

# Improving Bayesian parameter estimation with the latest RHIC and LHC data including a new initial conditions model

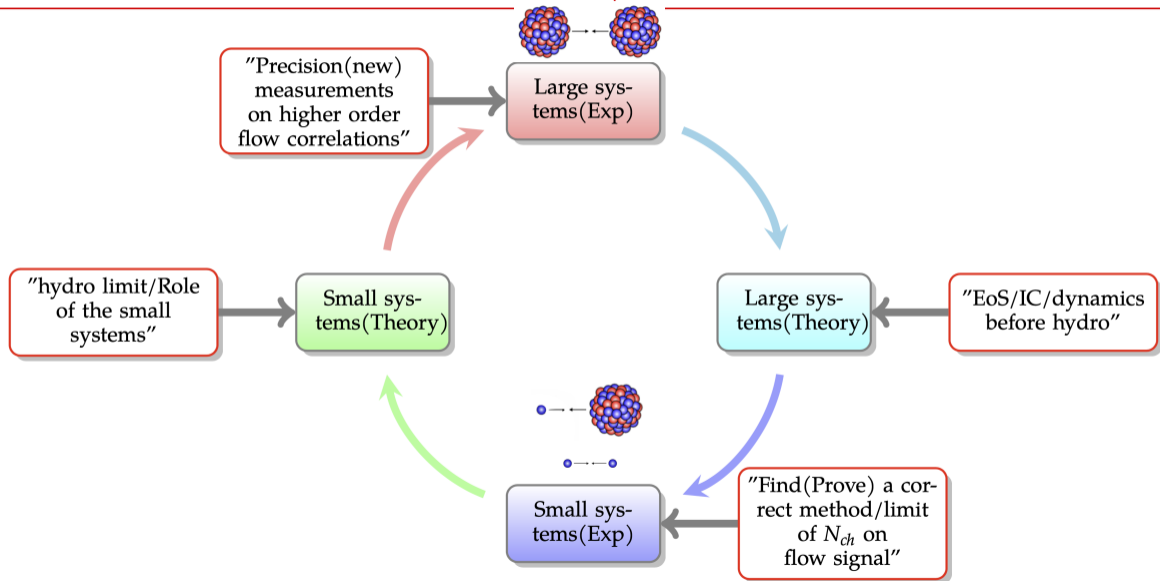
Dong Jo Kim<sup>1,2</sup>, Maxim Virta<sup>1</sup>, Jasper Parkkila<sup>3</sup>

1. University of Jyväskylä, Finland, 2. Helsinki Institute of Physics 3. CERN

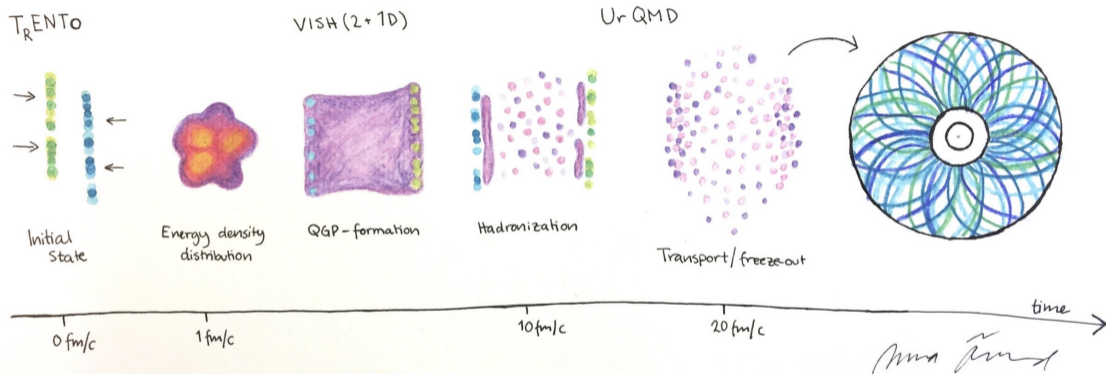
Wednesday 8<sup>th</sup> February, 2023

The 38th Winter Workshop on Nuclear Dynamics, Puerto Vallarta, Mexico

# CHALLENGES - UNDERSTANDING THE EXPERIMENTAL/THEORY UNCERTAINTIES



## THE DIFFERENT STAGES OF HEAVY-ION COLLISIONS

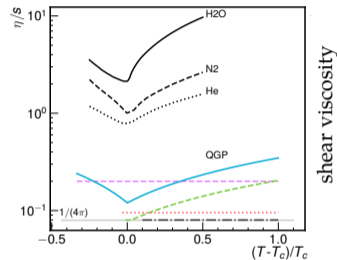
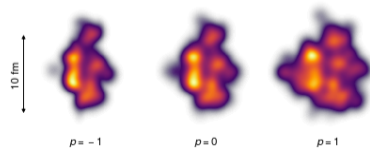


Credits to Anna Onnerstad

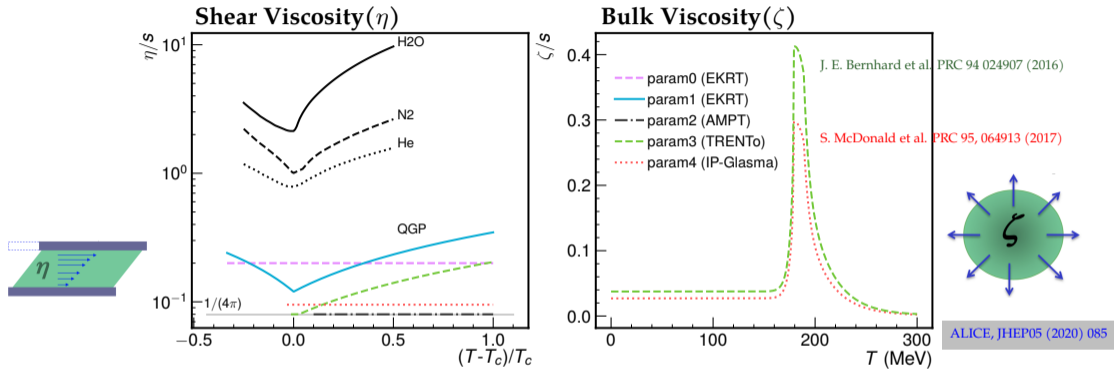
$$T^{\mu\nu} = e u^{\mu} u^{\nu} - (P + \Pi) \Delta_{\mu\nu} + \pi^{\mu\nu}, \quad \delta_{\mu} T^{\mu\nu} = 0$$

