Quarkonium transport in strongly coupled plasmas

and a comparison with heavy quark transport

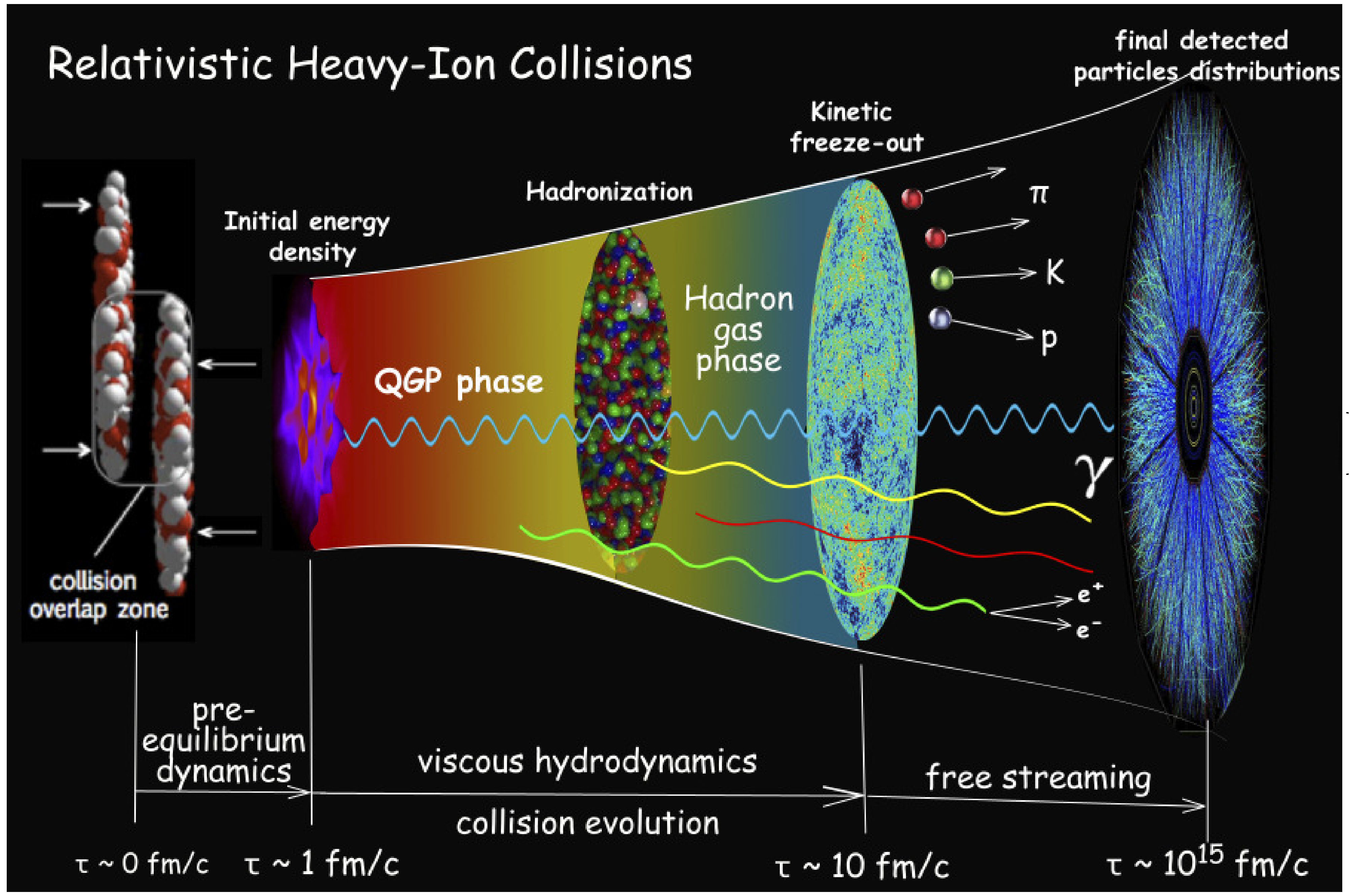
38th Winter Workshop on Nuclear Dynamics
Marriott Puerto Vallarta Resort & Spa
February 9, 2023

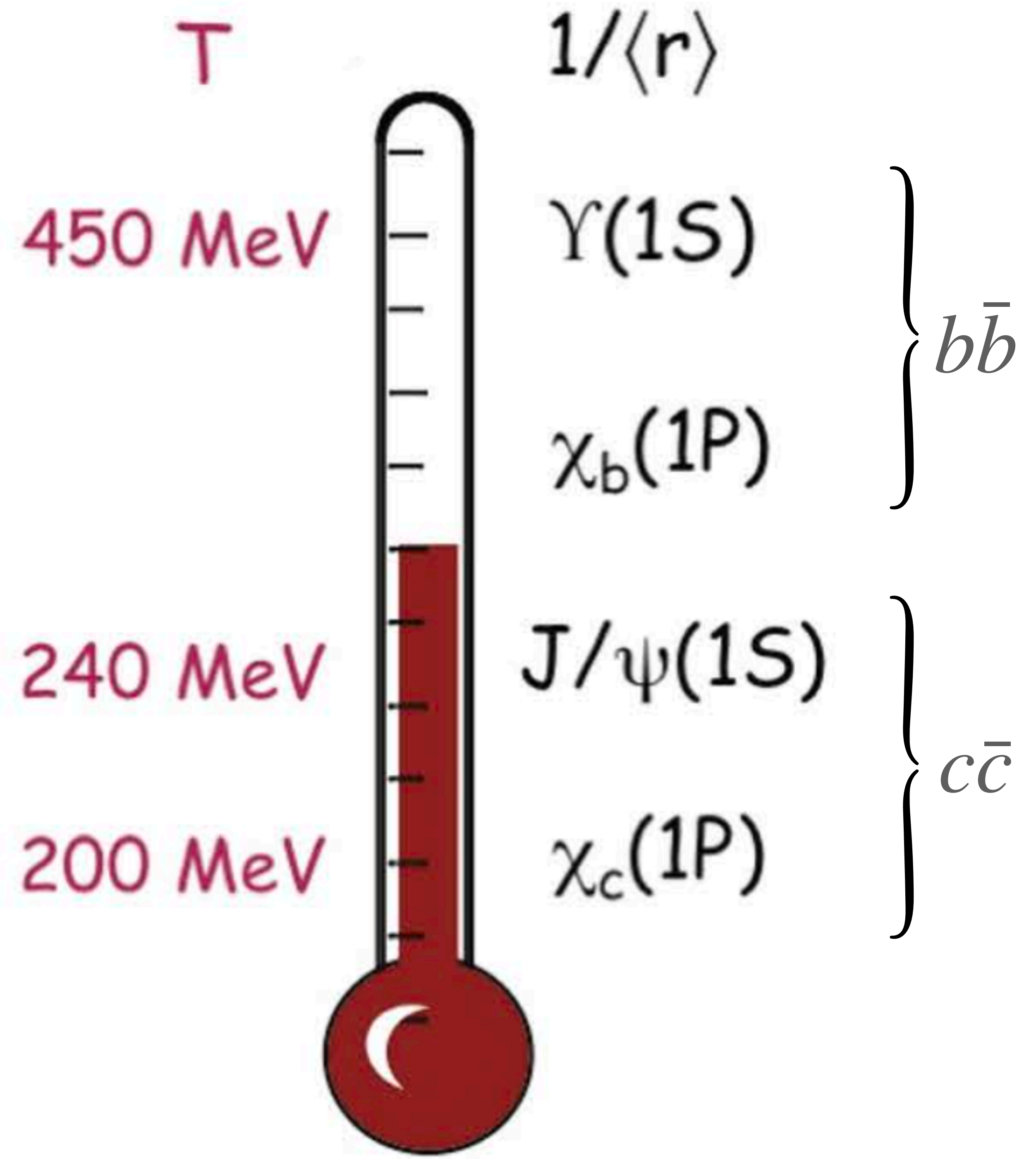
Bruno Scheihing-Hitschfeld (MIT)
with Xiaojun Yao (UW) and Govert Nijs (MIT)
based on 2107.03945, 2205.04477, 2302.XXXXX

Quarkonium in Heavy-Ion Collisions

- Heavy quarks and quarkonia are amongst the most informative probes of the QGP.
- To interpret the full wealth of data, we need a precise theoretical understanding of heavy quarks in a thermal medium,
  - as single open heavy flavors, and
  - as pairs of heavy flavors that can bind into quarkonia.

