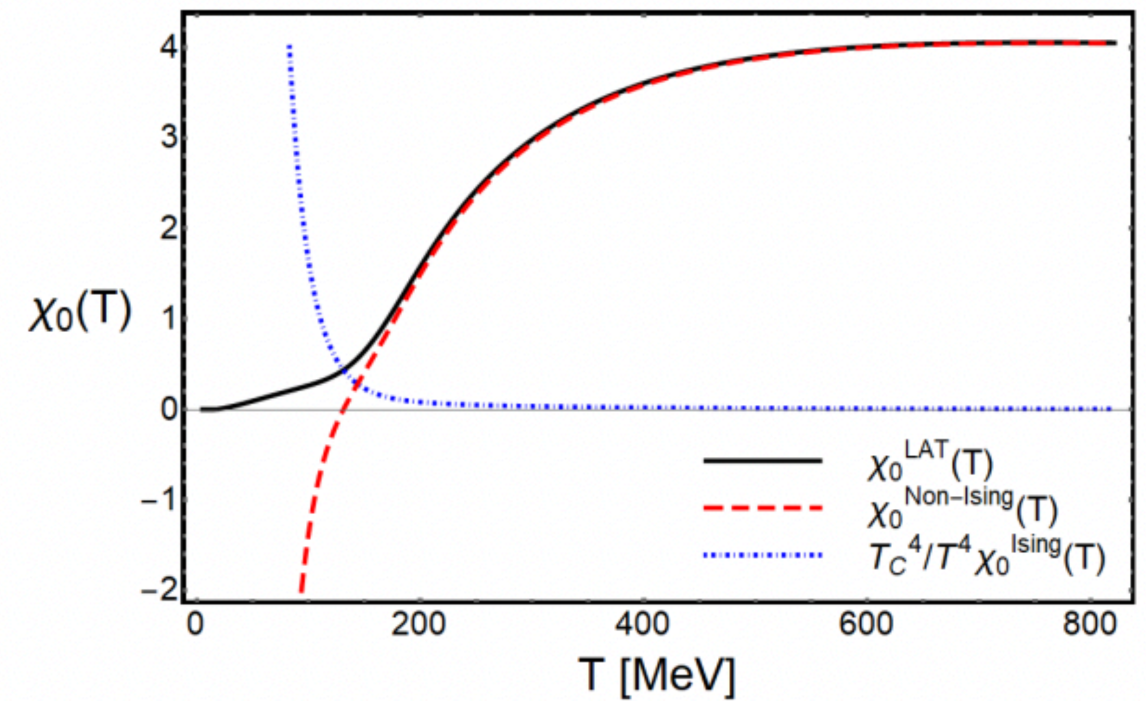
Wuppertal-Budapest

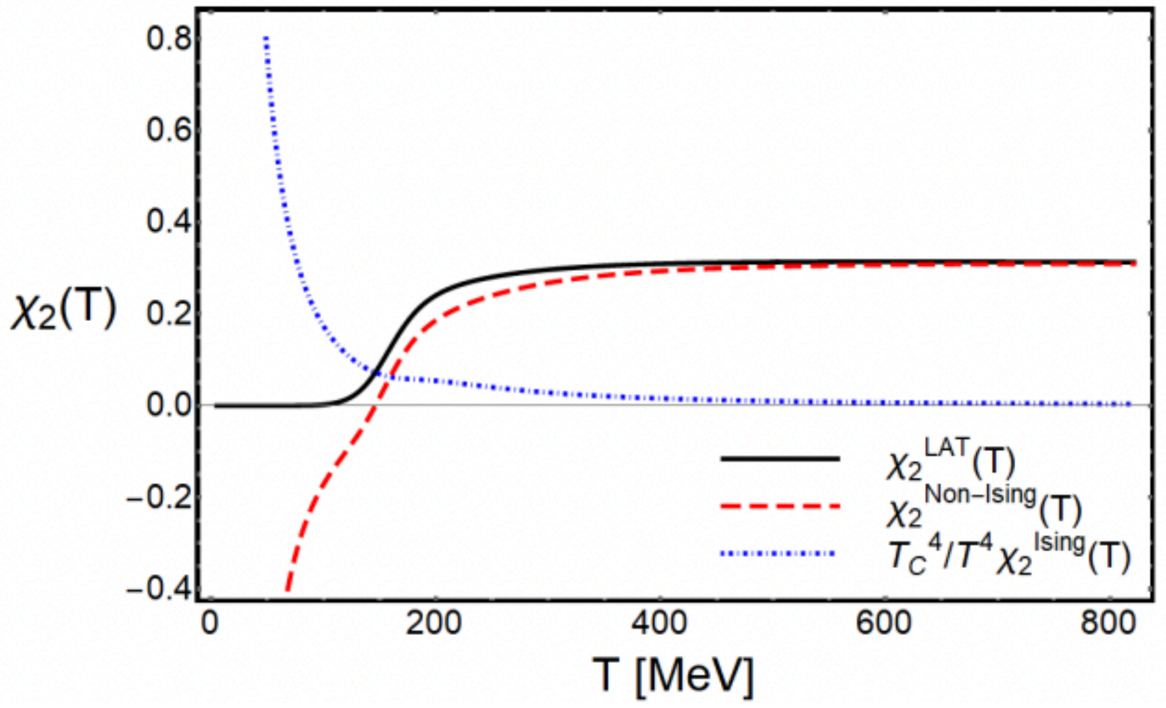
[Borsányi, S. et al. Journal of high energy physics (2018)(10), 1-29]

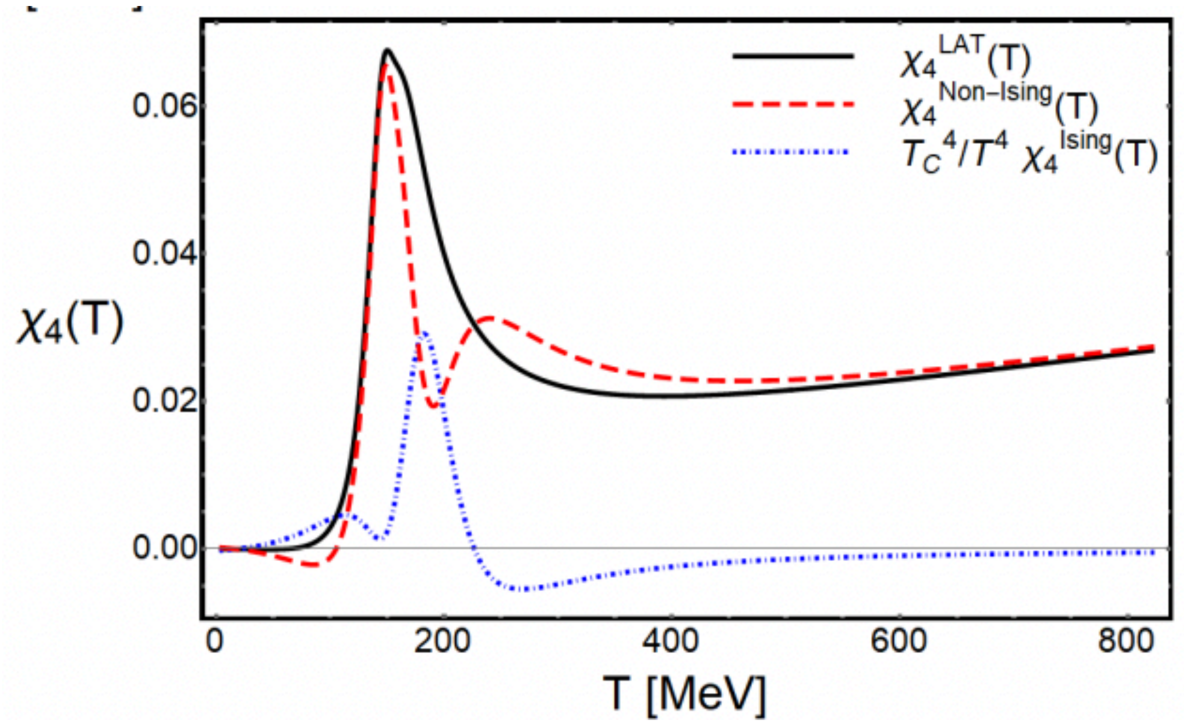
Taylor EoS with Critical point

$$P(T,\mu_B) = T^4 \sum_{n=0}^{2} \frac{1}{(2n)!} \chi_{2n}^{Non-Ising}(T) \left(\frac{\mu_B}{T}\right)^{2n} + T_C^4 P_{symm}^{Ising}(T,\mu_B)$$

$$\chi_n^{Lat}(T) = \chi_n^{Non-Ising}(T) + \frac{T_C^4}{T^4} \chi_n^{Ising}(T)$$

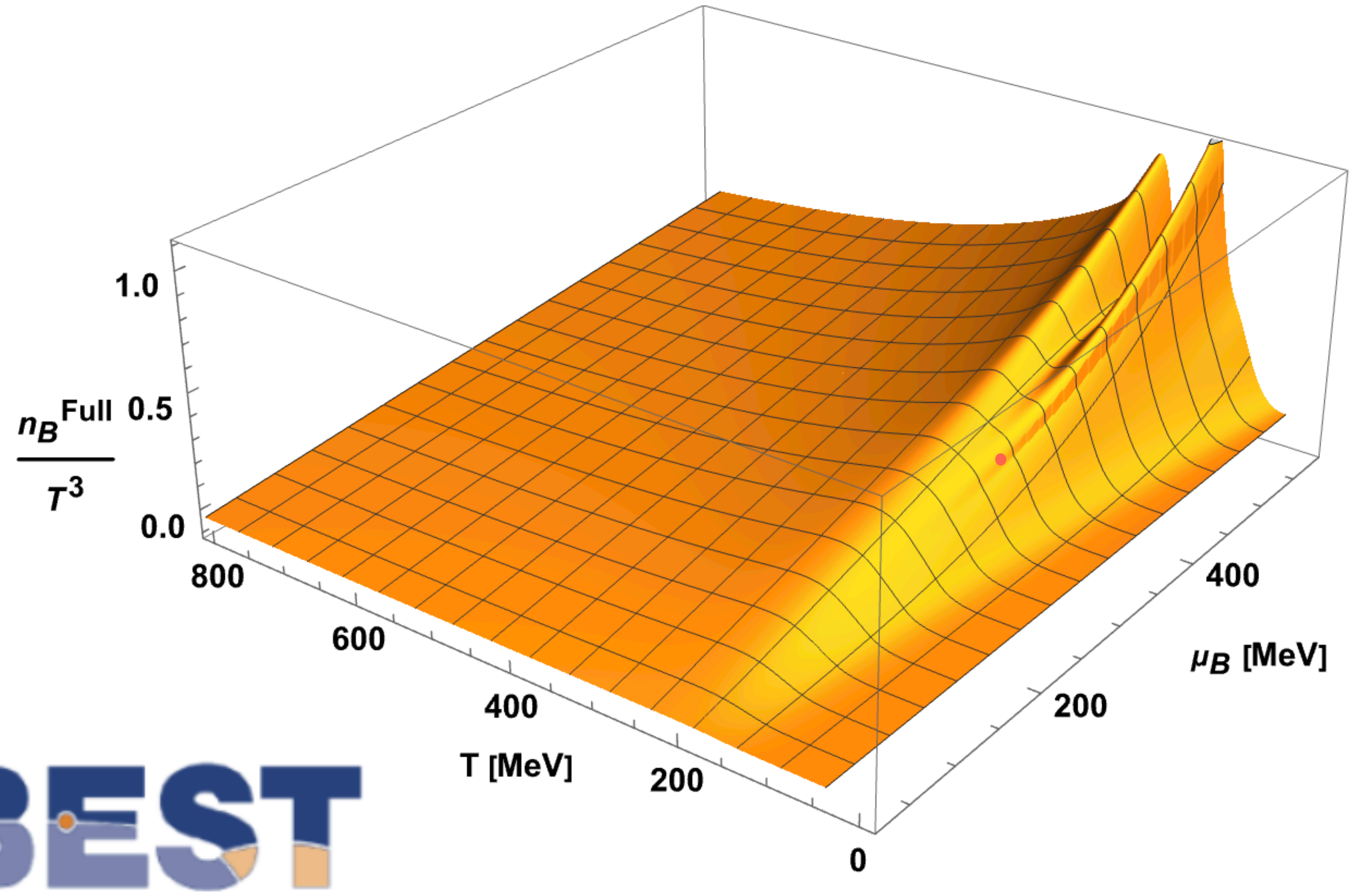






3D Diagram

Baryon density



$$\frac{n_B(T,\mu_B)}{T^3} = \frac{\partial(P/T^4)}{\partial(\mu_B/T)}$$

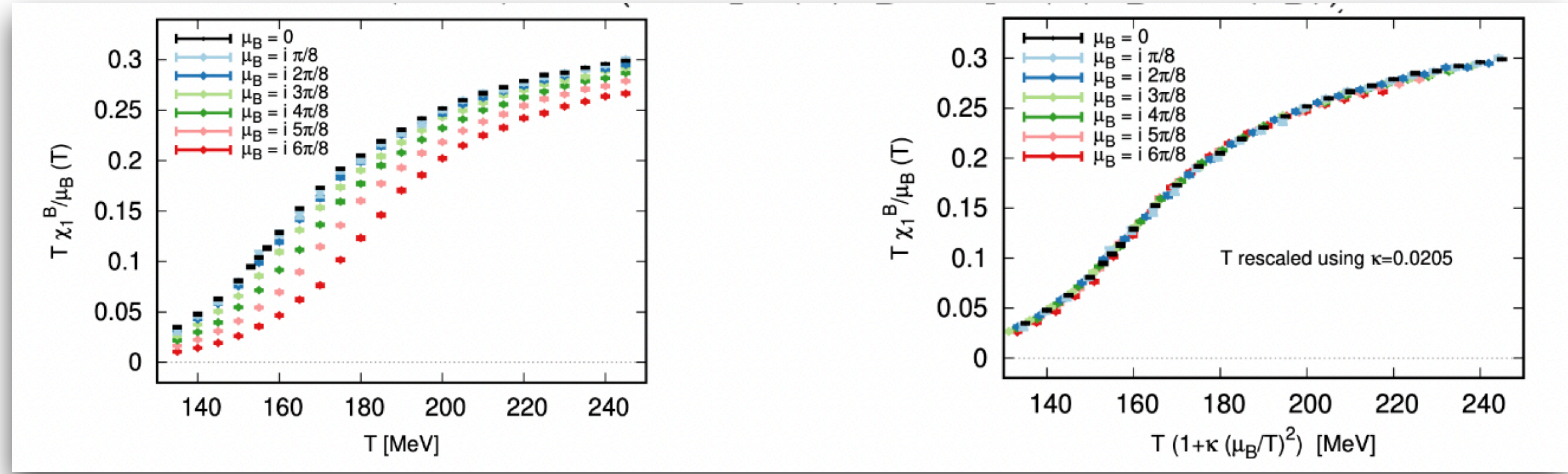
Critical Point at $\mu_{BC} = 350\ [MeV]$

- **Thermodynamic Observables at $\mu_B \geq 450\ MeV$ show unphysical behavior**