## 8 x 7 PARAMETERS, JYVASKYLA(2022)

Parameter	Description
$T_c$	Temperature of const. $\eta/s(T)$ , $T < T_c$
$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{\text{slope}}$	Slope of $\eta/s(T)$ above $T_c$
$(\eta/s)_{\text{curve}}$	Curvature of $\eta/s(T)$ above $T_c$
$(\zeta/s)_{\text{peak}}$	Temperature of $\zeta/s(T)$ maximum
$(\zeta/s)_{\text{max}}$	Maximum $\zeta/s(T)$
$(\zeta/s)_{\text{width}}$	Width of $\zeta/s(T)$ peak
$T_{\text{switch}}$	Switching / particlization temperature
$N(2.76 \text{ TeV})$	Overall normalization (2.76 TeV)
$N(5.02 \text{ TeV})$	Overall normalization (5.02 TeV)
$p$	Entropy deposition parameter
$w$	Nucleon width
$\sigma_k$	Std. dev. of nucleon multiplicity fluctuations
$d_{\text{min}}^3$	Minimum volume per nucleon
$\tau_{\text{fs}}$	Free-streaming time

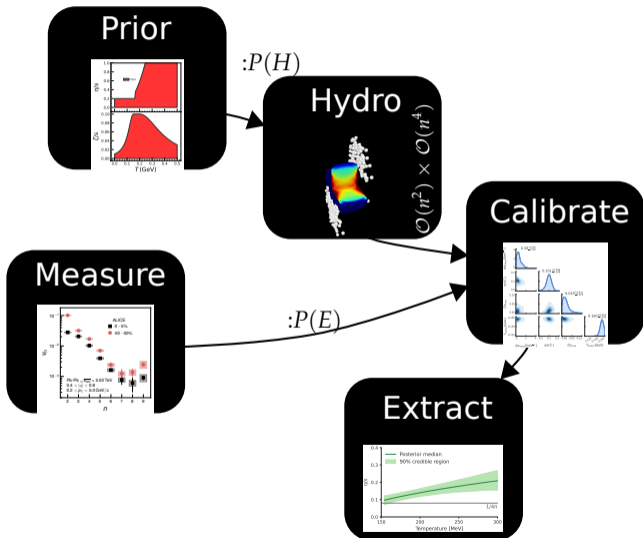
Trento p-value, <http://qcd.phy.duke.edu/trento/>

## TRANSPORT PROPERTIES IN HEAVY-ION COLLISIONS



$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\text{slope}}(T - T_c) \left( \frac{T}{T_c} \right)^{(\eta/s)_{\text{curve}}}, \quad (\zeta/s)(T) = \frac{(\zeta/s)_{\text{max}}}{1 + \left( \frac{T - (\zeta/s)_{T_{\text{peak}}}}{(\zeta/s)_{\text{width}}} \right)^2}$$

## BAYESIAN PARAMETER ESTIMATION



Bayes' theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

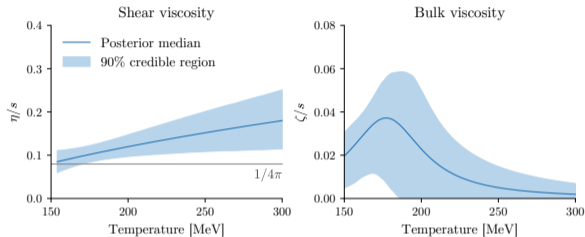
$$P(E) = \sum_{i=1}^n P(E|H_i)P(H_i)$$

- Find optimal set of model parameters that best reproduce the experimental data
- Utilize constraints, such as flow observables, to help narrow down the  $\eta/s(T)$  and such.

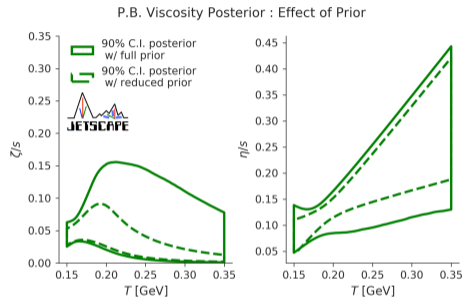
Testing a single set of parameters requires  $\mathcal{O}(10^4)$  hydro events, and evaluating eight different parameters five times each requires  $5^8 \times 10^4 \approx 10^9$  hydro events.

**That's roughly  $10^5$  CPU years!**

## BAYESIAN PARAMETER ESTIMATION

Duke T<sub>R</sub>ENTo+VISH(2+1D)+UrQMD

Steffen A. Bass *et al.*, Nature Physics (2019)

JETSCAPE T<sub>R</sub>ENTo+MUSIC+SMASH

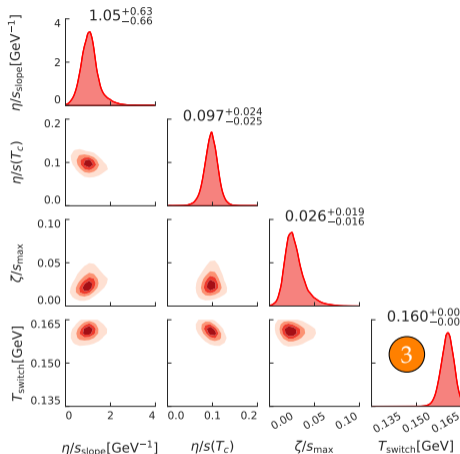
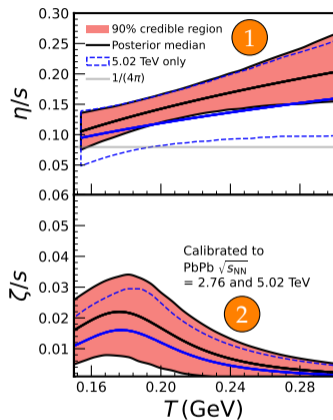
JETSCAPE Collaboration, PRC **103** (2021) 054904

- Large uncertainty for both  $\eta/s(T)$  and  $\zeta/s(T)$ .
- Subsequent studies with still limited observables:
  - J. Auvinen *et al.* PRC. **102**, 044911 (2020)
  - G. Nijs *et al.* PRL. **126**, 202301 (2021)

Uncertainties need to and can be further improved.

Only low-order harmonic  $v_n$  was used, including a limited set of mostly 2.76 TeV observables.

# RESULT: JYVASKYLA (2022) – COMBINED COLLISION ENERGY ANALYSIS (2.76 + 5.02 TeV)



- 1 Significantly improved  $\eta/s(T)$  uncertainty
- 2 Non-zero  $\zeta/s(T)$
- 3 Higher switching temperature  $T_{\text{switch}}$
- 4 Overall better convergence for parameter components

Together with two collision energies and added observables, the uncertainty has reduced!