credit: Paul Sorensen and Chun Shen, 1304.3634





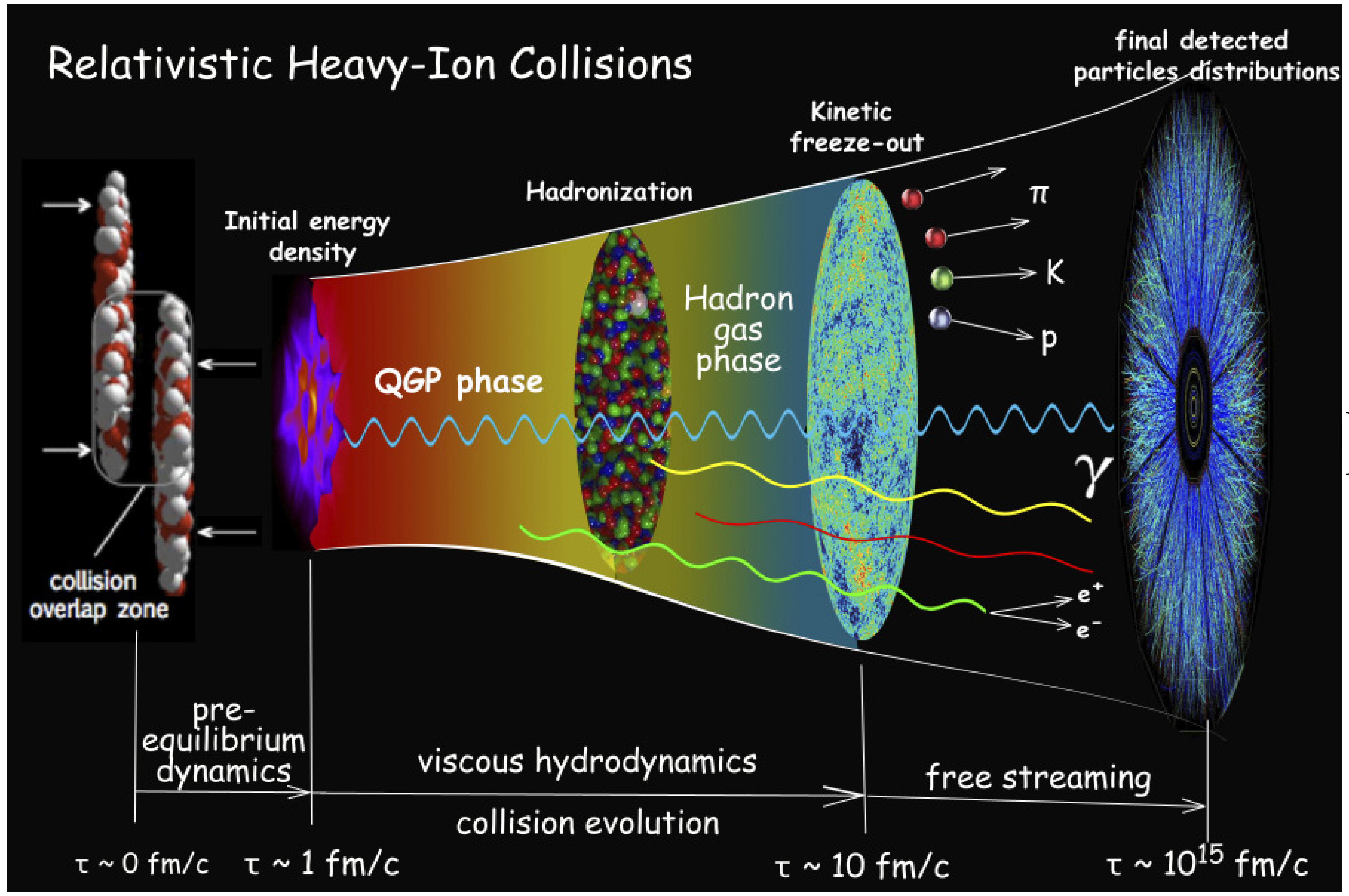
credit: Mocsy, Petreczky, Strickland, 1302.2180

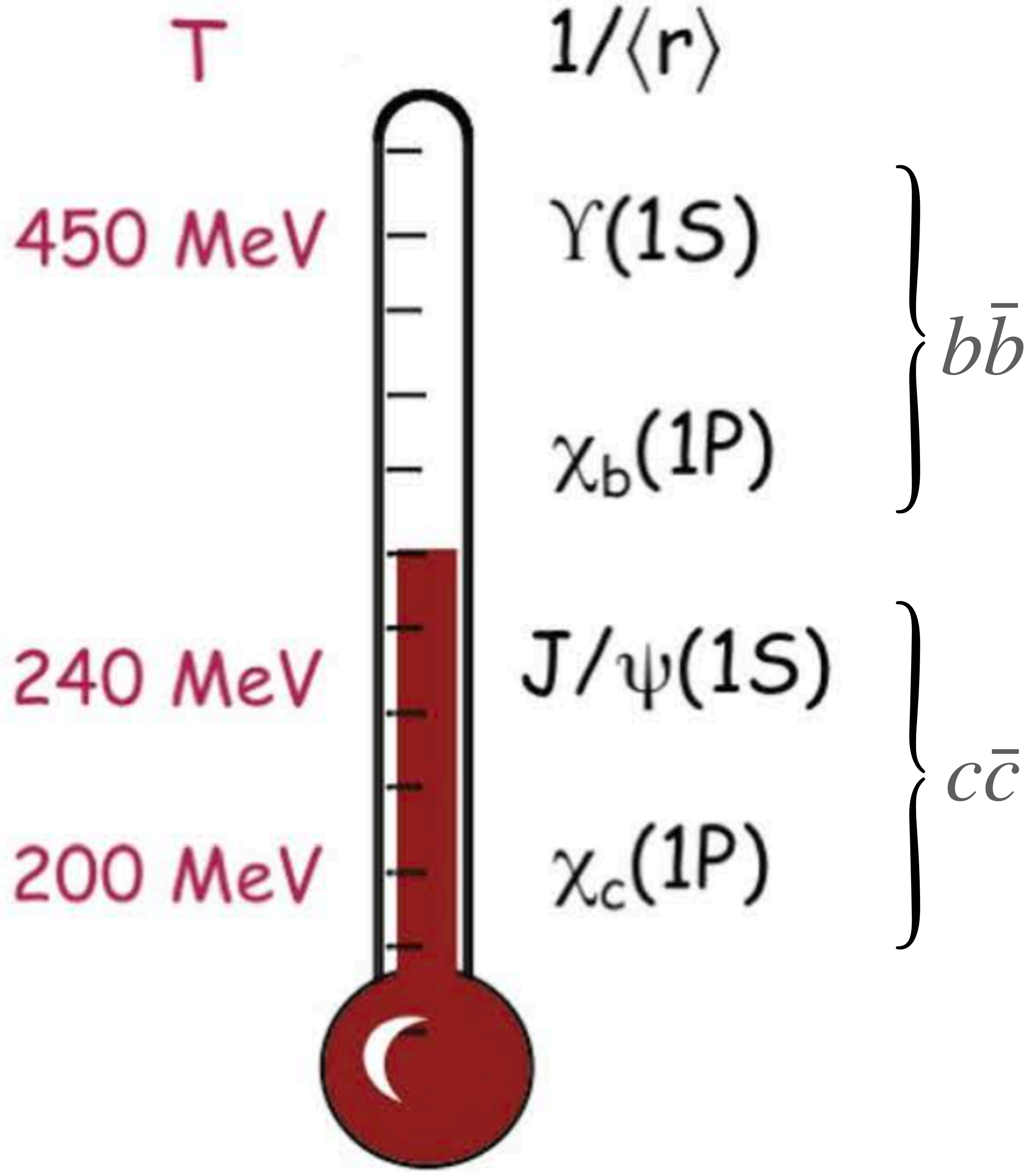
Quarkonium in Heavy-Ion Collisions

- Heavy quarks and quarkonia are amongst the most informative probes of the QGP.
- To interpret the full wealth of data, we need a precise theoretical understanding of heavy quarks in a thermal medium,
  - as single open heavy flavors, and
  - as pairs of heavy flavors that can bind into quarkonia.

