Shear viscosity of parton matter under 2-body scatterings

Zi-Wei Lin
East Carolina University (ECU)

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Outline

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- Isotropic versus forward-angle scatterings
- Comparison of η and η/s from different methods
- Application to parton matter in the AMPT model
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National
Science
Foundation

Based on
Noah MacKay & ZWL,
Eur Phys J C 82, 918 (2022)

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Motivation

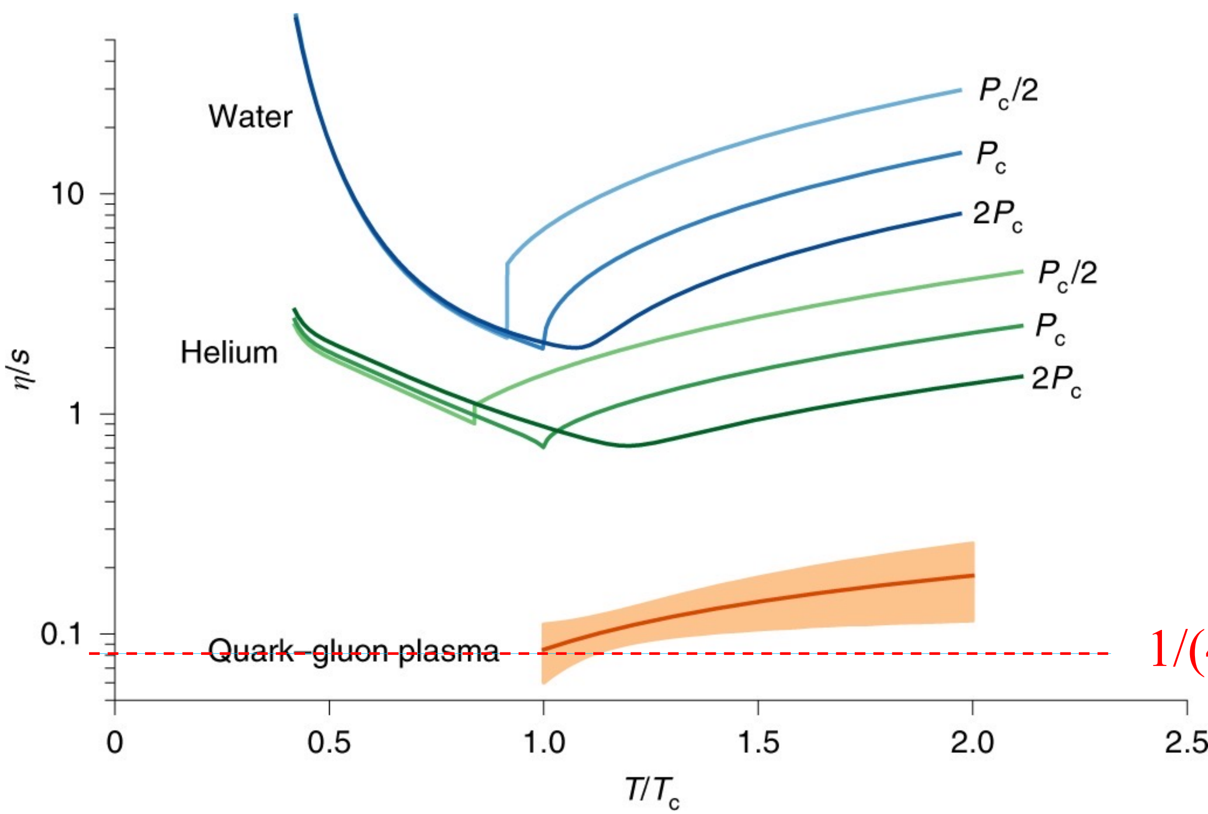
Shear viscosity η is an important property of the quark–gluon plasma:

$$T^{\mu\nu} = eu^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

η *or* η/s: is an input function to viscous hydrodynamics;

is generated by interactions in transport models: relation?

η/s can be extracted from data/model comparisons:



Bernhard, Moreland & Bass, Nature Phys (2019)

$1/(4\pi)$ bound from AdS/CFT

Kovtun, Son & Starinets, Phys Rev Lett (2005)

Isotropic versus forward-angle two-body scatterings

In this study, we only consider a massless parton matter in thermal equilibrium under 2-to-2 elastic scatterings.

- Isotropic scattering: $$\frac{d\sigma}{d\Omega} = constant = \frac{\sigma}{4\pi}$$

- Forward-angle scattering:
  As the example, we take the parton cross section used in AMPT/ZPC/MPC:

$$\frac{d\sigma}{d\hat{t}} = \frac{9\pi\alpha_s^2}{2}(1+a)\frac{1}{(\hat{t}-\mu^2)^2}$$

$a \equiv \frac{\mu^2}{\hat{s}}$ is added to obtain a $\hat{s}$-independent cross section $\sigma = \frac{9\pi\alpha_s^2}{2\mu^2}$

Based on the pQCD gg-gg cross section:

$$\frac{d\sigma}{d\hat{t}} \propto 3 - \frac{\hat{t}\,\hat{u}}{\hat{s}^2} - \frac{\hat{s}\,\hat{u}}{\hat{t}^2} - \frac{\hat{s}\,\hat{t}}{\hat{u}^2}$$

+ screening mass μ

Isotropic versus forward-angle two-body scatterings

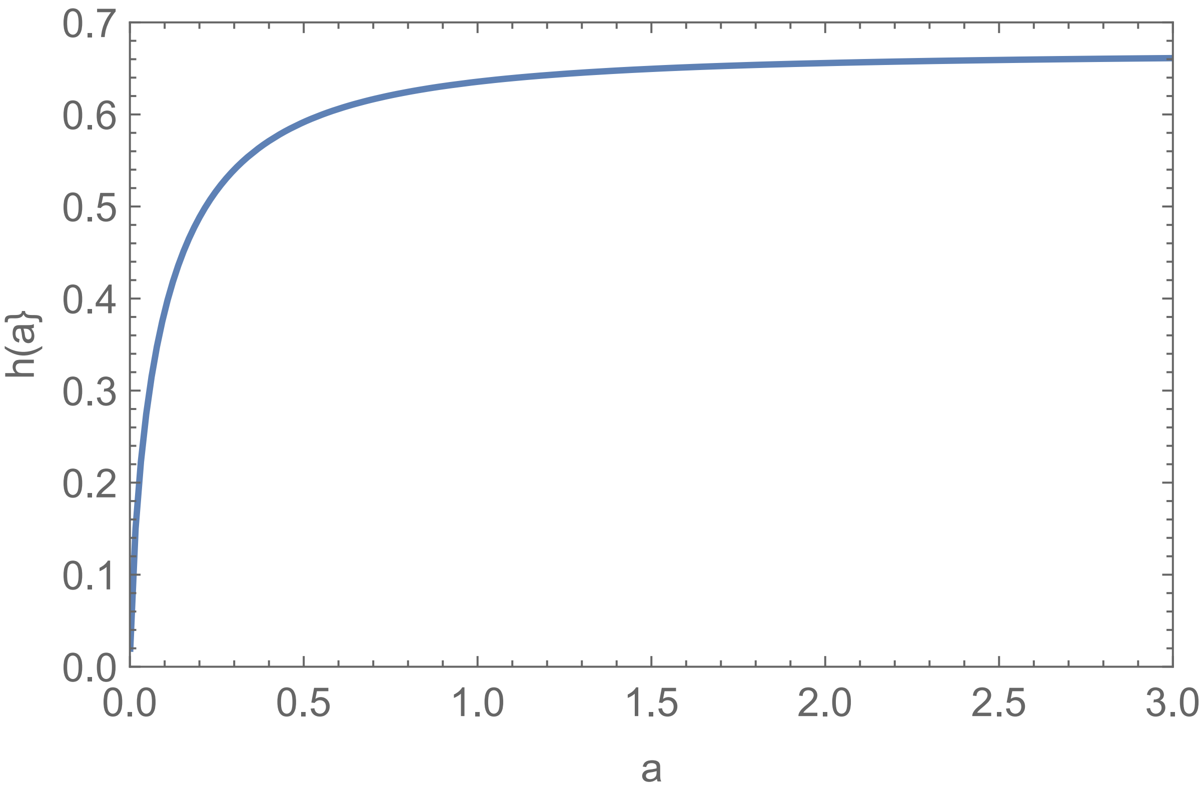
Transport cross section σ_{tr} often appears in shear viscosity expressions:

$$\sigma_{tr} = \int d\sigma \sin^2\theta_{cm}$$

- Isotropic scattering: $\sigma_{tr} = \frac{2}{3}\sigma$
- Forward-angle scattering: $\sigma_{tr} = 4a(1+a)\left[(1+2a)\ln\left(1+\frac{1}{a}\right) - 2\right]\sigma$

$$\equiv h(a)\,\sigma$$

the anisotropy function



$a \equiv \frac{\mu^2}{\hat{s}}$:

$\gg 1$ ~isotropic, *h(a)*→2/3

$\ll 1$ very forward

h(a)→0*: scatterings are less effective*

Isotropic versus forward-angle two-body scatterings

Thermal average:

even if σ is a constant, σ_{tr} is not since it depends $a \equiv \frac{\mu^2}{\hat{s}}$.