Part 2: Lattice EoS: Alternative Expansion Scheme

Alternative Expansion Scheme: EoS

Simulating at Imaginary μ_B



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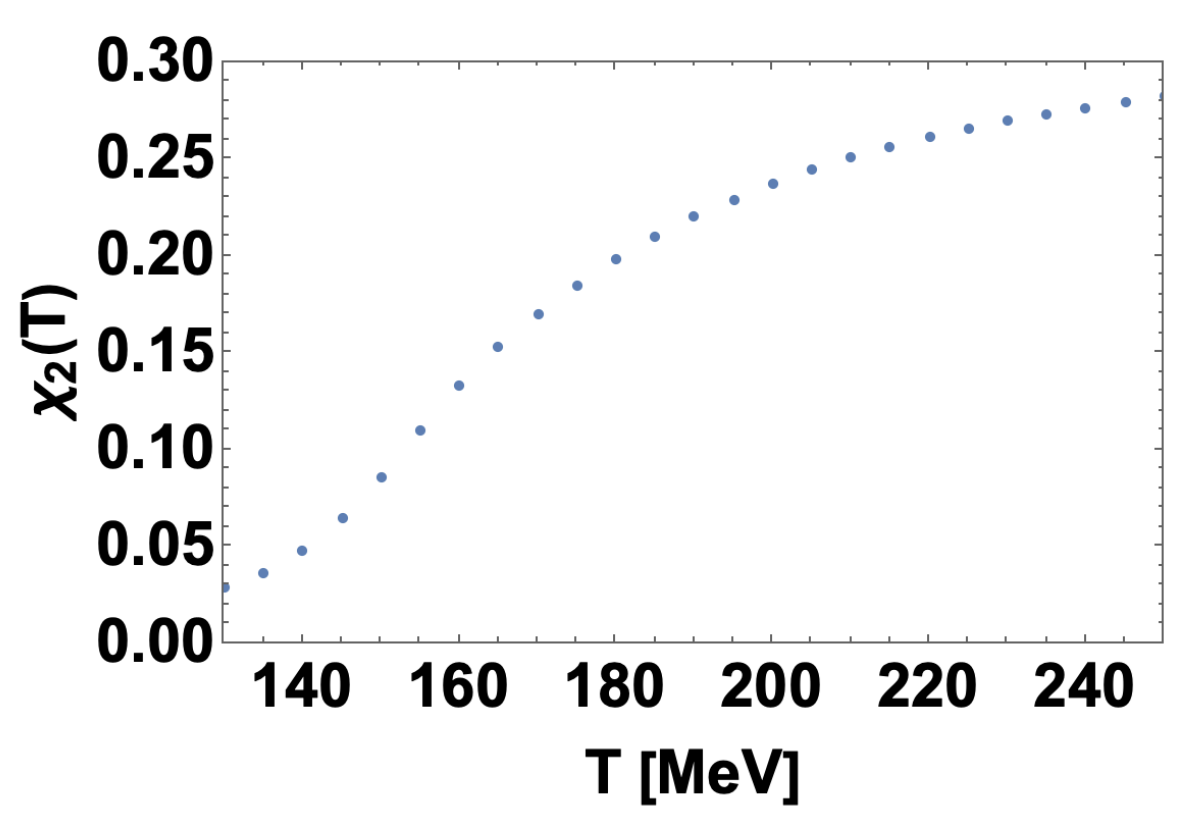


[Borsányi, S. et al. PRL (2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$\chi_n^B(T,\mu_B) = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$

$$T'[T,\mu_B] = T\left[1+\kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2+\kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4+\mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

- **μ_B dependence is captured in T-rescaling.**

- **Trusted up to** $\frac{\mu_B}{T} = 3.5$

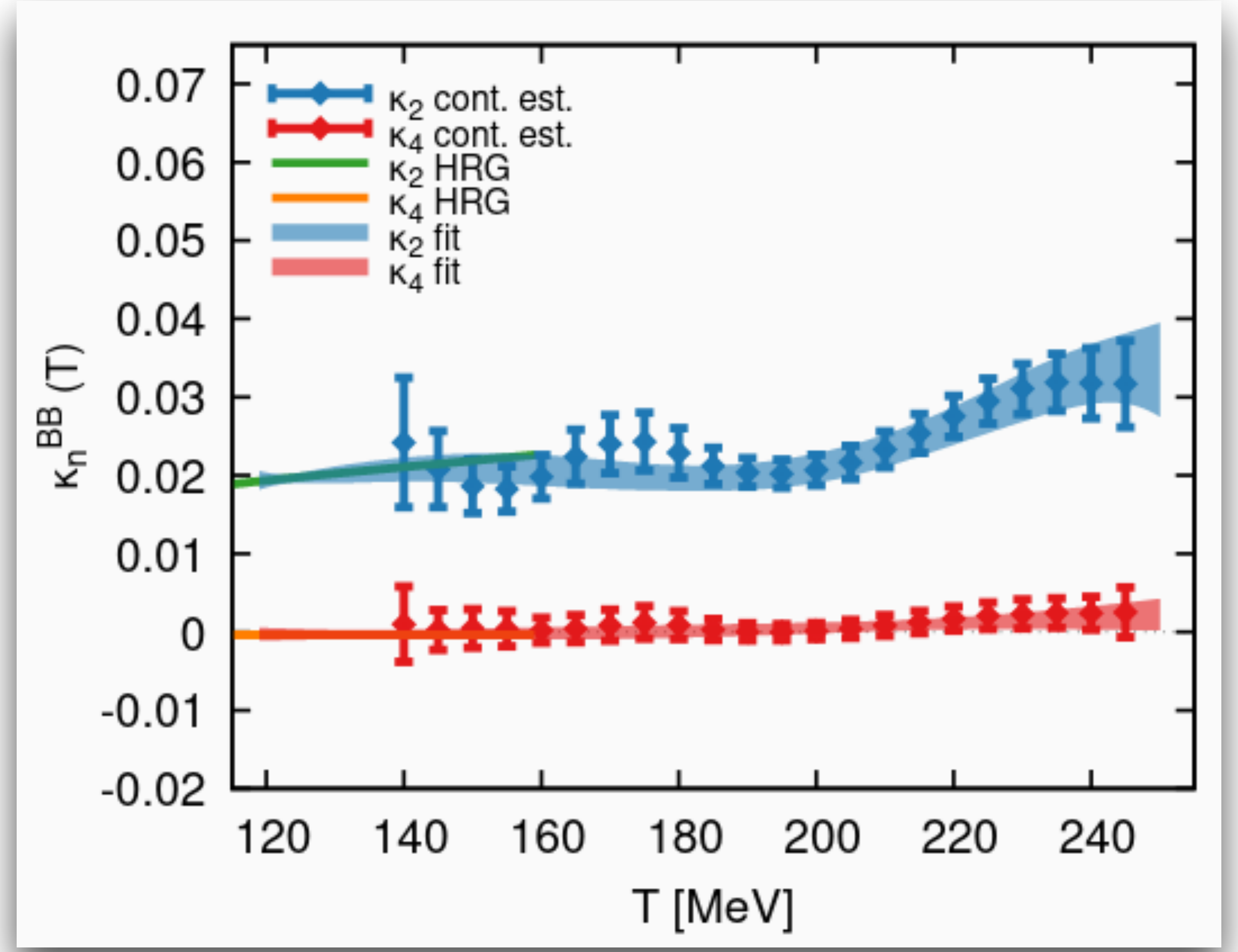
[Borsányi, S. et al. PRL (2021)]

Alternative expansion scheme

Comparing Taylor expansion and Alternative expansion

- $$\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\left(\frac{\partial \chi_2^B(T)}{\partial T}\right)}$$

- $$\kappa_4^{BB}(T) = \frac{1}{360\chi_2^B(T)^3}\left(3\chi'^{B2}_2\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$$



[Borsányi, S. et al. PRL (2021)]

Pros

- $\kappa_2(T)$ **is fairly constant over a large T-Range**
- **There is a separation of scale between** $\kappa_2(T)$ **and** $\kappa_4(T)$
- $\kappa_4(T)$ **is almost zero** $\rightarrow$ **faster convergence**
- **A good agreement with HRG results at Low Temperature**

Part 3: Putting Critical Point into Alternative Expansion: EoS

EoS with a Critical point

Strategy

Scale 3D-Ising model EoS → **Map Ising to QCD** → **Reconstruct the Full Baryon Density** → **Estimate the critical Contribution**

Scale 3D-Ising Model EoS

Close to the critical point, we define a parametrization for Magnetization M, Magnetic field h, and reduced temperature

QCD Critical point is in the 3D-Ising model Universality class

$$r = \frac{T - T_C}{T_C}$$

$$M = M_0 R^{\beta} \theta$$

$$h = h_0 R^{\beta\delta} \tilde{h}(\theta)$$

$$r = R(1 - \theta^2)$$

$$(R, \theta) \longmapsto (r, h)$$

- M_0, h_0 **are normalization constants**
- $\beta \approx 0.326$ **and** $\delta = 4.8$ **are the 3D Ising model critical exponents**
- $\tilde{h}(\theta) = (\theta + a\theta^3 + b\theta^5)$ **with** $a = -0.76201, b = 0.00804$
- **The parameters take on the values** $R \geq 0, |\theta| \leq \theta_0 \approx 1.154, \theta_0$ **being the first non-trivial zero of** $\tilde{h}(\theta)$

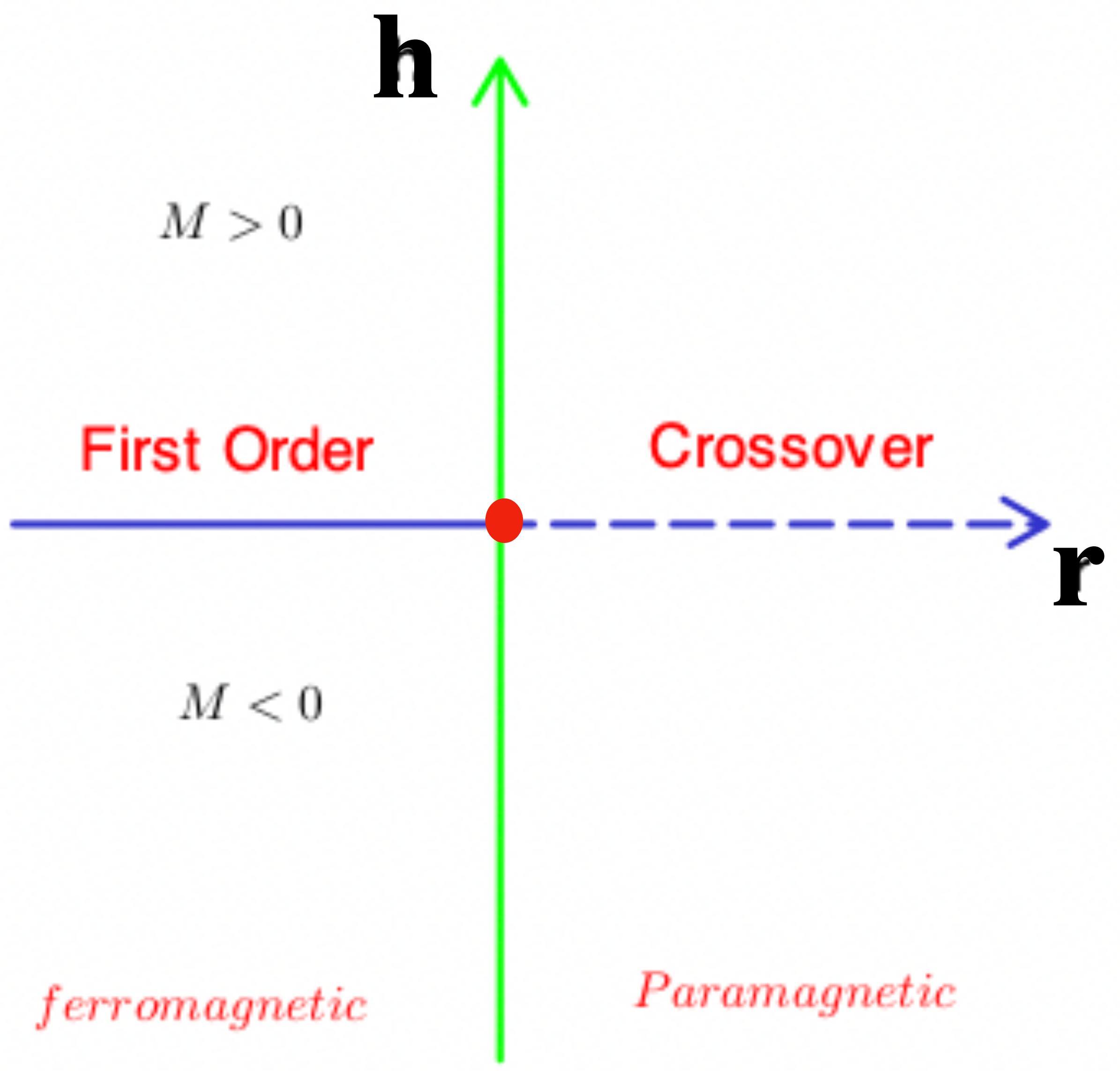
[Parotto et al PhysRevC.101.034901(2020)]

[Nonaka et al Physical Review C, 71(4), 044904.(2005)]

[Guida et al Nuclear Physics B, 489(3), 626-652.(1997)]

