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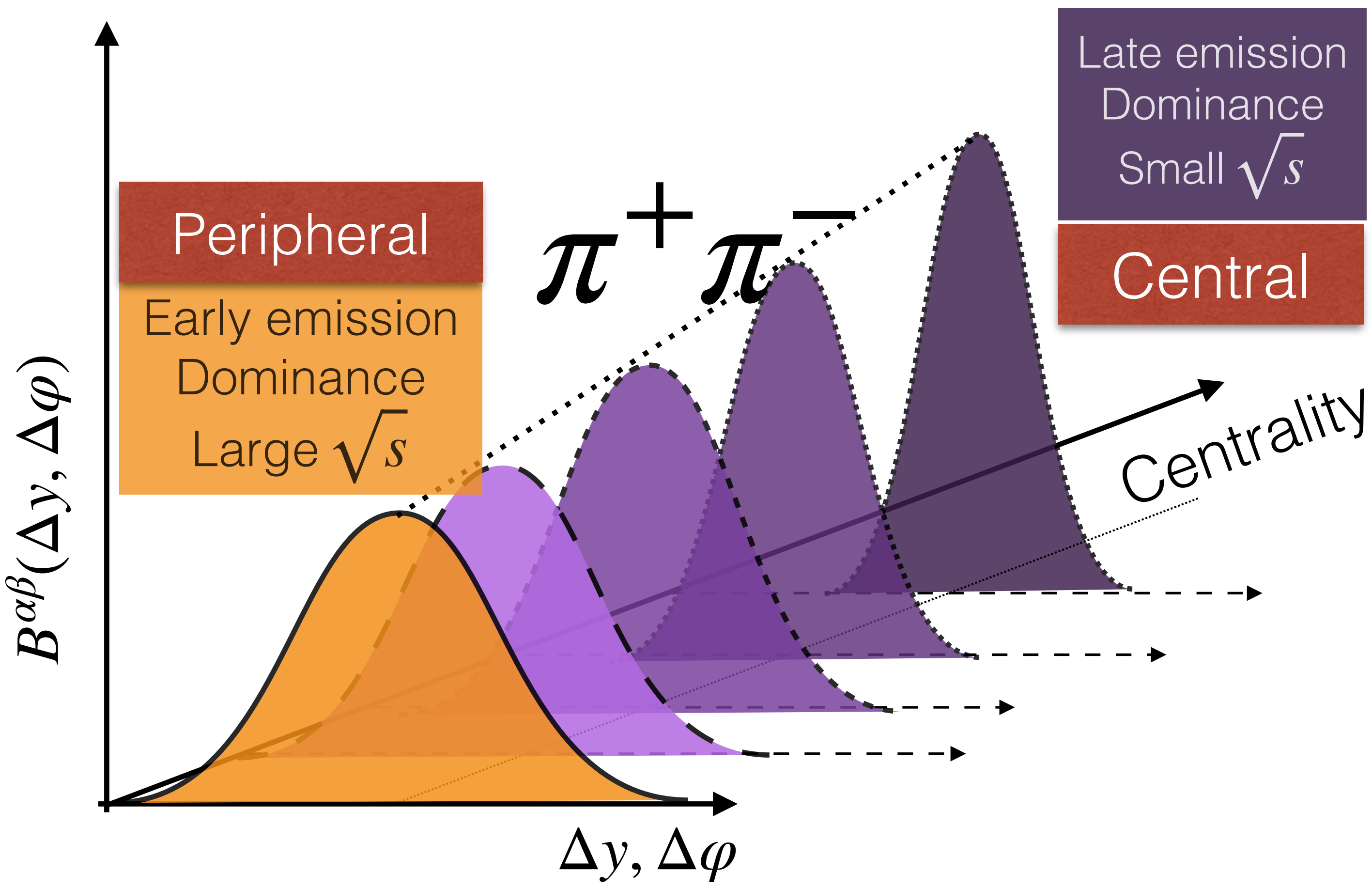

Measuring the properties of Quark Gluon Plasma with Unified Balance Functions

Wayne State University
Feb 8, 2023
Puerto Vallarta, Mexico

Work in collaboration w/
Victor Gonzalez
Sumit Basu
Ana Marín
Pedro Ladrón
Brian Hanley

This talk based on recent papers:

- Accounting for non-vanishing net-charge with unified balance functions, ***Phys.Rev.C*** **107 (2023) 1, 014902**
- Effects of Non-Vanishing Net Charge in Balance Functions, e-Print: **2211.10770 [hep-ph]**
- Work in progress

Outline

- Why/what are “unified” balance functions
- Sum-rule
- Studies with PYTHIA8

Measuring QGP Properties!

- System dynamic
  - Fast (local) thermalization, ▶ Relative species abundances
  - Isentropic expansion,
  - **Two stage quark production** ▶ **General balance functions**
- Equation of state
  - Susceptibilities ▶ Net charge/baryon fluctuations; **General balance functions**
- Transport properties
  - **Shear viscosity** ▶ Anisotropic flow; **Transverse momentum correlations, G_2**
  - Bulk viscosity ▶ Multiplicity fluctuations
  - Compressibility
  - **Quark diffusivity** ▶ **General balance functions**
  - Heat capacity ▶ Temperature fluctuations; pT fluctuations
  - Conductivity ▶
  - Stopping $\hat{q}$ ▶ Jet quenching

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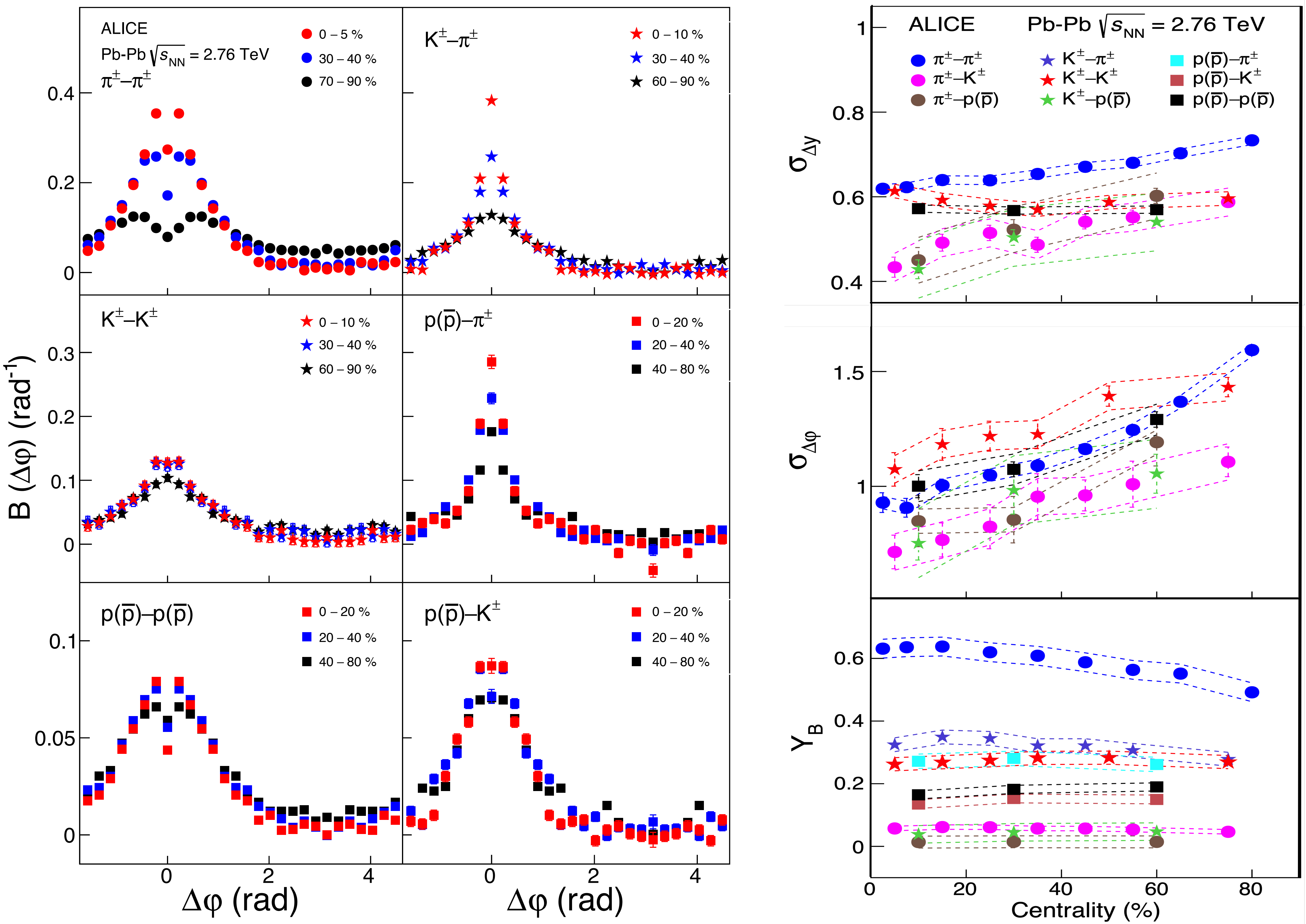
PID Balance Functions

J. Pan, PhD, Wayne State (2019)

$(\pi, K, p) \otimes (\pi, K, p)$

Phys.Lett.B 833 (2022) 137338

ALICE



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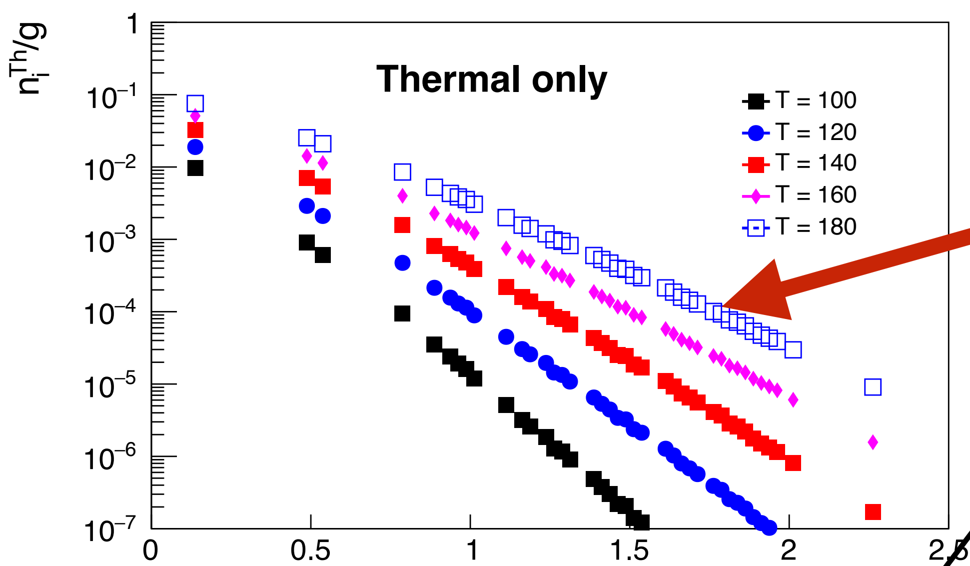
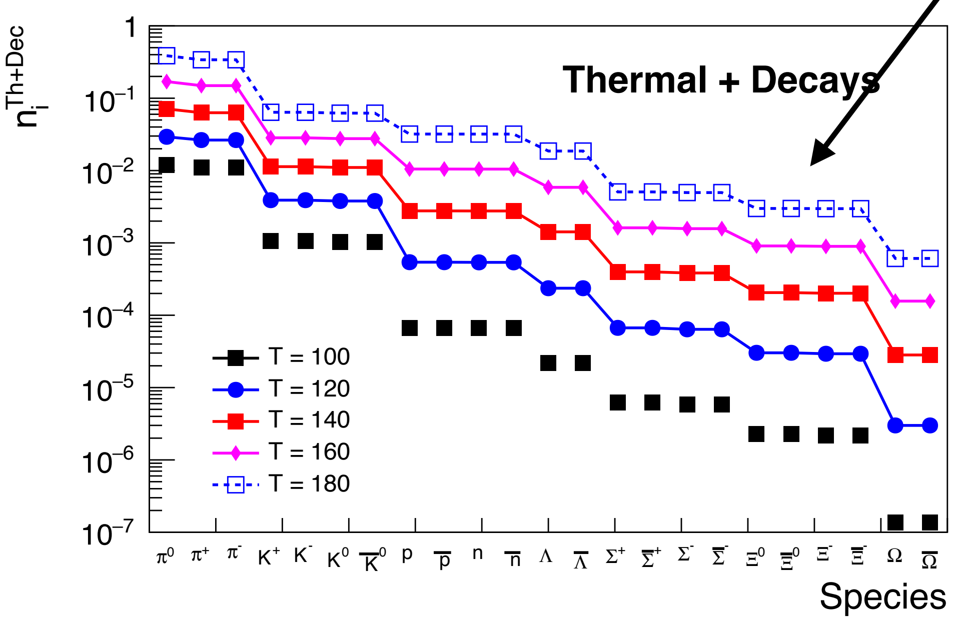
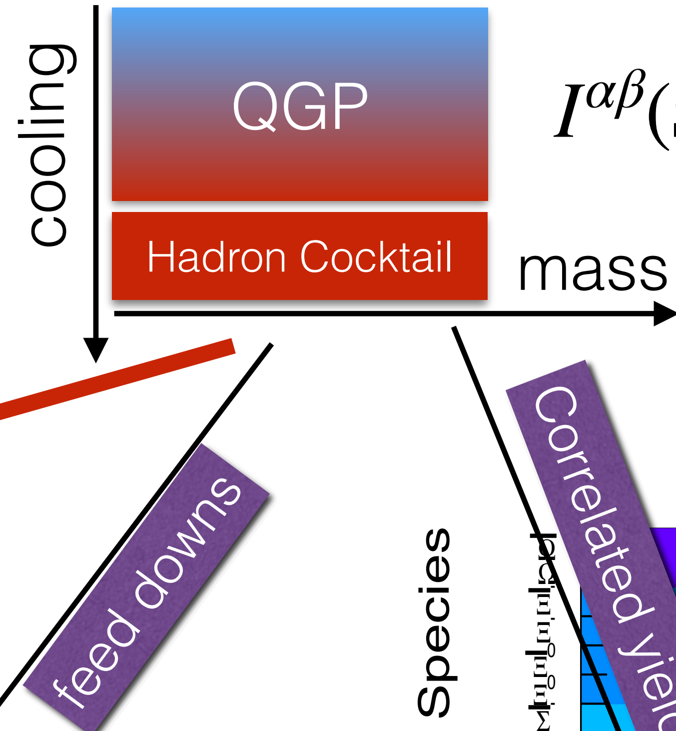
Hadron Chemistry & Balance Functions

QGP susceptibilities determine fluctuations and correlations (R_2, $B_2^{\alpha\beta}$) of charge, strangeness, and baryon number.