→ **This talk**

credit: Paul Sorensen and Chun Shen, 1304.3634





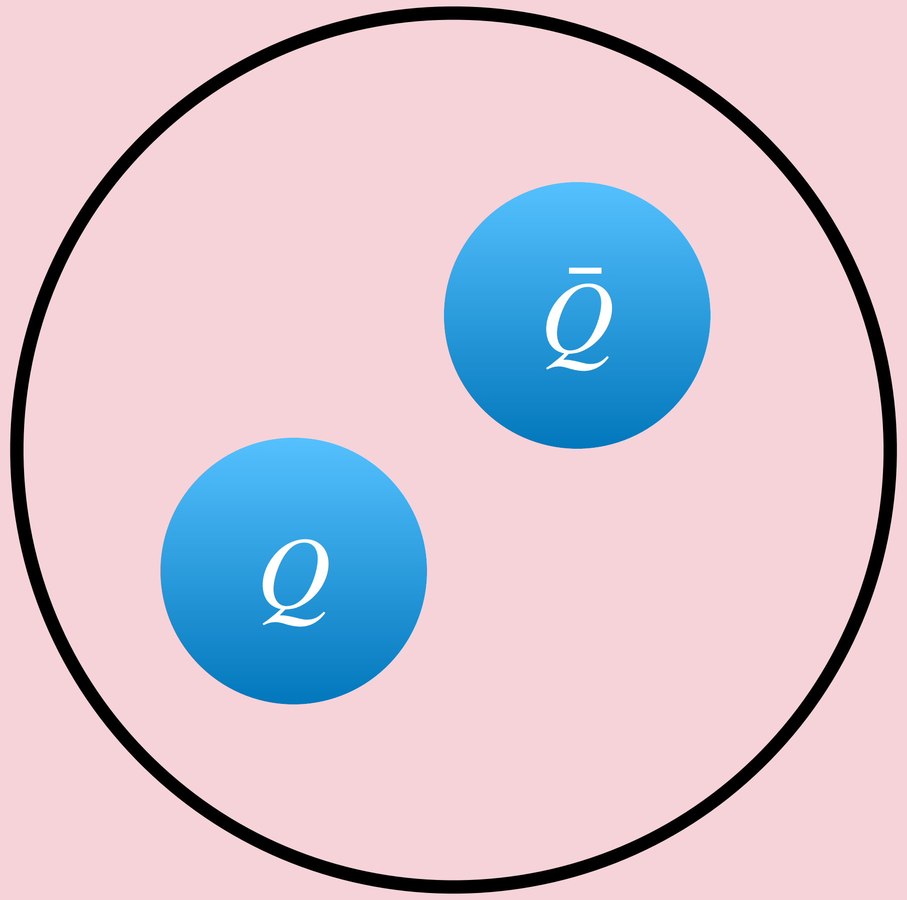
credit: Mocsy, Petreczky, Strickland, 1302.2180

Quarkonium in medium

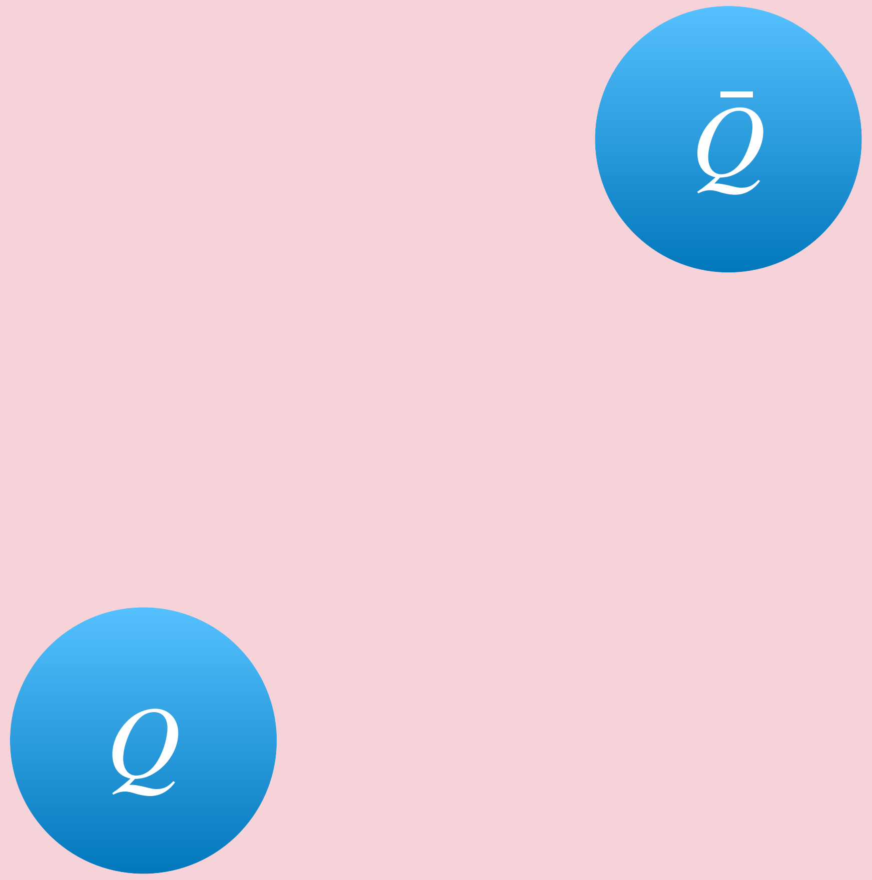
$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