EoS with a Critical point

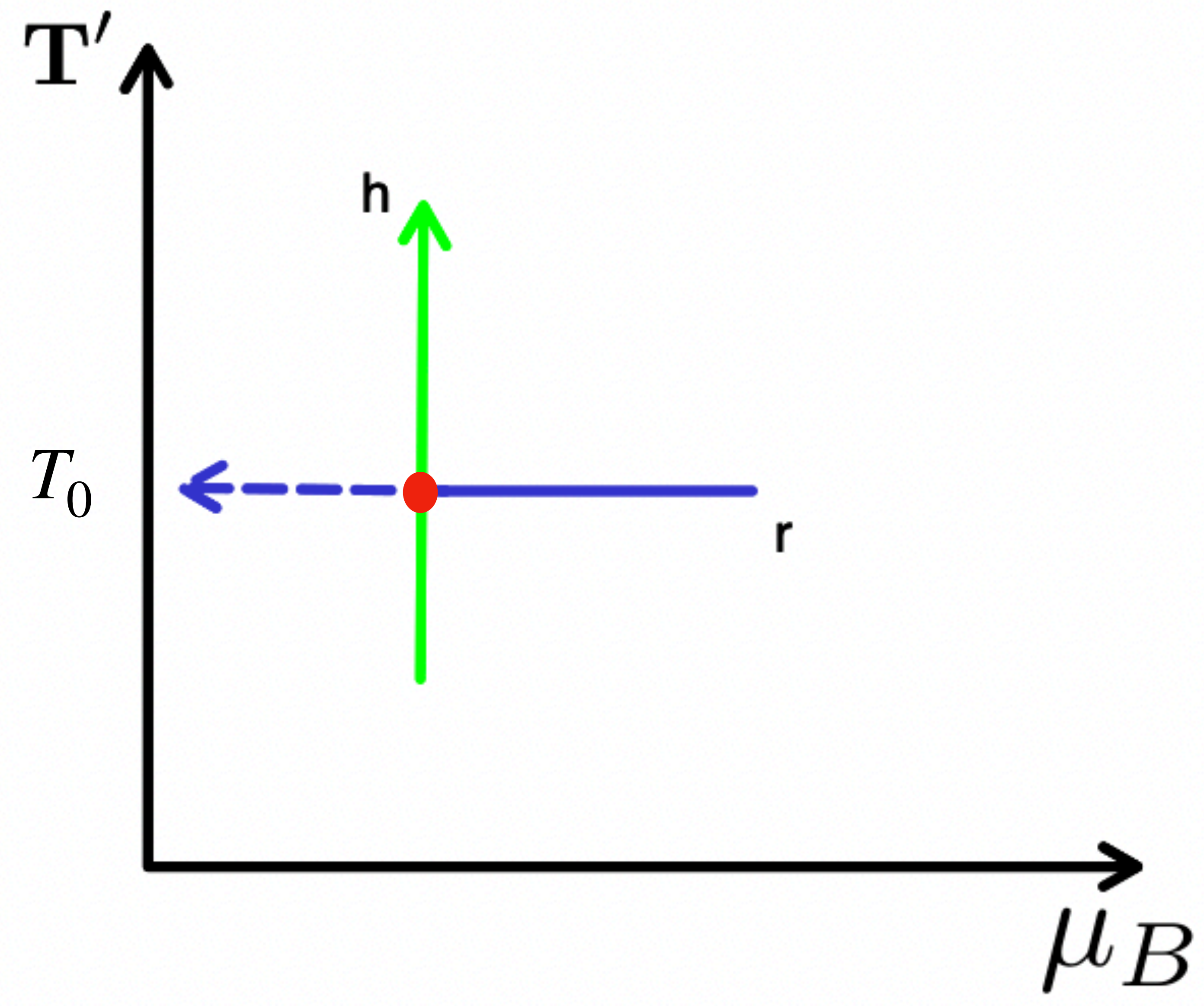
3D Ising to QCD Mapping



3D-Ising Co-ordinates

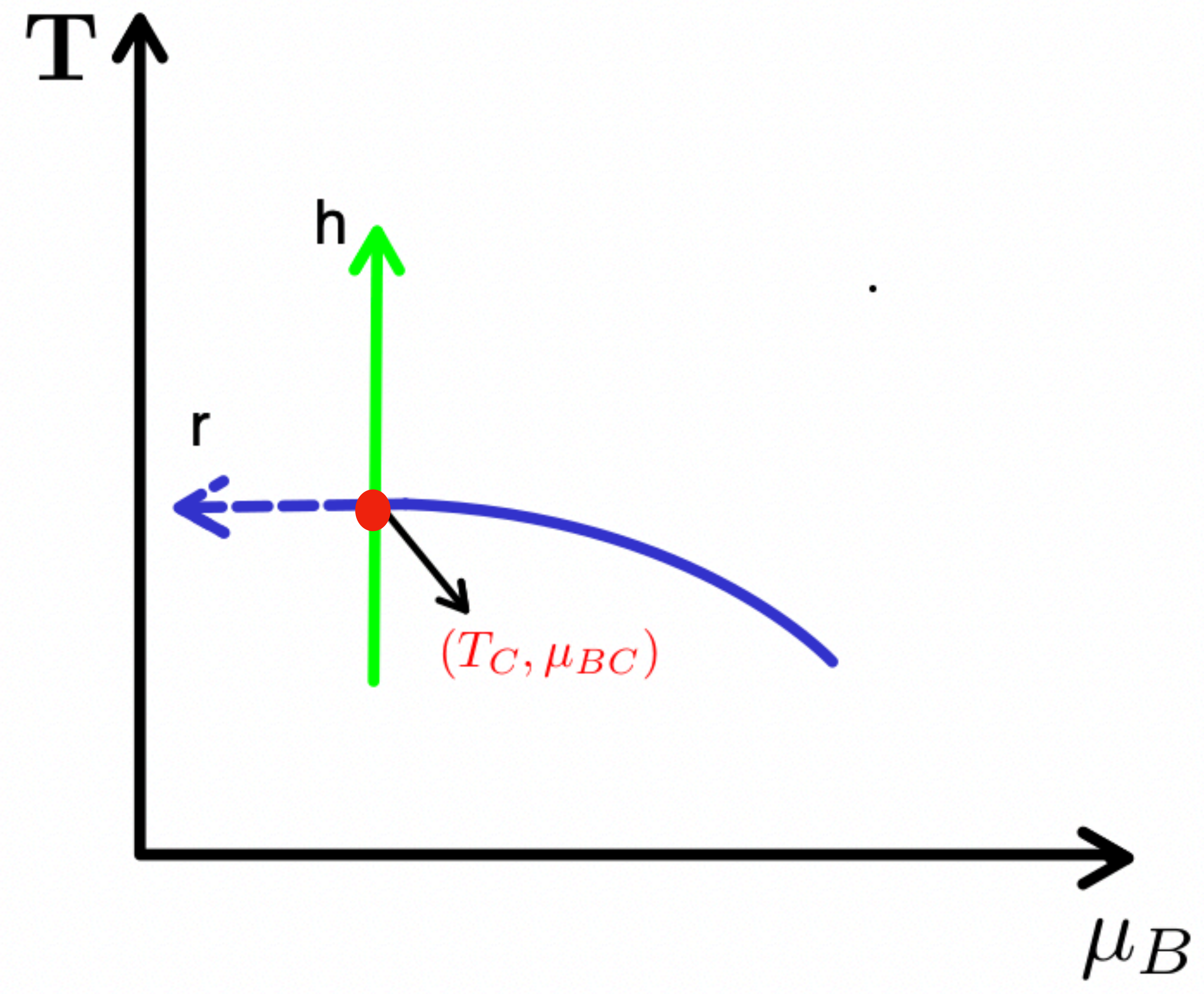
$$\frac{T' - T_0}{T_0} = w \; h \sin \alpha_{12}$$

$$\frac{\mu_B - \mu_{BC}}{T_0} = w(-r\rho - h \cos \alpha_{12})$$



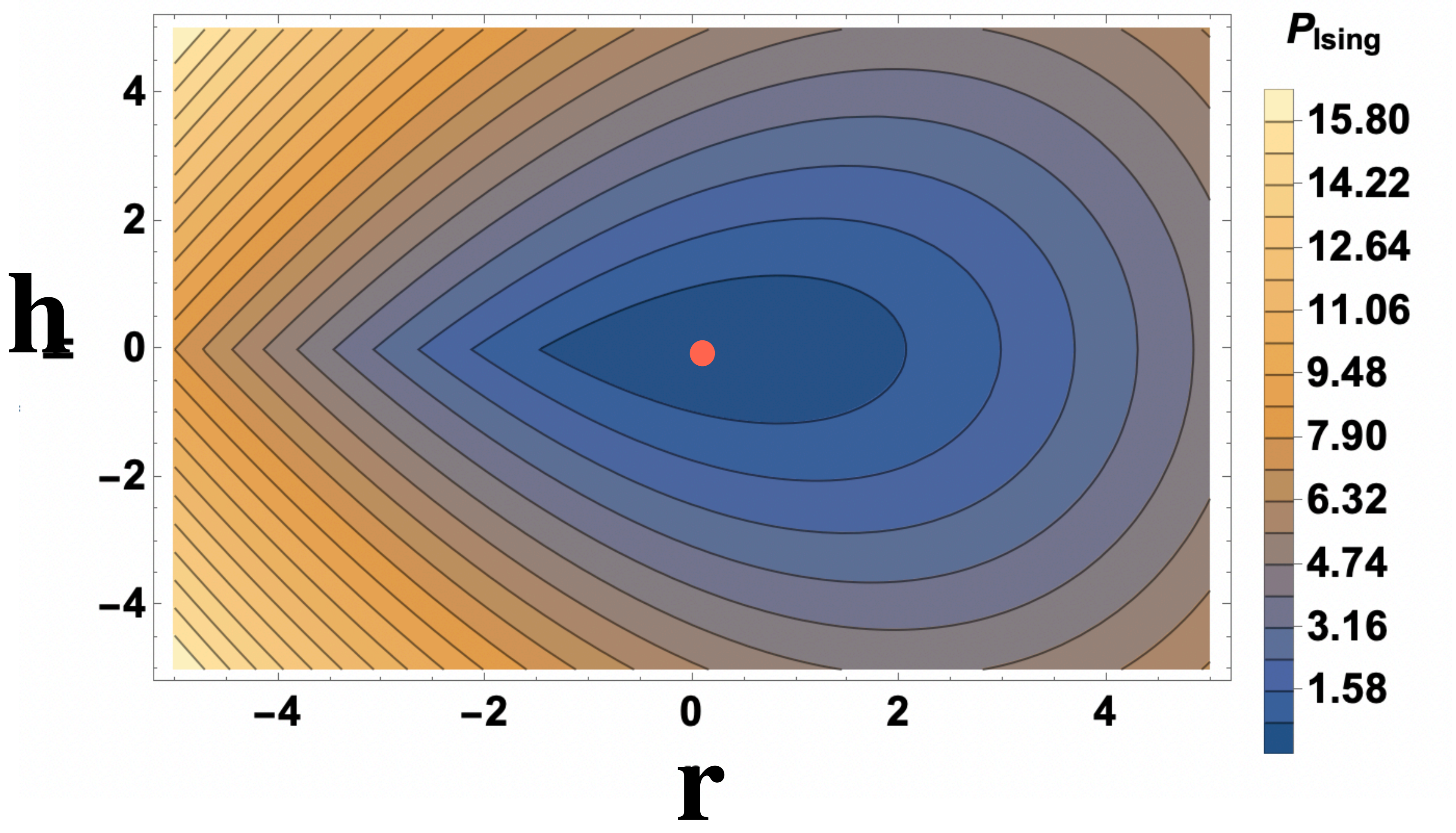
(T', μ_B)-Co-ordinates

$$T' = T\left[1 + \left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4\right]$$

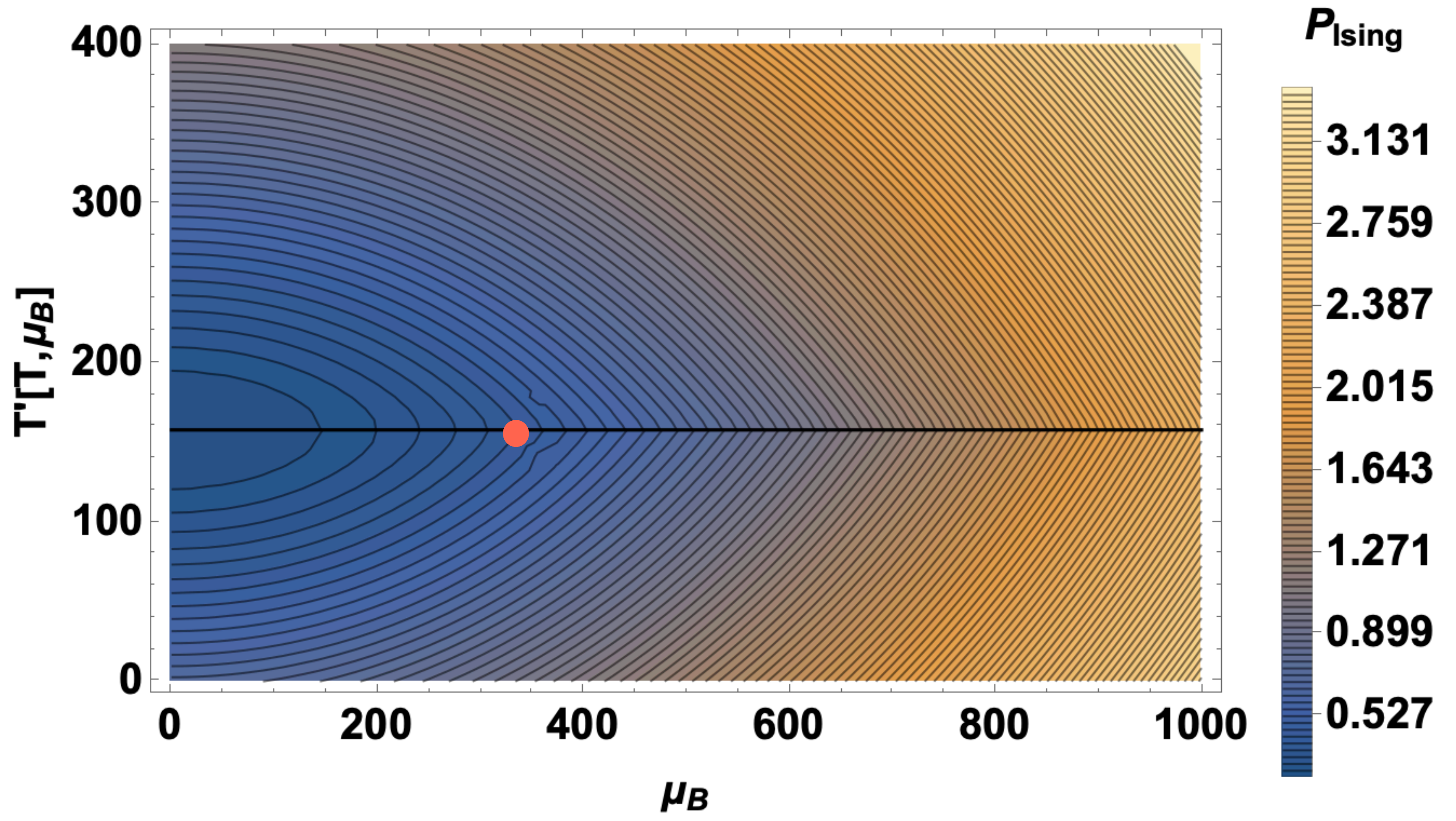


QCD - Co-ordinates

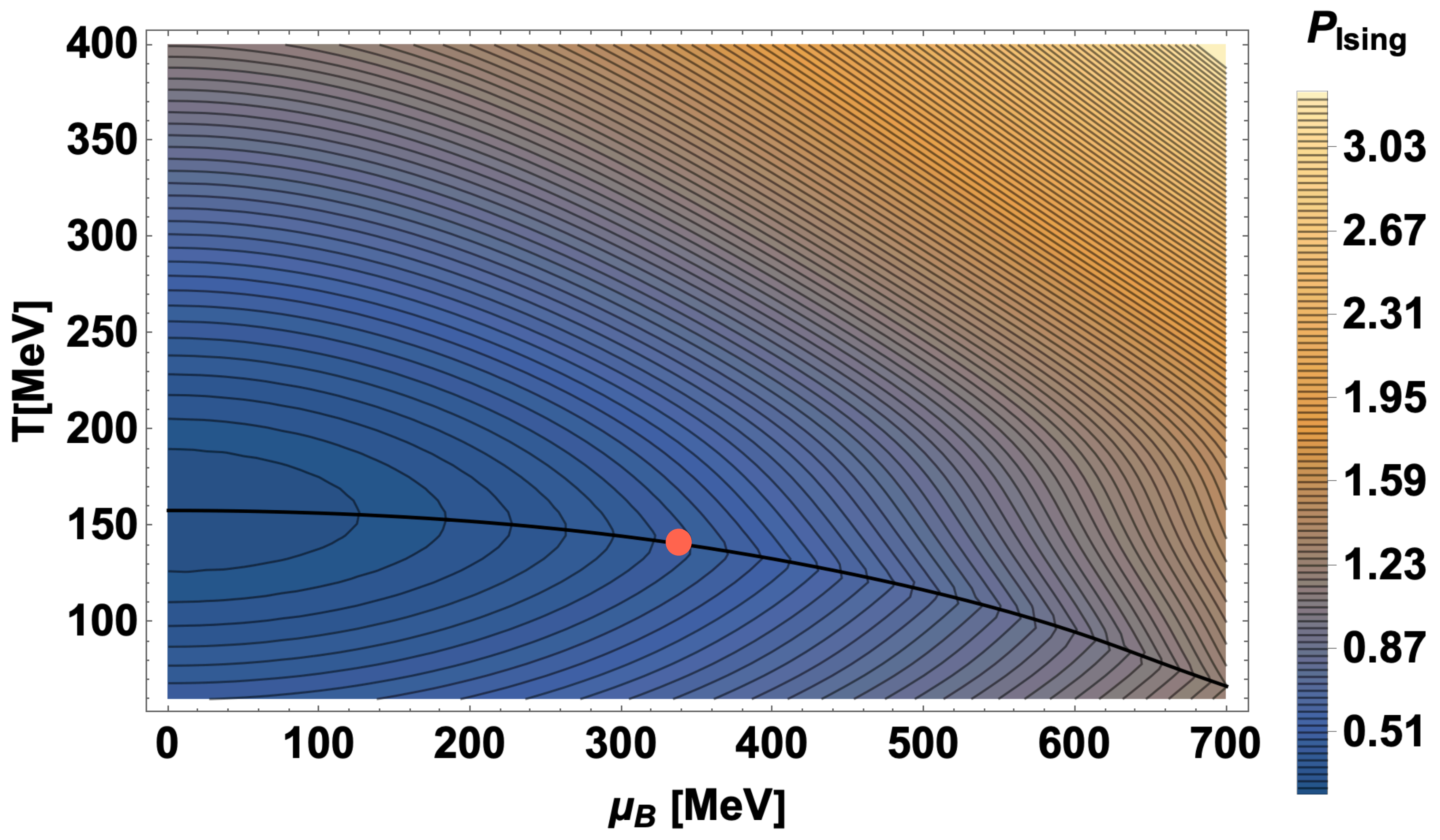
Ising Pressure



Ising Coordinates



(μ_B, T') **Coordinates**



QCD Coordinates

Parameters

$$w = 1,\ \rho = 2,\ \mu_{BC} = 350\ MeV,\ T_0 = 158\ [MeV]$$

$$T_C(\mu_B) = T_0\left[1 - \kappa_2(T_0)\left(\frac{\mu_B}{T_0}\right)^2\right]$$

Re-Constructing the Full Baryon Density

$$\frac{n_B^{full}(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi^B_{2,Lattice}(T'_{full}(T,\mu_B),0)$$

$$T'_{full}(T,\mu_B) = \underbrace{T'_{Lattice}(T,\mu_B)}_{\text{lowest order in } (\frac{\mu_B}{T})} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } (\frac{\mu_B}{T})}$$