Single Spectra

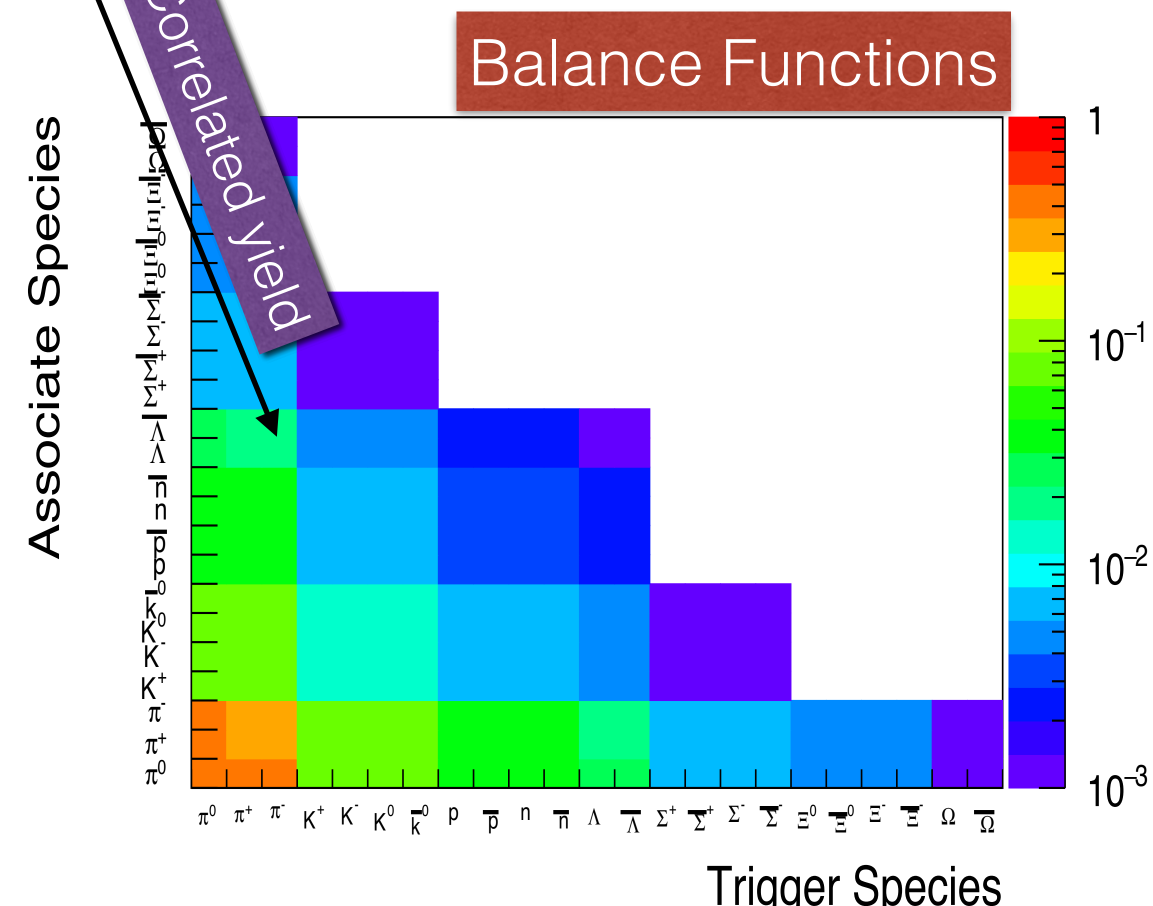






$$I^{\alpha\beta}(\Omega) = \int_{\Omega} d\vec{p}_1 d\vec{p}_2 B^{\alpha\beta}(\vec{p}_1, \vec{p}_2)$$

function of centrality ???



Fractional balance functions: $$f^{\alpha\beta}(\Omega) = \frac{I^{\alpha\beta}(\Omega)}{\sum_{\beta} I^{\alpha\beta}(\Omega)}$$

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CP, Gonzalez, Hanley, et al., *PRC* 107 (2023) 1, 014902

Notation and Definitions

Labels α and β: + or - or specific hadrons, etc,

Densities: $\rho_1^{\alpha}(\vec{p}_1) \equiv \dfrac{d^3N_1^{\alpha}}{dy_1 d\varphi_1 dp_{\mathrm{T},1}}$; $\rho_2^{\alpha\beta}(\vec{p}_1,\vec{p}_2) \equiv \dfrac{d^6N_2^{\alpha\beta}}{dy_1 d\varphi_1 dp_{\mathrm{T},1} dy_2 d\varphi_2 dp_{\mathrm{T},2}}$

N_1^{α} and $N_2^{\alpha\beta}$: numbers of particles of species α and pairs of species α and β.

Measurement acceptance $\mathbf{\Omega}$; Phase Space Volume: $V = \int_{\Omega} dy d\varphi dp_{\mathrm{T}}$

Average yields….

Singles: $\langle N_1^{\alpha} \rangle = \int_{\Omega} \rho_1^{\alpha}(\vec{p}) dy d\varphi dp_{\mathrm{T}} = V\bar{\rho}_1$;

Pairs: $\langle N_2^{\alpha\beta} \rangle = \left\langle N_1^{\alpha} \left(N_1^{\beta} - \delta_{\alpha\beta} \right) \right\rangle$

$$= \int_{\Omega} dy_1 d\varphi_1 dp_{\mathrm{T},1} \int_{\Omega} dy_2 d\varphi_2 dp_{\mathrm{T},2}\ \rho_2^{\alpha\beta}(\vec{p}_1,\vec{p}_2)$$

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Integral Balance Functions (I)

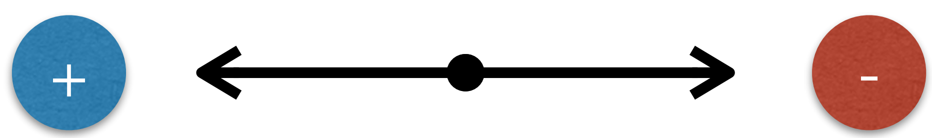
Consider:

$$I^{+-} = \frac{\langle N_2^{+-}\rangle}{\langle N_1^{-}\rangle} - \frac{\langle N_2^{--}\rangle}{\langle N_1^{-}\rangle} \qquad I^{-+} = \frac{\langle N_2^{-+}\rangle}{\langle N_1^{+}\rangle} - \frac{\langle N_2^{++}\rangle}{\langle N_1^{+}\rangle}$$

These correlators measure how many particles of type $\alpha(\bar{\alpha})$) balance each ``**trigger**" or "**reference**" particle $\bar{\beta}(\beta)$

CHARGE CONSERVATION:

Creation of $+$ **must be accompanied** by the production of $-$:



In 4π, **full** $p_{\rm T} > 0$ **acceptance**, for charged particles, one expects (for vanishing net charge)

$$I^{+-} \rightarrow 1 \qquad I^{-+} \rightarrow 1$$

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Integral of Balance Functions (II)

If the number of (+,-) pair creations (i.e., sources) is N_s in an event, then the **total number** of produced singles and pairs are

$$N_1^+ = N_s$$

$$N_1^- = N_s$$

$$N_2^{+-} = N_s^2$$

$$N_2^{-+} = N_s^2$$

$$N_2^{++} = N_s(N_s - 1)$$

$$N_2^{--} = N_s(N_s - 1)$$

.

$$I^{-+}(4\pi) = I^{+-}(4\pi) = \frac{\langle N_s^2 \rangle}{\langle N_s \rangle} - \frac{\langle N_s^2 - N_s \rangle}{\langle N_s \rangle} = 1$$

As indeed expected!

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Integral of Balance Functions (II)

Assuming **incoming net charge** is: Q

$$N_1^{+} = N_s + Q$$
$$N_1^{-} = N_s$$
$$N_2^{+-} = (N_s + Q)N_s$$
$$N_2^{+-} = (N_s + Q)N_s$$
$$N_2^{++} = (N_s + Q)(N_s + Q - 1)$$
$$N_2^{--} = N_s(N_s - 1)$$

.

$$I^{-+}(4\pi) = \frac{\langle N_s(N_s+Q)\rangle}{\langle N_s+Q\rangle} - \frac{\langle (N_s+Q)(N_s+Q-1)\rangle}{\langle N_s+Q\rangle}$$
$$= 1 - Q$$

$$I^{+-}(4\pi) = \frac{\langle (N_s+Q)N_s\rangle}{\langle N_s\rangle} - \frac{\langle N_s(N_s-1)\rangle}{\langle N_s\rangle}$$
$$= 1 + Q$$

Integral dominated by the “incoming particles” not the collisions of interest…

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Integral of Balance Functions w/ $Q \neq 0$

An easy fix…

$$I^{+-} \equiv \frac{\langle N_2^{+-}\rangle}{\langle N_1^{-}\rangle} - \frac{\langle N_2^{--}\rangle}{\langle N_1^{-}\rangle} - \left(\langle N_1^{+}\rangle - \langle N_1^{-}\rangle\right) \rightarrow 1$$

$$I^{-+} \equiv \frac{\langle N_2^{-+}\rangle}{\langle N_1^{+}\rangle} - \frac{\langle N_2^{++}\rangle}{\langle N_1^{+}\rangle} + \left(\langle N_1^{+}\rangle - \langle N_1^{-}\rangle\right) \rightarrow 1$$

…where the added terms contribute $\mp Q$.

Also useful to define:

$$I^{s} = \frac{1}{2}\left(I^{+-} + I^{-+}\right) \rightarrow 1$$

… which is evidently independent of Q…

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Differential BFs — "Pratt et al" - General BFs

General Balance Function:

$$B^{\alpha|\bar{\beta}}(y_1|y_2) = \rho_2^{\alpha|\bar{\beta}}(y_1|y_2) - \rho_2^{\bar{\alpha}|\bar{\beta}}(y_1|y_2) = \frac{\rho_2^{\alpha\bar{\beta}}(y_1,y_2)}{\rho_1^{\bar{\beta}}(y_2)} - \frac{\rho_2^{\bar{\alpha}\bar{\beta}}(y_1,y_2)}{\rho_1^{\bar{\beta}}(y_2)}$$

$\alpha|\beta$ pronounced α "given" β…

Conditional densities: $$\rho_2^{\alpha|\beta}(y_1|y_2) = \frac{\rho_2^{\alpha\beta}(y_1,y_2)}{\rho_1^{\beta}(y_2)}$$

Density of a species α at y_1 **given** a particle of species β is emitted at y_2.