For a parton matter in thermal equilibrium at temperature T,
the thermal average for Boltzmann statistics is Kolb & Raby, Phys Rev D (1983)

$$\langle \sigma_{tr} \rangle = \frac{\sigma}{32} \int_0^\infty du \, [\, u^4 K_1(u) + 2u^3 K_2(u)] \, h\left(\frac{w^2}{u^2}\right)$$
$$\equiv \sigma \, \boldsymbol{h_0(w)}$$

$$w \equiv \frac{\mu}{T}$$
$$u \equiv \frac{\sqrt{\hat{s}}}{T}$$

$h_0(w)$ is just an average of the anisotropy function *h(a)*

Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{6T}{5\sigma}$ for isotropic scatt.

Huovinen & Molnar, Phys Rev C (2009)

- Navier–Stokes (NS) method: $\eta^{NS} \approx 1.2654 \frac{T}{\sigma}$ for isotropic scatt.
  de Groot, van Leeuwen & Weert book (1980)
- Relaxation time approximation (RTA) & modified version (RTA*):

$$\eta^{RTA} = \frac{4T}{5\sigma} \qquad \eta^{RTA*} = \frac{6T}{5\sigma}$$ for isotropic scatt.

Anderson & Witting, Physica (1974) Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\,c_{00}}, \quad \ldots$

Wiranata & Prakash, Phys Rev C (2012)

Numerical:

- Green–Kubo relation: $\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$

τ: relaxation time extracted from correlation <…>

Comparison of η and η/s from different methods

Analytical:

- Israel–Stewart (IS) method: $\eta^{IS} = \frac{4T}{5\langle\sigma_{tr}\rangle} = \frac{4T}{5\sigma\, h_0(w)}$

generalized to anisotropic scatt.
Huovinen & Molnar, Phys Rev C (2009)

- Navier–Stokes (NS) method: $\eta^{NS} \approx 0.8436 \frac{T}{\langle\sigma_{tr}\rangle}$

- Relaxation time approximation (modified version RTA*):

$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr} v_{rel}\rangle}$$

Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)

- Chapman–Enskog (CE) method: $\eta^{CE} = \frac{T\,\gamma_0^2}{10\,c_{00}}, \quad \ldots$

Wiranata & Prakash, Phys Rev C (2012)

Numerical:

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τ: relaxation time extracted from correlation <…>

Comparison of η and η/s from different methods

More on analytical methods:

MacKay & ZWL, Eur Phys J C (2022)

- Relaxation time approximation (modified version RTA*):

$$\eta^{RTA*} = \frac{4T}{5\langle\sigma_{tr} v_{rel}\rangle} \rightarrow \frac{4T}{5\sigma h_1(w)}$$

$$\langle\sigma_{tr} v_{rel}\rangle = \frac{8z}{K_2^2(z)} \int_1^\infty dy\, y^2(y^2-1) K_1(2zy) \int d\sigma \sin^2\theta_{cm}$$ in general ($z=m/T$)

$$\rightarrow \frac{\sigma}{16} \int_0^\infty du\, u^4 K_1(u)\, h\left(\frac{w^2}{u^2}\right) \equiv \sigma h_1(w)$$ for massless partons & AMPT $\frac{d\sigma}{d\hat{t}}$

- Chapman–Enskog (CE) method: $$\eta^{CE} = \frac{T\gamma_0^2}{10\, c_{00}} \rightarrow \frac{4T}{5\sigma h_2(w)}$$

$$\frac{8c_{00}}{\gamma_0^2} = \frac{32z^3}{25K_3^2(z)} \int_1^\infty dy (y^2-1)^3 \left[\left(y^2 + \frac{1}{3z^2}\right) K_3(2zy) - \frac{y}{z} K_2(2zy)\right] \int d\sigma \sin^2\theta_{cm}$$ in general

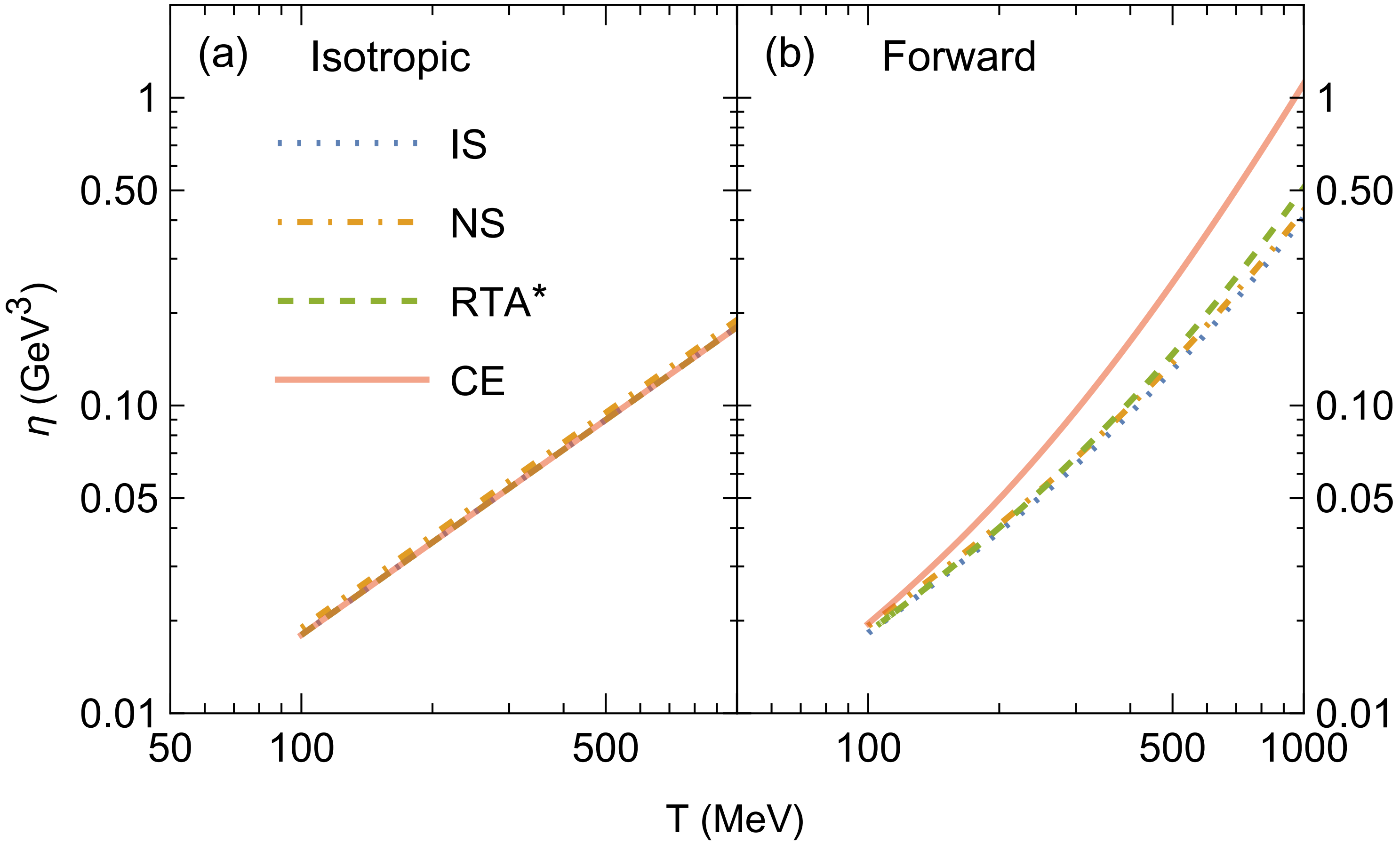
$$\rightarrow \frac{\sigma}{6400} \int_0^\infty du\, u^6 \left[\left(\frac{u^2}{4} + \frac{1}{3}\right) K_3(u) - \frac{u}{2} K_2(u)\right] h\left(\frac{w^2}{u^2}\right) \equiv \sigma h_2(w)$$ for massless partons & AMPT $\frac{d\sigma}{d\hat{t}}$