color singlet;
bound state



color octet;
unbound state

Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

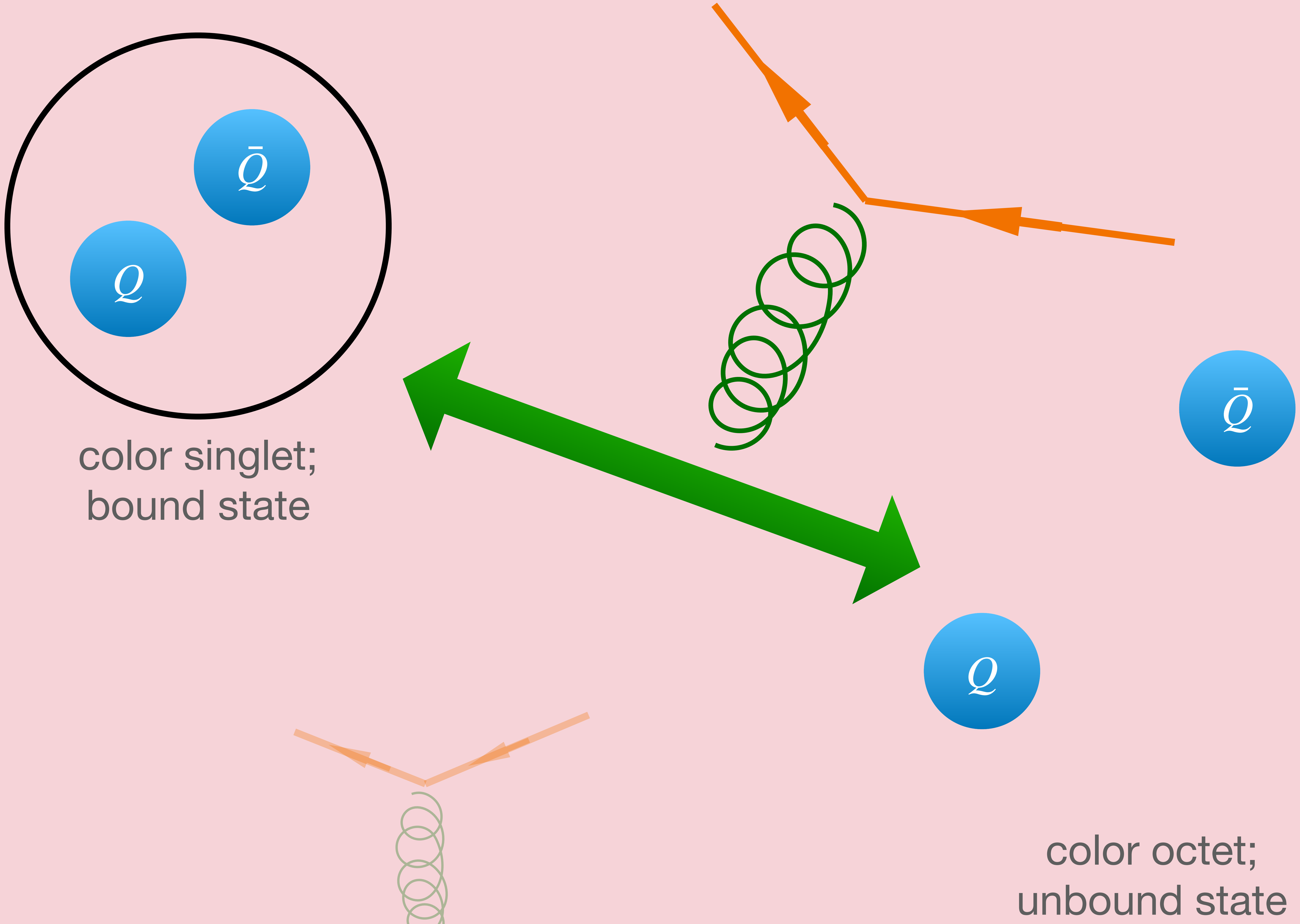
$T > 0$

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

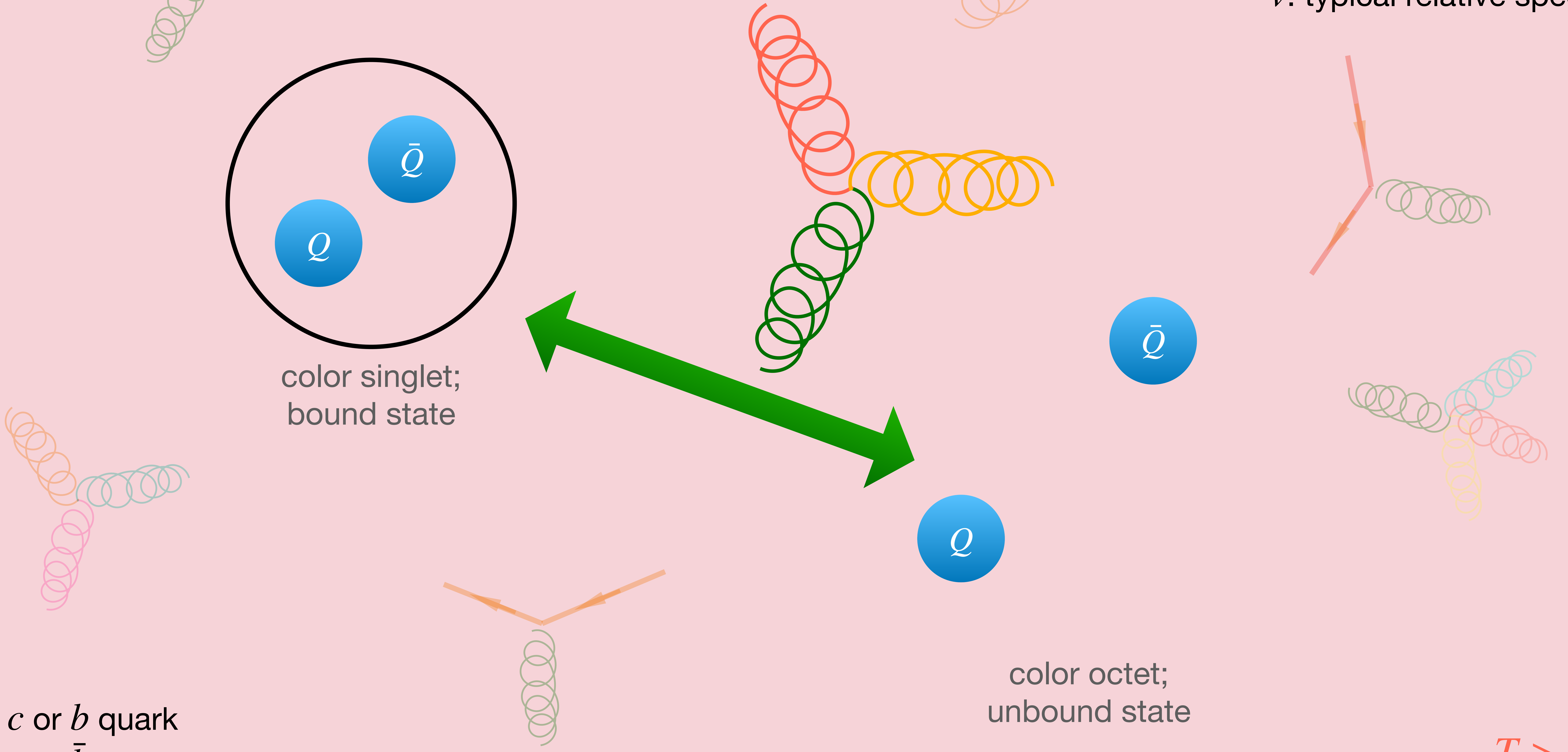
$T > 0$

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

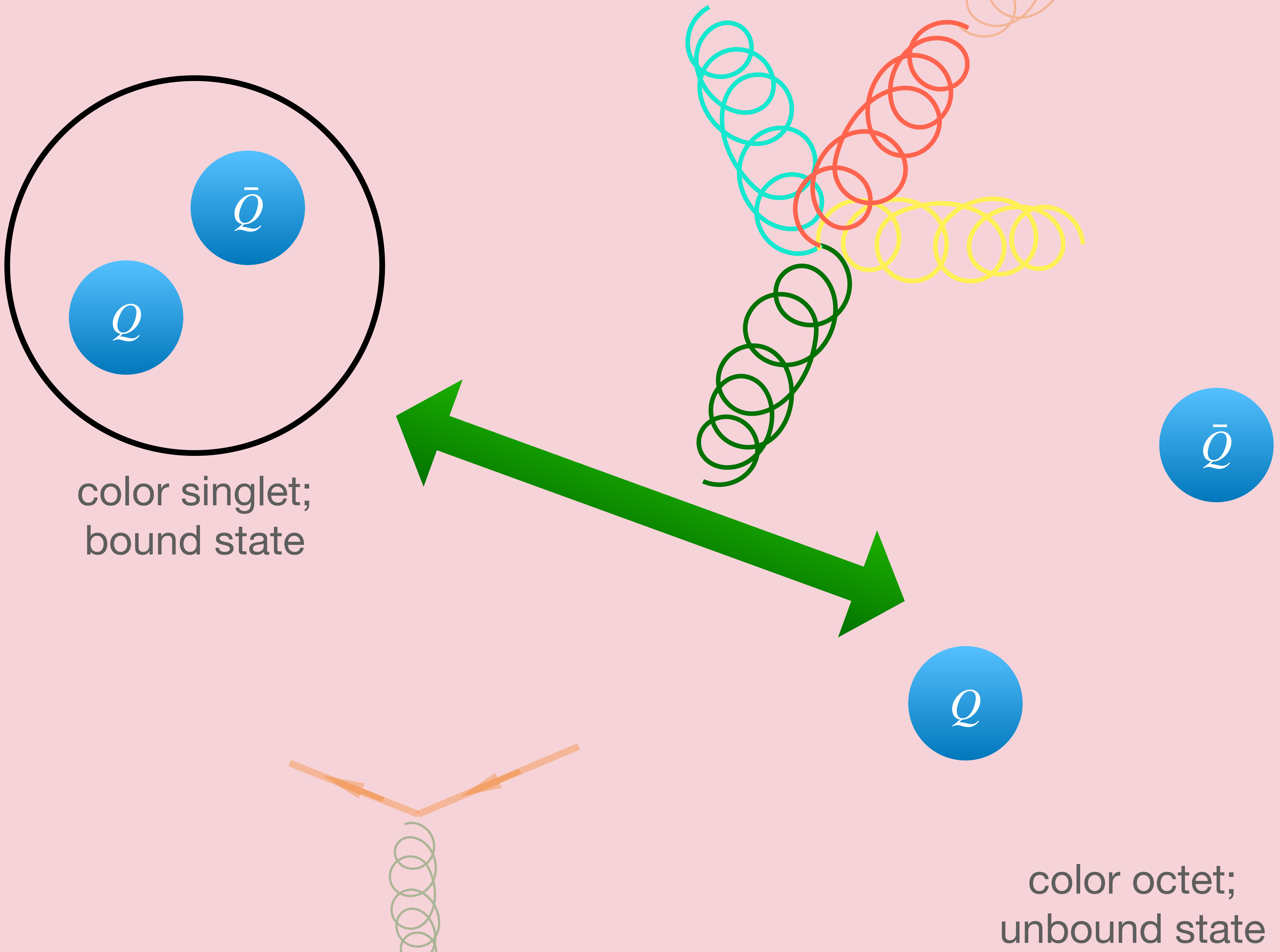
$T > 0$

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

$M \gg Mv \gg Mv^2$

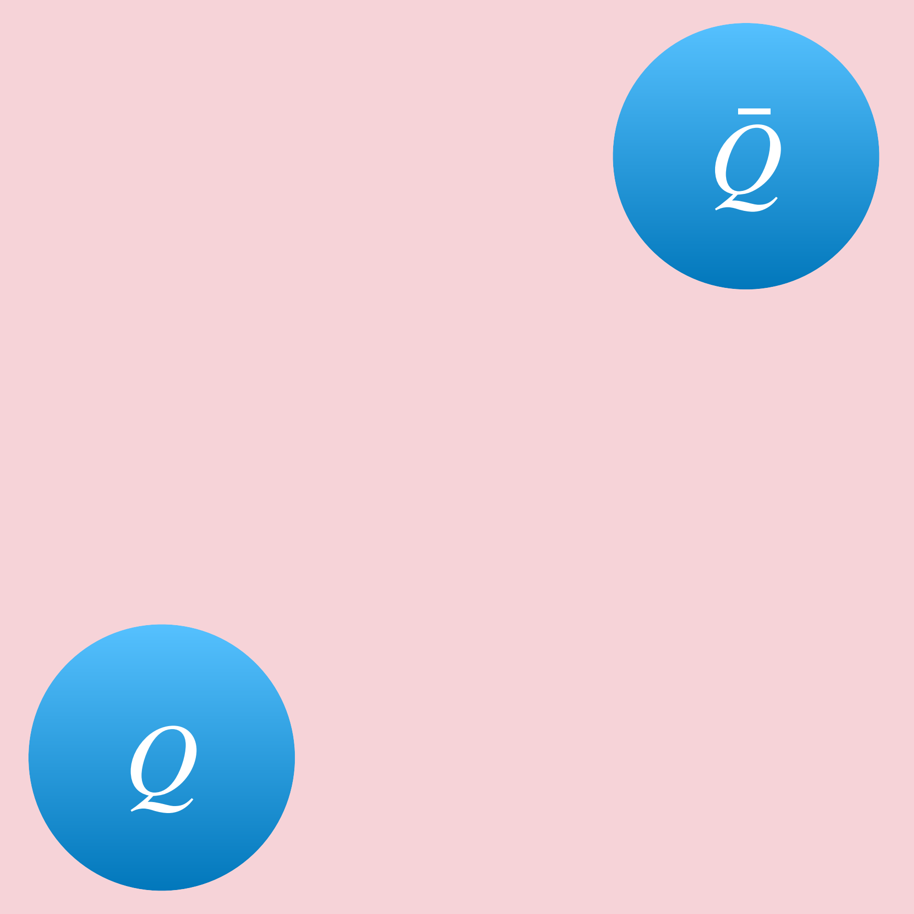
M: heavy quark mass