$B^{\alpha|\bar{\beta}}(y_1|y_2)$: **function of y_1 only** since y_2 is ``given" (i.e., a parameter), for particle β, the reference, while particle α is the associate (the one that balances the charge of the trigger)

Note: $B^{\alpha|\bar{\beta}}(y_1|y_2)$ is not a density— it can be negative in specific ranges of "y"

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Inclusive Charge BFs and their Integrals

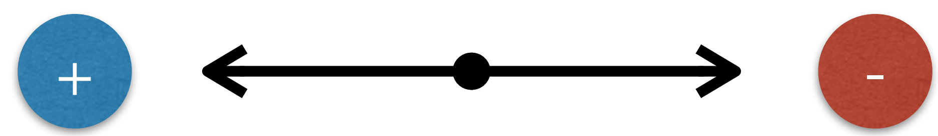
Let $\alpha = \beta = +$; $\bar{\alpha} = \bar{\beta} = -$

$$B^{+|-}(y_1|y_2) = \frac{\rho_2^{+-}(y_1, y_2)}{\rho_1^{-}(y_2)} - \frac{\rho_2^{--}(y_1, y_2)}{\rho_1^{-}(y_2)}$$

CHARGE CONSERVATION:

Creation of $\alpha = +$ **must be accompanied** by the production of $\bar{\alpha} = -$:

Integral of $B^{+|-}(y_1|y_2)$:



$$I^{+|-}(y_2|\Omega) \equiv \int_{\Omega} dy_1 B^{+|-}(y_1|y_2)$$

In the 4π , full p_{T} acceptance limit yields.

$$\lim_{\Omega \to 4\pi} I^{+|-}(y_2|\Omega) \to 1$$

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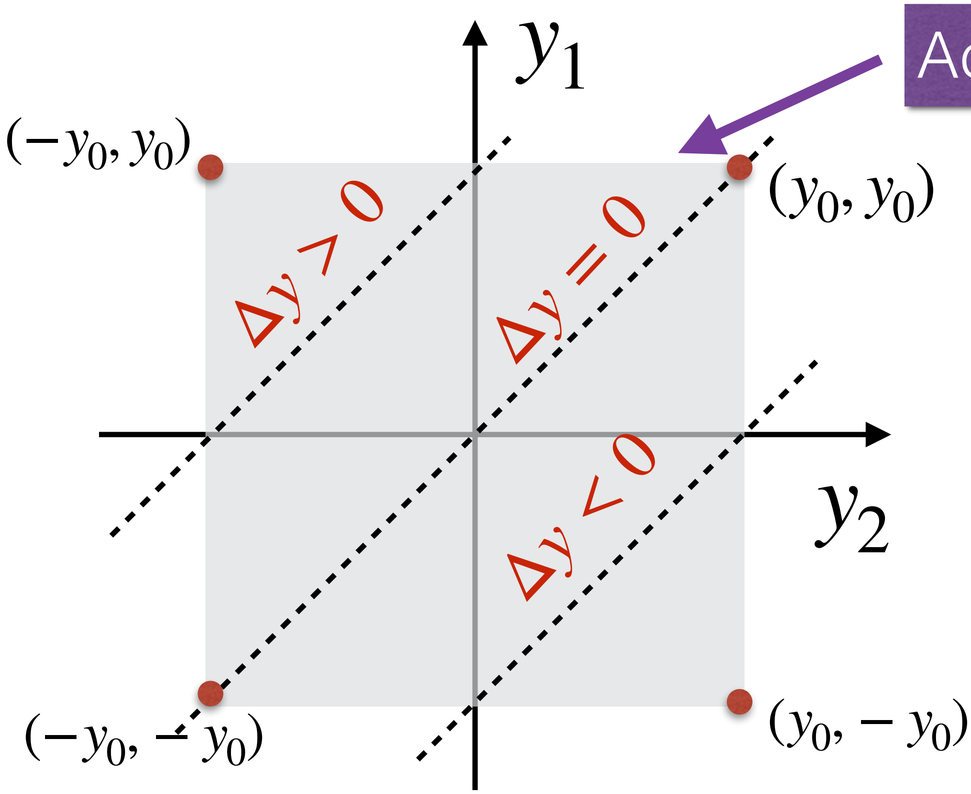
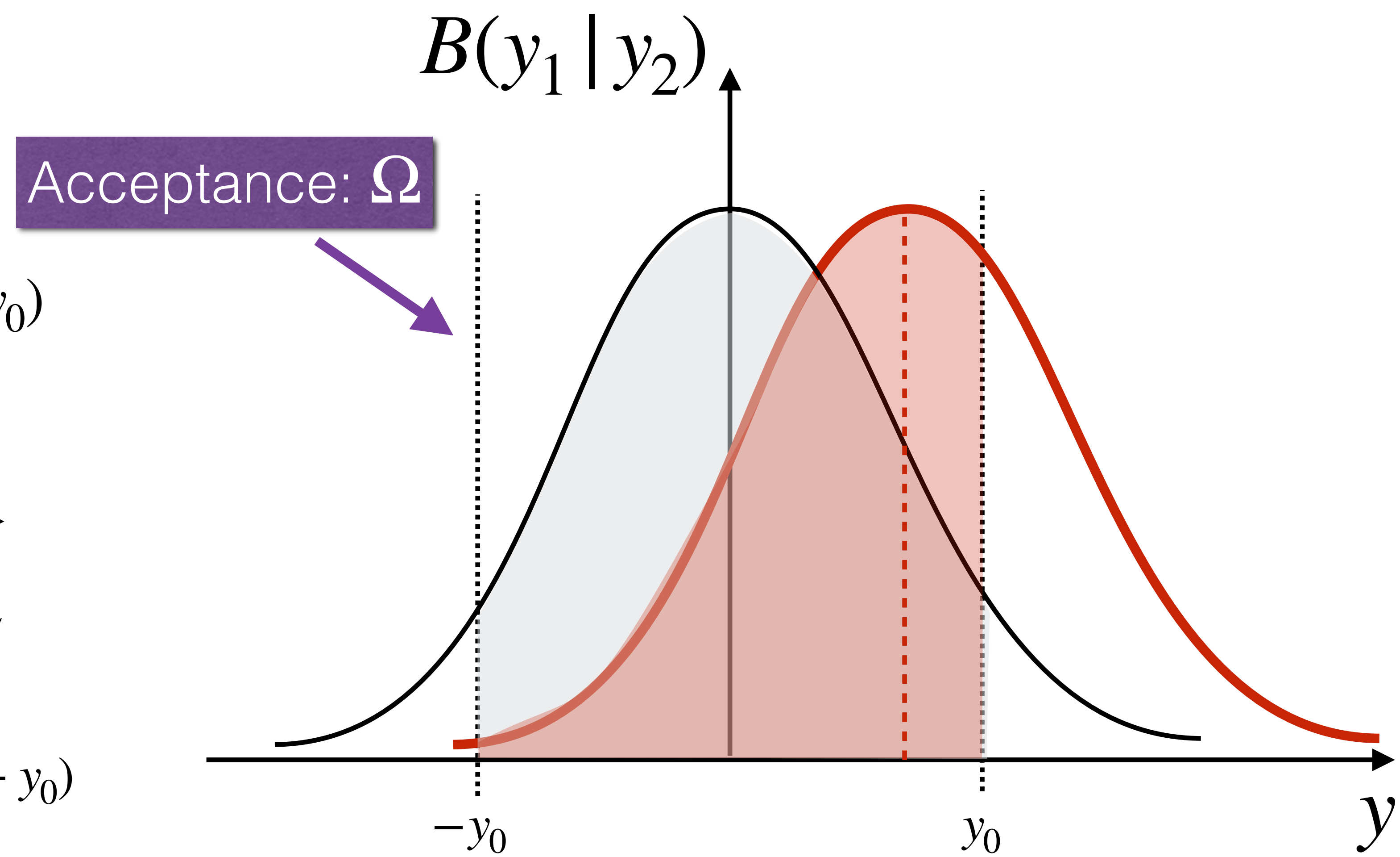
$$B^{+|-}(y_1|y_2) = \frac{\rho_2^{+-}(y_1,y_2)}{\rho_1^{-}(y_2)} - \frac{\rho_2^{--}(y_1,y_2)}{\rho_1^{-}(y_2)}$$

Accepted BFs

$$\Delta y \equiv y_1 - y_2$$





We must average across and (ideally) correct for the acceptance.

$$\bar{I}^{+-}(y_0) \equiv \int_{-y_0}^{y_0} dy_2 P_1^{-}(y_2) I^{+|-}(y_2)$$

$$P_1^{-}(y_2) = \frac{1}{\langle N_1^{-}\rangle(y_0)}\rho_1^{-}(y_2)$$ i.e., the probability of finding the -ve at y_2.

$$\bar{I}^{+-}(y_0) = \frac{1}{\langle N_1^{-}\rangle}\int_{-y_0}^{y_0} dy_2 \int_{-y_0}^{y_0} dy_1 \left[\rho_2^{+-}(y_1,y_2) - \rho_2^{--}(y_1,y_2)\right] = \frac{\langle N_2^{+-}\rangle - \langle N_2^{--}\rangle}{\langle N_1^{-}\rangle}$$

But these also depend on Q!

CP, Gonzalez, Hanley, et al., *Phys.Rev.C* 107 (2023) 1, 014902

“Bound” Balance Function

Added piece

Associated particle functions: $A_2^{\alpha|\beta}(y_1|y_2) = \frac{C_2^{\alpha\beta}(y_1,y_2)}{\rho_1^{\beta}(y_2)} = \frac{\rho_2^{\alpha\beta}(y_1,y_2)}{\rho_1^{\beta}(y_2)} - \rho_1^{\alpha}(y_1)$

Unified” general balance functions:

$$B^{\alpha|\bar{\beta}}(y_1|y_2) = A_2^{\alpha|\bar{\beta}}(y_1|y_2) - A_2^{\bar{\alpha}|\bar{\beta}}(y_1|y_2)$$

$$B^{\bar{\alpha}|\beta}(y_1|y_2) = A_2^{\bar{\alpha}|\beta}(y_1|y_2) - A_2^{\alpha|\beta}(y_1|y_2)$$

Bound balance functions

$$B^{\alpha\bar{\beta}}(y_1,y_2|\Omega) = \frac{1}{\langle N_1^{\bar{\beta}}\rangle}\left[C_2^{\alpha\bar{\beta}}(y_1,y_2) - C_2^{\bar{\alpha}\bar{\beta}}(y_1,y_2)\right]$$

$$B^{\bar{\alpha}\beta}(y_1,y_2|\Omega) = \frac{1}{\langle N_1^{\beta}\rangle}\left[C_2^{\bar{\alpha}\beta}(y_1,y_2) - C_2^{\alpha\beta}(y_1,y_2)\right],$$

Differences of 2-cumulants

For systems involving multiply charged particles, strangeness or baryon balance functions, one must use **charge or baryon or strangeness densities instead of number densities**.

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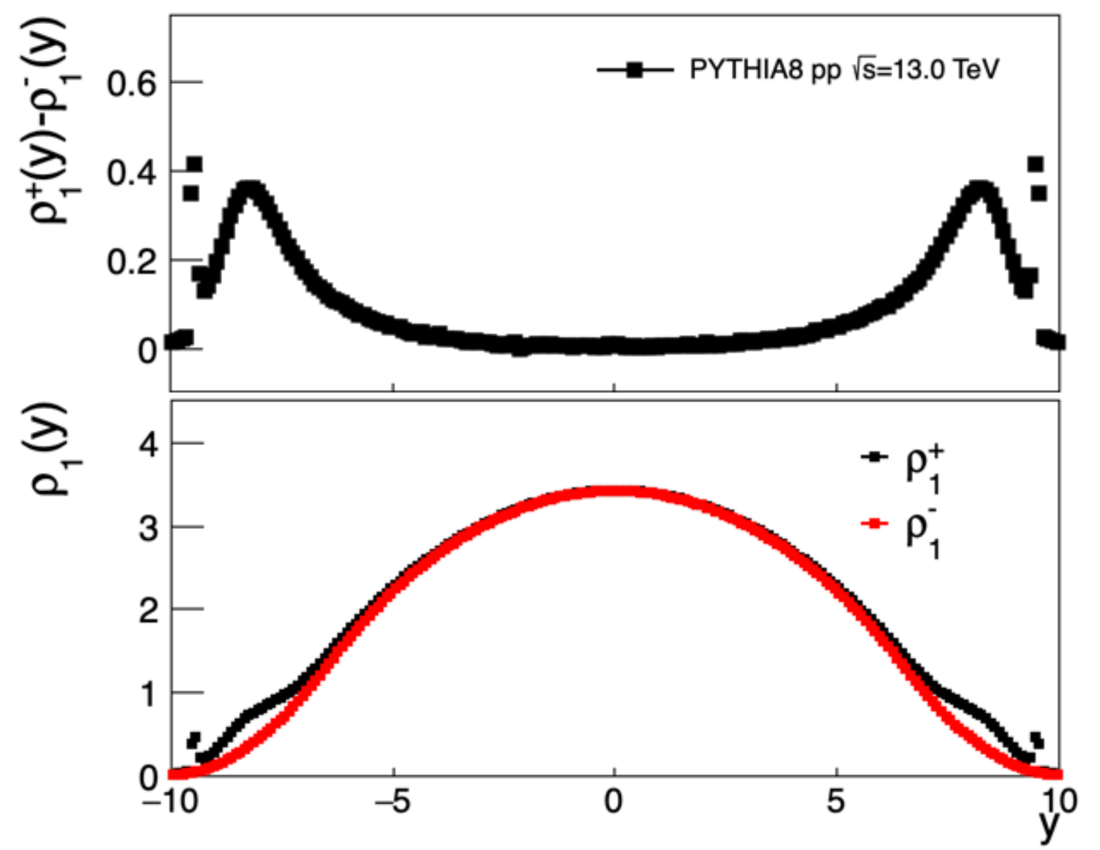
CP, Gonzalez, Hanley, et al., 2211.10770 [hep-ph]

Exploring BF measurements based on Simulations

- Using **pp collisions** at various $\sqrt{s}$ simulated w/ PYTHIA
  - Mostly MONASH tune but some others as well.
- **Why PYTHIA?**
  - Reproduces measured data.
  - **Locally conserves $E, \vec{p}$ + quantum numbers**.
  - Easy to use & fast.
- Use a **simulation frame work (CAP)**
  - Multiple models & types of analysis tasks,
  - Automated sub-sample statistical uncertainty determination, closure tests, and more.
- Compute BFs on grid.wayne.edu w/ CAP
  - Typically **20 jobs, 50 sub-jobs, >200,000 events each — Total 200 millions**
  - Enables easy subsample analysis for statistical uncertainties.