# FIRST WHY THE UNCERTAINTY WAS REDUCED?

	Duke (2019) [3]	Jyvaskyla (2021) [1]	Jyvaskyla (2022) [2]
2.76 TeV	PID <sup>1</sup> mult. $N_{ch}$ PID <sup>1</sup> $\langle p_T \rangle$ Transverse energy $E_T$ $\delta p_T / \langle p_T \rangle$ $v_2$ to $v_4$		$N_{ch}$ PID <sup>1</sup> $\langle p_T \rangle$ $v_2$ to $v_4$ NSC(3,2), NSC(4,2) NSC(2,3,4), NSC(2,3,5) $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$
5.02 TeV	$N_{ch}$ $v_2$ to $v_4$	PID <sup>2</sup> mult. $N_{ch}$ PID <sup>1</sup> $\langle p_T \rangle$ $v_2$ to $v_4$ $v_5$ to $v_7$ NSC(3,2) to NSC(4,2) $\chi_{4,22}$ to $\chi_{6,mk}$	PID <sup>2</sup> mult. $N_{ch}$ PID <sup>1</sup> $\langle p_T \rangle$ $v_2$ to $v_4$ $v_5$ to $v_7$ NSC(3,2) to NSC(4,2) $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$

All reference data based on ALICE measurements.

Red: Missing from other group (Duke, etc)  
 Blue: New since our PRC.  
 Orange: Not used in our studies.

[1]. J.E. Parkkila, A. Onnerstad, D.J. Kim, PRC 104 (2021) 054904

<sup>1</sup> $\pi^\pm, K^\pm$  and  $p^\pm$

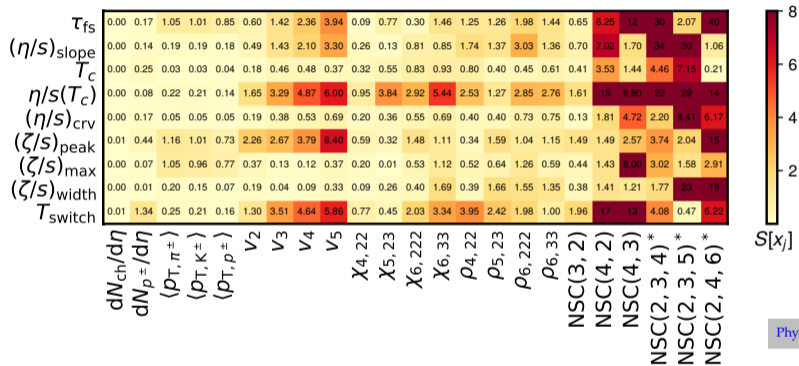
[2]. J.E. Parkkila, A. Onnerstad, M. Virta, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, Phys. Lett. B 835 (2022) 137485

<sup>2</sup> $p^\pm$

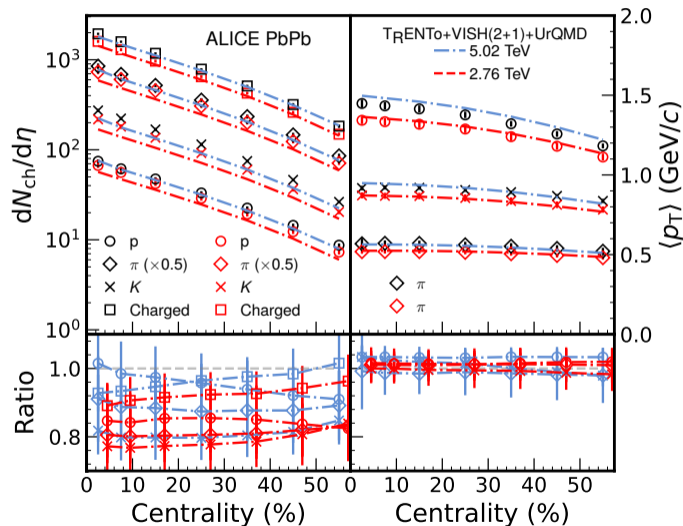
[3]. J.E. Bernhard et al, Nature Phys. 15, 1113-1117 (2019)

## SECOND WHY THE UNCERTAINTY WAS REDUCED?

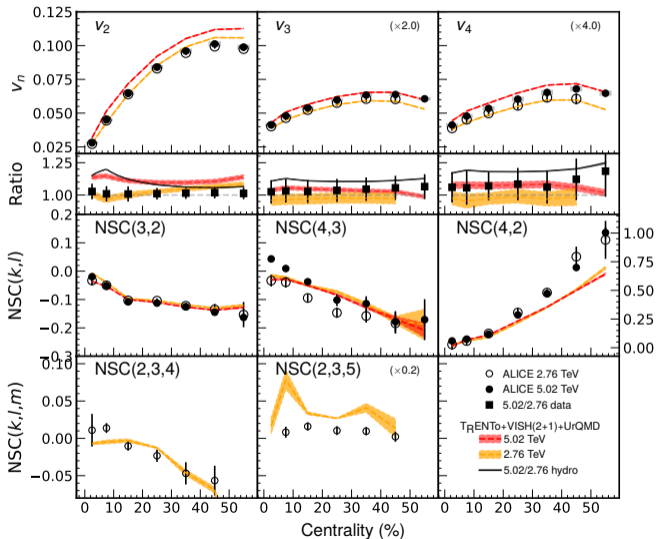
Sensitivity of the observables:  $S[x_j] = \Delta/\delta.$ , where  $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$ .



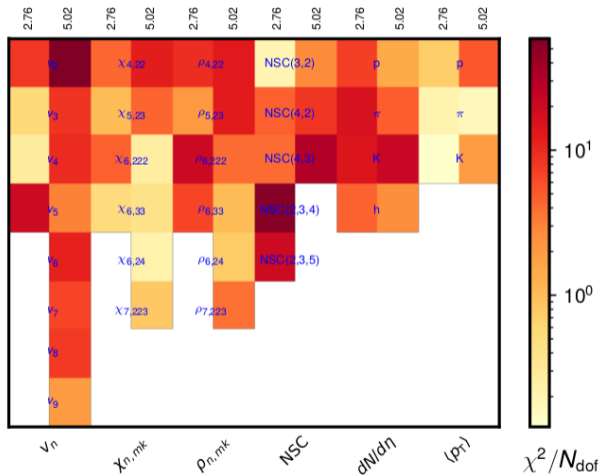
- $N_{p^\pm}/d\eta$  is sensitive to  $T_{switch}$  and  $\langle p_T \rangle$  is sensitive to  $\tau_{fs}$ .
- NSC( $m,n$ ) and NSC( $k,l,m$ ) are among the most sensitive observables followed by  $v_n$  and  $\chi_{n,mk}$ .
- The precision measurements of observables, reflecting mostly non-linear responses, are crucial.