v: typical relative speed

Quarkonium in medium

At high T, quarkonium "melts" because the medium screens the interactions between heavy quarks (Matsui & Satz 1986)

$$Q\bar{Q} \text{ melts if } r \sim \frac{1}{Mv} \gg \frac{1}{T}$$



color octet;
unbound state

$T > 0$

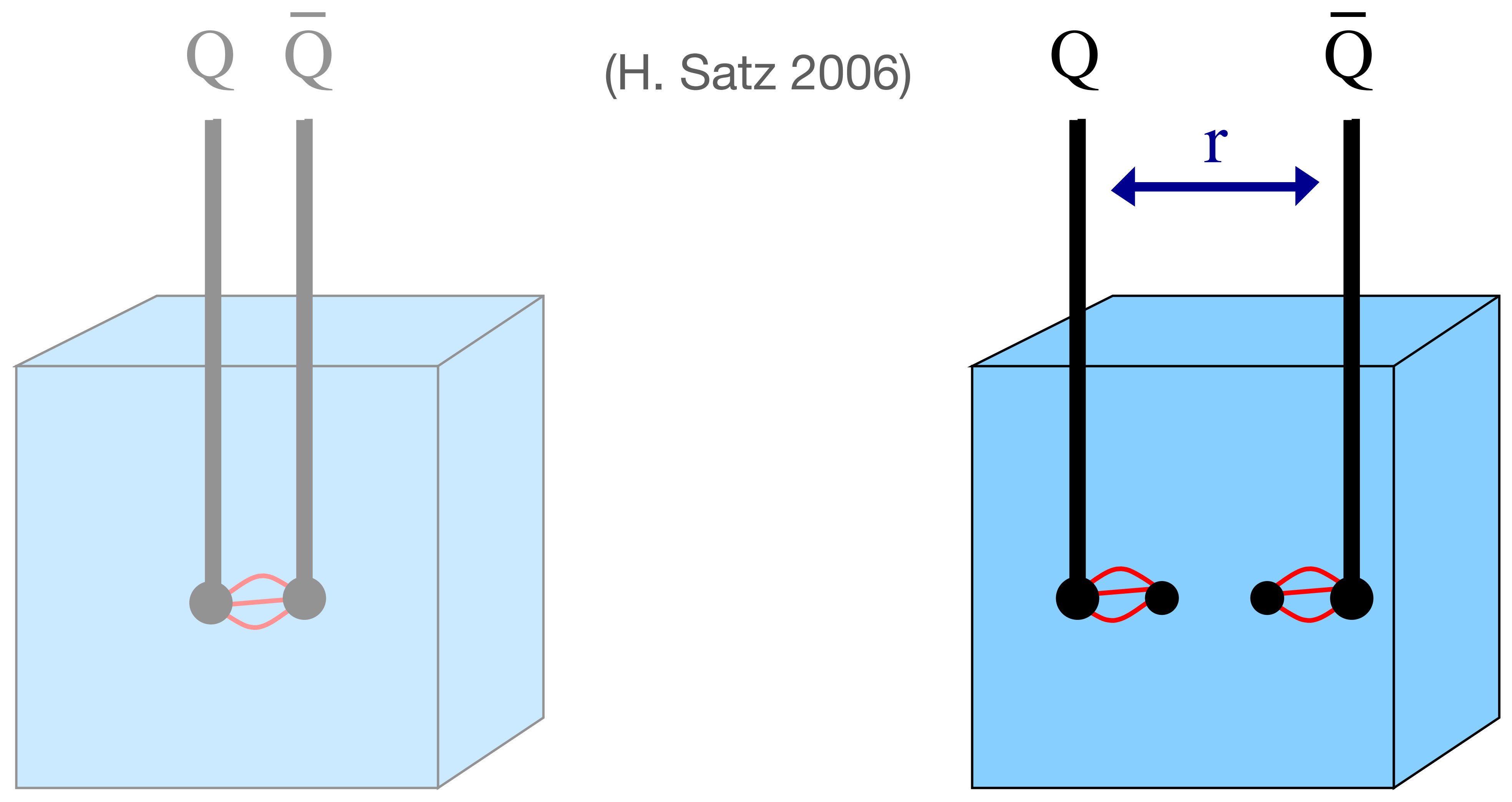
Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

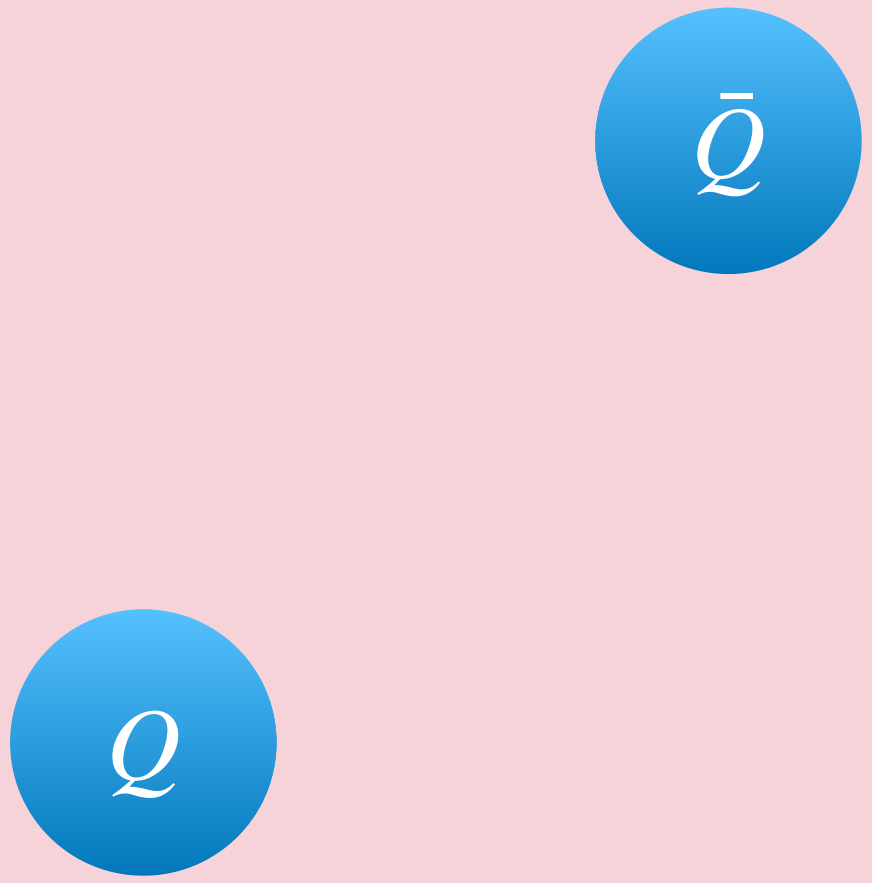
Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass
v: typical relative speed