$h_1(w)$ & $h_2(w)$ are different averages of the anisotropy function *h(a)*

Comparison of η and η/s from different methods

Analytical results of η

for massless gluons & σ=2.6mb (or μ~0.7GeV):



- For isotropic scatterings:

IS=RTA*=CE

≈NS (~5% higher)

- For forward scatterings:

IS≈RTA*≈NS < CE mostly

T<<μ → almost isotropic

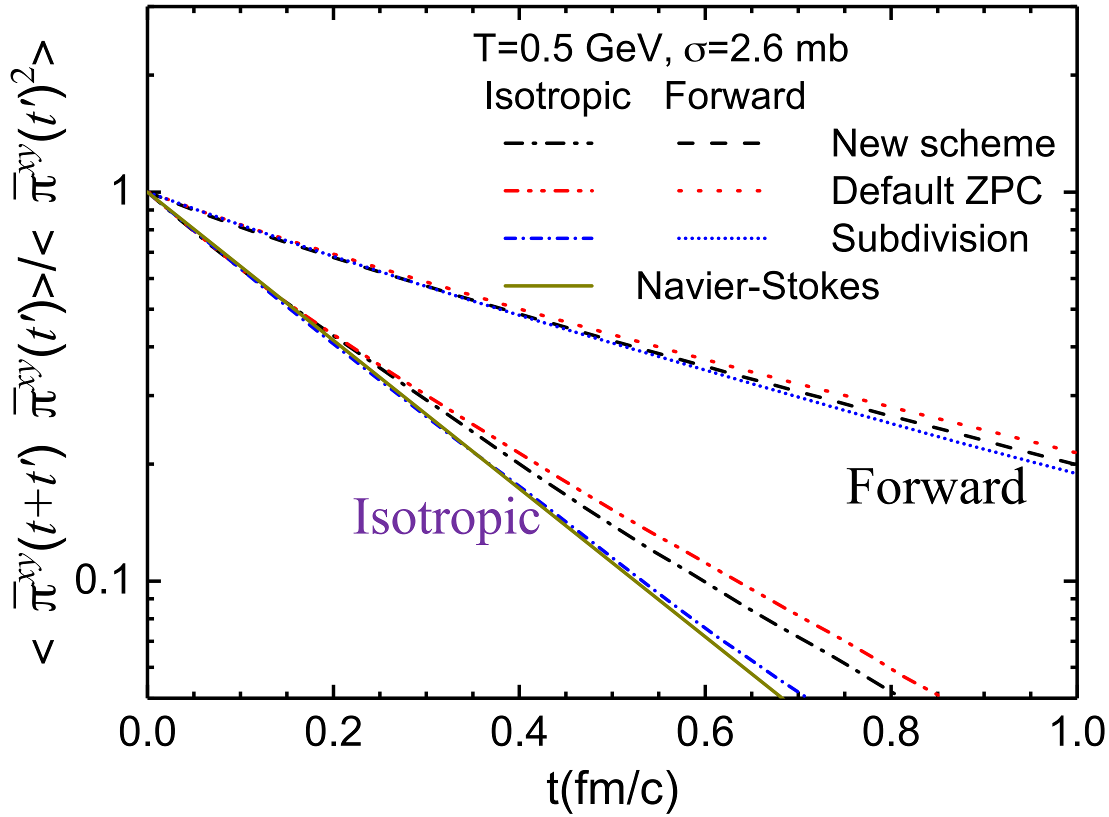
Comparison of η and η/s from different methods

Q: which analytical result of η is accurate?

A: compare with numerical results from Green-Kubo:

$$\eta = \frac{V}{T}\int_0^\infty dt < \bar{\pi}^{xy}(t+t')\bar{\pi}^{xy}(t') > = \frac{4}{15}\varepsilon\tau$$

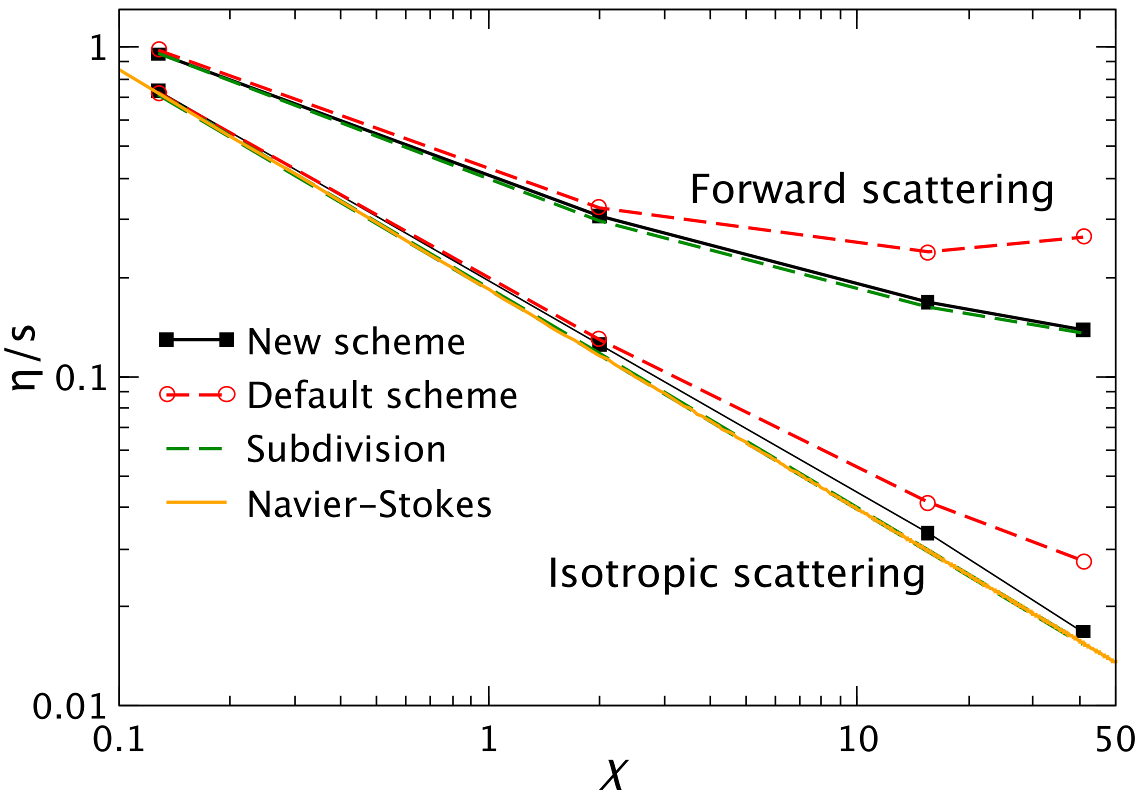
- With ZPC parton cascade, we calculated η of gluons in a box with the Green-Kubo relation.
- Subdivision method (with l=10^6) agrees well with NS expectation for isotropic scatterings.



Zhao, Ma, Ma & ZWL, Phys Rev C (2020)

Comparison of η and η/s from different methods

We extracted η/s of gluons in a box versus χ with Green-Kubo relation:



Zhao, Ma, Ma & ZWL, Phys Rev C (2020)

χ (opacity parameter):
= radius of interaction / mean free path

$$\chi = \sqrt{\frac{\sigma}{\pi}}/\lambda = n\sqrt{\frac{\sigma^3}{\pi}}$$

Zhang, Gyulassy & Pang, Phys Rev C (1998)

For fixed α_s,
η/s is only a function of χ.