PID MULTIPLICITY AND  $\langle p_T \rangle$ 

- Agreement for charged particle yield in 2.76 TeV and 5.02 TeV
- 10–20% difference for PID multiplicity
- Qualitative agreement for  $\langle p_T \rangle_{\pi,K}$

$v_n$  AND SYMMETRIC CUMULANTS

- Good agreement for 2.76 TeV  $v_n$ , overestimated  $v_2$  for 5.02 TeV by  $\sim 10\%$
- Magnitude and centrality dependence of NSC well captured. Further improved estimate for NSC(4,2).
- Good agreement for NSC(2,3,4). NSC(2,3,5) overestimated.

REMAINING CONCERNS?  $\chi^2$ -TEST

## NEW DEVELOPMENTS ON EXPERIMENTS

The precision measurements of observables, reflecting mostly non-linear responses, are crucial  
**New and Independent information!**

$$V_n \equiv v_n \{ \Psi_n \} e^{in(\Psi_n - \phi)}$$

1 Symmetric cumulants	$\langle v_m^2 v_n^2 \rangle_c, \langle v_k^2 v_l^2 v_m^2 \rangle_c, \text{"SC}(m,n)\text{"}$	additional information
2 Non-linear flow modes	relations of flow obs, $\text{"}\chi_{n,kl}\text{"}$	decomposition
3 Asymmetric cumulants	$\langle v_m^{2 \cdot a} v_n^{2 \cdot b} \rangle_c, \text{"AC}_{a,b}(m,n)\text{"}$	new moments
4 Symmetric plane Corr.	$\langle \cos(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k}) \rangle$	independent
5 Small systems	$v_n$ only from 2PC <div style="background-color: #008080; color: white; padding: 2px; display: inline-block;">Su-Jeong Ji On Thur</div>	jets are dominant

- Thanks to the higher order flow measurements!

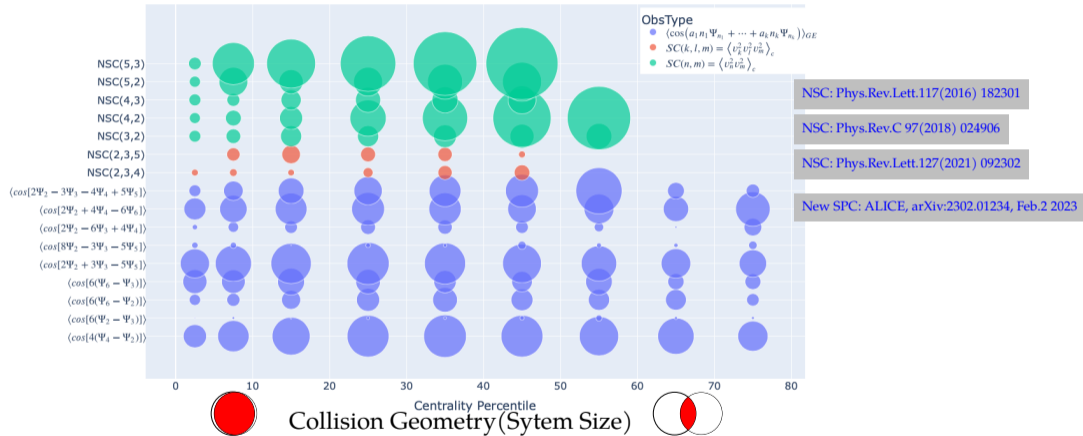
ALICE, Phys.Rev.Lett.116 (2016) 132302, JHEP05 (2020) 085

- Thank you: Flow Fluctuation!

ALICE, JHEP 07(2018) 103, 2018

## RECENT FLOW OBSERVABLES IN ONE FIGURE, 1 AND 4

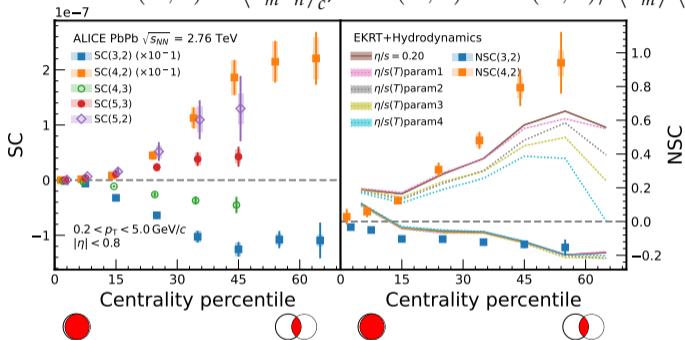
Pb–Pb 2.76 TeV



- Bubble size  $\propto$  correlation strength : few selected observables from 2.76TeV
- Varies with different harmonic orders and system size - Different sensitivities with independent information!
- Pb–Pb 5.02 TeV, [NL mode SC: JHEP05 \(2020\) 085, Phys.Lett.B818 \(2021\) 136354](#)

## HIGH PRECISION FLOW RESULTS AND NEW DEVELOPMENTS- 1 SYMMETRIC CUMULANTS

$$SC(m, n) = \langle v_m^2 v_n^2 \rangle_c, \quad NSC(m, n) = SC(m, n) / \langle v_m^2 \rangle \langle v_n^2 \rangle$$



$$SC(k, l, m) = \langle v_k^2 v_l^2 v_m^2 \rangle_c$$

ALICE, Phys. Rev. Lett. 127 (2021) 092302

- $\eta/s(T)$  and accessing  $\zeta/s(T)$
- Very challenging measurements because of their required high precisions (i.e  $10^{-6}$   $SC(m, n)$ ,  $10^{-12}$  for  $SC(k, l, m)$ ) and difficulties in correcting experimental biases.
- Accessing the temperature dependence of  $\eta/s(T)$