e-by-e: Q = +2
Average: density difference integrates to Q = +2

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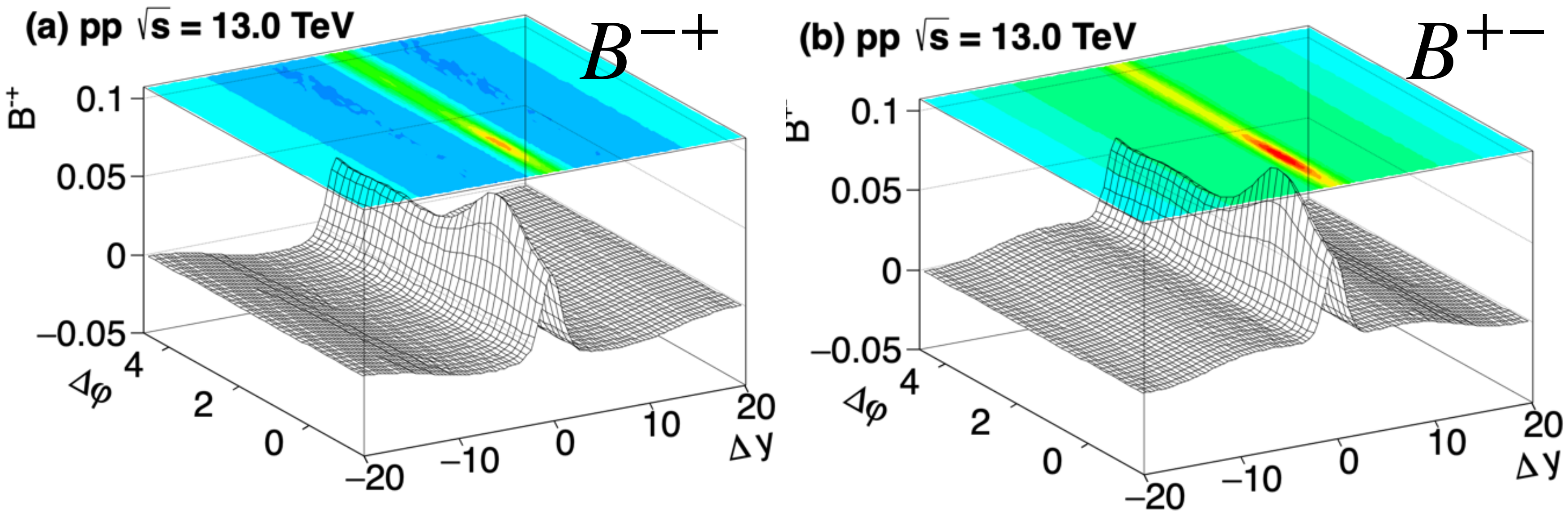
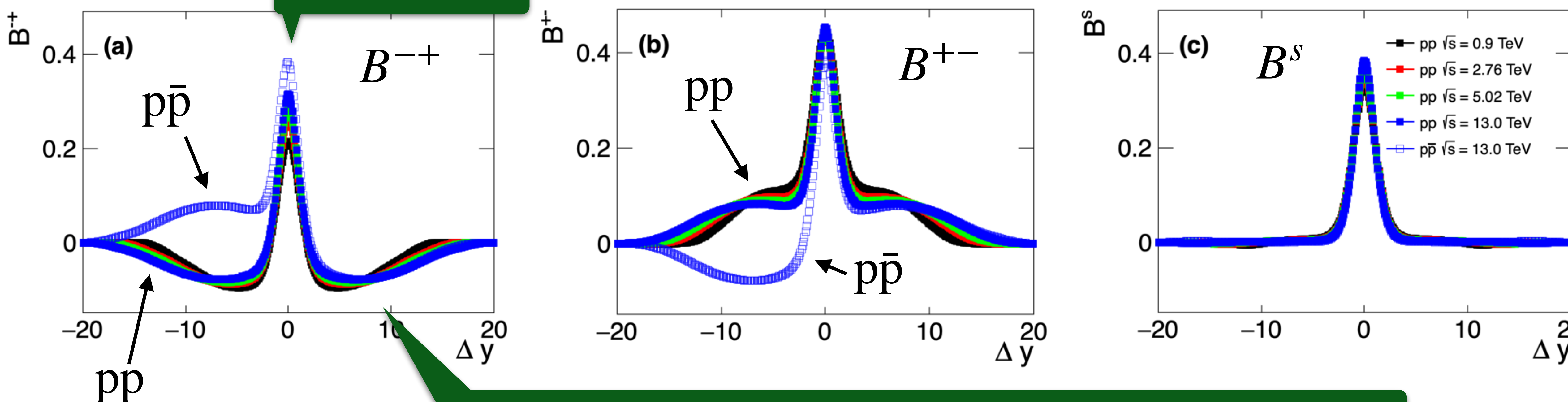
General Balance Functions (no compensation for Q)

$$B^{\bar{\alpha}\beta}_{Q=0}(y_1, y_2|y_0) = \frac{1}{\langle N_1^{\beta}\rangle(y_0)}\left[\rho_2^{\bar{\alpha}\beta}(y_1,y_2) - \rho_2^{\alpha\beta}(y_1,y_2)\right]$$

$$B^{s} \equiv (B^{\alpha\bar{\beta}} + B^{\bar{\alpha}\beta})/2$$





Huge over- and undershoots — due to singles

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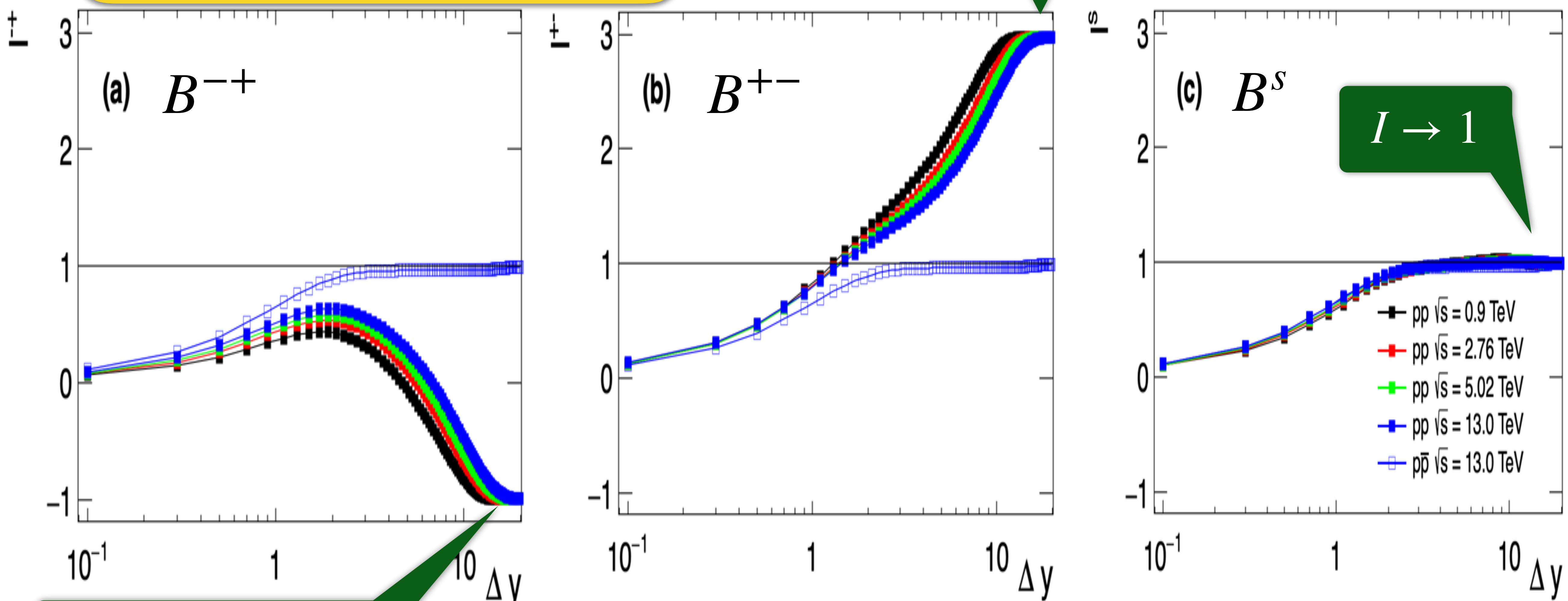
Cumulative Integrals of General Balance Functions

$$I^{\pm|\mp}(y_2|\Omega) \equiv \int_\Omega dy_1 B^{\pm|\mp}(y_1|y_2)$$

$I \rightarrow 1 + 2 = 3$



$I \rightarrow 1 - 2 = -1$

Only the integral of B^s properly converges to unity

B^{-+} and B^{+-} **do not** and are not suitable as BFs

Can this be fixed?

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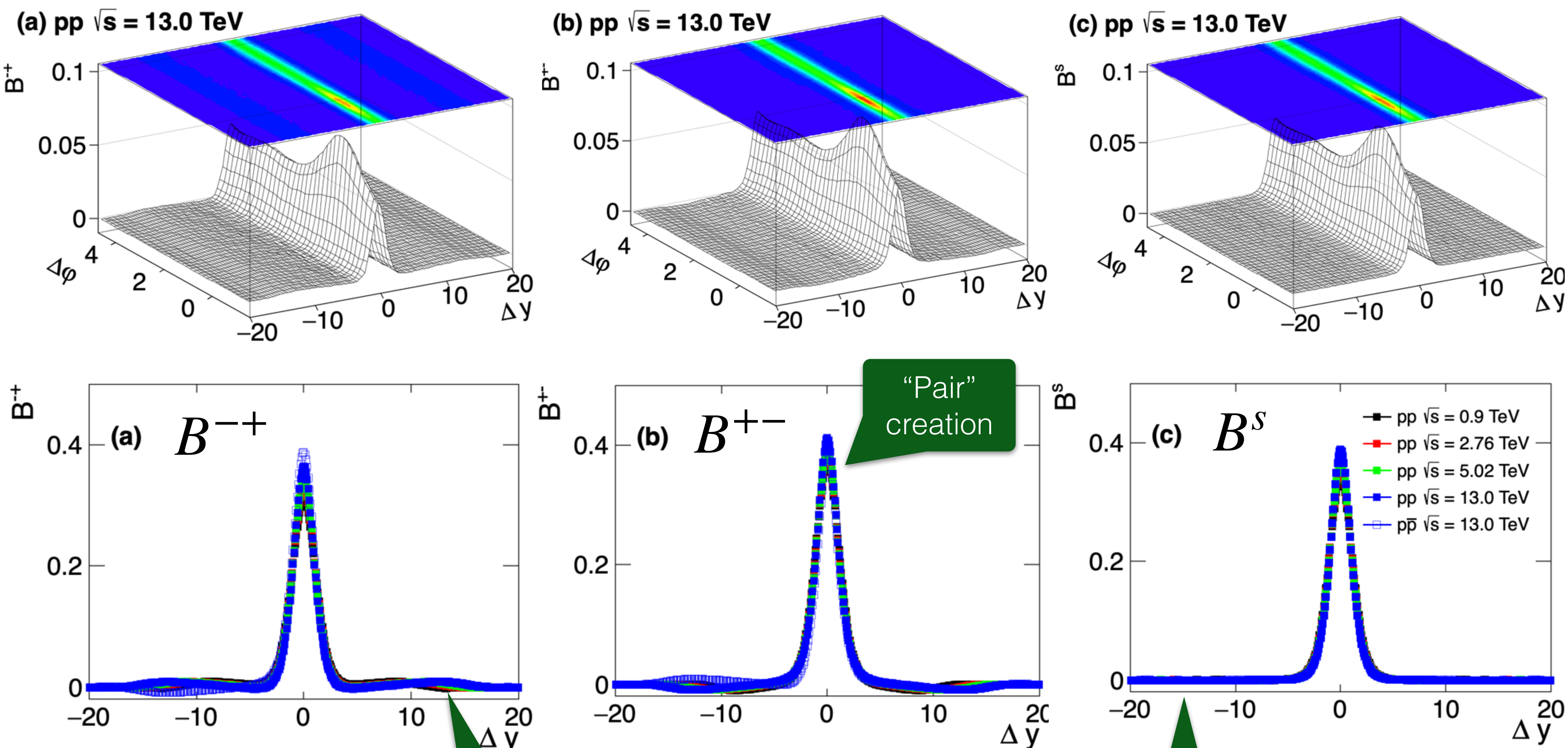
CP, Gonzalez, Hanley, et al, e-Print: 2211.10770 [hep-ph]