Lattice Term

Ising Term

Introducing a Critical Point

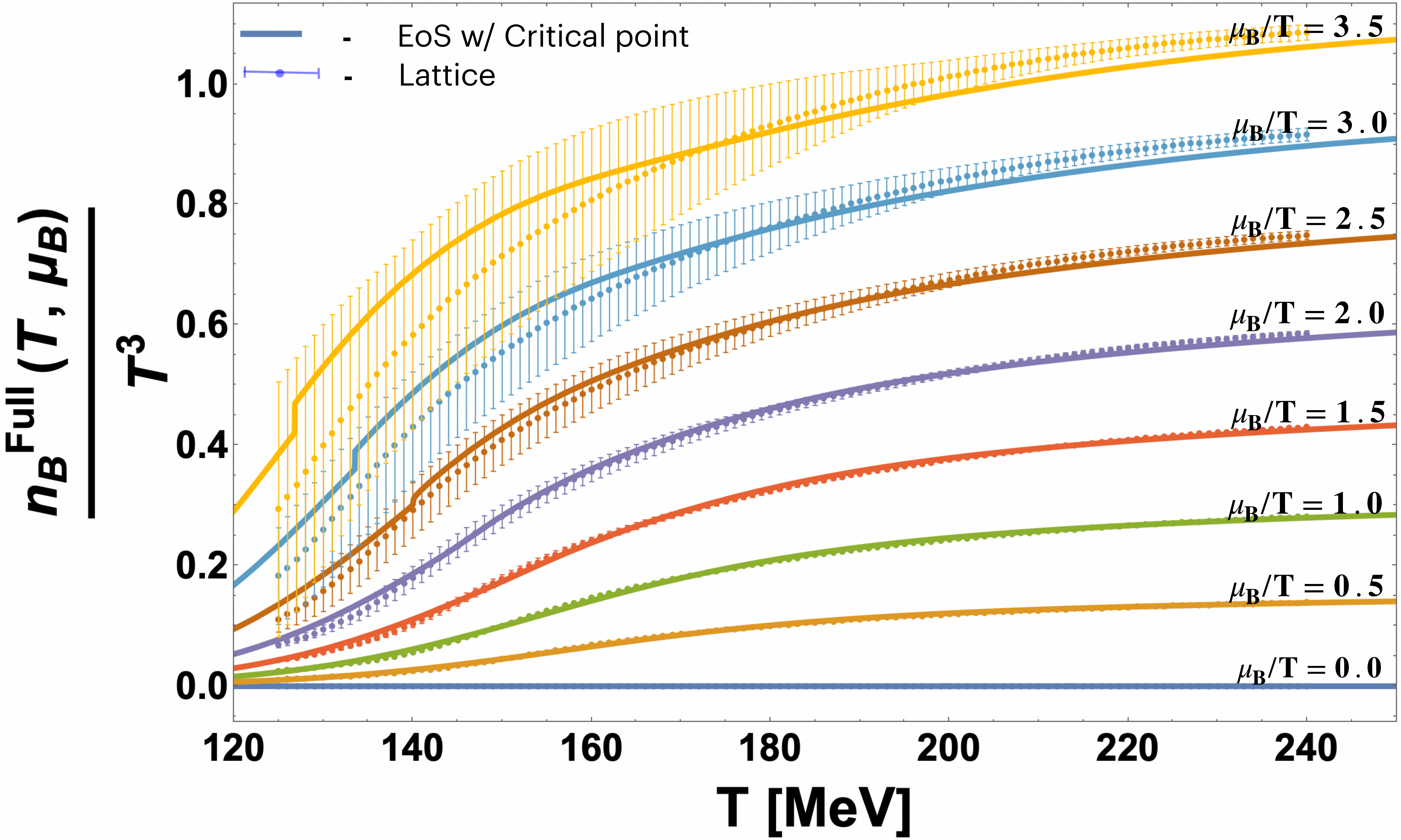
$$T'_{crit}(T,\mu_B) \approx T_0 + \left(\frac{\partial\chi^B_{2,lattice}(T,0)}{\partial T}\bigg|_{T=T_0}\right)^{-1}\frac{n_B^{crit}(T,\mu_B)}{T^3(\mu_B/T)} + \ldots$$

$$\chi_1^{crit}(T,\mu_B) = \frac{n_B^{crit}(T,\mu_B)}{T^3} = \frac{\partial(P^{crit}(T,\mu_B)/T^4)}{\partial(\mu_B/T)}$$

Baryon density results

Full Baryon Density at a constant $\frac{\mu_B}{T}$ compared with Lattice

$\mu_B = 350$ [MeV], $\alpha_{12} = 90$, $\rho = 2$, $w = 2$



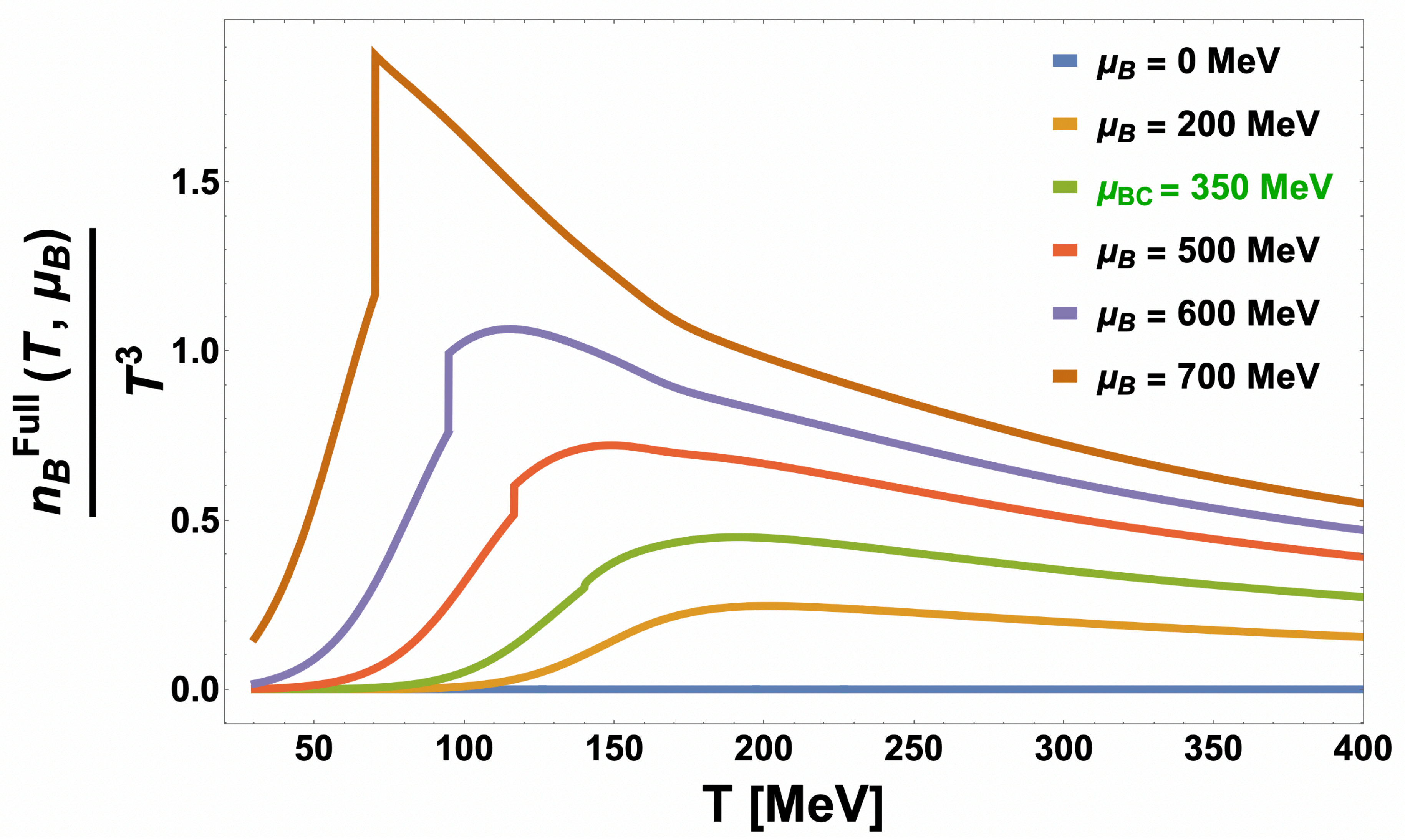
[Borsányi, S. et al. PRL (2021)]

Baryon density results

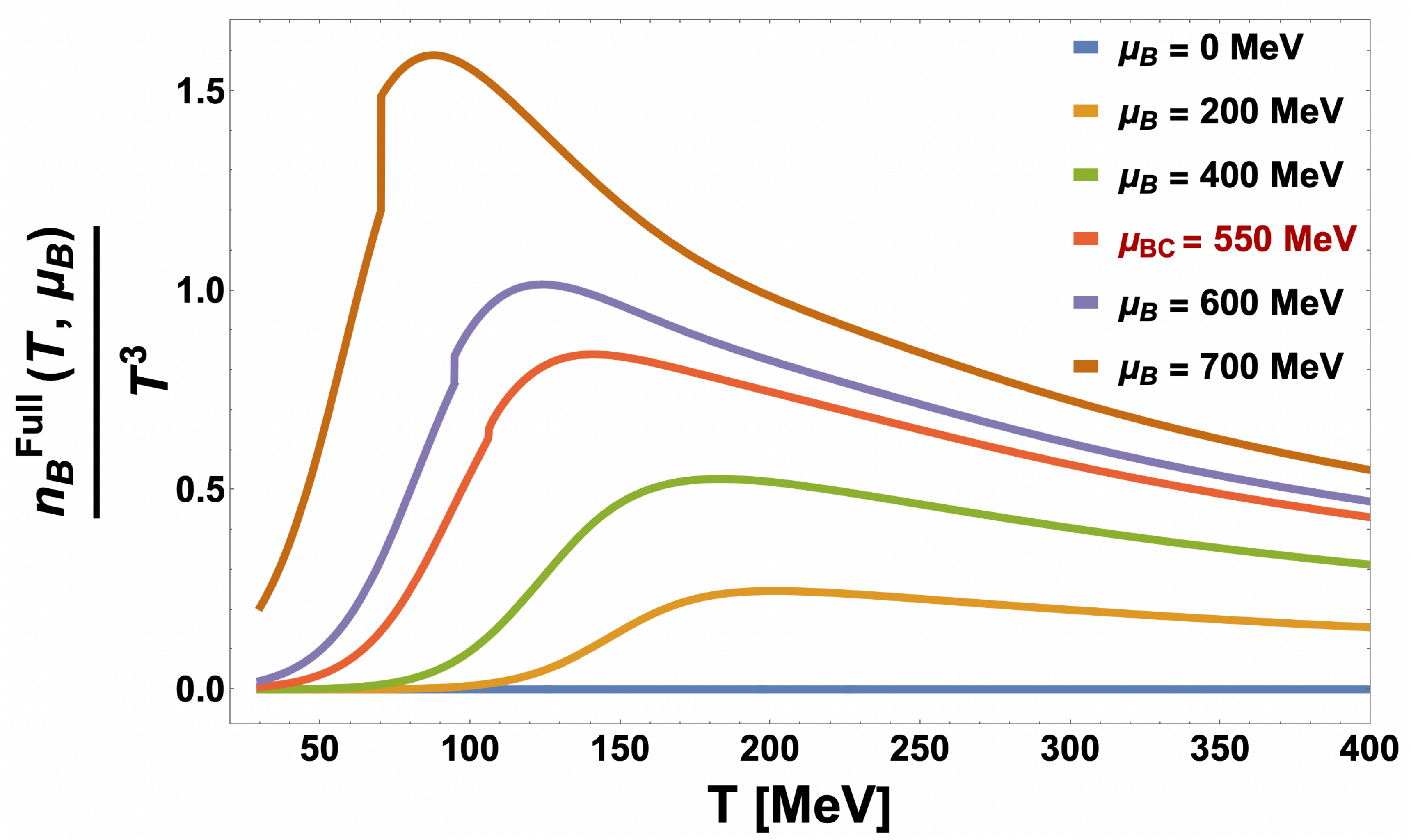
Baryon Density at a constant μ_B for different μ_{BC}