ALICE, Phys. Rev. Lett. 117 (2016) 182301, Editors' Suggestion

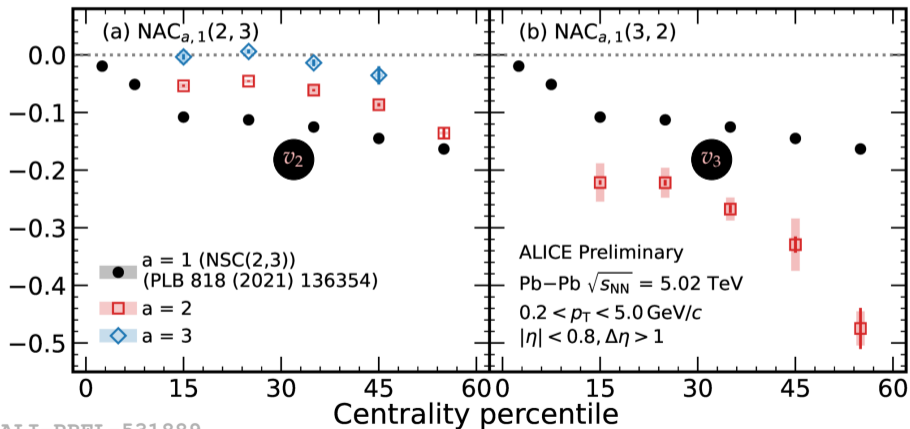
ALICE, Phys. Rev. C 97 no. 2, (2018) 024906



# ASYMMETRIC CUMULANTS: 3 WHAT ABOUT DIFFERENT MOMENTS OF $v_n$ ?

- Generalized symmetric cumulants with different moments - additional information! (Skewness, Kurtosis)
- First measurements in Pb-Pb collisions at 5.02 TeV.

$$\text{NAC}_{a,1}(m, n) = \langle v_m^{2 \cdot a} v_n^{2 \cdot 1} \rangle_c / \langle v_m^2 \rangle^a \langle v_n^2 \rangle^1, \quad \text{NAC}_{1,1}(m, n) = \text{NSC}(m, n)$$



ALI-PREL-531889

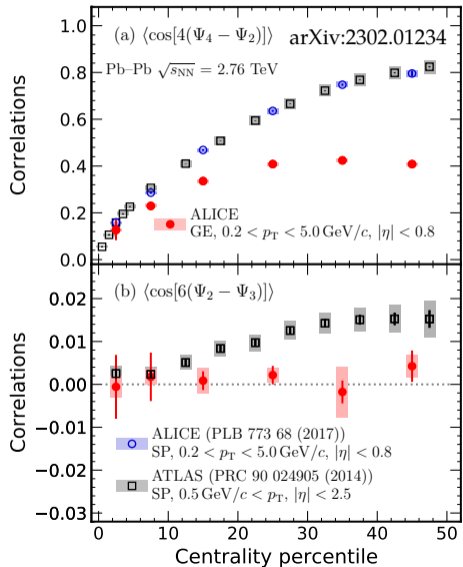
## 4 IMPROVED SYMMETRY PLANES CORRELATIONS (SPC)

- Paper for 2.76 TeV ALICE, arXiv:2302.01234, Feb.2 2023
- Previously done with scalar product (SP) method (ATLAS: PRC 90, 024905 (2014), ALICE, PLB 773: 68 (2017) [J.E. Parkkila, D.J. Kim])
- Separating effects from  $v_m$ - $v_n$  correlations

$$\langle \cos(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k}) \rangle_{\text{GE,SP}}$$

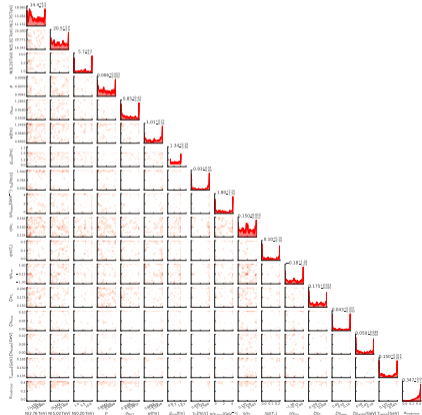
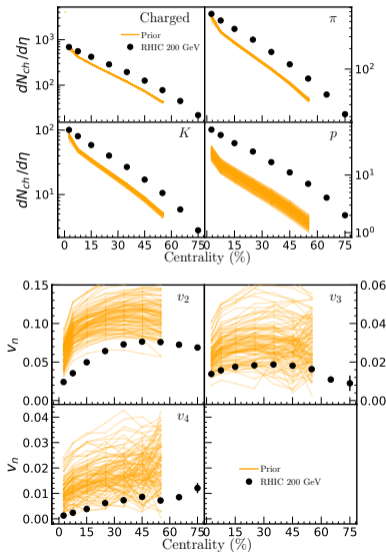
$$\approx \sqrt{\frac{\pi}{4}} \frac{\langle v_{n_1}^{a_1} \dots v_{n_k}^{a_k} \cos(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k}) \rangle}{\sqrt{\langle v_{n_1}^{2a_1} \dots v_{n_k}^{2a_k} \rangle}},$$

$$= \frac{\langle v_{n_1}^{a_1} \dots v_{n_k}^{a_k} \cos(a_1 n_1 \Psi_{n_1} + \dots + a_k n_k \Psi_{n_k}) \rangle}{\sqrt{\langle v_{n_1}^{2a_1} \rangle \dots \langle v_{n_k}^{2a_k} \rangle}},$$

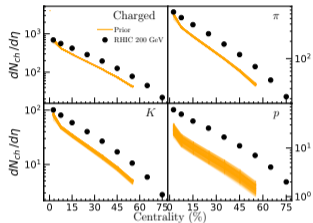
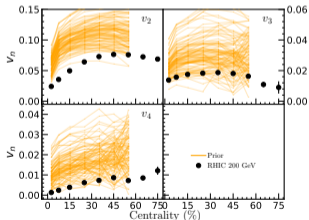
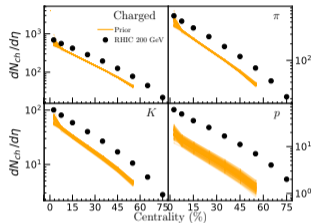
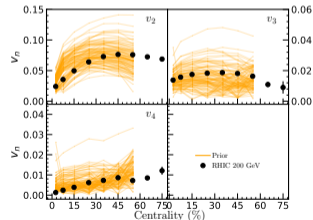


## INCLUSION OF RHIC DATA

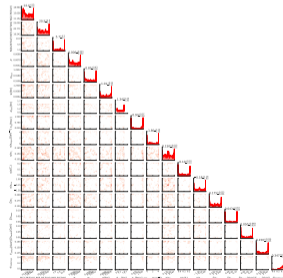
- Included observables :  $v_2 - v_4$ ,  $\langle p_T \rangle$  and  $N_{ch}$  for charged and identified particles
- Prior doesn't extend well over the data points
- Posterior distributions don't converge