"Unified" Balance Functions

$$B^{\bar{\alpha}\beta}(y_1, y_2|y_0) = \frac{1}{\langle N_1^{\beta}\rangle}\left[C_2^{\bar{\alpha}\beta}(y_1,y_2) - C_2^{\alpha\beta}(y_1,y_2)\right]$$

$$B^{s} \equiv (B^{\alpha\bar{\beta}} + B^{\bar{\alpha}\beta})/2$$



"Pair" creation

Finite long range correlations

Information loss

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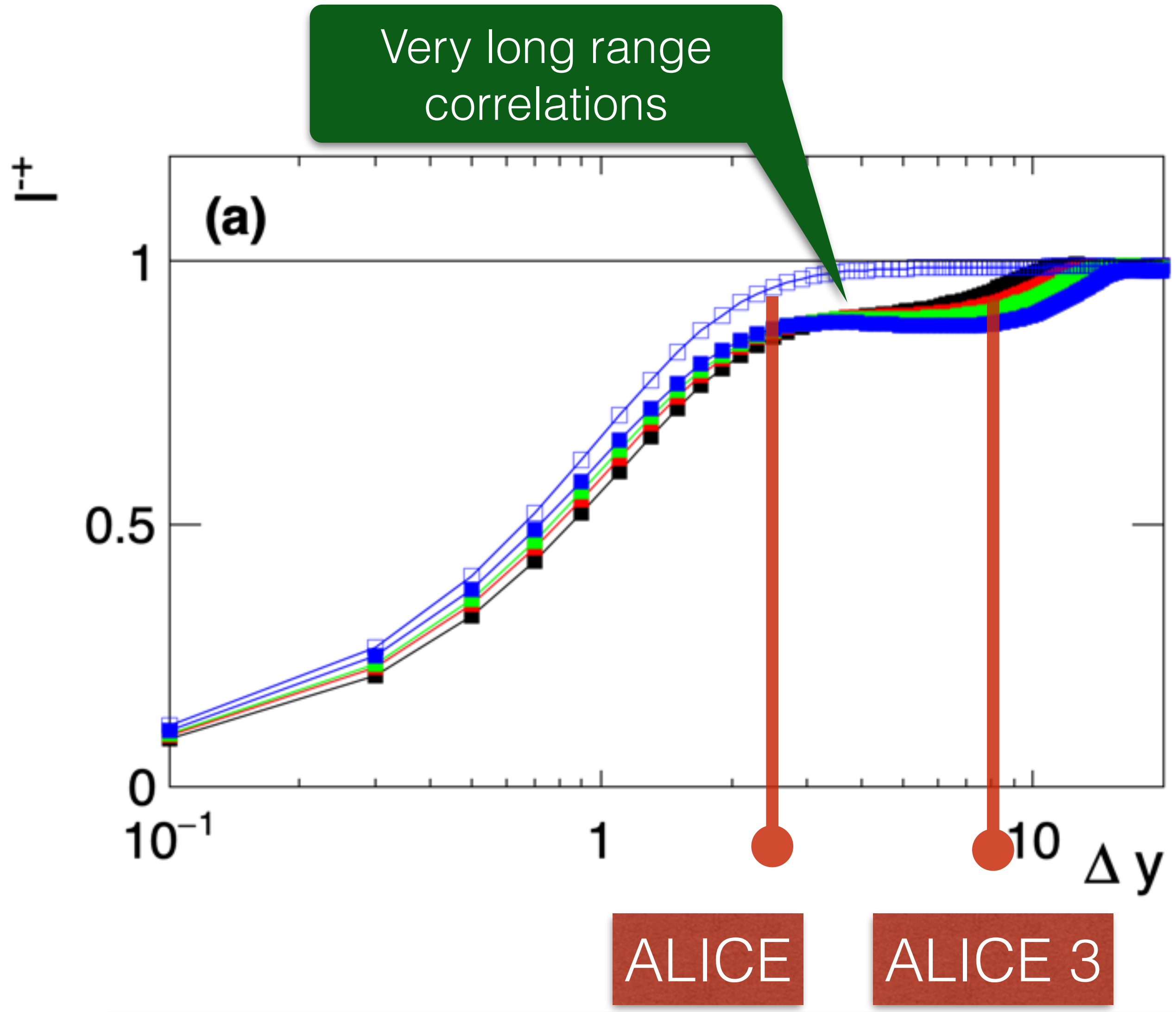
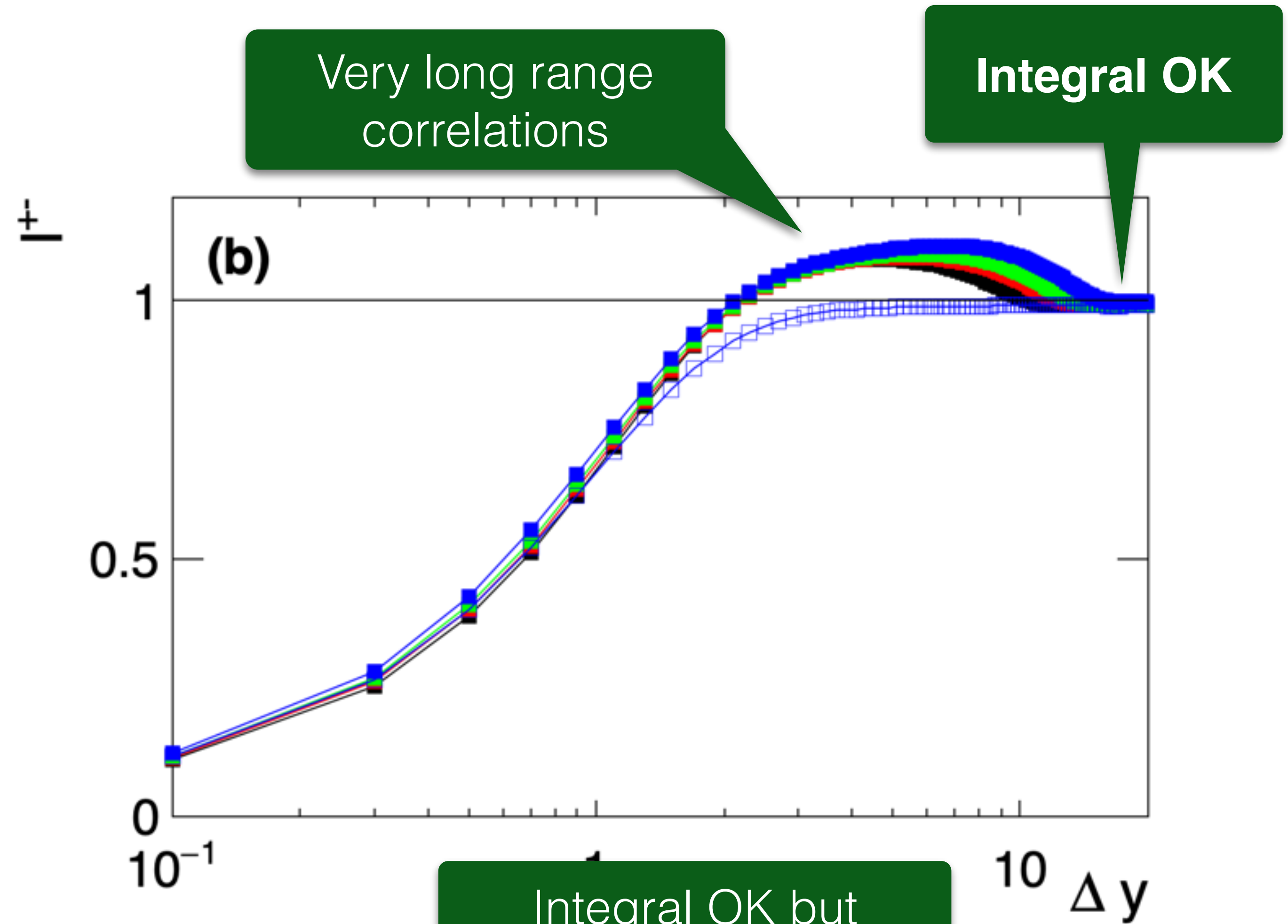
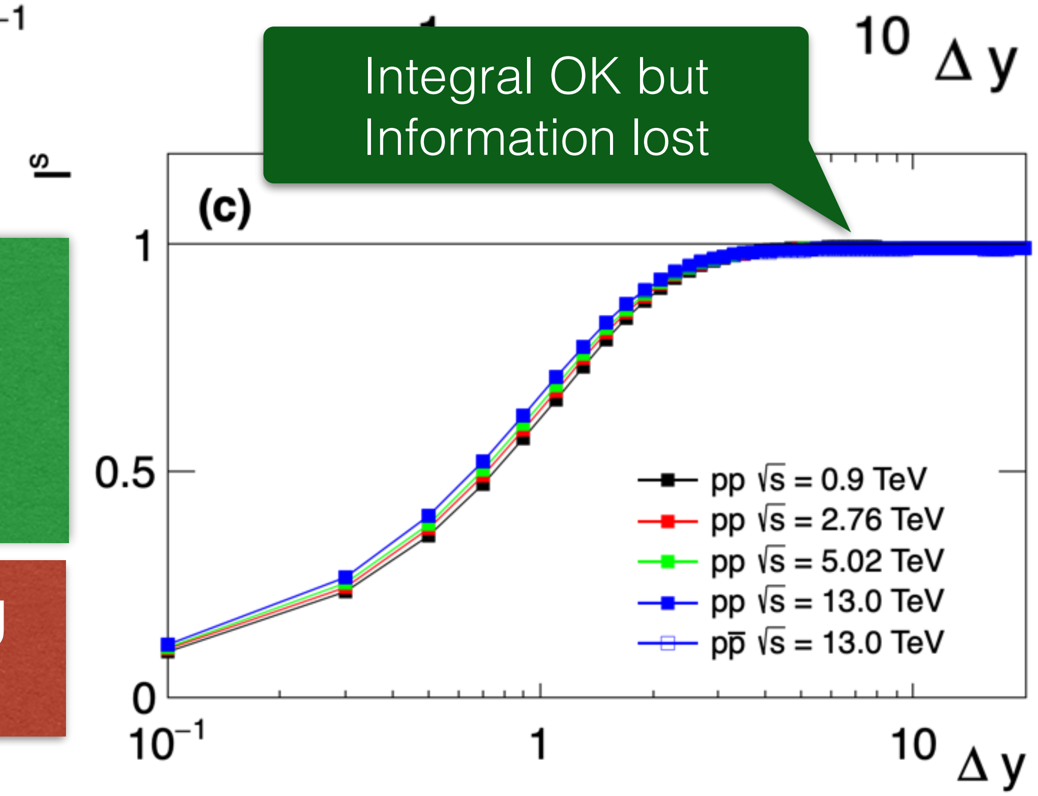
Integrals of Unified Balance Functions





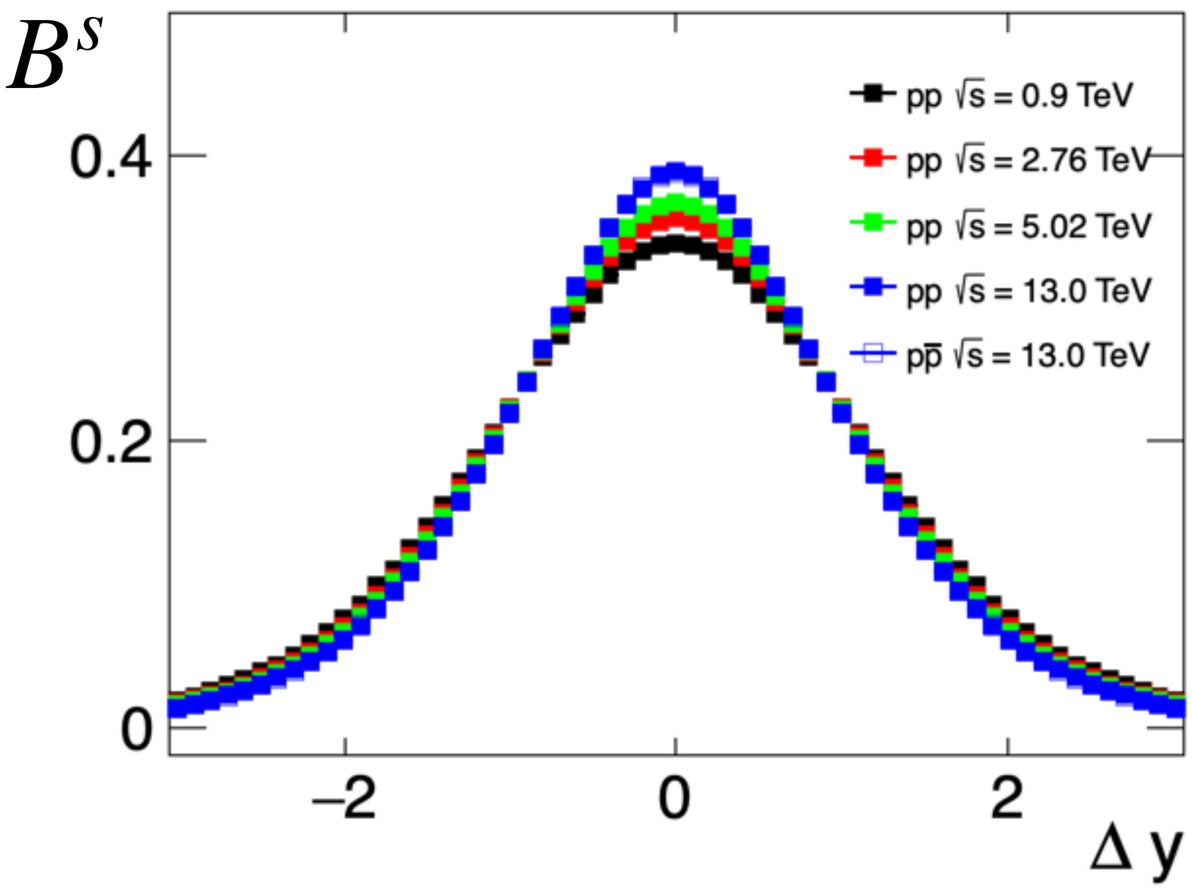
Width of acceptance + BF determines the integral. Measured values of κ_2 by STAR/ALICE not a good measure of susceptibilities because they explicitly depend on the width of the acceptance.