$\alpha_{12} = 90, \quad \rho = 2, \quad w = 2$

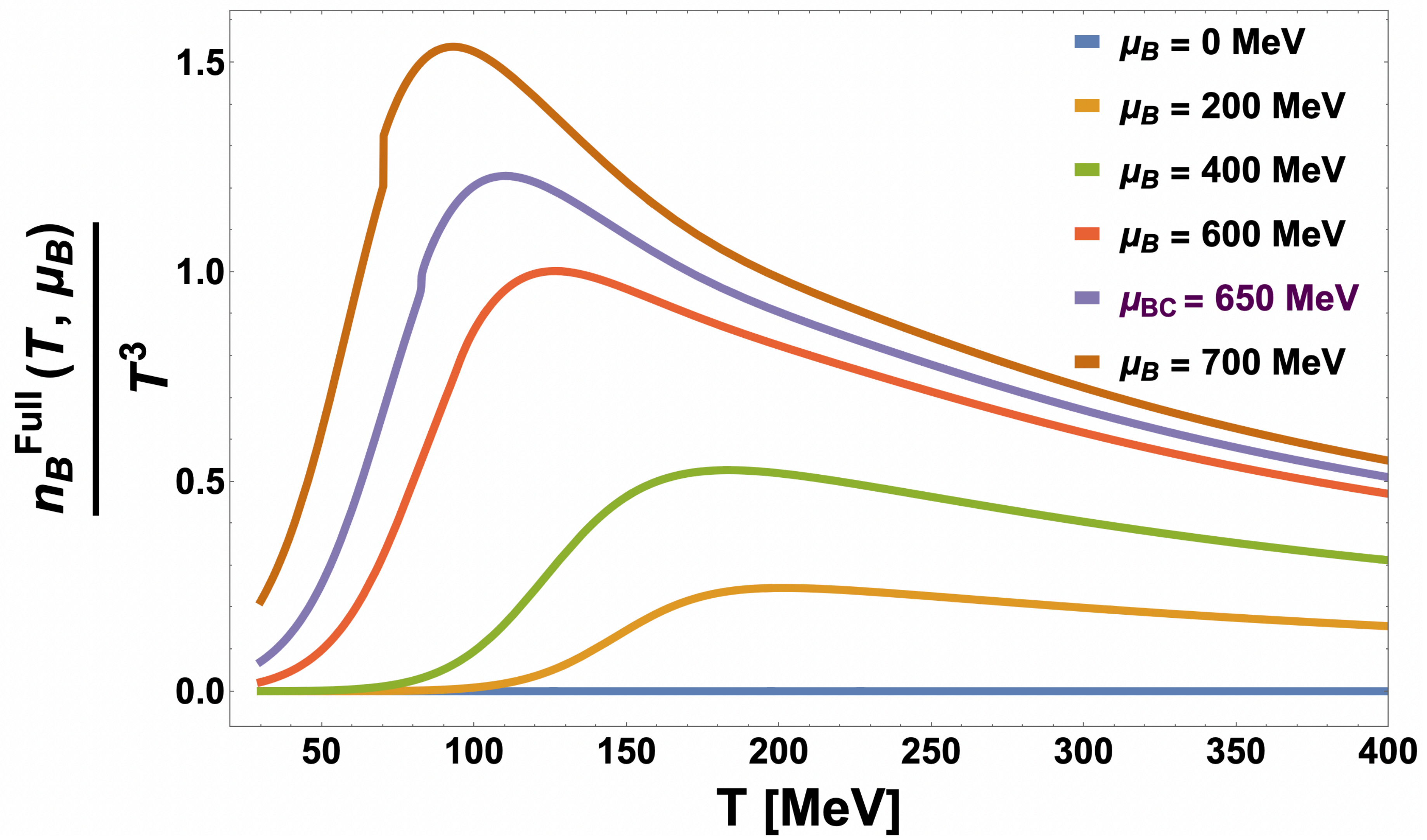
$\mu_{BC} = 350 \quad MeV$



$\mu_{BC} = 550 \quad MeV$



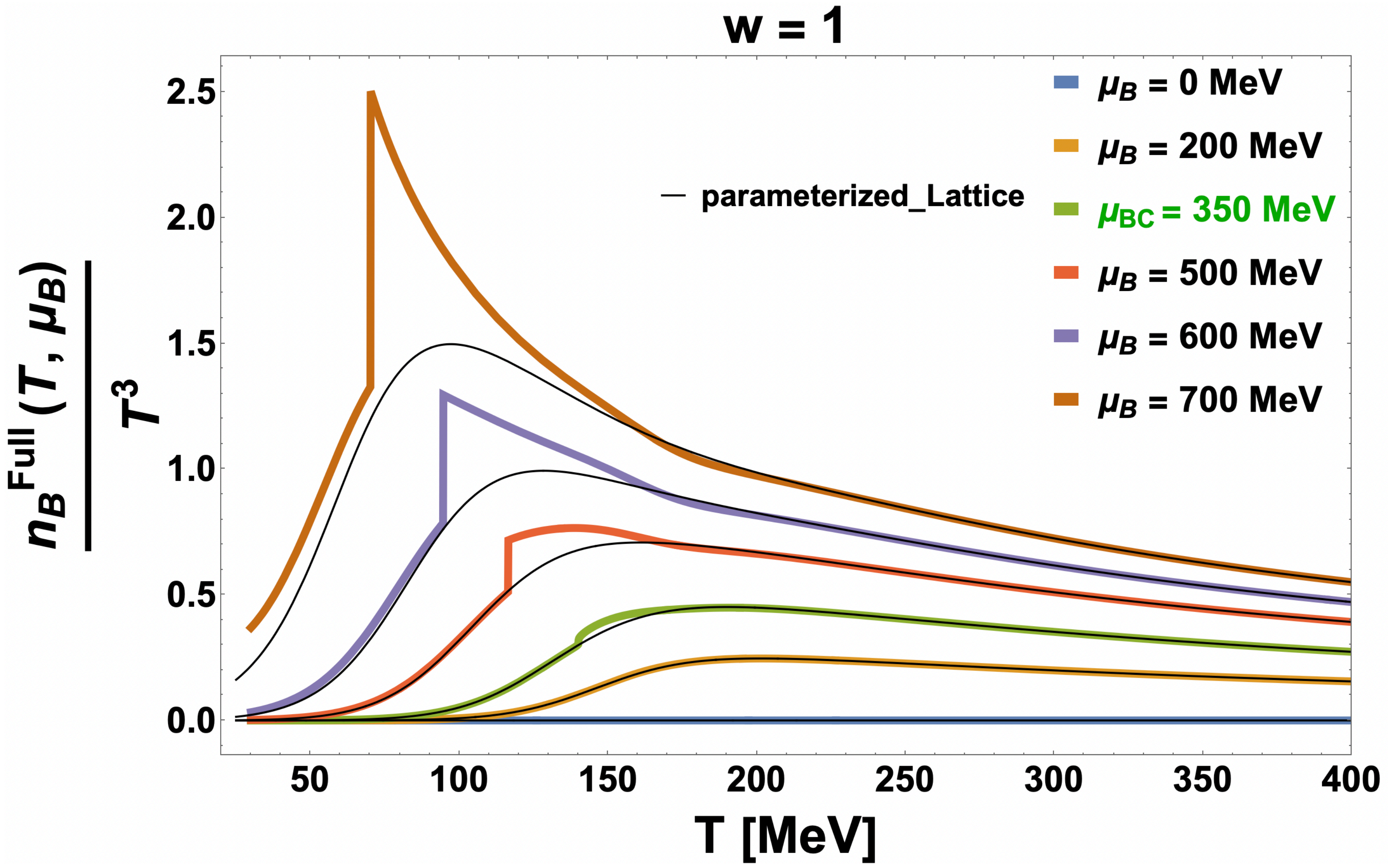
$\mu_{BC} = 650 \quad MeV$

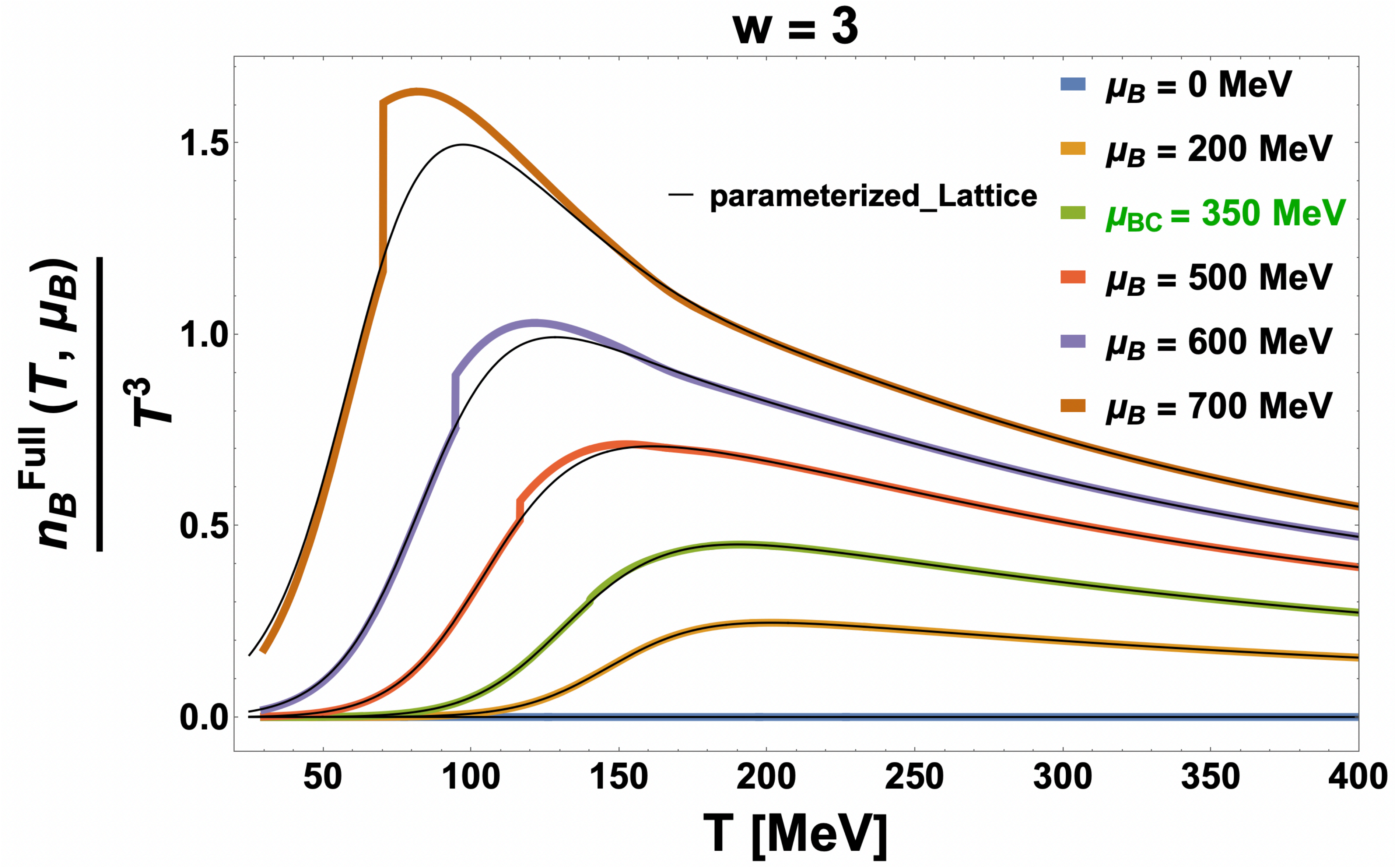


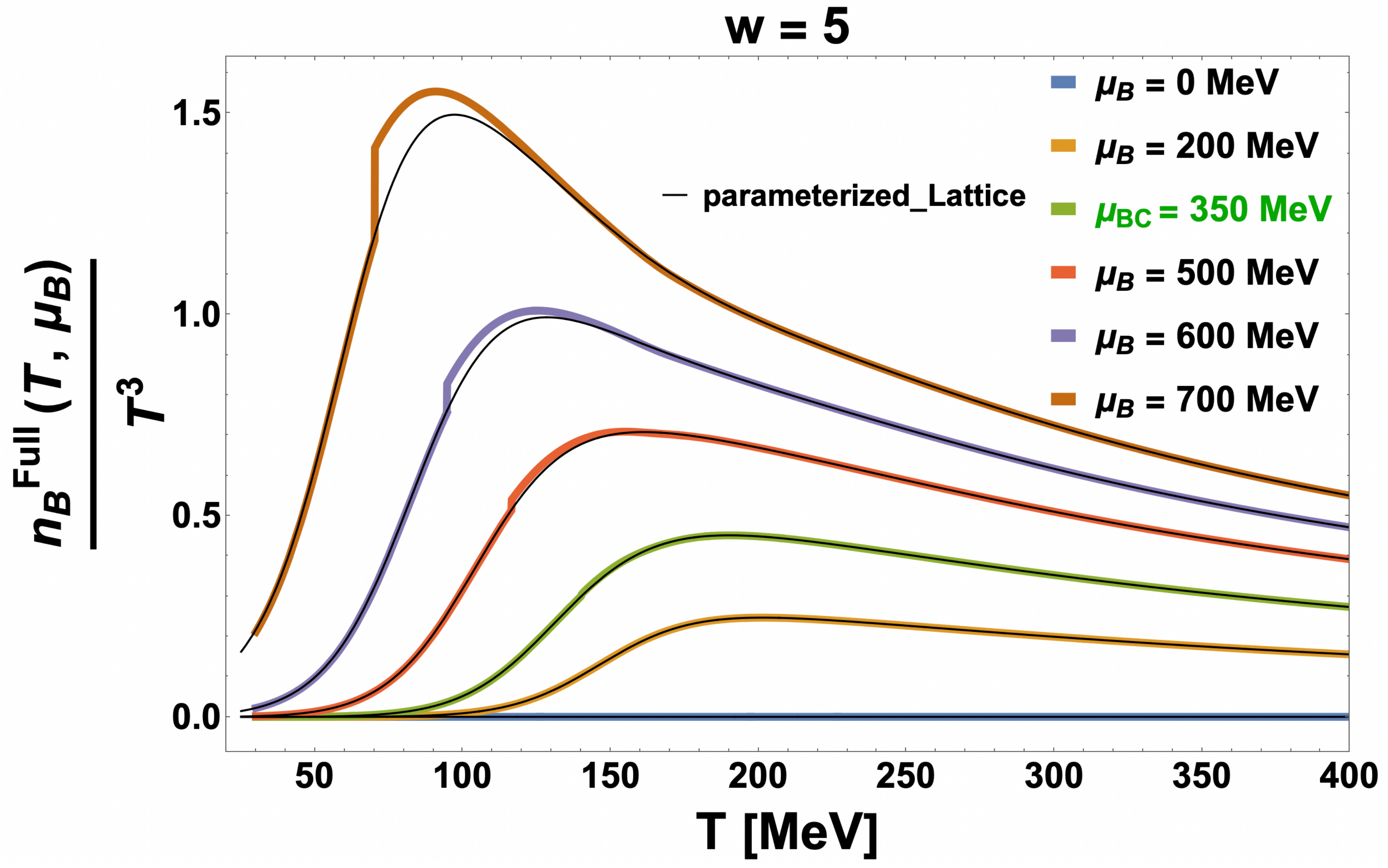
Estimating the Critical contribution

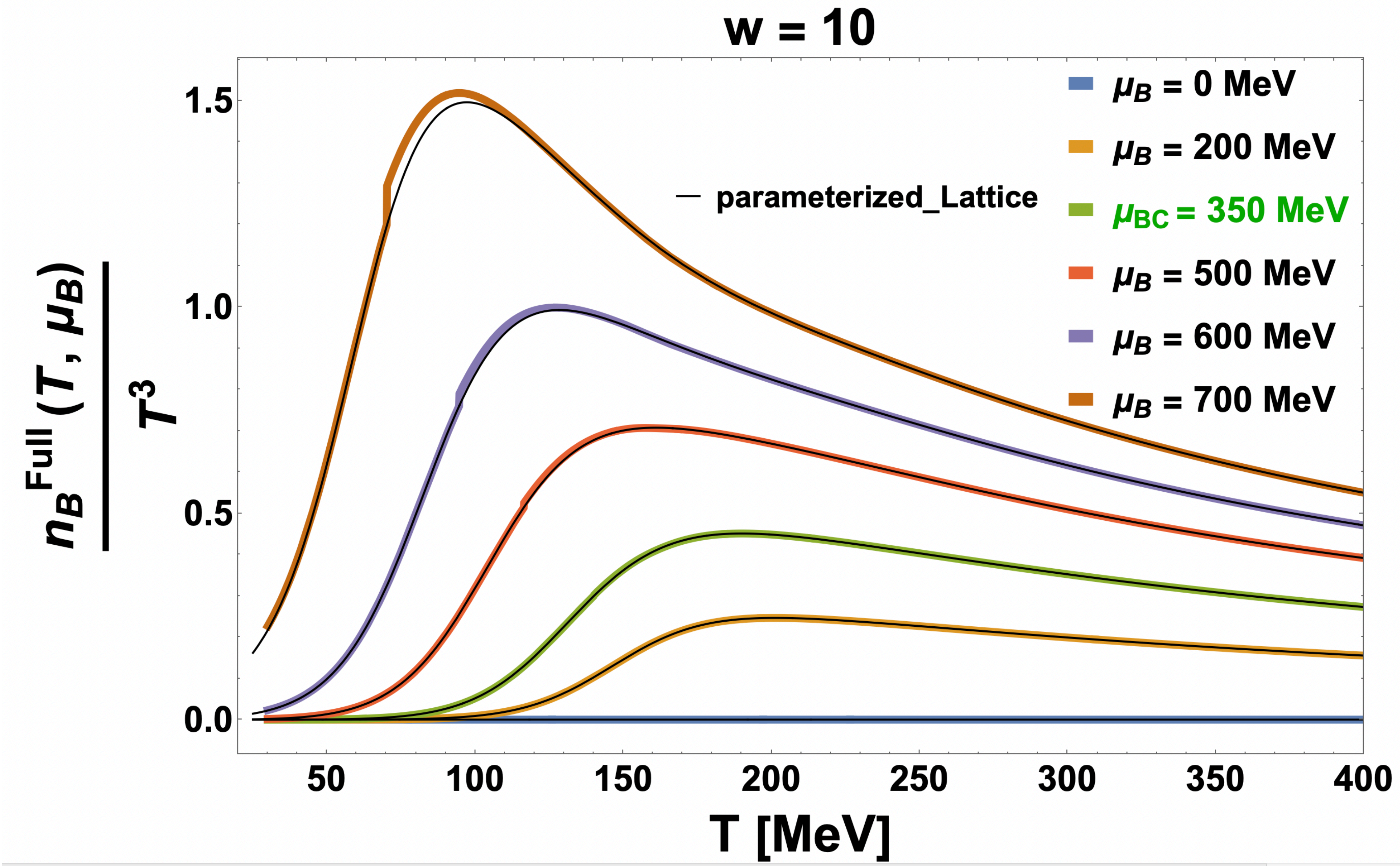
Baryon Density at a constant μ_B for 1 to w=10

$\mu_{\mathrm{BC}} = 350\ [\mathrm{MeV}],\ \ \alpha_{12} = 90,\ \ \rho = 2$









Thermodynamic Relations

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \int_0^{\mu_B} d\hat{\mu}'_B \frac{n_B(T,\hat{\mu}'_B)}{T^3}$$