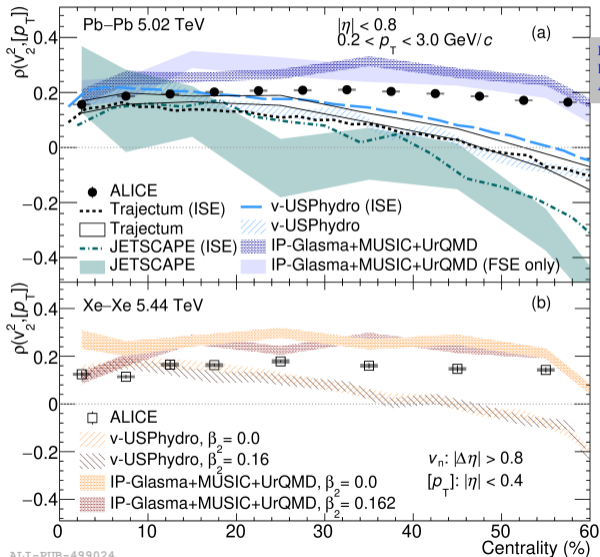


## INCLUSION OF RHIC DATA

Fixed  $w_{nucl} = 0.5$  $w_{nucl} = [0.67-1.24]$ Fixed  $w_{nucl} = 0.5$  $w_{nucl} = [0.67-1.24]$ 

- Included observables :  $v_2 - v_4$ ,  $\langle p_T \rangle$  and  $N_{ch}$  for charged and identified particles
- Fixed nuclen width  $w = 0.5$  and now relaxed (0.67-1.24)
- small improvements for  $N_{ch}$  and better for  $v_n$
- Posterior distributions don't converge still



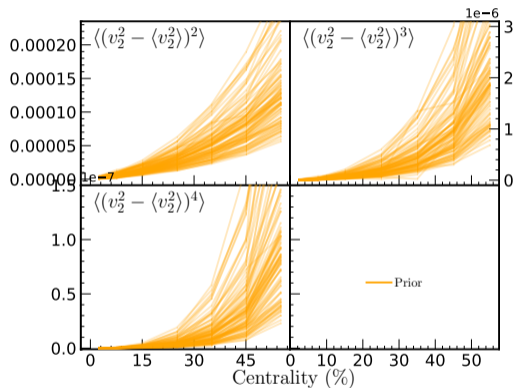
$v_n, [p_T]$  CORRELATION - SHORTAGE OF  $T_{RENT0}$  MODEL

P. Bozek, R. Samanta, Phys. Rev. C 102, 034905  
 B. Schenke, C. Shen, D. Teanev, Phys. Rev. C 102, 034905  
 ALICE, Phys.Lett.B 834 (2022) 137393

$$\rho(v_2^2, [p_T]) = \frac{\langle \delta v_2^2 \delta [p_T] \rangle}{\sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta [p_T])^2 \rangle}} \quad (1)$$

- Correlation between  $[p_T]$  and  $v_2$ :
  - can be used to differentiate initial state models
  - More peripheral  $\rightarrow$  best described by models with IP-Glasma
  - strong centrality dependence on the models with Trento
- Ongoing progress:
  - Calculate sensitivity
  - Adapt it to the Bayesian Analysis

# MOMENTS OF $\delta v_n$ AND $\delta p_T$

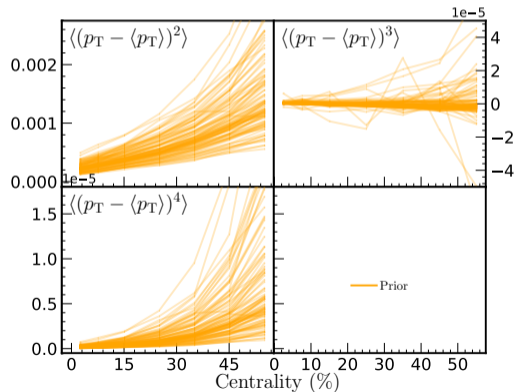


■ Characterizes the fluctuation of different order

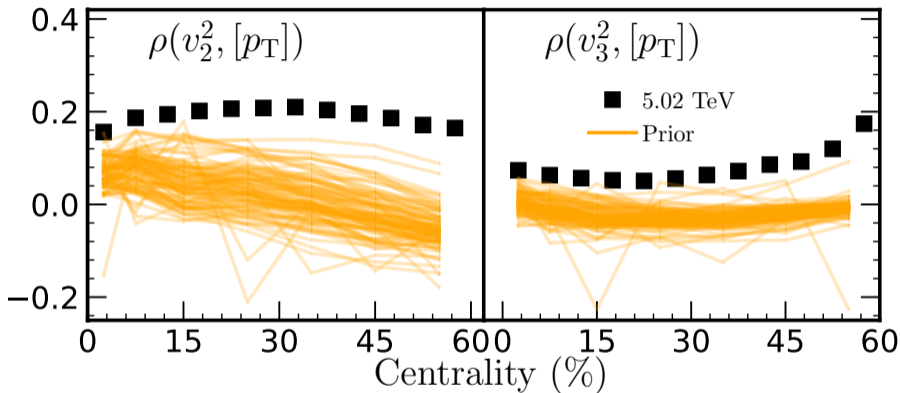
■  $|\eta| < 0.8$  and  $0.2 \text{ GeV} < p_T < 5.0 \text{ GeV}$

## Moments of a distribution

**Variance:**  $(X - \mu)^2$ , **skewness:**  $(X - \mu)^3$   
**and kurtosis:**  $(X - \mu)^4$



# PRIOR DISTRIBUTION OF $\rho(v_n^2, [p_T])$



## $v_n^2, [p_T]$ correlation

$$\rho(v_n^2, [p_T]) = \frac{\langle (v_n^2 - \langle v_n^2 \rangle) ([p_T] - \langle [p_T] \rangle) \rangle}{\sqrt{\langle (v_n^2 - \langle v_n^2 \rangle)^2 \rangle \langle ([p_T] - \langle [p_T] \rangle)^2 \rangle}}$$

- Clear centrality dependence
- Gains negative values - what is going on?