$Q\bar{Q}$ melts if $r \sim \dfrac{1}{Mv} \gg \dfrac{1}{T}$



color octet;
unbound state

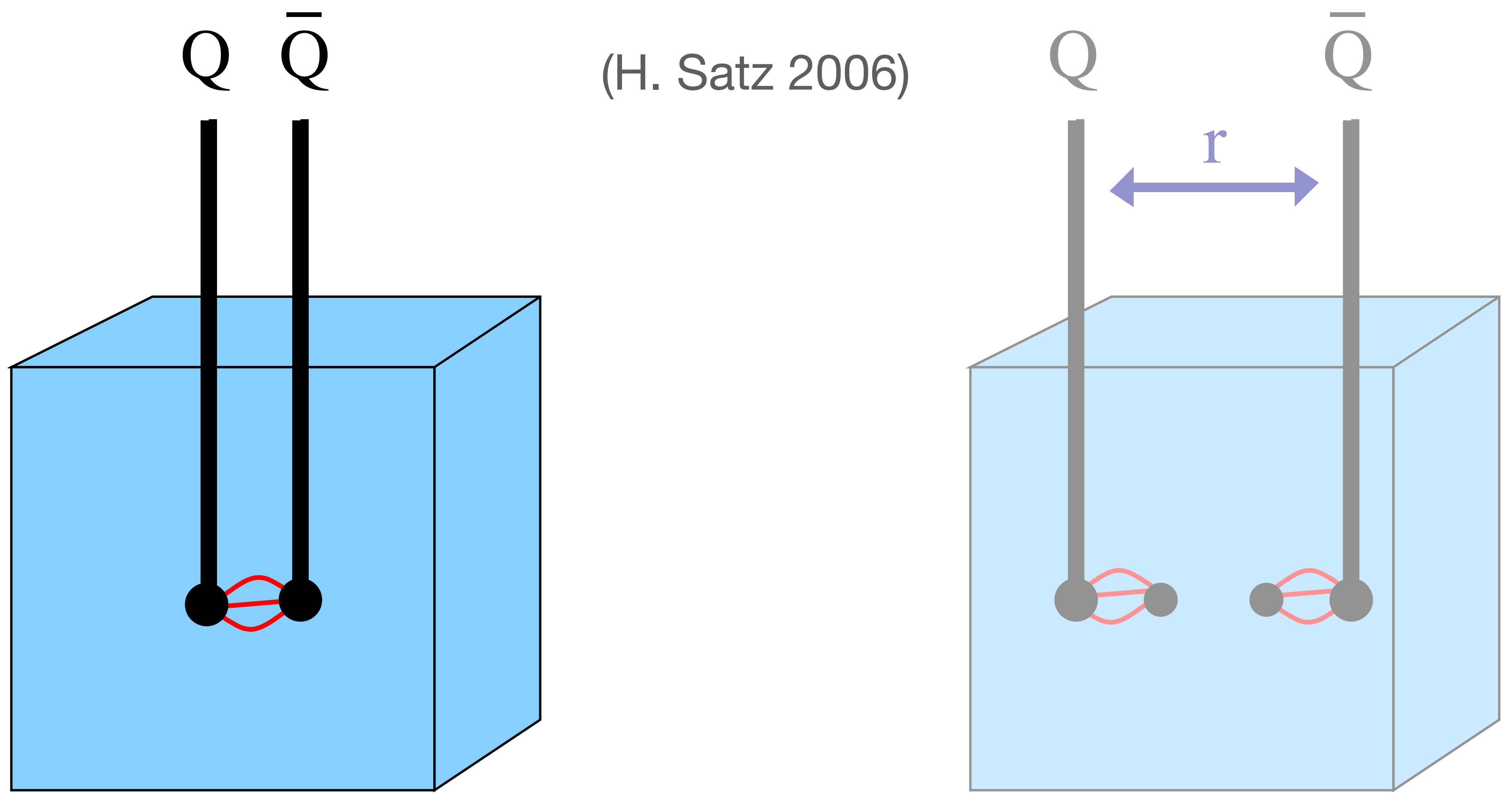
Q: c or b quark
$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

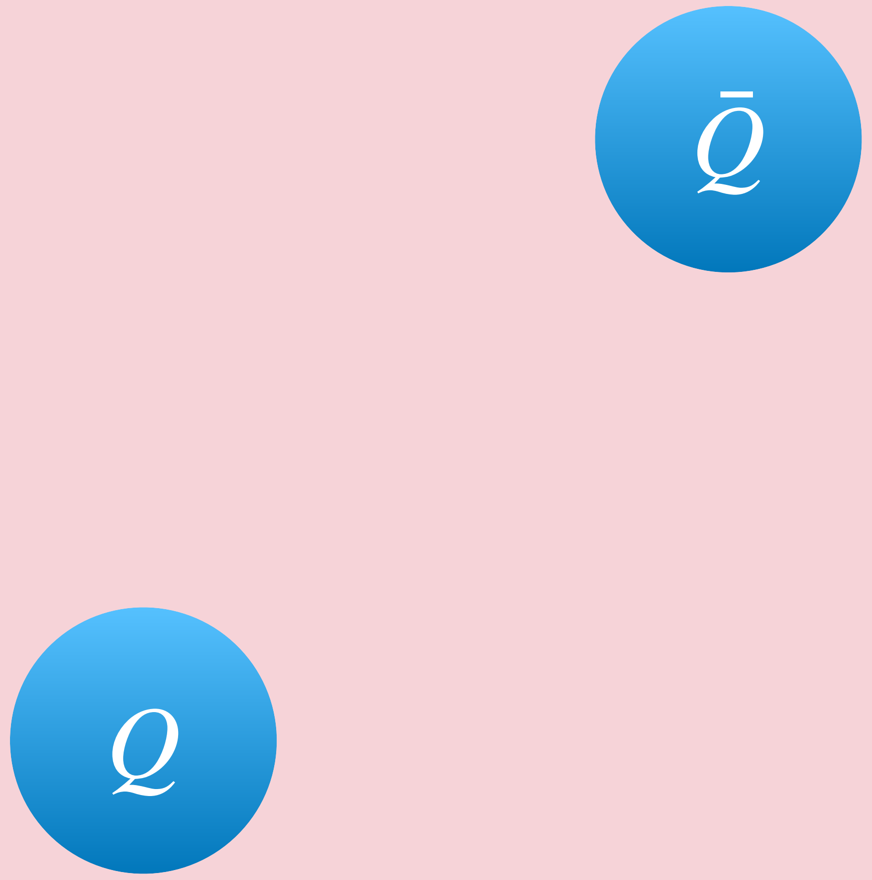
Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass
v: typical relative speed