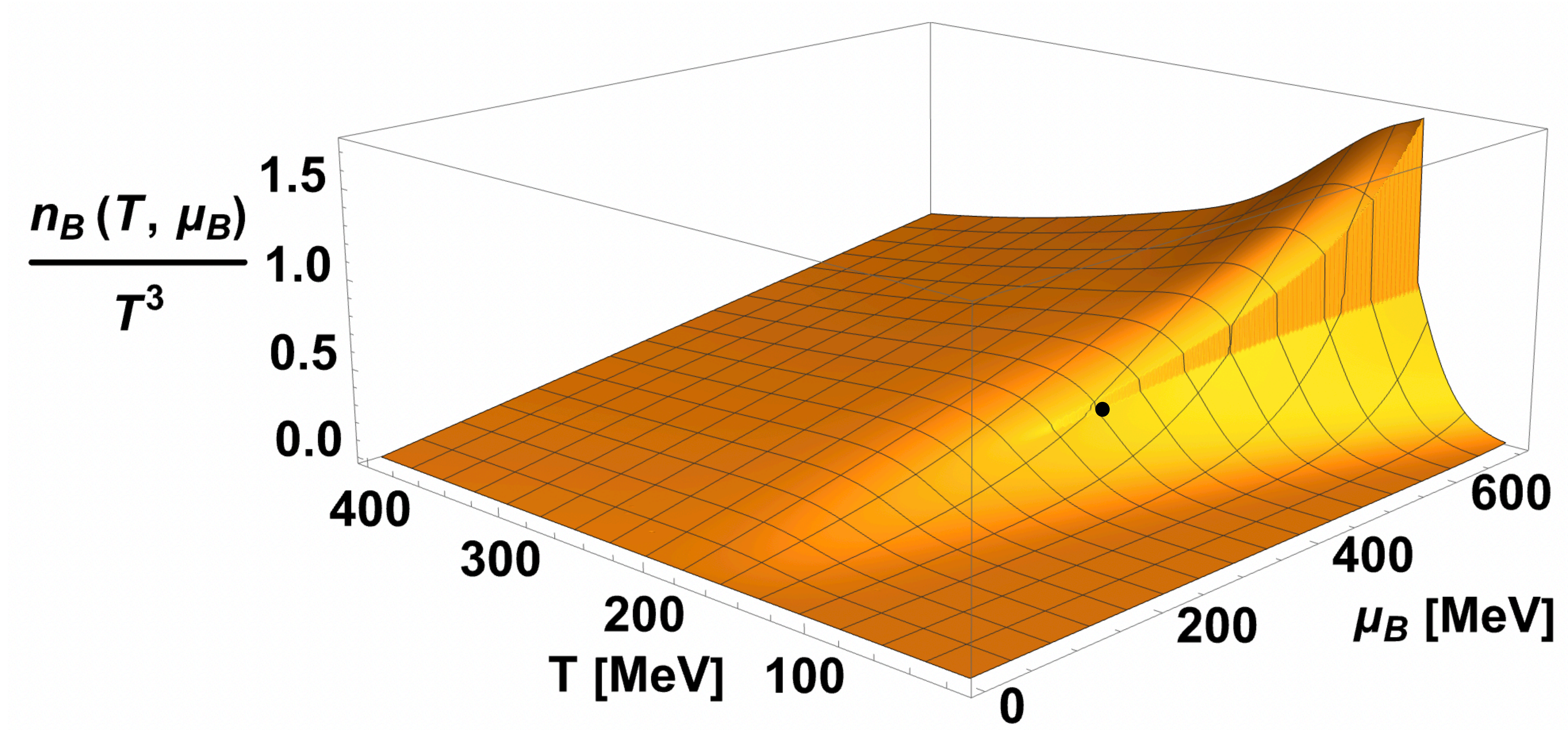
$$\frac{S(T,\mu_B)}{T^3} = \frac{1}{T^3}\left(\frac{\partial P}{\partial T}\right)\Bigg|_{\mu_B}$$

$$\frac{\epsilon(T,\mu_B)}{T^3} = \frac{S}{T^3} - \frac{P}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3}$$

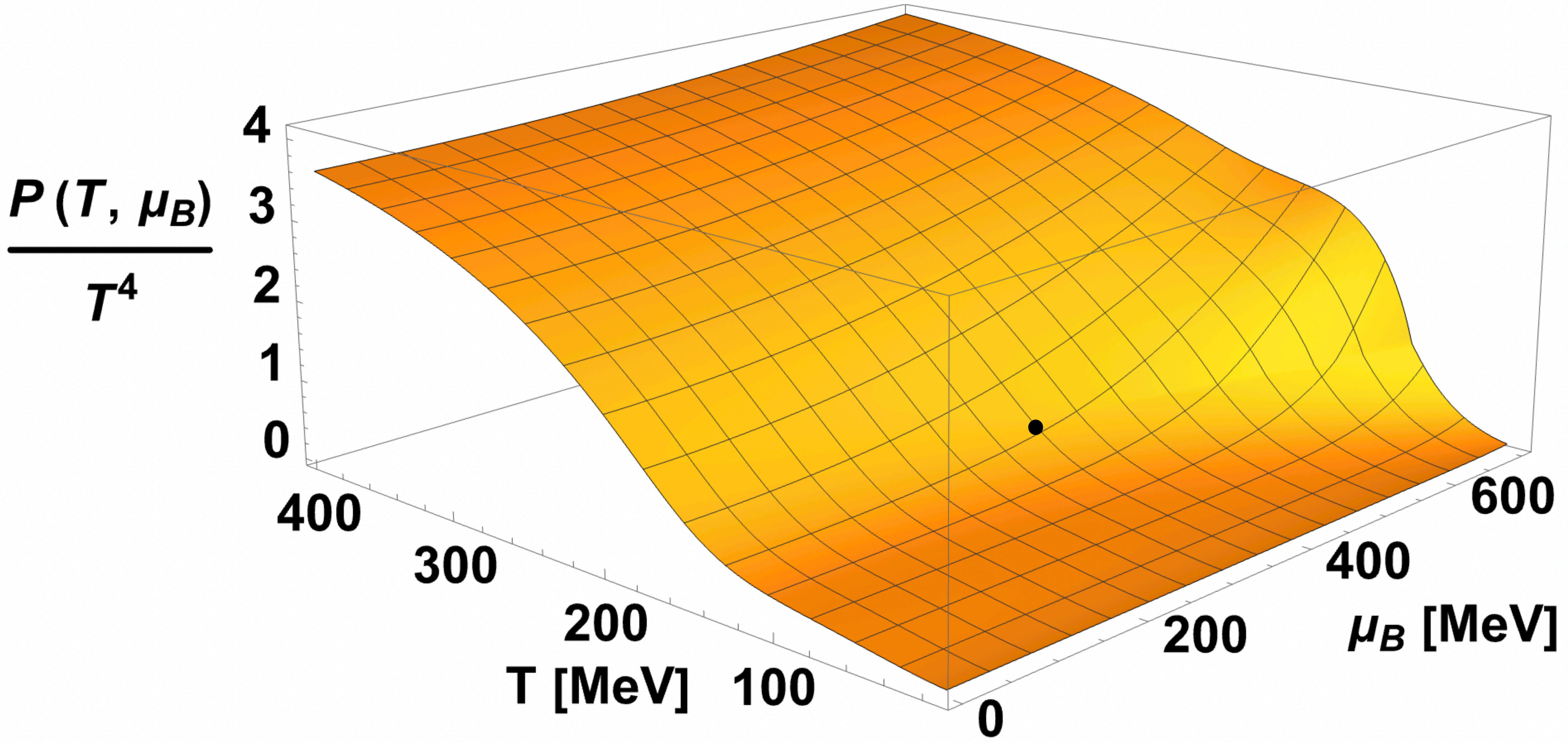
$$\frac{\chi_2(T,\mu_B)}{T^2} = T\left(\frac{\partial(n_B/T^3)}{\partial \mu_B}\right)\Bigg|_{\mu_B}$$