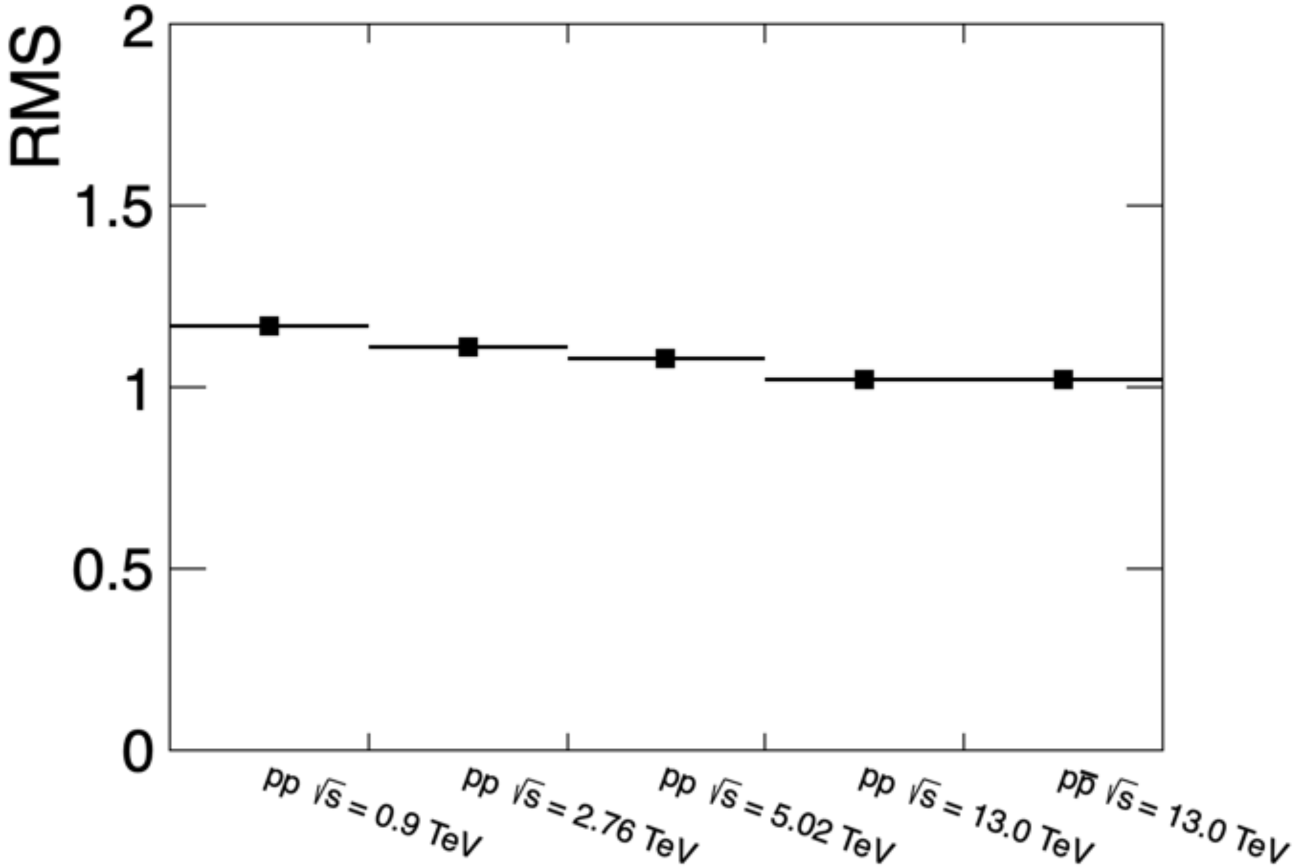


ALICE 3 should be able to measure these long ranges correlations

CP, Gonzalez, Hanley, Basu, e-Print: 2211.10770 [hep-ph]

Width of Balance Functions vs. $\sqrt{s}$





Modest narrowing vs. $\sqrt{s}$

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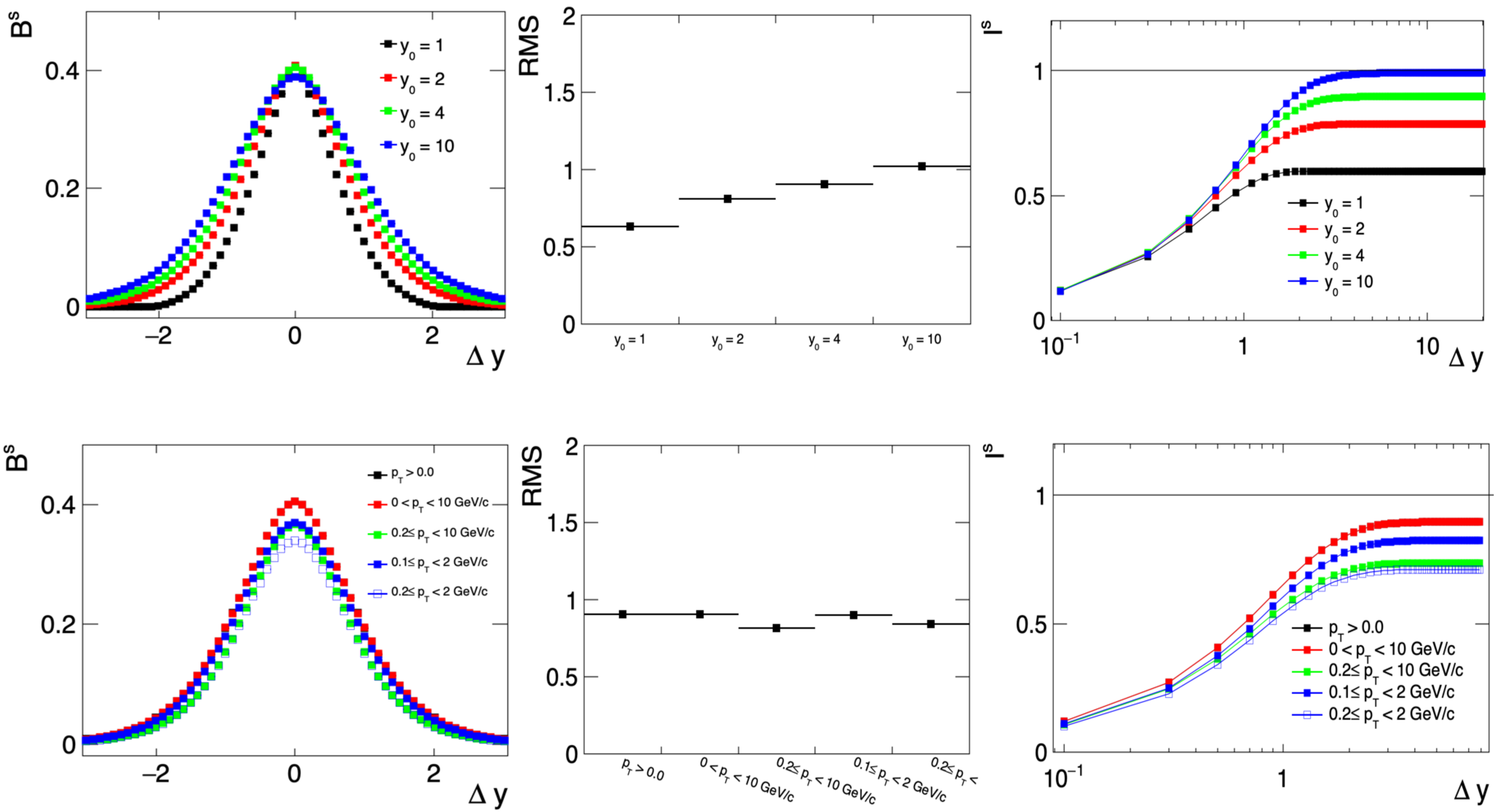
Impact of Acceptance in rapidity & pT



y, pT acceptance impact width & integral

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C.P., Phys. Rev. C 100, 034905 (2019)

Sum Rules

CP, Gonzalez, Hanley, Basu, in progress

“Unified” Balance Functions obey simple sum-rules

Charge Balance Functions

$$B^{+|\bar{\beta}}(y_1|y_2) = \sum_{\alpha} B^{\alpha|\bar{\beta}}(y_1|y_2) \qquad B^{-|\beta}(y_1|y_2) = \sum_{\alpha} B^{\bar{\alpha}|\beta}(y_1|y_2)$$

α,$\bar{\alpha}$ span all particles (anti-) that balance the charge of particles $\bar{\beta}$ and β

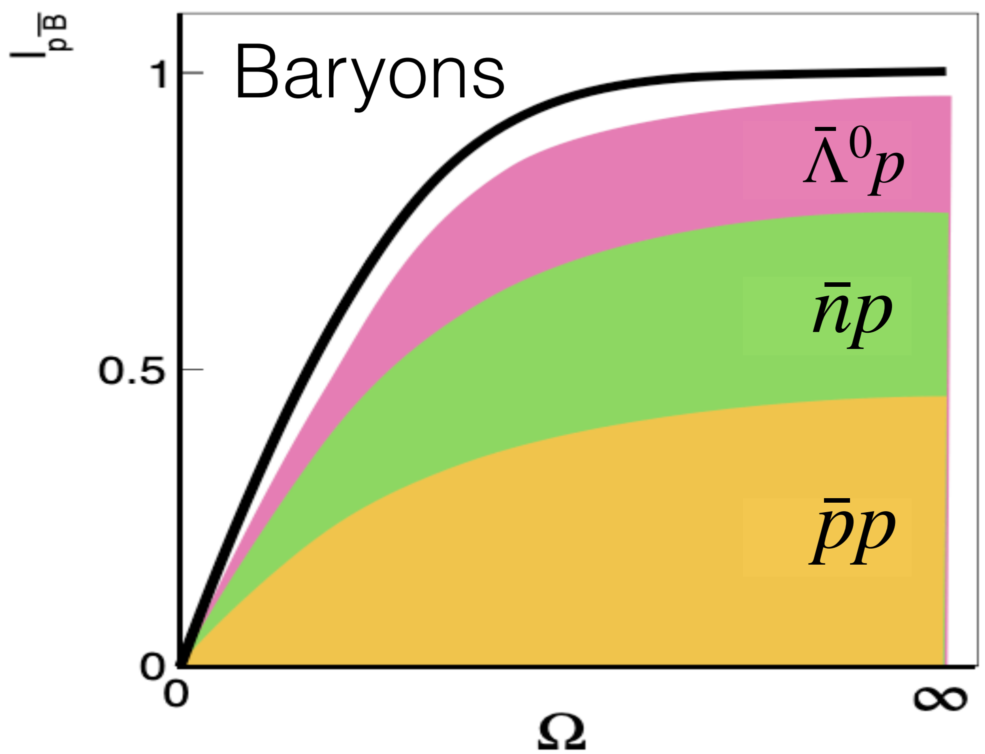
Baryon Balance Functions

$$B^{B|\bar{\beta}}(y_1|y_2) = \sum_{\alpha} B^{\alpha|\bar{\beta}}(y_1|y_2)$$

$$B^{\bar{B}|\beta}(y_1|y_2) = \sum_{\bar{\alpha}} B^{\bar{\alpha}|\beta}(y_1|y_2)$$

$B, \bar{B}$ indices: Baryon and Anti-baryon

$\alpha, \bar{\alpha}$ span all baryons (anti-baryons)



$$1 \equiv I_{4\pi}^{\bar{B}p} = I_{4\pi}^{\bar{p}p} + I_{4\pi}^{\bar{n}p} + I_{4\pi}^{\bar{\Lambda},p} + \cdots = \sum_{\bar{\beta}} I_{4\pi}^{p\bar{\beta}}$$

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Charged Hadron UBFs: π, K, p

Identified Particles: charged pions, kaons, protons

Pairs $\alpha\beta$:

β **: trigger (reference)**

α **: associate**

$\pi\pi$	$K\pi$	$p\pi$
πK	KK	pK
πp	Kp	pp

Findings:

GBFs do not integrate to unity — violate sum rules

But UBFs DO satisfy sum-rules

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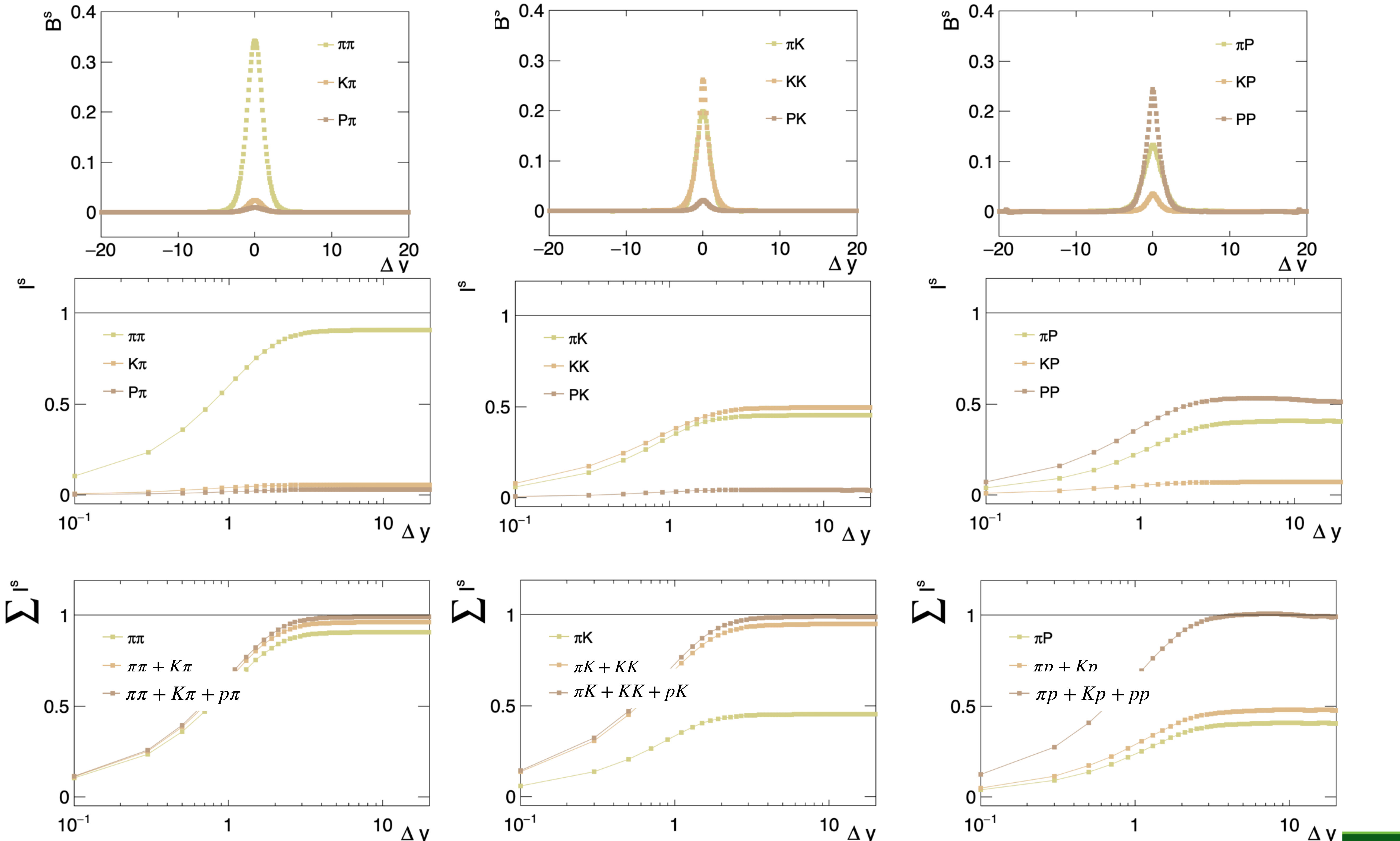
Examples based on PYTHIA