$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$



color octet;
unbound state

Q: c or b quark
$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

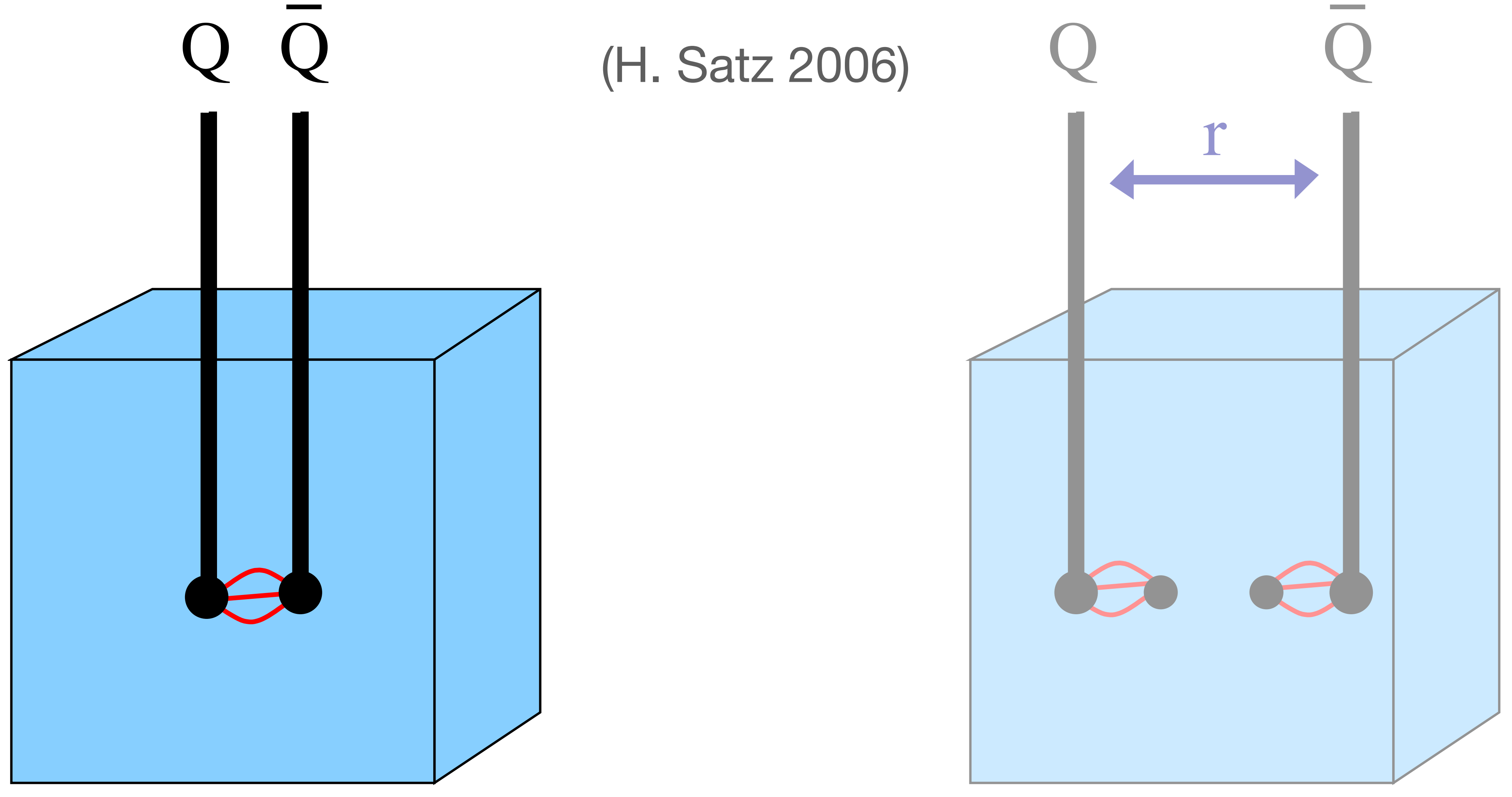
$T > 0$

$M \gg Mv \gg Mv^2$

M: heavy quark mass

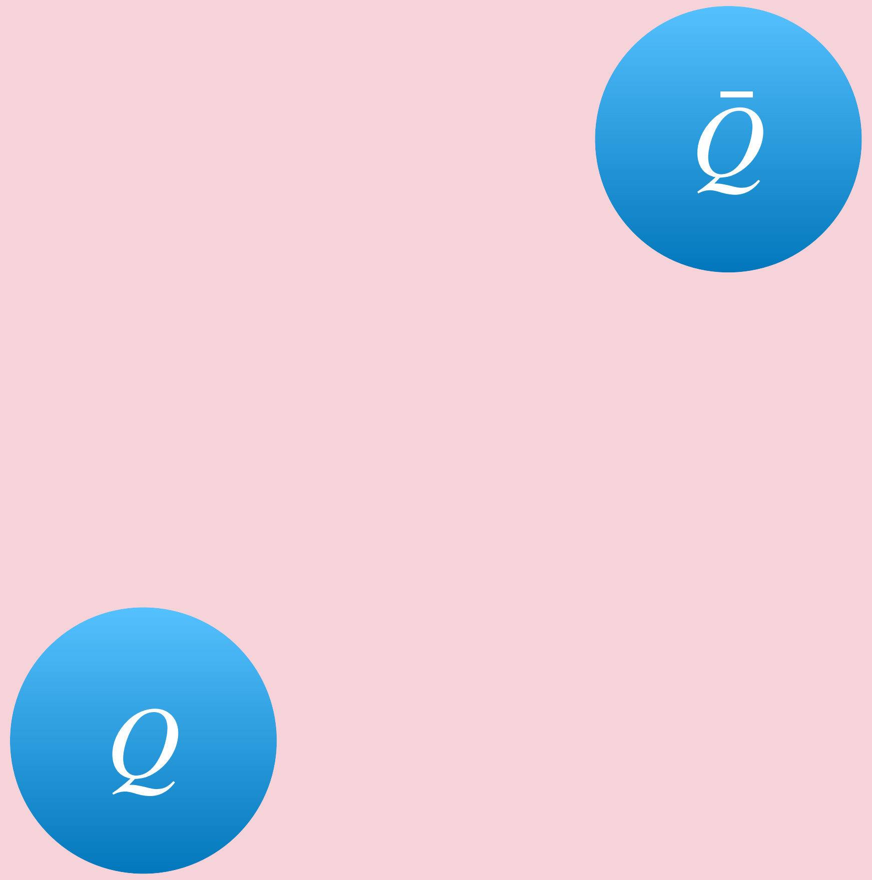
v: typical relative speed

Quarkonium in medium



$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$

$\Longrightarrow$ most of quarkonium starts to form when $Mv \gtrsim T$



color octet;
unbound state

Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

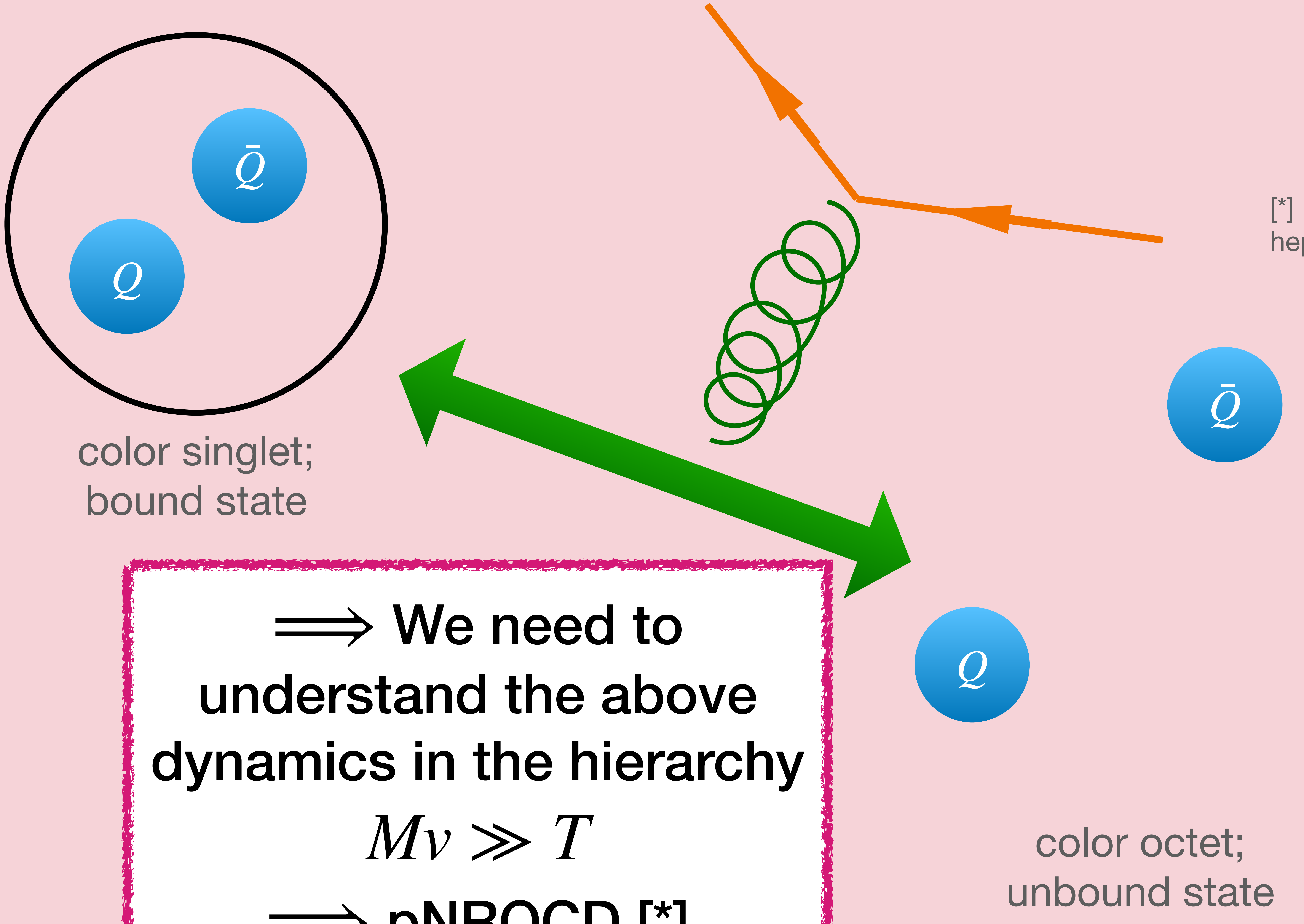
$T > 0$

Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass

v: typical relative speed



[*] N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

$\Longrightarrow$ **We need to understand the above dynamics in the hierarchy**