Thermodynamic observables

Baryon Density



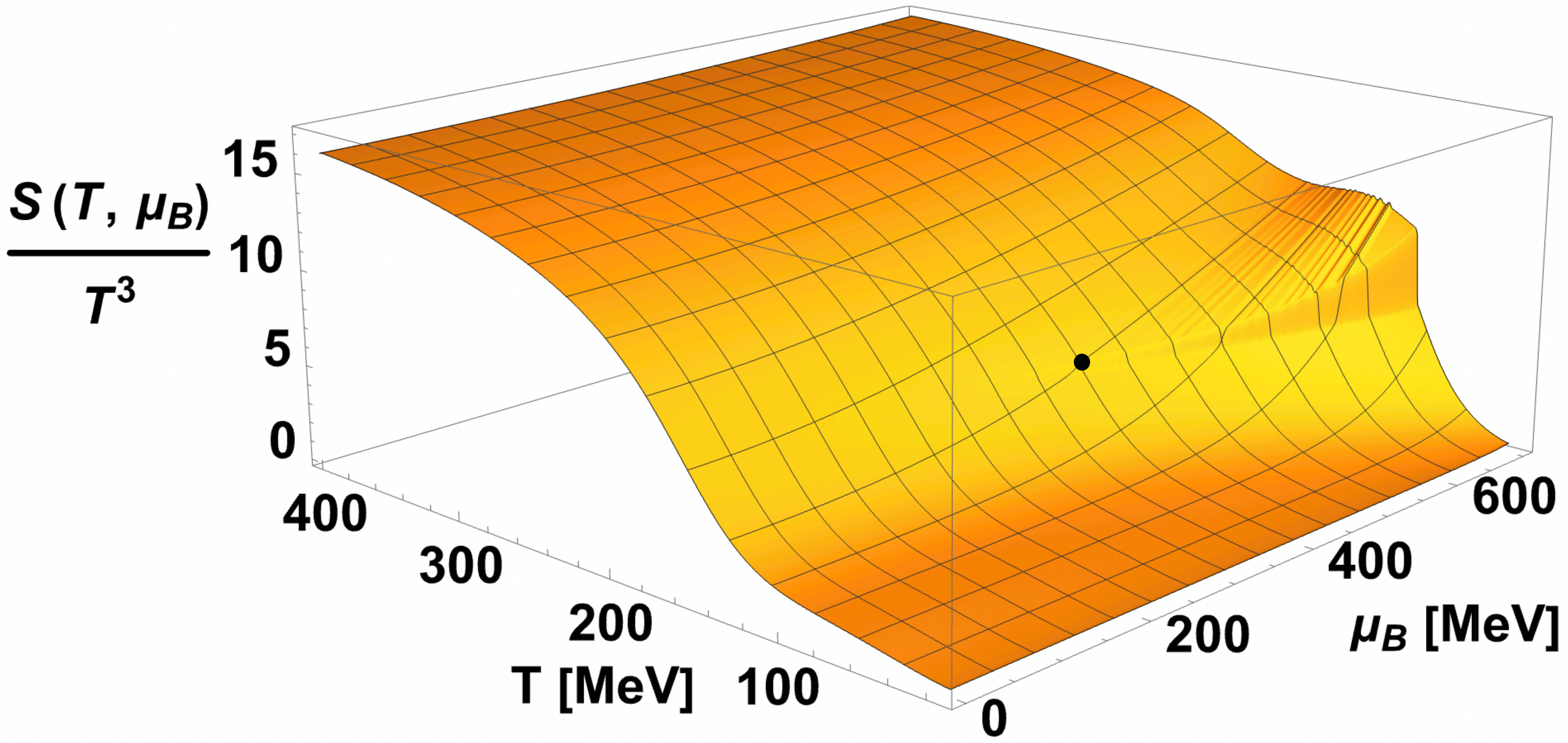
Pressure

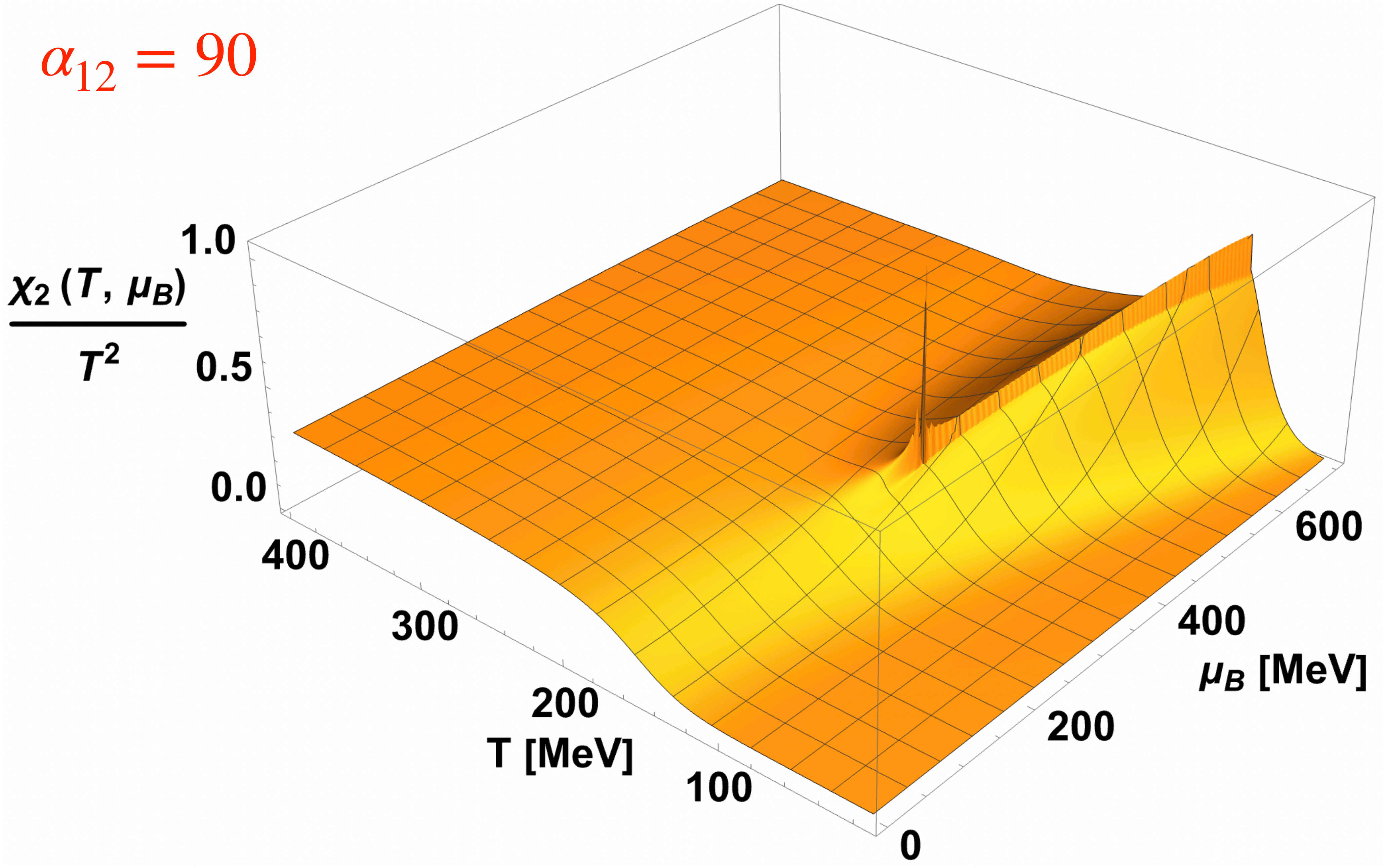


● **Critical Point**

$w = 2 \;,\; \rho = 2 \;\&\; \alpha_{12} = 90$

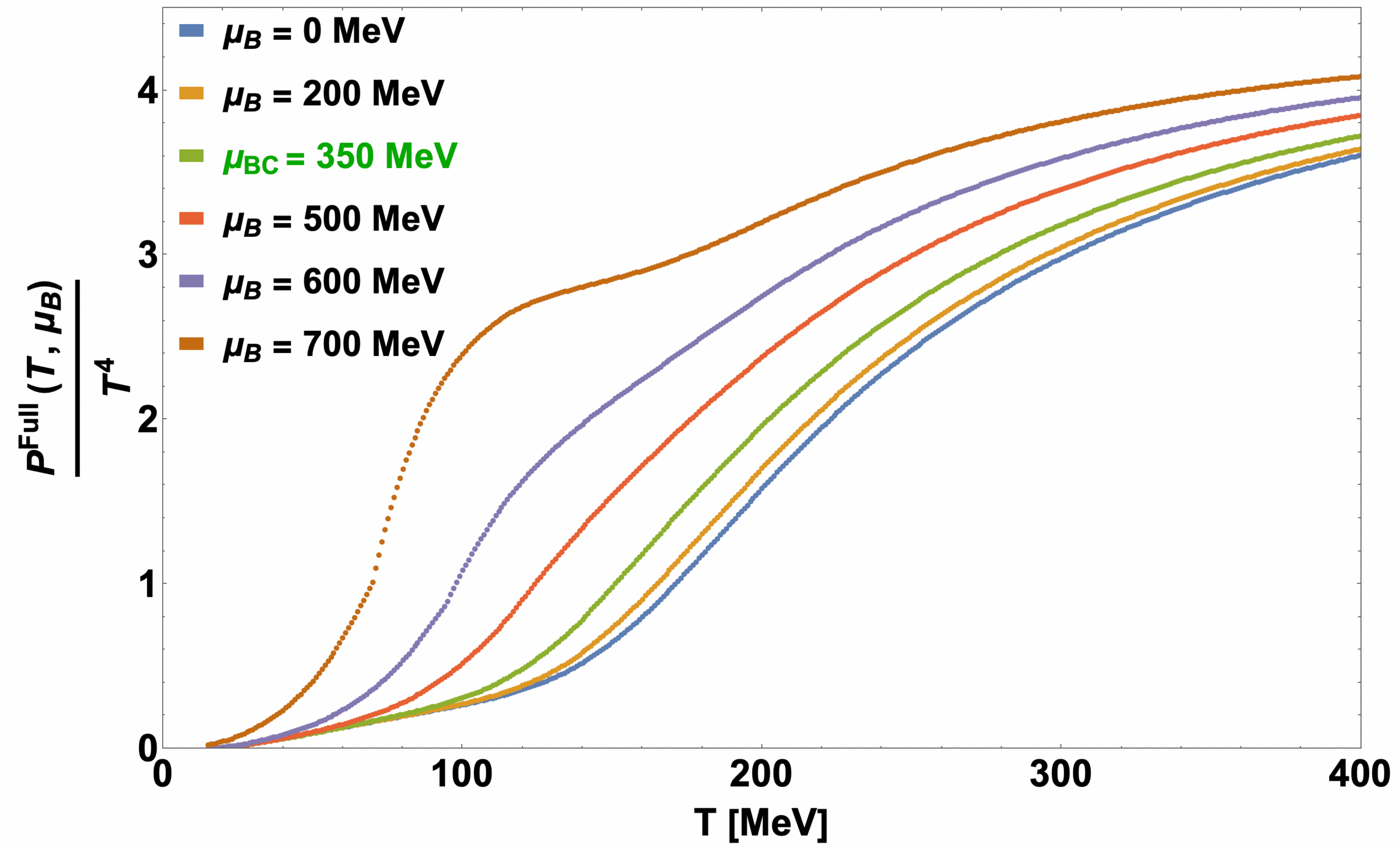
Entropy Density



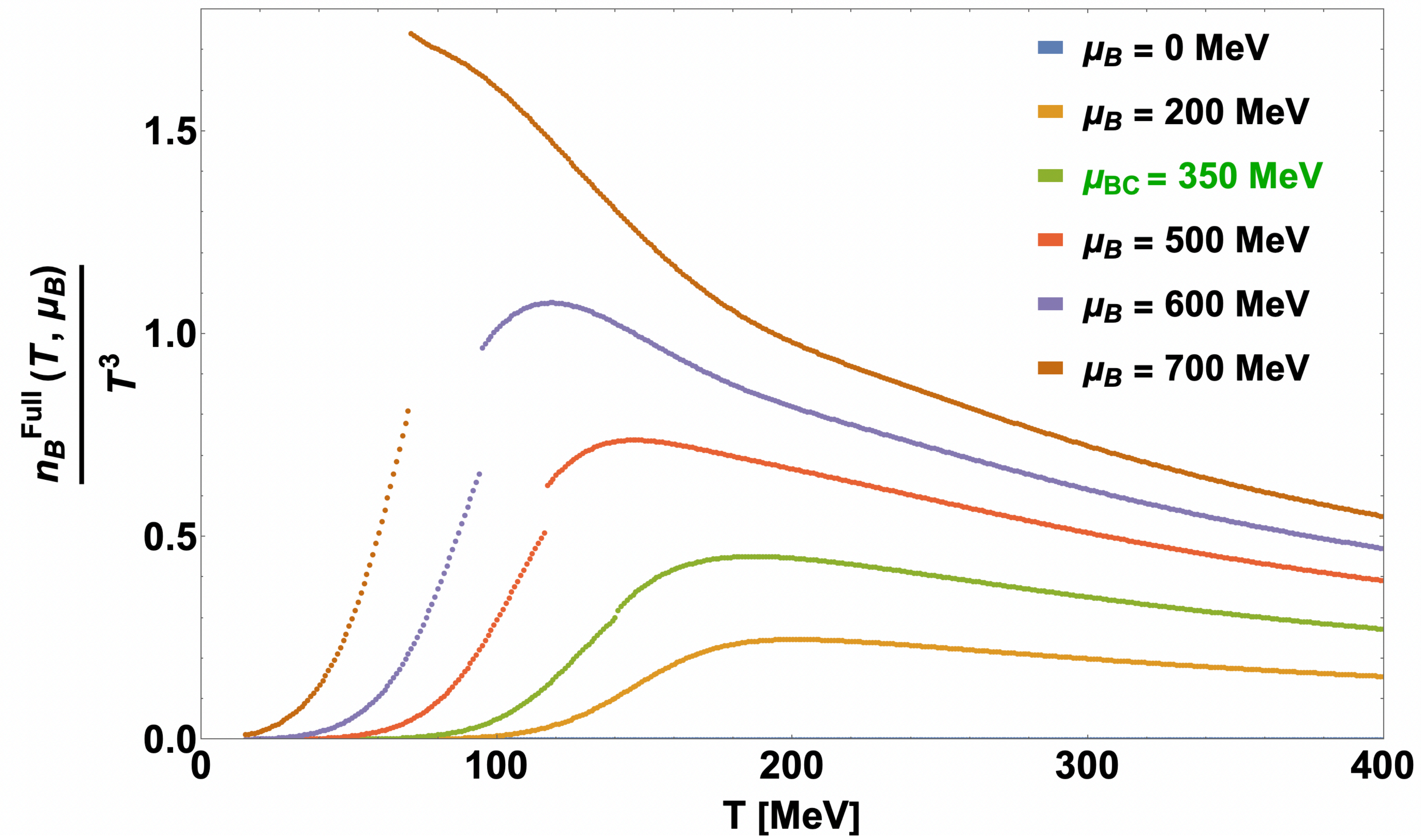


Thermodynamic observables

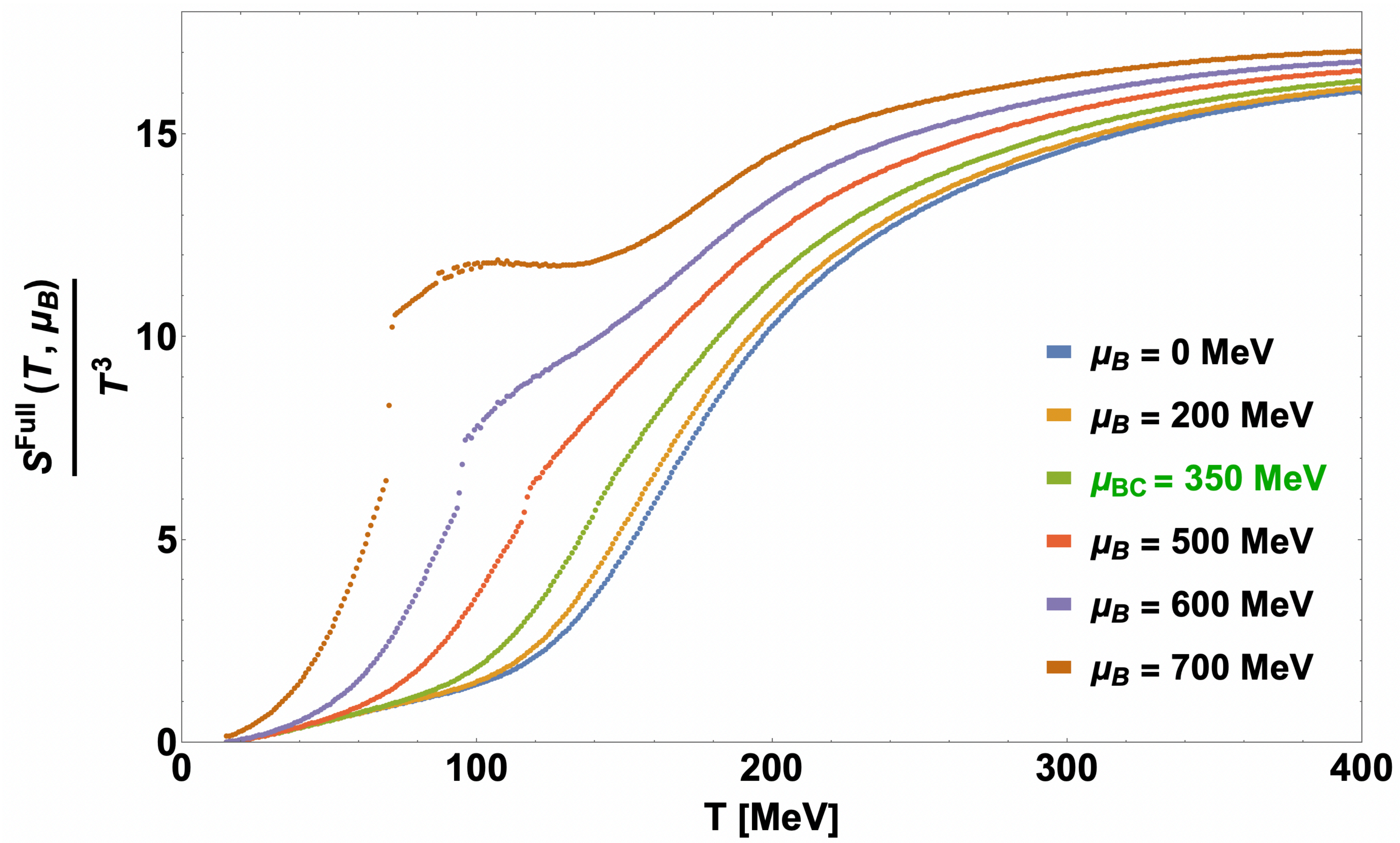
Pressure



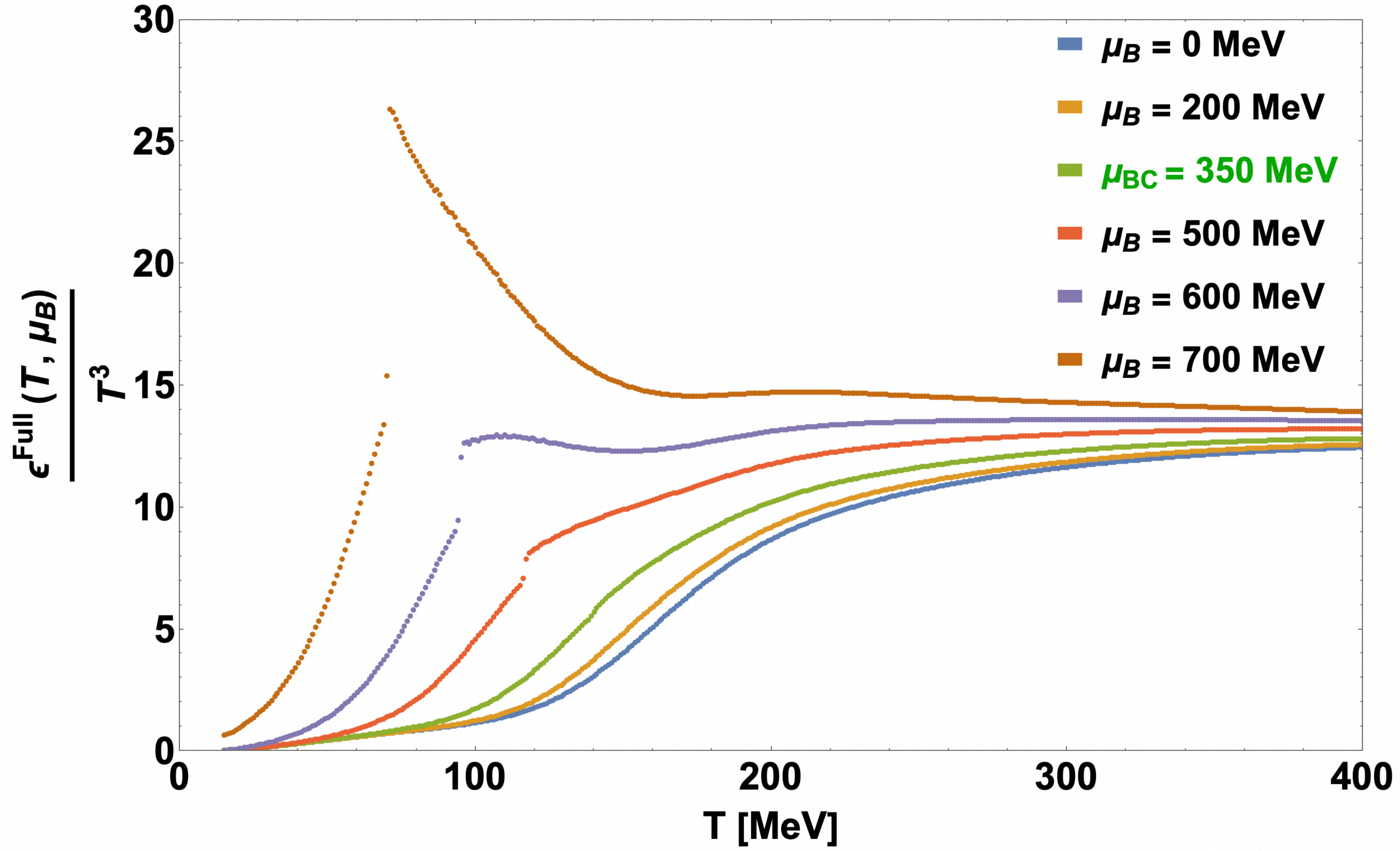
Baryon density



Entropy density

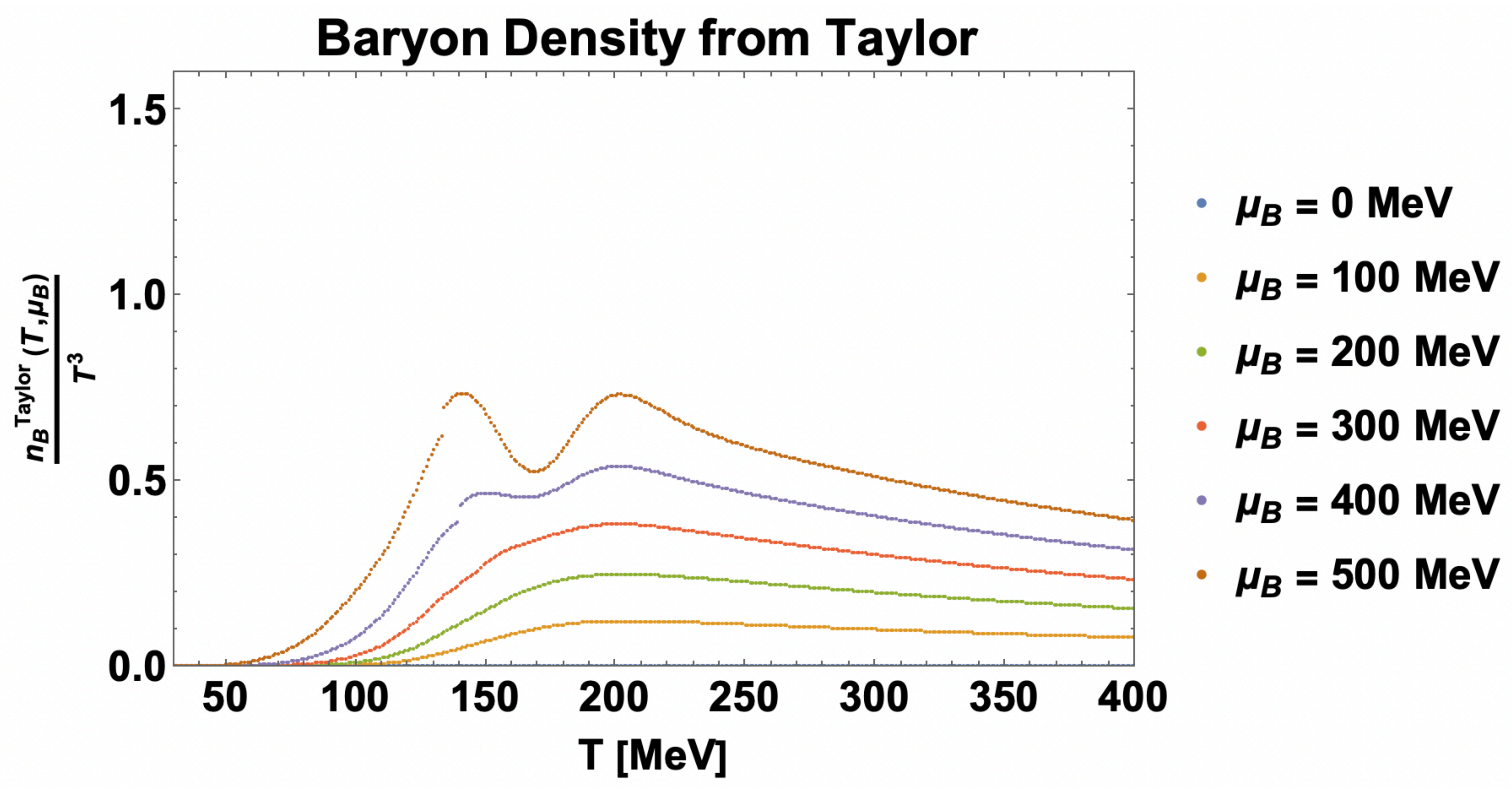


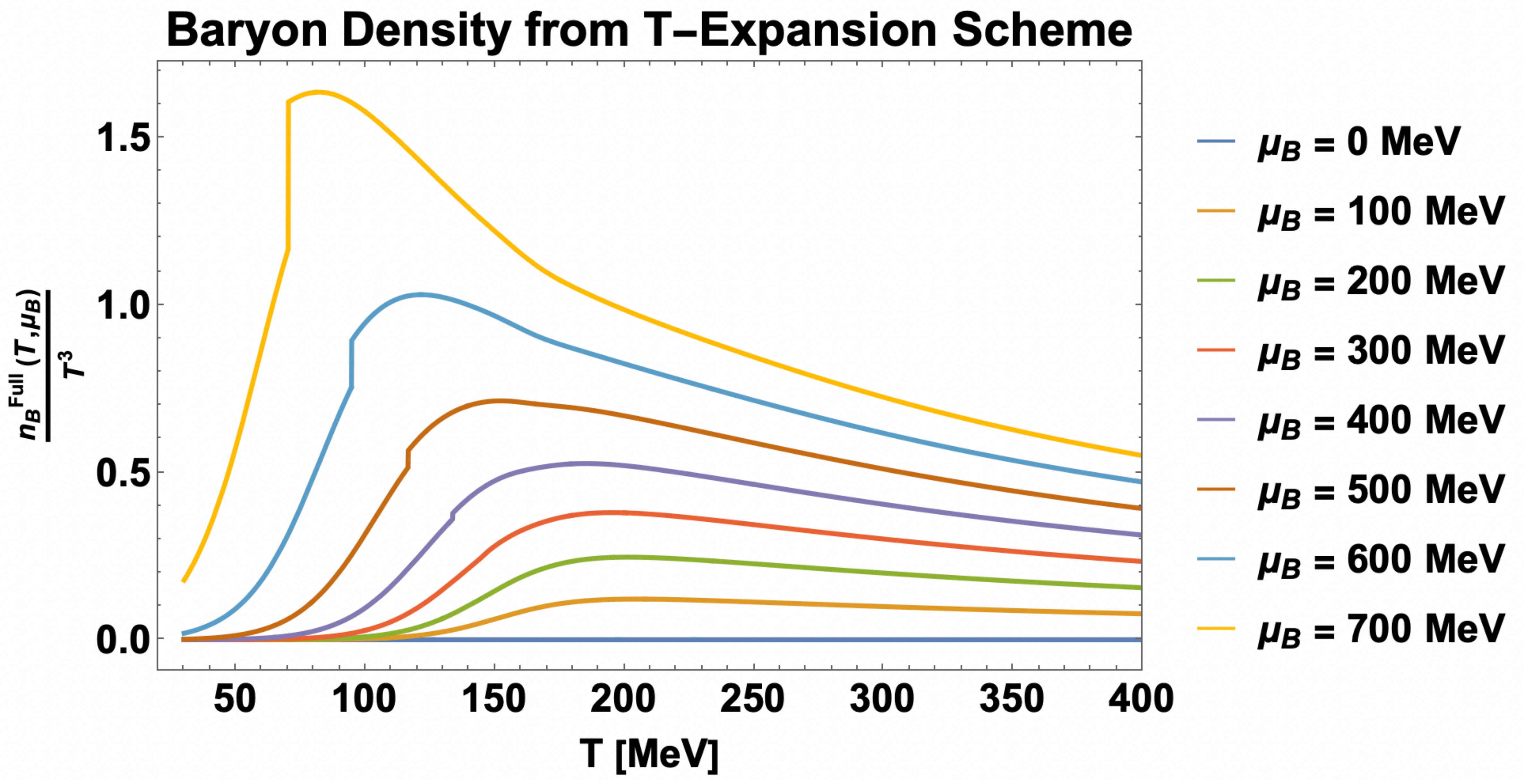
Energy density



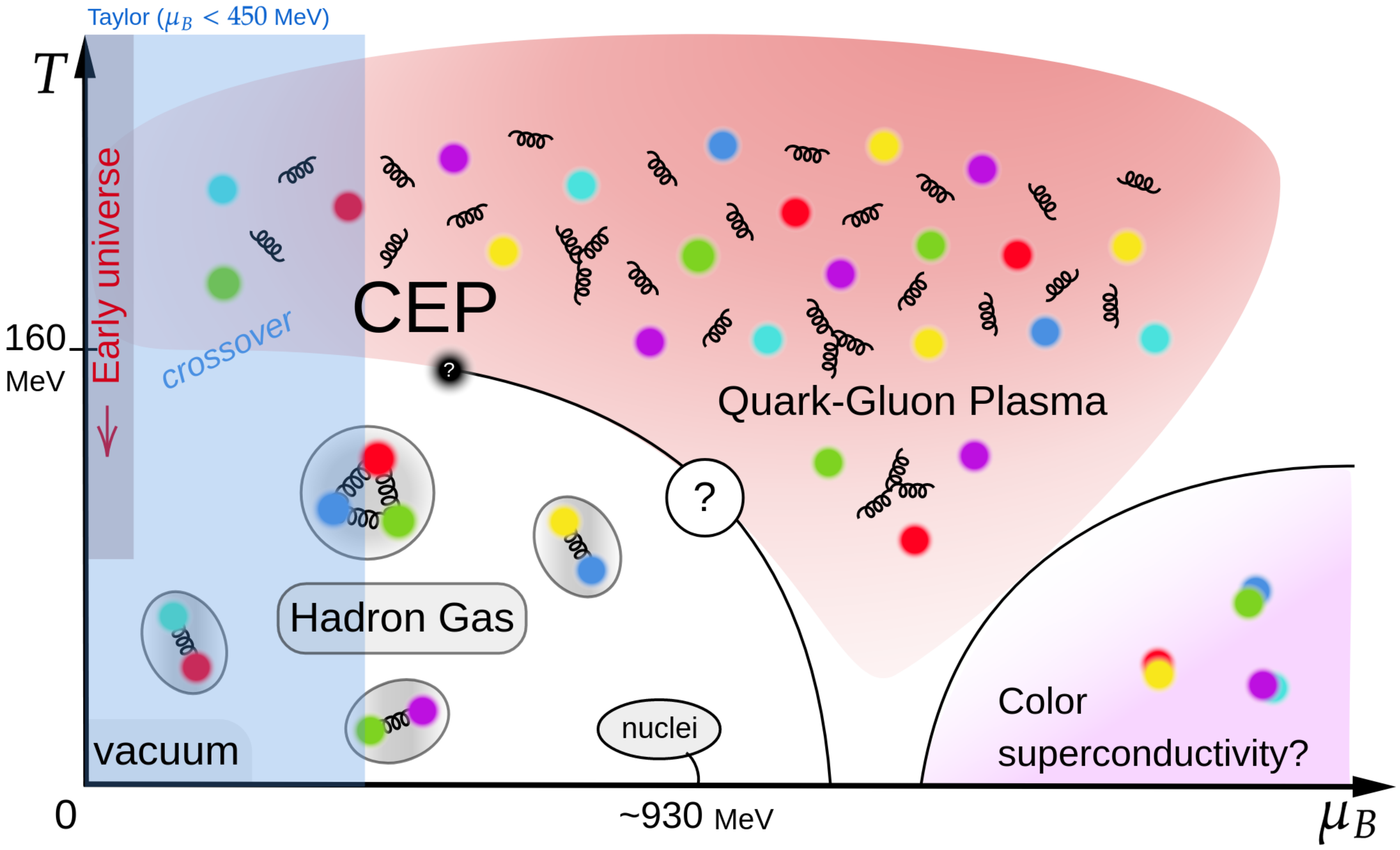
Summary

$\mu_B = 350$ [MeV], $\alpha_{12} = 90$, $w = 3$, $\rho = 2$

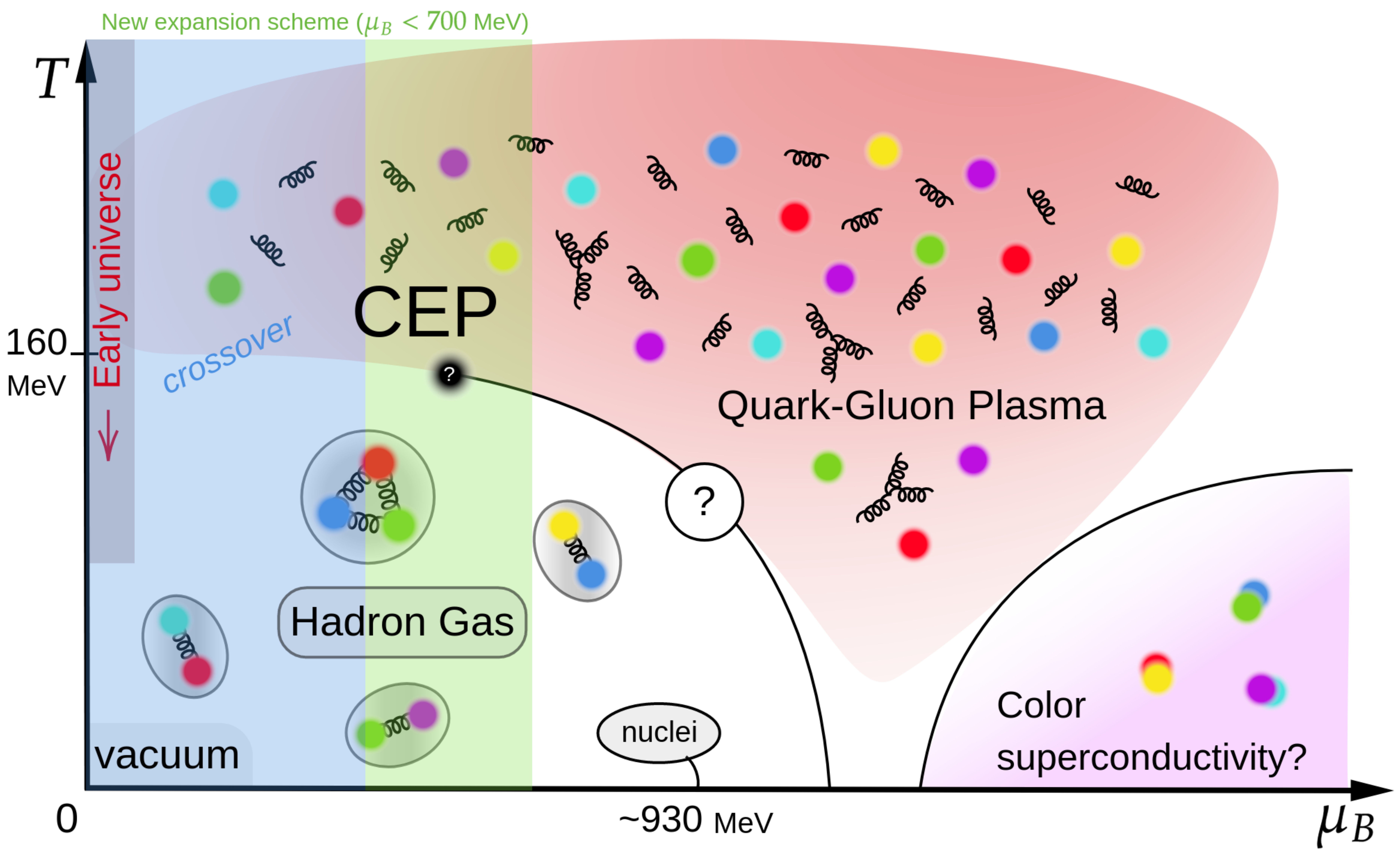




- **A more physical EoS that captures a large part of the Phase diagram is required.**
- **We provide a family of EoS with a correct Critical point up** $\mu_B = 700\ MeV$.