Light hadron UBFs: π, K, p



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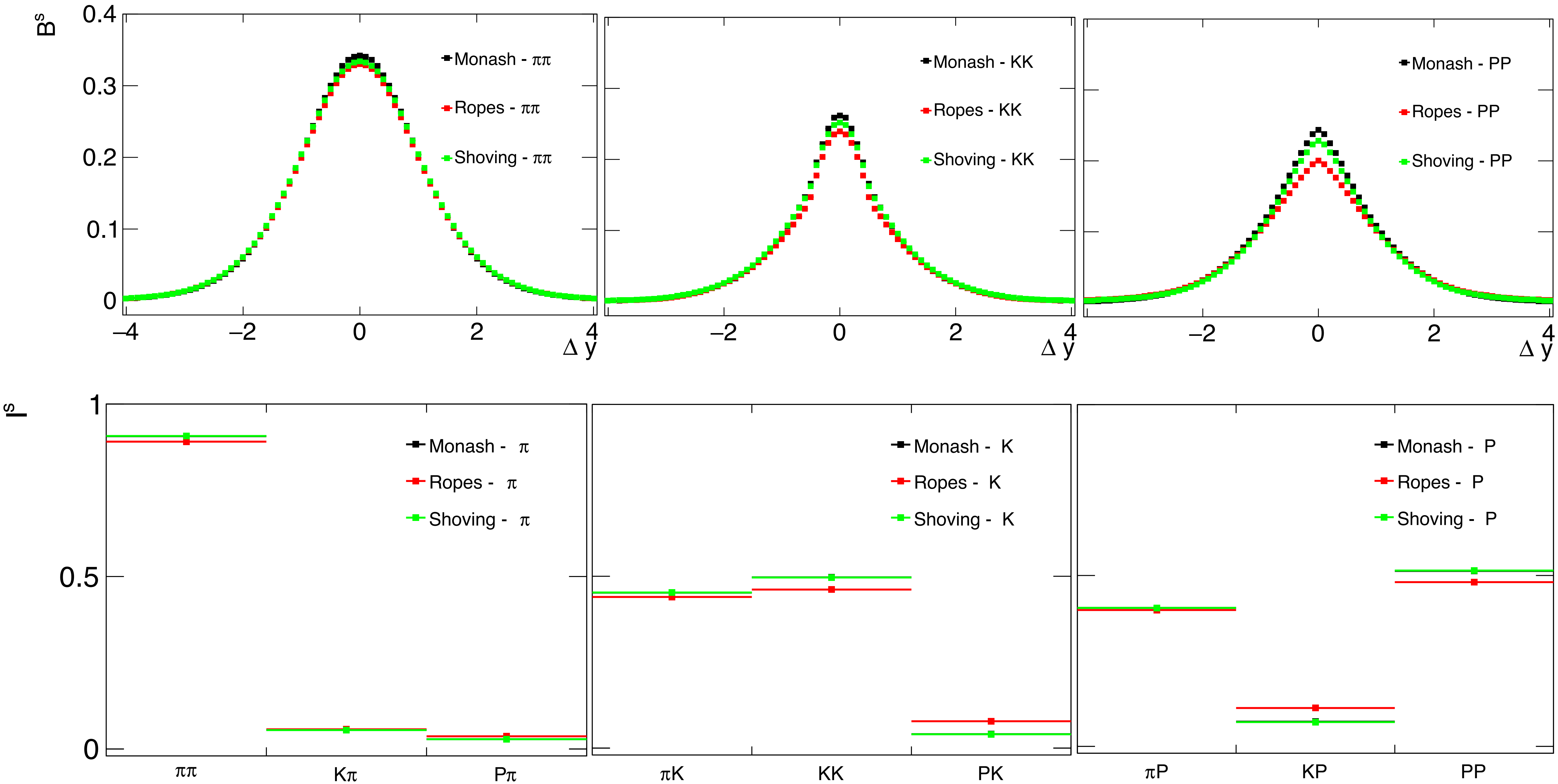
PYTHIA $\sqrt{s} = 13$ TeV — π, K, p

Fractional Integrals: Monash vs Ropes vs Shoving



Fractions w/ Shoving "identical" to MONASH
Finite change observed w/ Ropes
Relative fractions indeed dependent on production mechanisms!
(Small effect in PYTHIA8)

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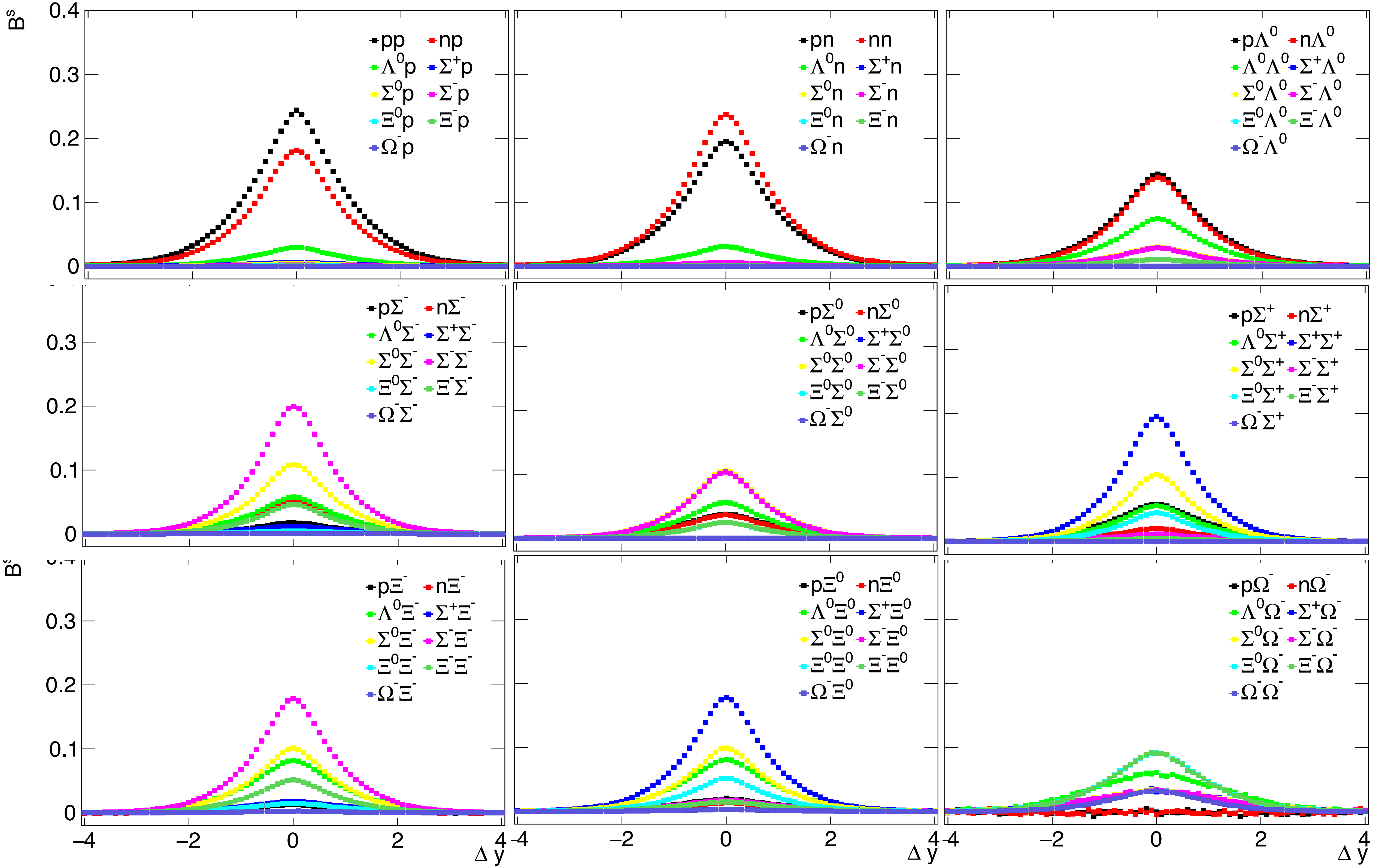
Baryon UBFs

- PYTHIA: Disable weak decays of low mass states
- Baryons included (and their anti-particles)
  - p: proton
  - n: neutron — only measurable in practice at >> 1 GeV/c
  - Λ^0: "easy" to observe: $\Lambda^0 \rightarrow \mathrm{p} + \pi^-$
  - Σ^-: hard to observe: $\Sigma^- \rightarrow \mathrm{n} + \pi^-$
  - Σ^0: hard to observe: $\Sigma^0 \rightarrow \Lambda^0 + \gamma$
  - Σ^+: hard to observe: $\Sigma^+ \rightarrow \mathrm{p} + \pi^0$; $\Sigma^0 \rightarrow \mathrm{n} + \pi^+$
  - Ξ^-: measurable from: $\Xi^- \rightarrow \Lambda^0 + \pi^-$
  - Ξ^0: hard to observe: $\Xi^0 \rightarrow \Lambda^0 + \pi^0$
  - Ω^-: measurable from: $\Omega^- \rightarrow \Lambda^0 + K^-$