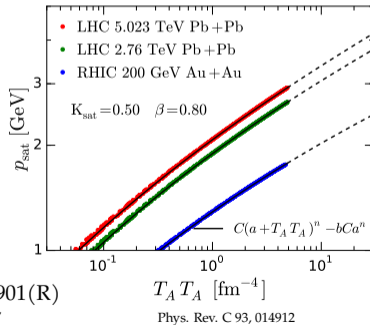
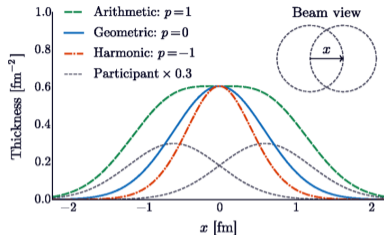
ALICE, Phys. Lett. B 834 (2022) 137393

INITIAL STATE MODELS, T<sub>R</sub>ENTo vs. EKRT: ONGOINGT<sub>R</sub>ENTo[1]

- Flexibility to produce some other models
- Unable to predict ( $\sqrt{s_{NN}}$  - Cent) dependence
- Has six free parameters

## EKRT [2]

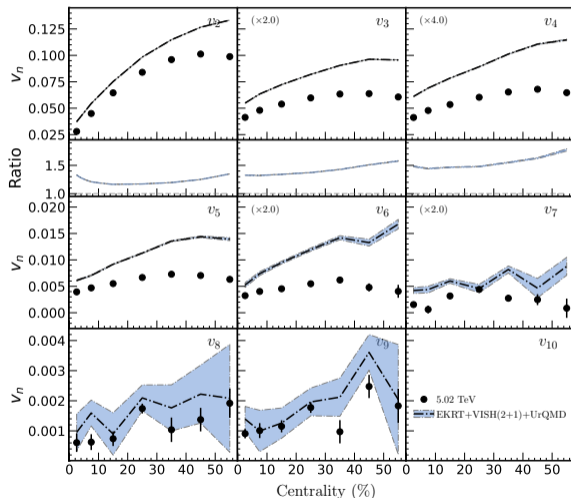
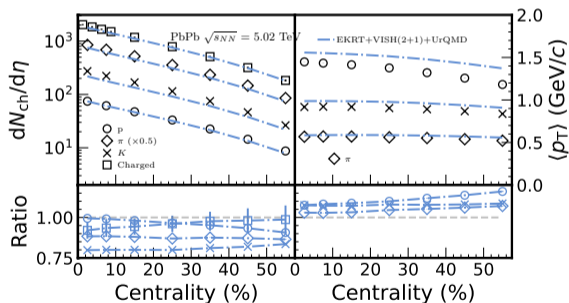
- Only two free parameters,  $K_{sat}$  and  $\beta_{sat}$
- ( $\sqrt{s_{NN}}$  - Cent) dependence comes automatically from the gluon saturation and mini-jet production
- Computationally a bit heavier  $\rightarrow$  much improved via ML recently

[1]. J. S. Moreland, J. E. Bernhard, and S. A. Bass, PRC **92** 011901(R)[2]. H. Niemi, K. J. Eskola, and R. Paatelainen, PRC **93** 024907 $T_A T_A$  [fm<sup>-4</sup>]Phys. Rev. C **93**, 014912



## EKRT PbPb 5.02 TeV RESULTS

- Particle distributions relatively well described
- $\langle p_T \rangle$  a bit overestimated
- Large overestimation in flow harmonics



## SUMMARY

### Success:

- More Beam energies and new sensitive flow data → better understanding of QCD matter
- Importance of measuring new independent observables - good progress

### Challenges:

- Current status
  - 10% difference for  $v_2$  (5.02 TeV) and  $\rho(v_2^2, [p_T])$
  - still lacking for NSC(4,2)
  - Remaining discrepancy for PID multiplicity (especially  $\pi^\pm$ ) ...
  - better understanding/constraints on initial conditions are challenged!
- Improving the initial conditions with
  - EKRT, IP+Glasma
  - or nucleon size
  - better understanding of proton

Thank you for your attention!

## TOUGH QUESTIONS TO ANSWER BUT WE DO HAVE ANSWERS

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1. Q — What is the use of measuring correlations?

A — Additional and independent information than  $v_n$  (event averaged) because of the flow fluctuations

2. Q — Even if each observable provide independent information, how do we know it is useful information?

A — Based on the known constraints, sensitivities are quantified. More sensitive, better constraints!

3. Q — Is there a limit on how many independent observables we can add into the Bayesian analysis?

A — Please measure as many observables as you can as long as they are independent. Even though there are overlapping information, it is okay to include

# SUMMARY AND OUTLOOK

## Experiments

- RHIC data (AuAu collisions) - Energy and system size dependence
- LHC pPb and pp data - System size dependence but with improved method

A. Öennerstad

- Use new observables
  - Higher order ( $n > 5$ ) Symmetric cumulants
  - Improved Symmetric Plane Correlation (SPC) : independent from flow magnitude correlations and Asymmetric Cumulants (AC)

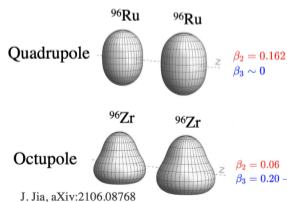
C. Mordasini

- Soft-Hard interaction
- What about isobar runs in LHC?

(WCPF2022, J, Jia)

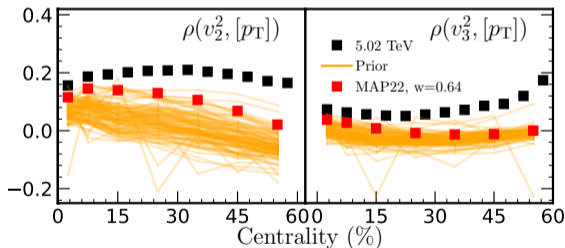
## Theory

- Improving the initial conditions with
  - EKRT, IP+Glasma
  - or nucleon size
  - better understanding of proton
- Testing hydro limit of small systems?
- Role of the small system for further

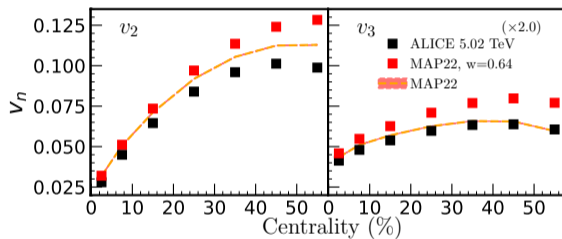


# THE EFFECTS OF NUCLEON WIDTH PARAMETER

- Nucleon width should be less than 0.8
- Jyväskylä 2022 estimates  $w > 0.8$
- Use Jyväskylä 2022 MAP with nucleon width  $w = 0.64$ , PhysRevLett.129.232301



## Validation of the parametrization

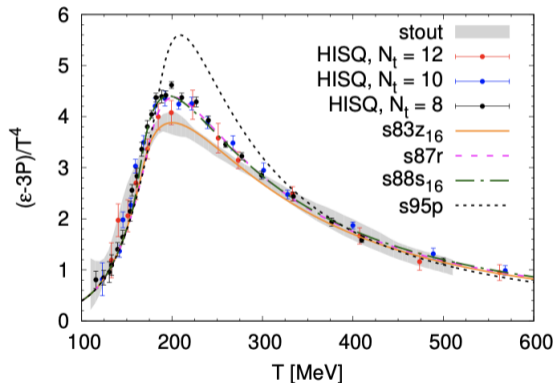


- Reduction only in  $w$  guides towards correct  $\rho$  but drifts away from experimental  $v_n$ s
- $|\eta| < 0.8$  and  $0.2 \text{ GeV} < p_T < 5.0 \text{ GeV}$

# QUESTIONS TO THINK ABOUT DURING THIS CONFERENCE?

## Effect of EoS to Bayesian analysis

- Uncertainties from the equation of state?