For example:

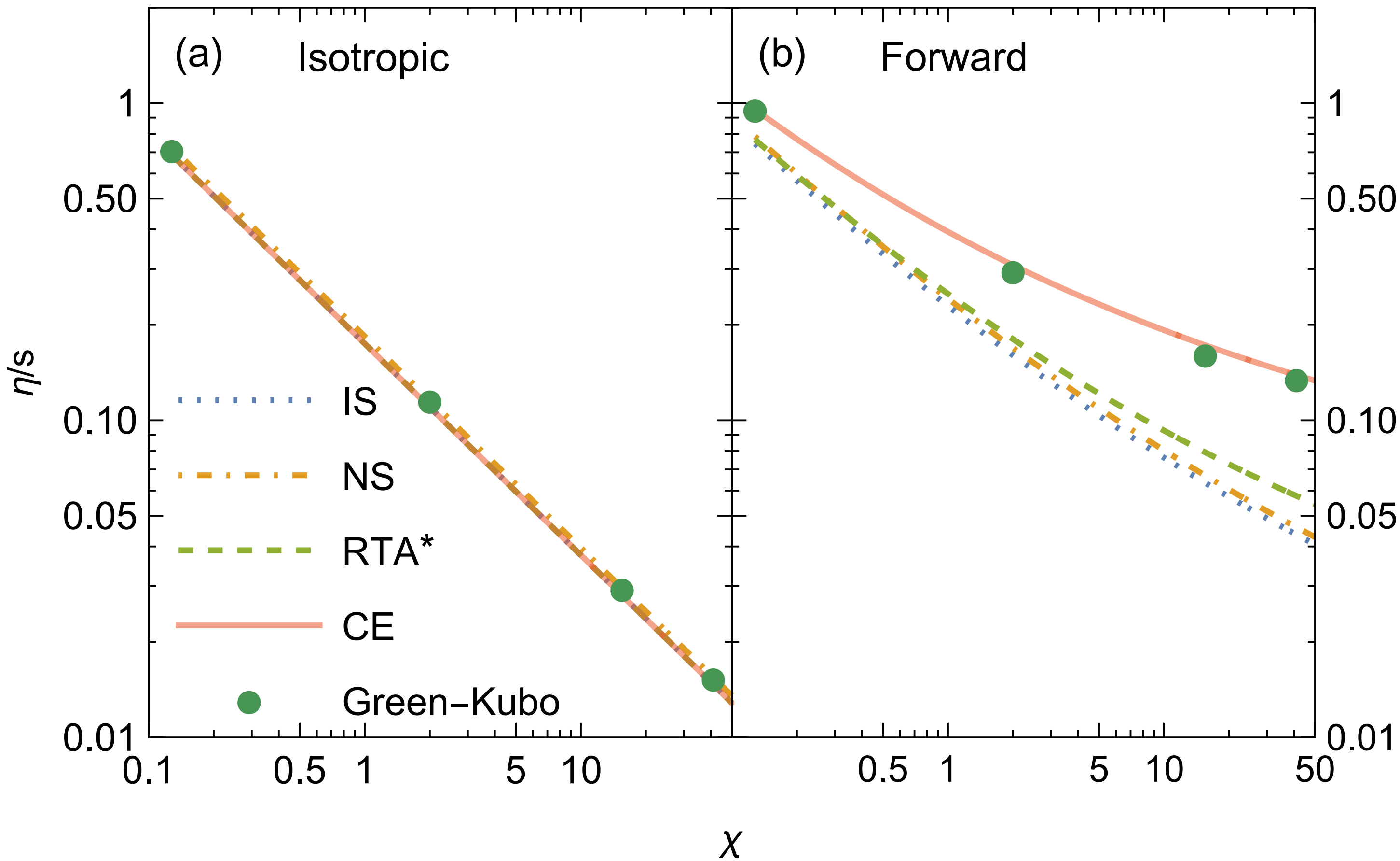
$$\left(\frac{\eta}{s}\right)^{NS} \simeq \frac{0.4633}{d_g^{1/3}\chi^{2/3}} = \frac{0.1839}{\chi^{2/3}}$$

for gluons (d_g=16)
under isotropic scatterings

Comparison of η and η/s from different methods

4 analytical methods vs Green-Kubo results of
η/s versus χ:

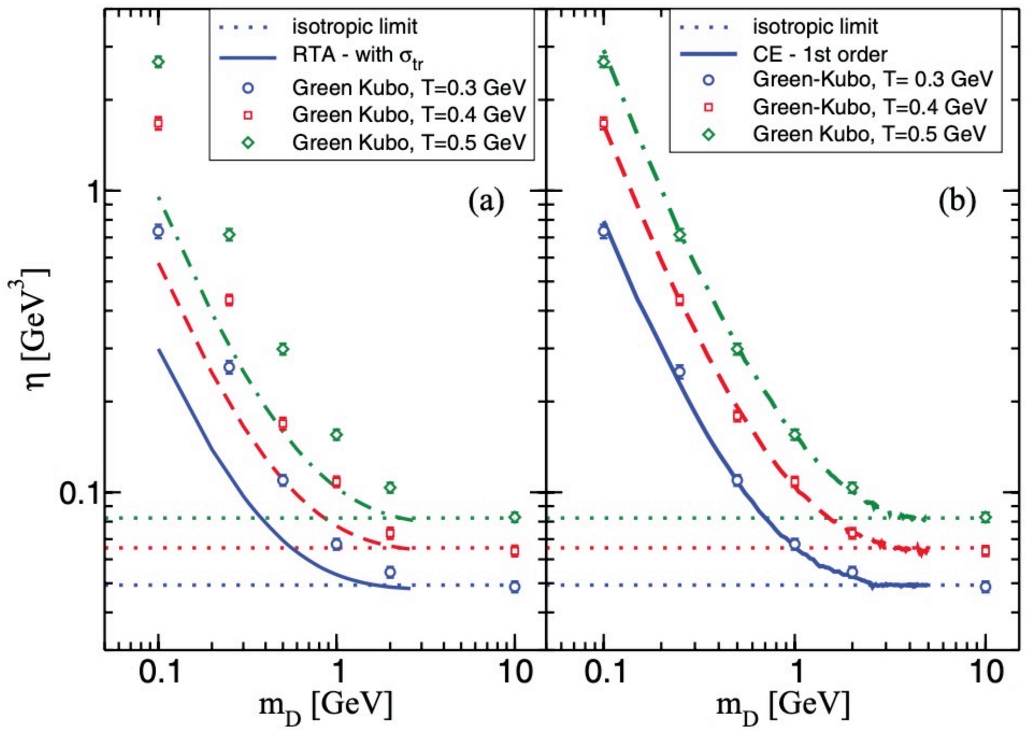
MacKay & ZWL, Eur Phys J C (2022)



- All methods agree well for isotropic scatterings
- For anisotropic scatterings: CE results agrees well with Green-Kubo; but the other analytical methods are not accurate
- η/s decreases with χ & T: due to constant σ

Comparison of η and η/s from different methods

The fact that Green-Kubo agrees with CE (but not RTA*) has been shown in
Plumari, Puglisi, Scardina & Greco, Phys Rev C (2012)



- Relaxation time approximation (modified version RTA*):

$$\langle h(a)\, v_{\rm rel}\rangle = \frac{8z}{K_2^2(z)} \int_1^\infty dy\, y^2 (y^2-1)\, h(2zy\,\bar{a})\, K_1(2zy) = f(z,\bar{a}), \quad (35)$$

$$\eta^*_{\rm RTA} = 0.8 \frac{1}{f(z, \frac{T}{m_D})} \frac{T}{\sigma_{\rm tot}}, \quad (36)$$

- Chapman–Enskog (CE) method:

$$[\eta_s]^I_{CE} = 0.8 \frac{1}{g(z,\bar{a})} \frac{T}{\sigma_{\rm tot}}, \quad (37)$$

with

$$g(z,\bar{a}) = \frac{32}{25} \frac{z}{K_3^2(z)} \int_1^\infty dy\, (y^2-1)^3\, h(2zy\,\bar{a}) \times [(z^2y^2+1/3)K_3(2zy) - zy\, K_2(2zy)]. \quad (38)$$

However, there are 2 typos in those η results, as pointed out in MacKay & ZWL, Eur Phys J C (2022):