UBFs — Baryons — pp @ $\sqrt{s} = 13$ TeV



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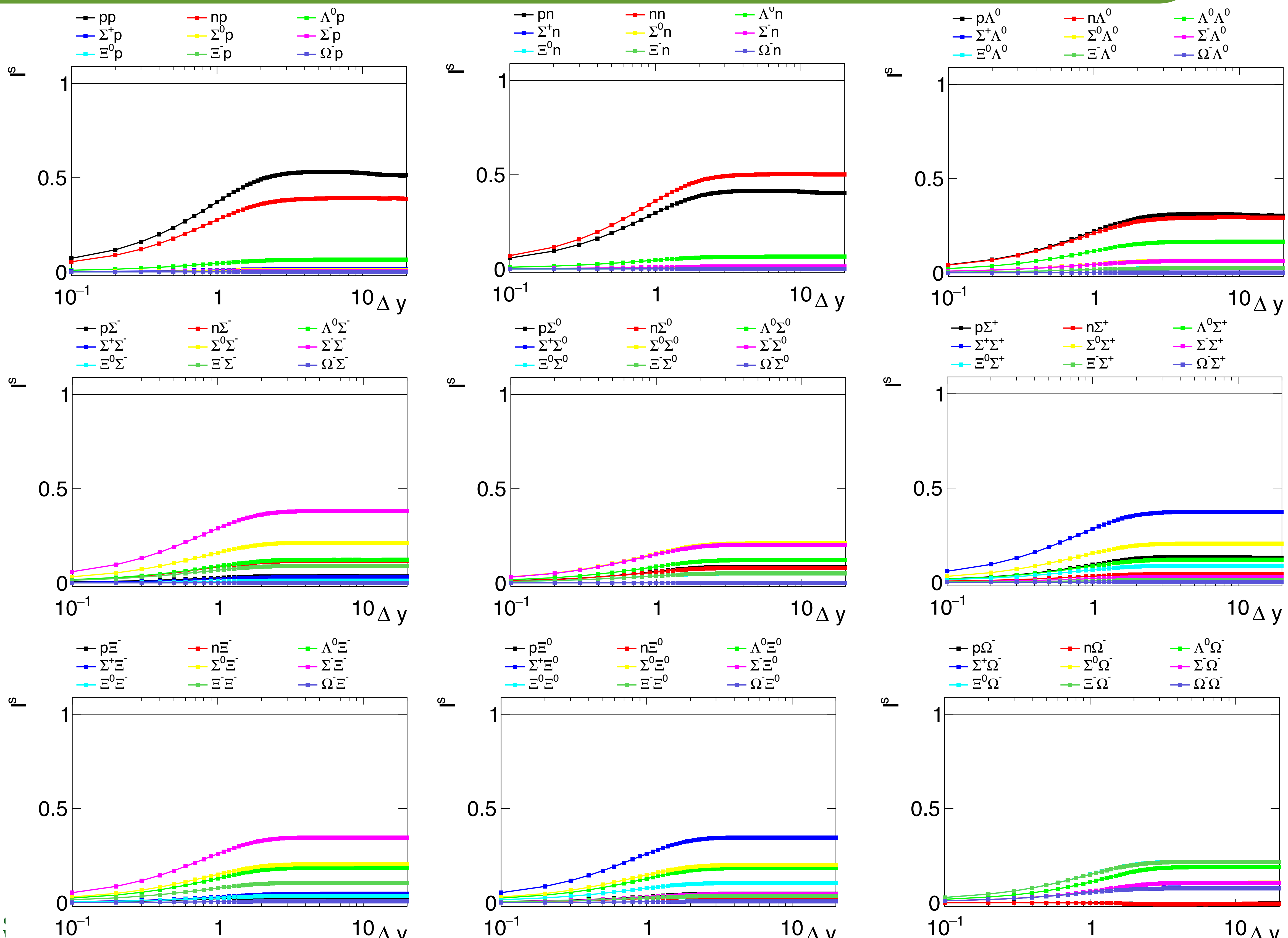
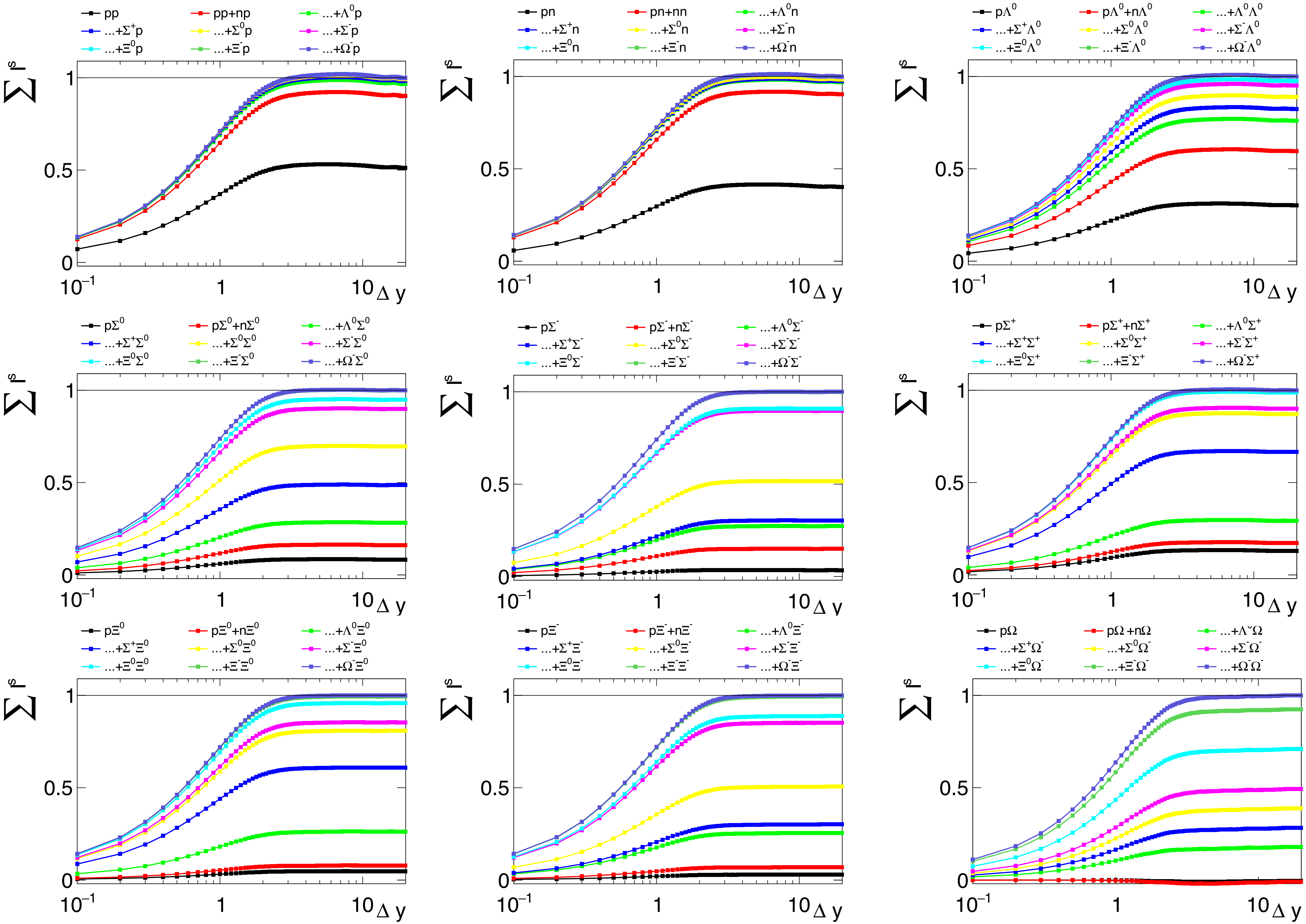
UBFs Integrals — Baryons — pp @ $\sqrt{s} = 13$ TeV



Examples based on PYTHIA (MONASH)

UBFs Integrals — Baryons — pp @ $\sqrt{s} = 13$ TeV

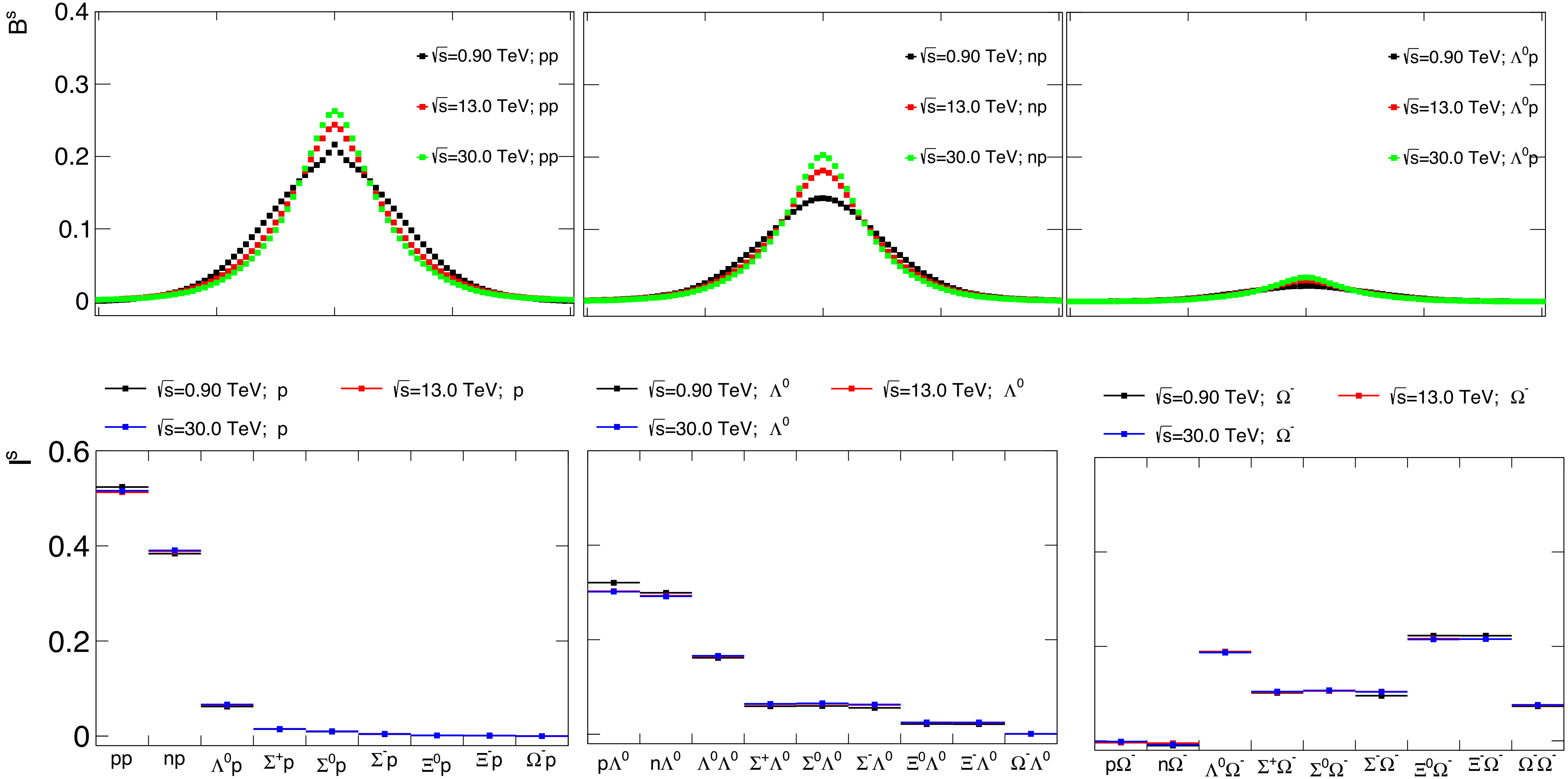


Summary

- Must use UBFs instead of general balance functions
  - Properly accounts for a system's net-charge, Q
  - UBF Integrals converge to unity in the full acceptance limit
  - Integrals and widths (shape) affected by acceptance
- "Triggered" UBFs
  - Obey a simple sum-rule
  - Have fractional integrals that depend on the particles and their production mechanism(s)
  - Will depend on transport when measured in a narrow acceptance.
- UBFs provide a tool to study long range quantum number conservation and transport.
- UBFs provide additional and stringent constraints on particle production models.

Examples based on PYTHIA (MONASH)

UBFs vs. Beam Energy: $\sqrt{s} = 0.9,\ 13.0,\ 30.0$ TeV



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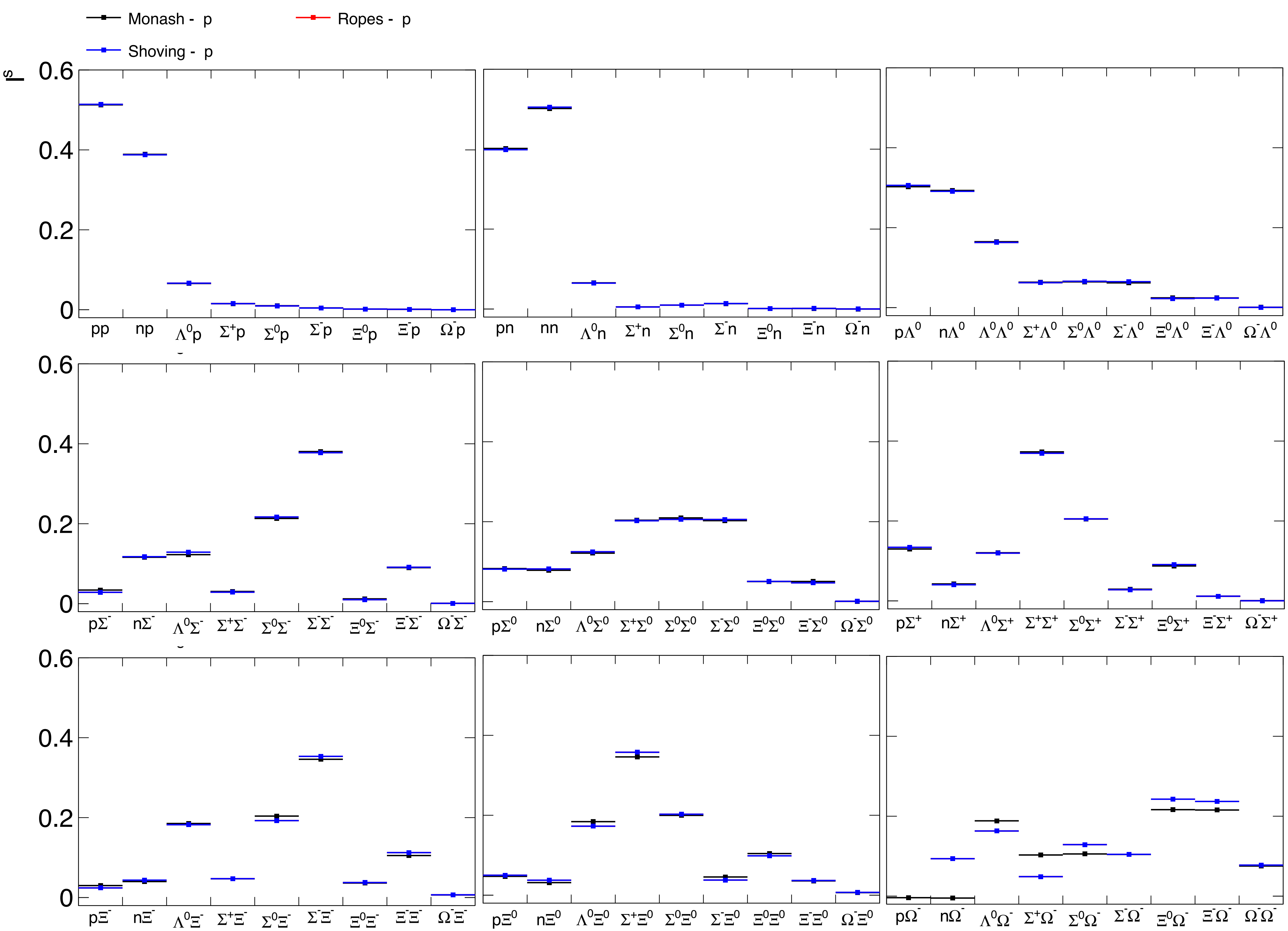
Fractional Integrals: Monash vs Ropes vs Shoving



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