- **Our EoS allows users to change parameters and compare with the data from the Experiment (Beam Energy Scan II)**



Taylor (μ_B < 450 MeV)
T
160 MeV
Early universe
crossover
CEP
?
Quark-Gluon Plasma
?
Hadron Gas
nuclei
vacuum
Color superconductivity?
0
~930 MeV
μ_B


New expansion scheme (μ_B < 700 MeV)
T
Early universe
160
MeV
crossover
CEP
?
Quark-Gluon Plasma
?
Hadron Gas
vacuum
nuclei
Color
superconductivity?
0
~930 MeV
μ_B

Future Work

- **Explore and constrain the Parameters space by requesting thermodynamics Stability and causality of our EoS**

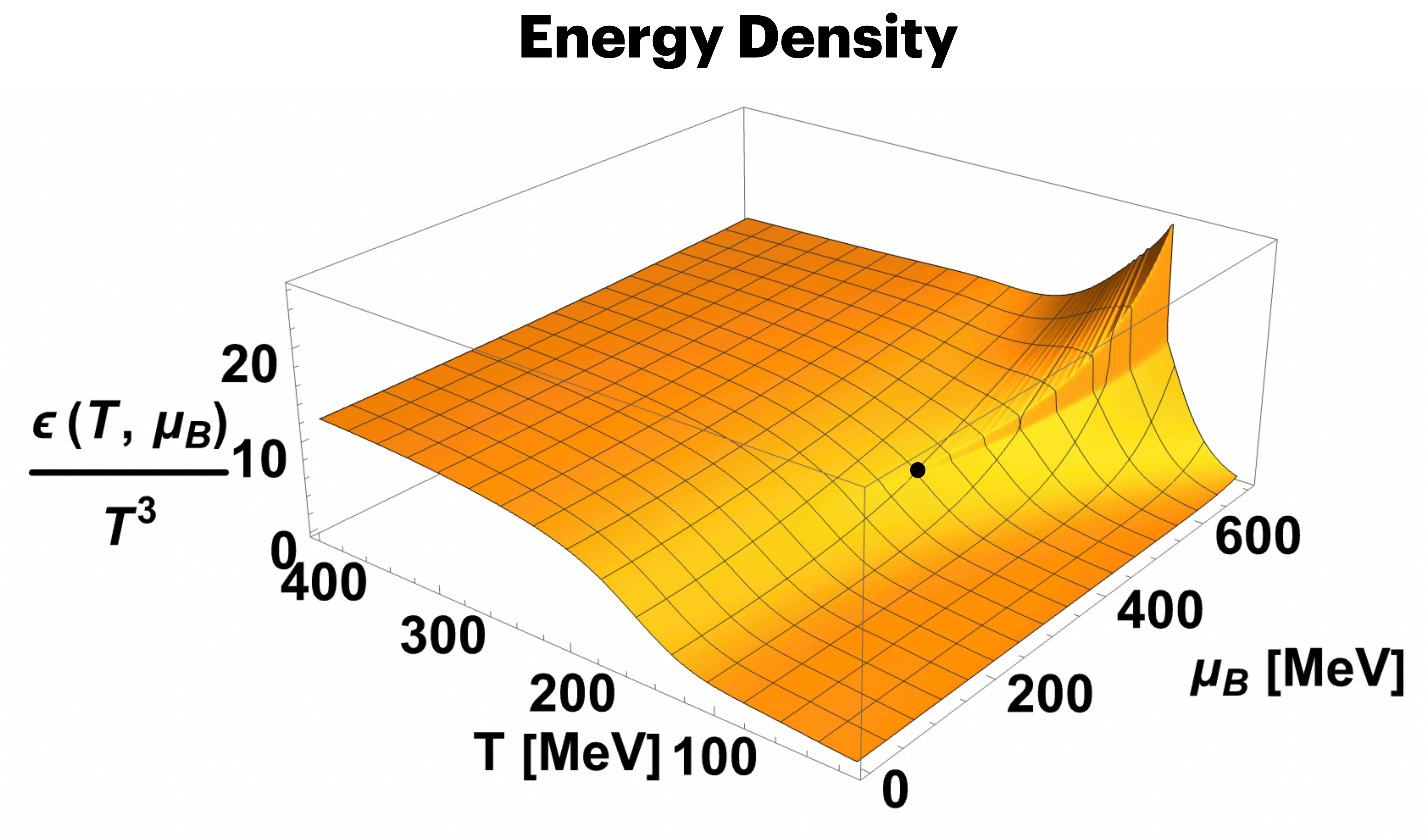
$$P(T,\mu_B) > 0, \quad n_B(T,\mu_B) > 0, \quad s(T,\mu_B) > 0 \quad \epsilon(T,\mu_B) > 0 \quad c_s^2(T,\mu_B) > 0$$

$$c_s^2(T,\mu_B) < 1$$

- **Merge with Nuclear Matter EoS at low Temperatures**

Thank you for your attention!

Back up!


Energy Density
$\frac{\epsilon(T, \mu_B)}{T^3}$
20
10
0
400
300
200
T [MeV]
100
600
400
200
μ_B [MeV]
0