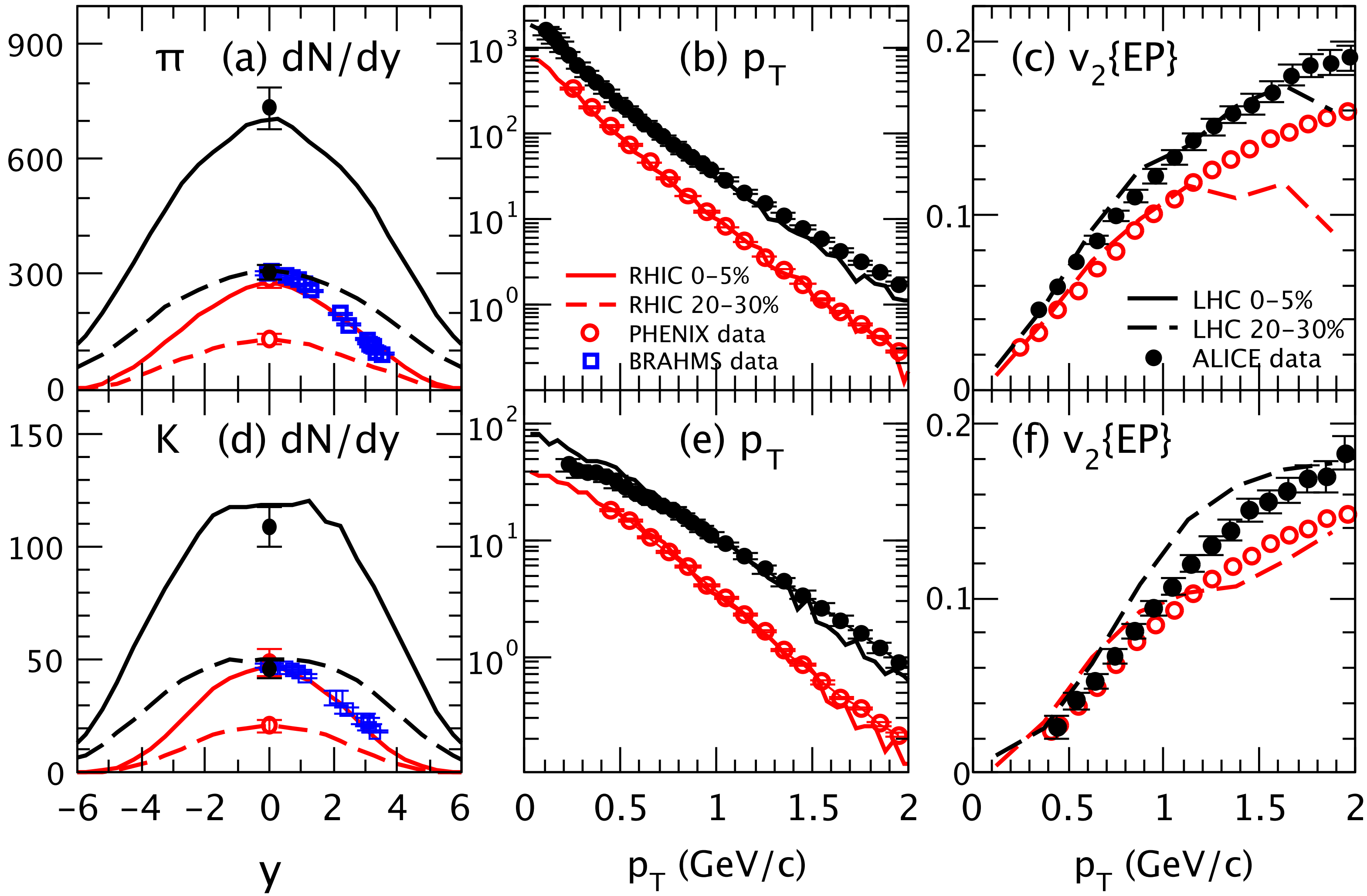
should be $h(1/(2zy\bar{a})^2)$

Application to parton matter in the AMPT model

We then apply the Chapman–Enskog (CE) method to study η and η/s of the parton matter in string melting AMPT for A+A.

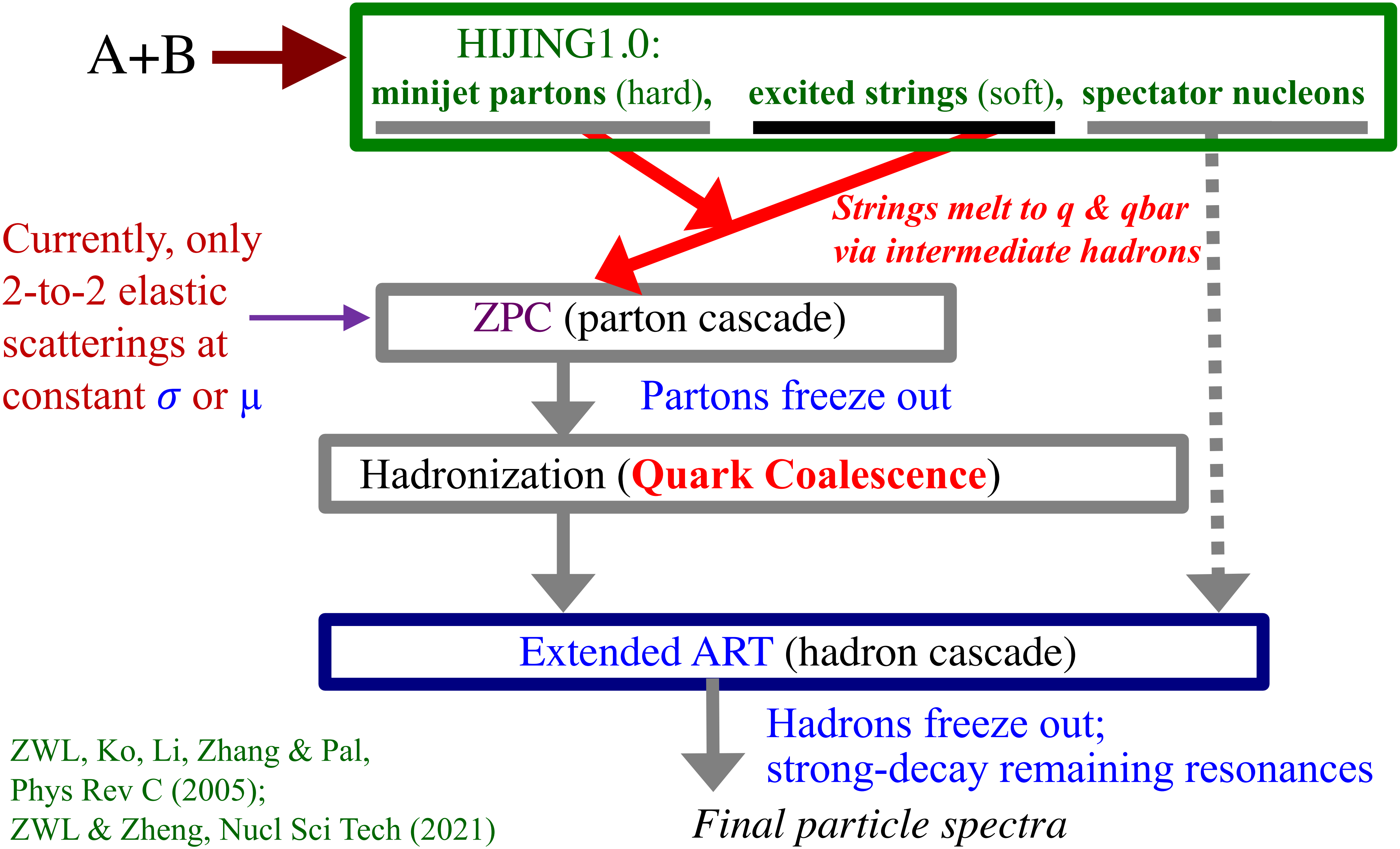
The AMPT model can reasonably describe the bulk matter properties at low p_T in A+A collisions:

ZWL, Phys Rev C (2014)



Application to parton matter in the AMPT model

The String Melting version of AMPT:



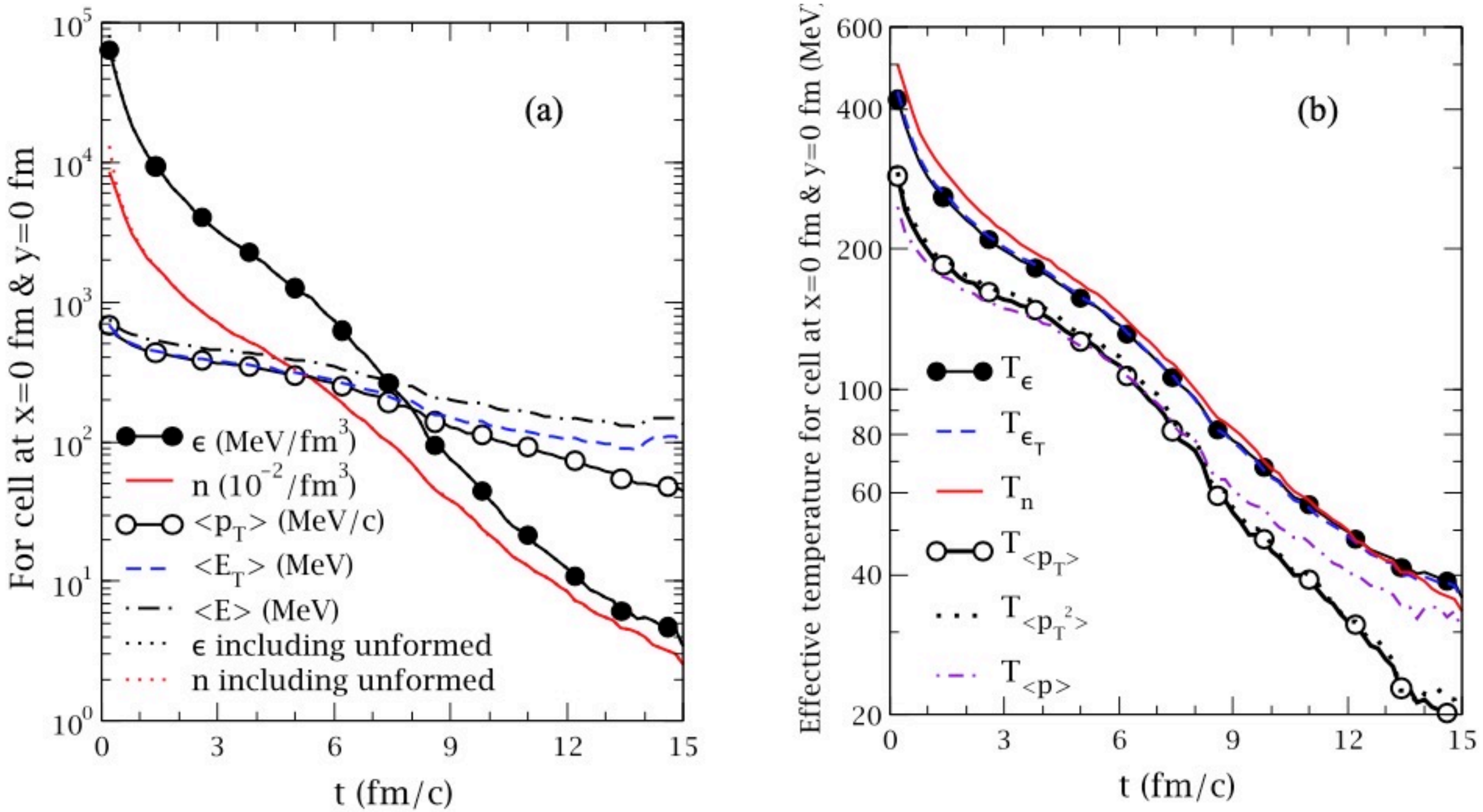
ZWL, Ko, Li, Zhang & Pal, Phys Rev C (2005);
ZWL & Zheng, Nucl Sci Tech (2021)

Application to parton matter in the AMPT model

For the parton matter in the center cell,
we extracted effective temperatures.

ZWL, Phys Rev C (2014)

For example, central Au+Au at 200A GeV:



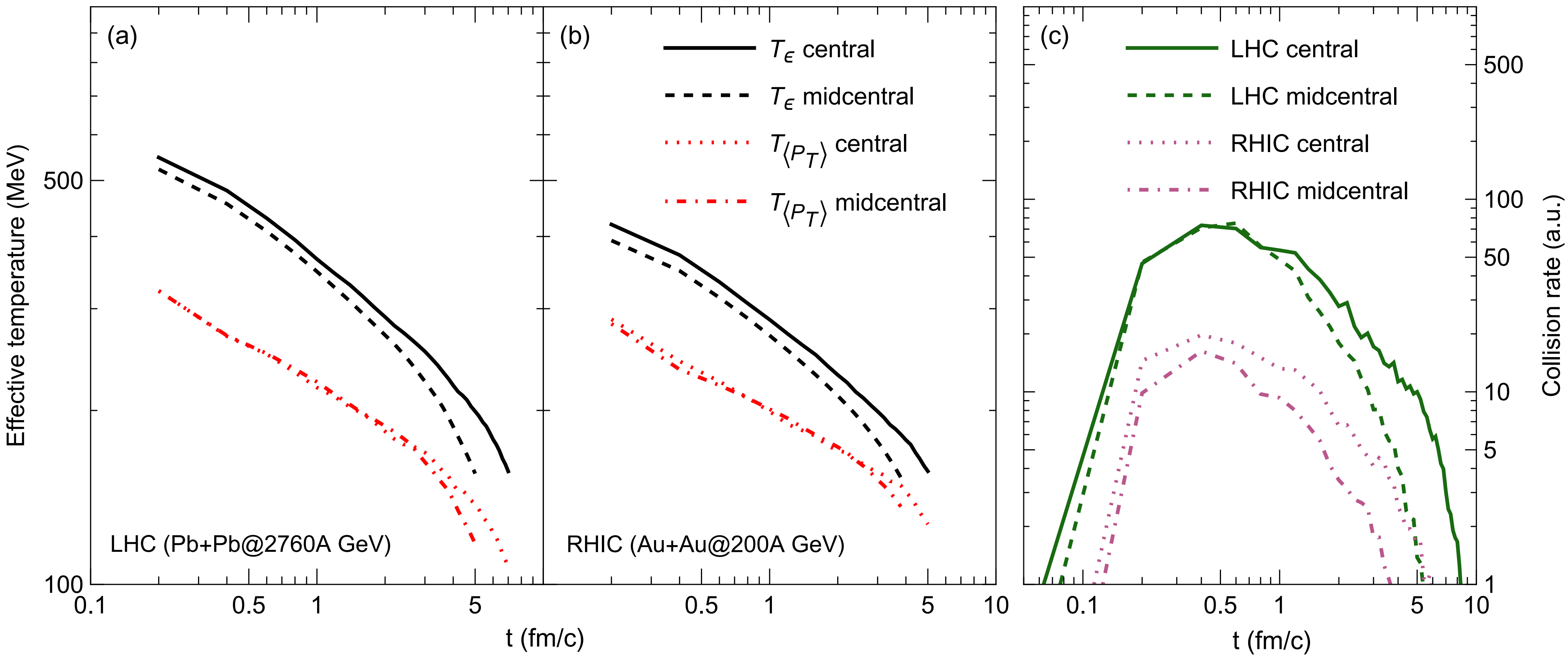
$$\varepsilon = \frac{3g_B}{\pi^2} T_\varepsilon^4, \quad T_{\langle p_T \rangle} = \frac{4}{3\pi}\langle p_T \rangle, \quad \ldots$$

$T_{\langle p_T \rangle} < T_\varepsilon$ → the parton matter is not in chemical equilibrium

Application to parton matter in the AMPT model

We have extracted effective temperatures $T_{\langle p_T \rangle}$ & T_ε for 4 different collision systems:

ZWL, Phys Rev C (2014)