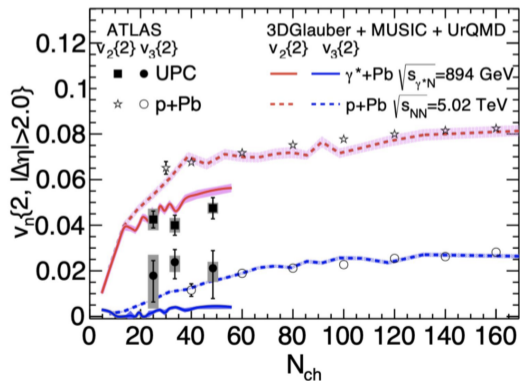


P. Huovinen, P. Petreczky, Nucl.Phys.A837:26-53,2010

PoS(Confinement2018)135

## Zero flow at zero multiplicity?

- What does it mean by seeing non-zero flow at zero multiplicity?

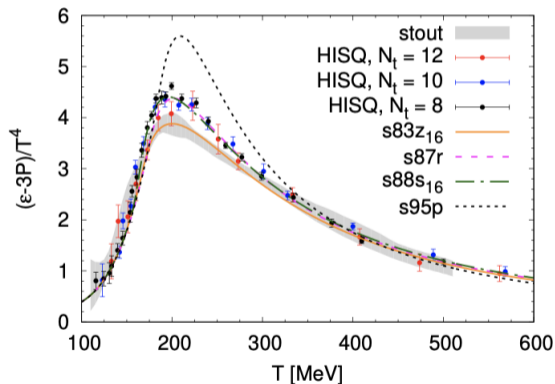


W. Zhao, C. Shen, B. Schenke, arXiv:2203.06094

## TWO QUESTIONS TO THINK ABOUT DURING THIS CONFERENCE?

### Effect of EoS to Bayesian analysis

- Uncertainties from the equation of state?

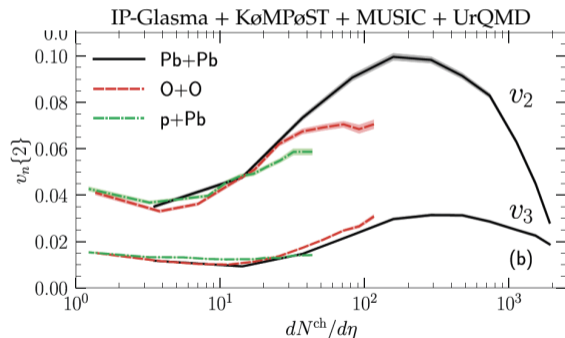


P. Huovinen, P. Petreczky, Nucl.Phys.A837:26-53,2010

PoS(Confinement2018)135

### Zero flow at zero multiplicity?

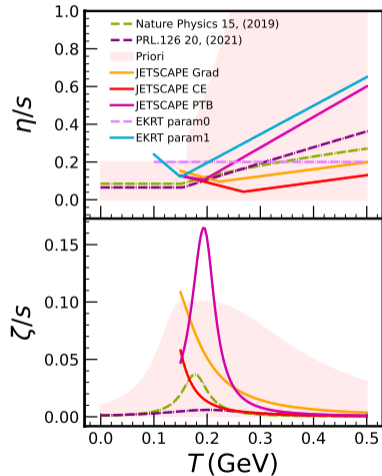
- What does it mean by seeing non-zero flow at zero multiplicity?



B. Schenke et. al, PRC, 105, 014909 (2022)

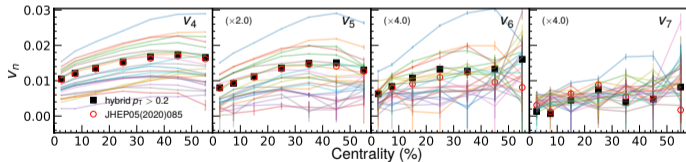


## ANALYSIS STEPS AND PRIORI

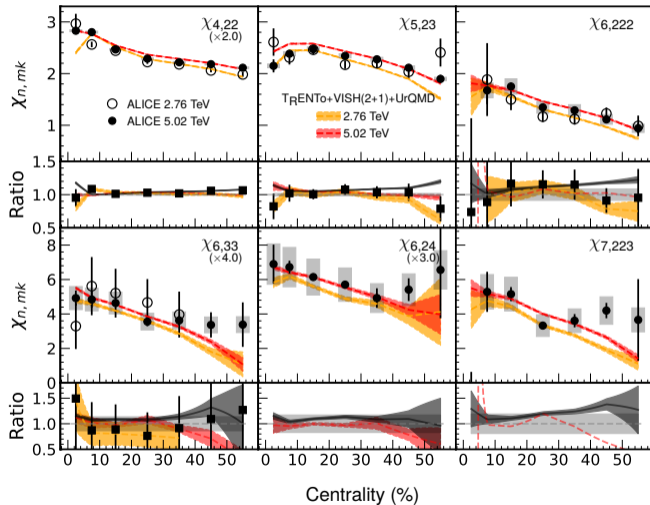


PRC 104 (2021) 054904

- 1 Choose prior parameter range based on results from 2019
- 2 Run hydro  $T_{\text{RENT}} + \text{VISH}(2+1\text{D}) + \text{UrQMD}$  for 500 parameterizations, 3-5 million events ( $\times 100$  previous).
- 3 Calculate observables using our experimental framework
- 4 Train emulator and setup/run Bayesian analysis



## NON-LINEAR FLOW COEFFICIENTS



- Qualitative agreement in both beam energies for all mode coupling coefficients.
- See arXiv:2111.08145 for all graphs.