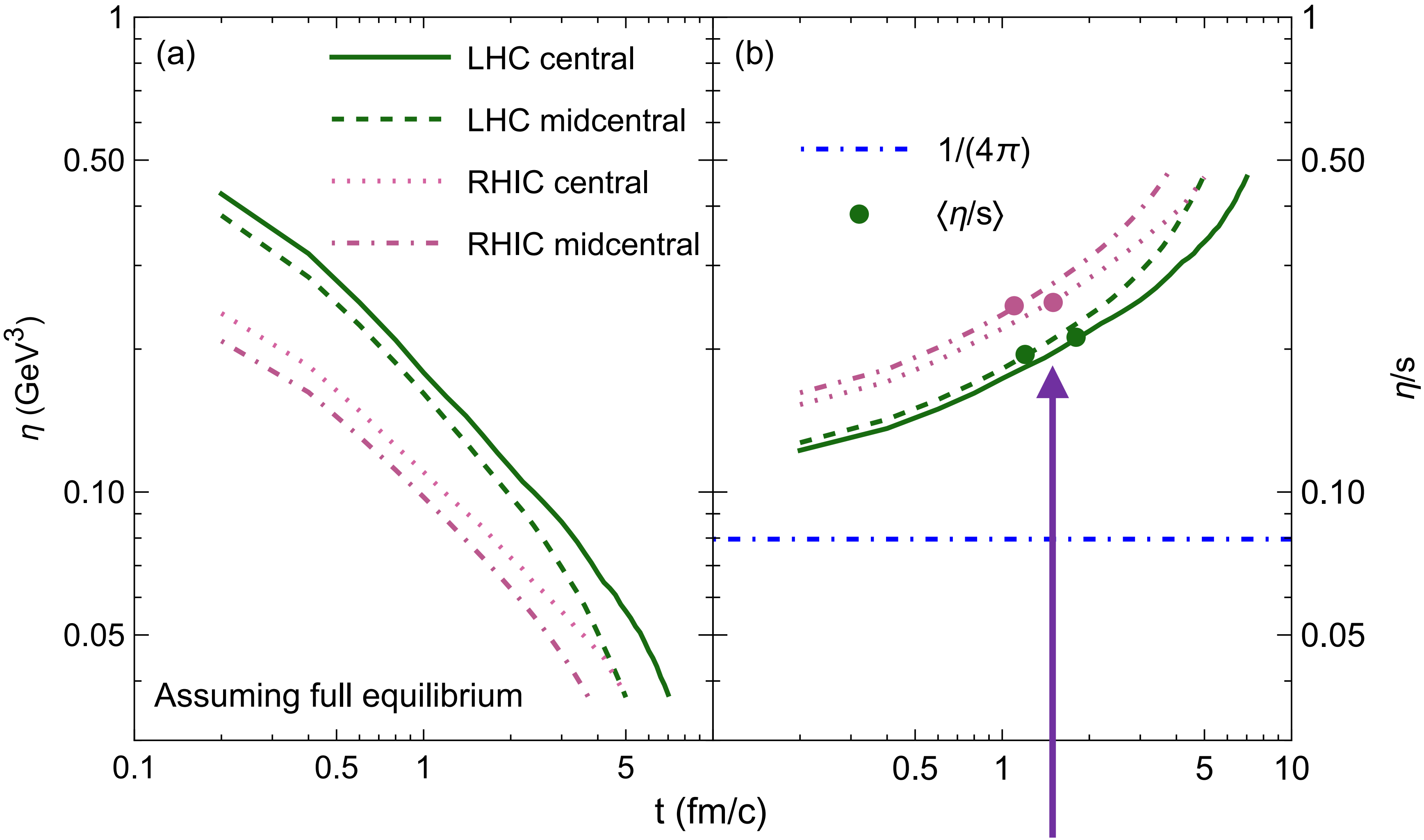


MacKay & ZWL, Eur Phys J C (2022)

We use these temperatures to calculate η and η/s of the center cell.

Application to parton matter in the AMPT model

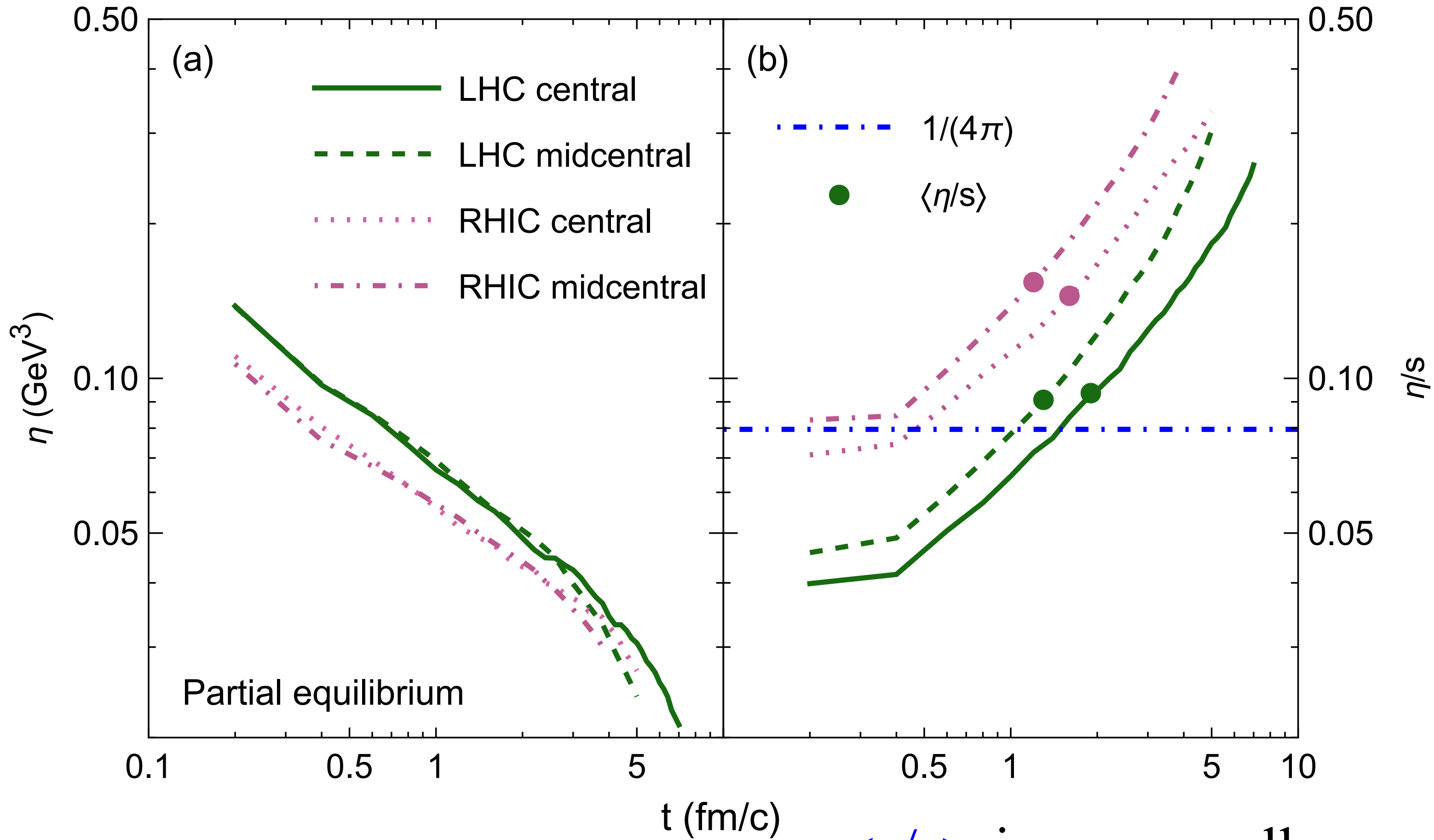
1) When treating the matter as a QGP in full equilibrium (N_f=3), we use temperature T_ε to calculate both η and s.



- $<\eta/s>$ *(time-averaged value weighted by collision rate)* is quite small
- Temperature dependence of η/s is "wrong", due to constant σ

Application to parton matter in the AMPT model

2) When treating the matter as a QGP in partial chemical equilibrium, we use temperature $T_{\langle p_T \rangle}$ to calculate η but use T_ε to calculate s, since η is determined by momentum transfer but not density:



$<\eta/s>$ is very small

- η is lower in partial equilibrium due to $T_{\langle p_T \rangle} < T_\varepsilon$:

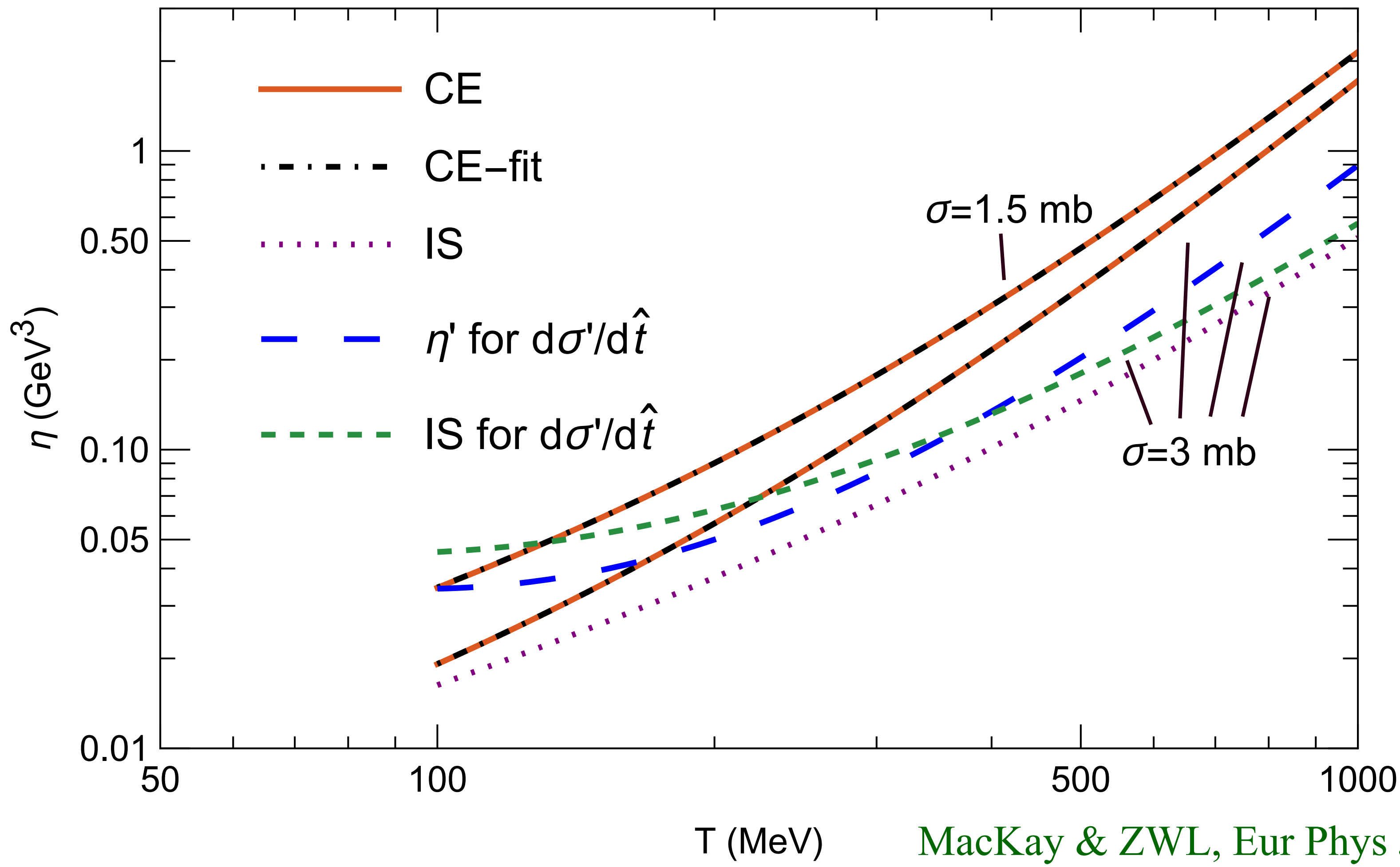
  lower T makes scattering more isotropic and effective

Application to parton matter in the AMPT model

- Our results improve previous calculations of η for parton matter, such as

$$\eta' = \frac{4\,T^3}{5\pi\alpha_s^2\left[\left(1+\frac{\mu^2}{9T^2}\right)\ln\left(1+\frac{18T^2}{\mu^2}\right)-2\right]}$$

based on IS: Magdy et al., Eur Phys J C (2021)



MacKay & ZWL, Eur Phys J C (2022)

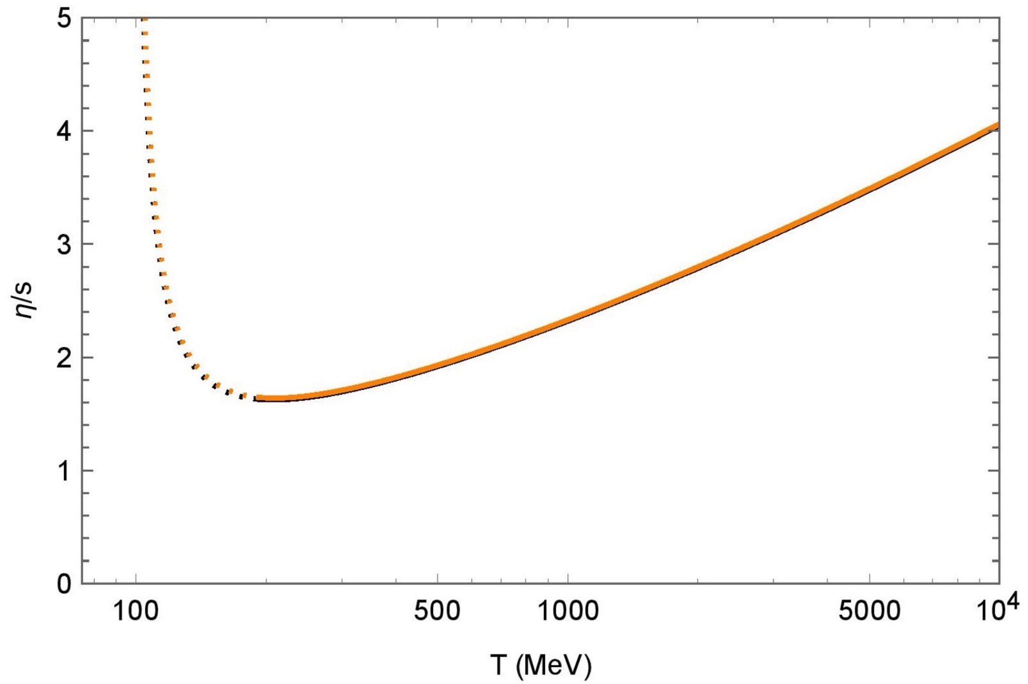
- We can use a $h_2(w)$ fit function for forward scattering $\eta^{CE} = \frac{4T}{5\sigma h_2(w)}$

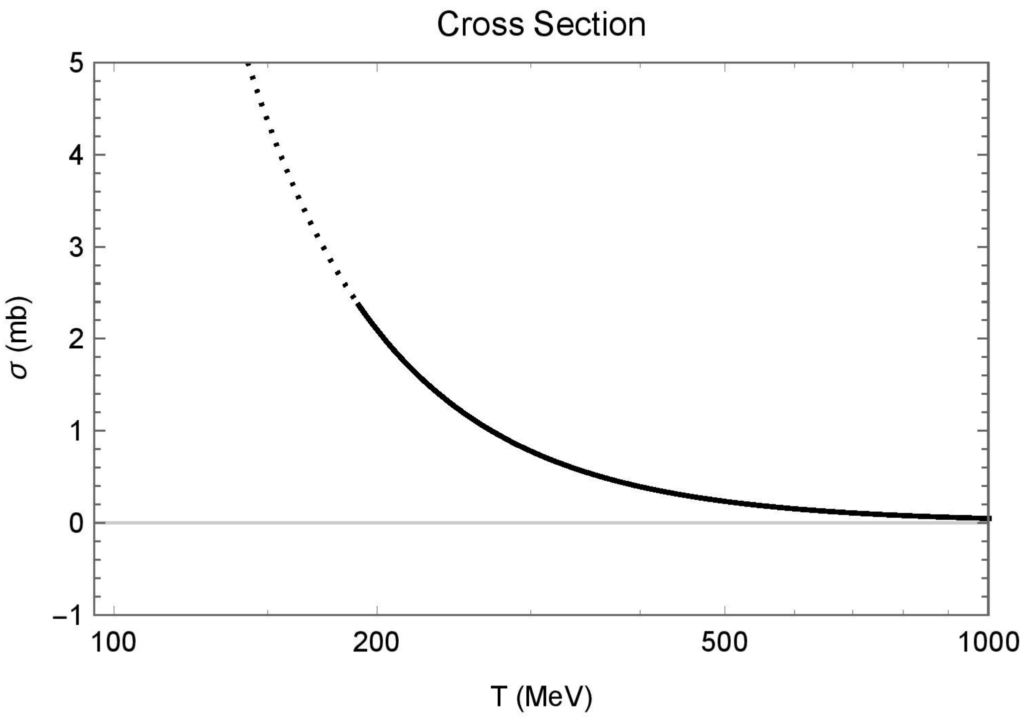
Application to parton matter in the AMPT model

pQCD
with $\mu \propto gT$:

$$\frac{\eta}{s} \approx \frac{5.12}{g^4 \ln(2.42/g)}$$

Arnold, Moore & Yaffe, JHEP (2003);
Csernai, Kapusta & McLerran, Phys Rev Lett (2006)





When using $\mu \propto gT$

→ $\sigma \propto 1/\mu^2$ will be larger at lower T

→ $\eta/s \propto T/\sigma$ will have the expected T- and t-dependences

→ a direction to improve ZPC/AMPT

Summary

- The Chapman–Enskog (CE) method gives accurate expression of η for parton matter under 2-to-2 scatterings

- The other analytical methods (IS, NS, RTA & RTA*) are not accurate for anisotropic scatterings as they disagree with Green-Kubo results

- Applying the CE method, $\langle\eta/s\rangle$ for parton matter in the center cell of high energy A+A collisions is found to be quite small at (1-3)/(4π)

- T-dependence or time-dependence of η/s in AMPT is opposite to pQCD expectation, because of the constant σ or screening mass μ
- This problem can be resolved by adopting $\mu \propto gT$;
  will lead to a better ZPC/AMPT
  as a dynamical model for non-equilibrium studies