$$Mv \gg T$$

$\Longrightarrow$ **pNRQCD [*]**

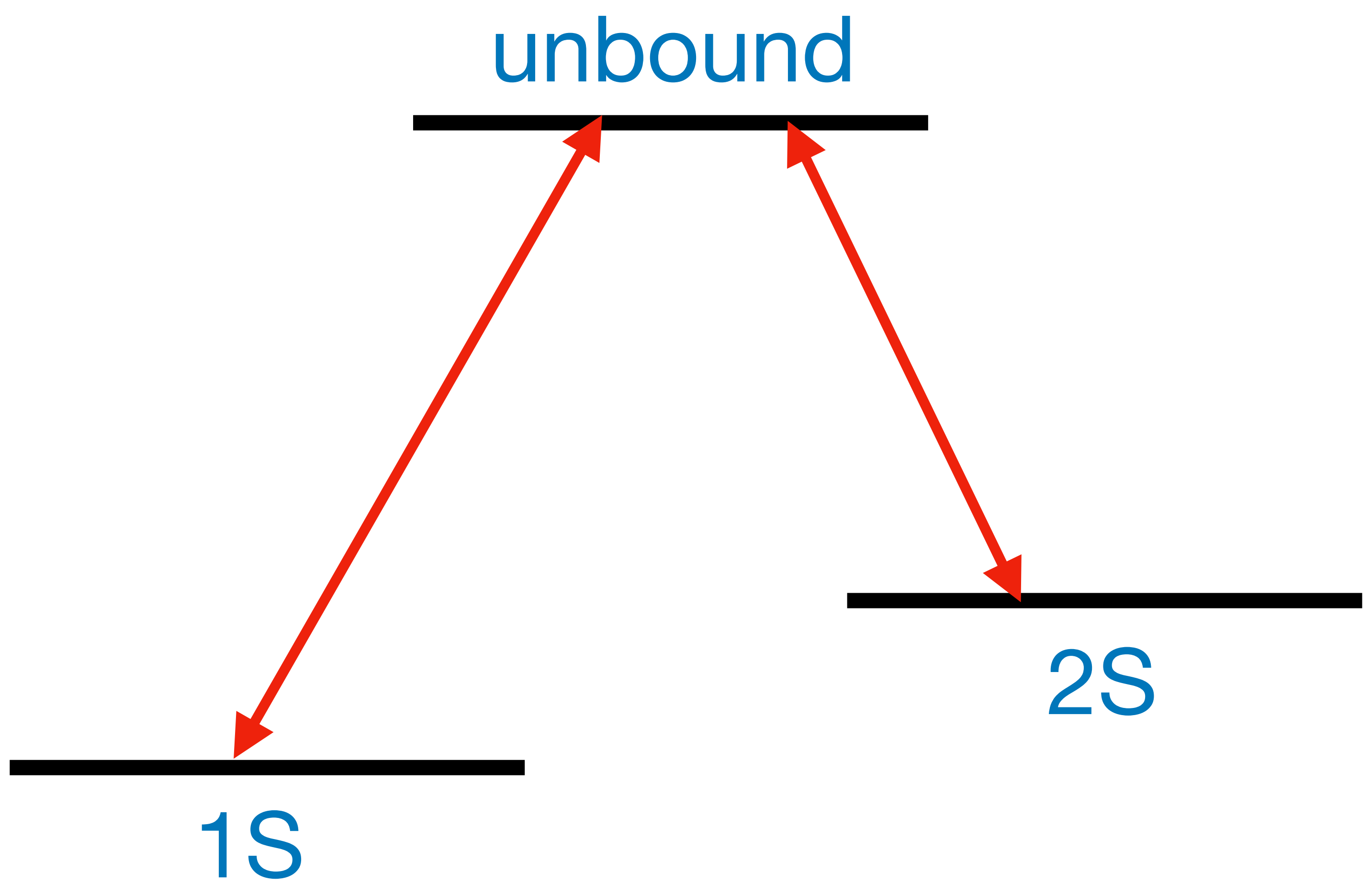
Q: c or b quark

$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

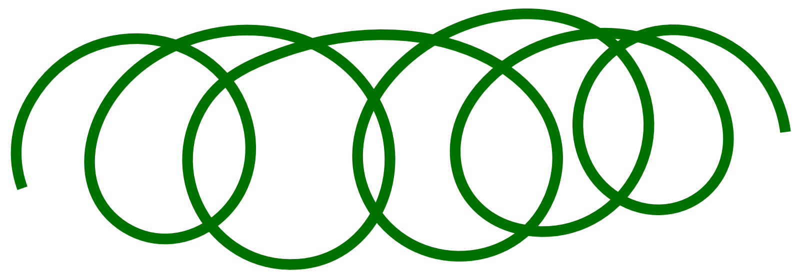
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r {\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$
$$+V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\right) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

[*] N. Brambilla, A. Pineda, J. Soto, A. Vairo hep-ph/9707481, hep-ph/9907240, hep-ph/0410047

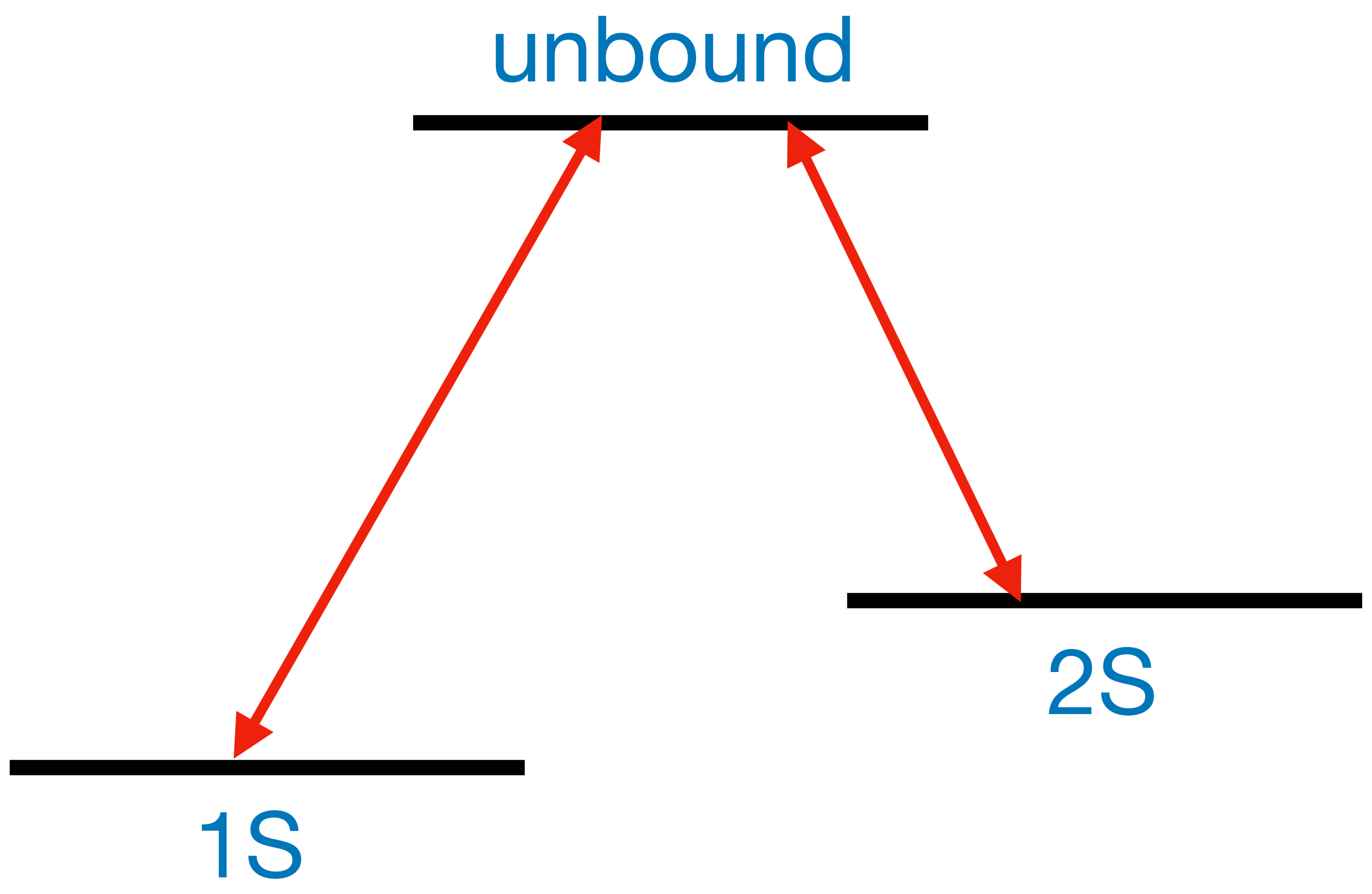
X. Yao, hep-ph/2102.01736

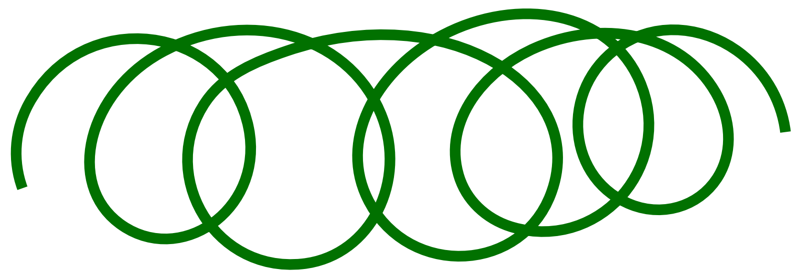
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)

Interaction with the environment

QGP (the environment)





$$\frac{1}{\tau_S} \sim \Delta E_n \sim M v^2$$

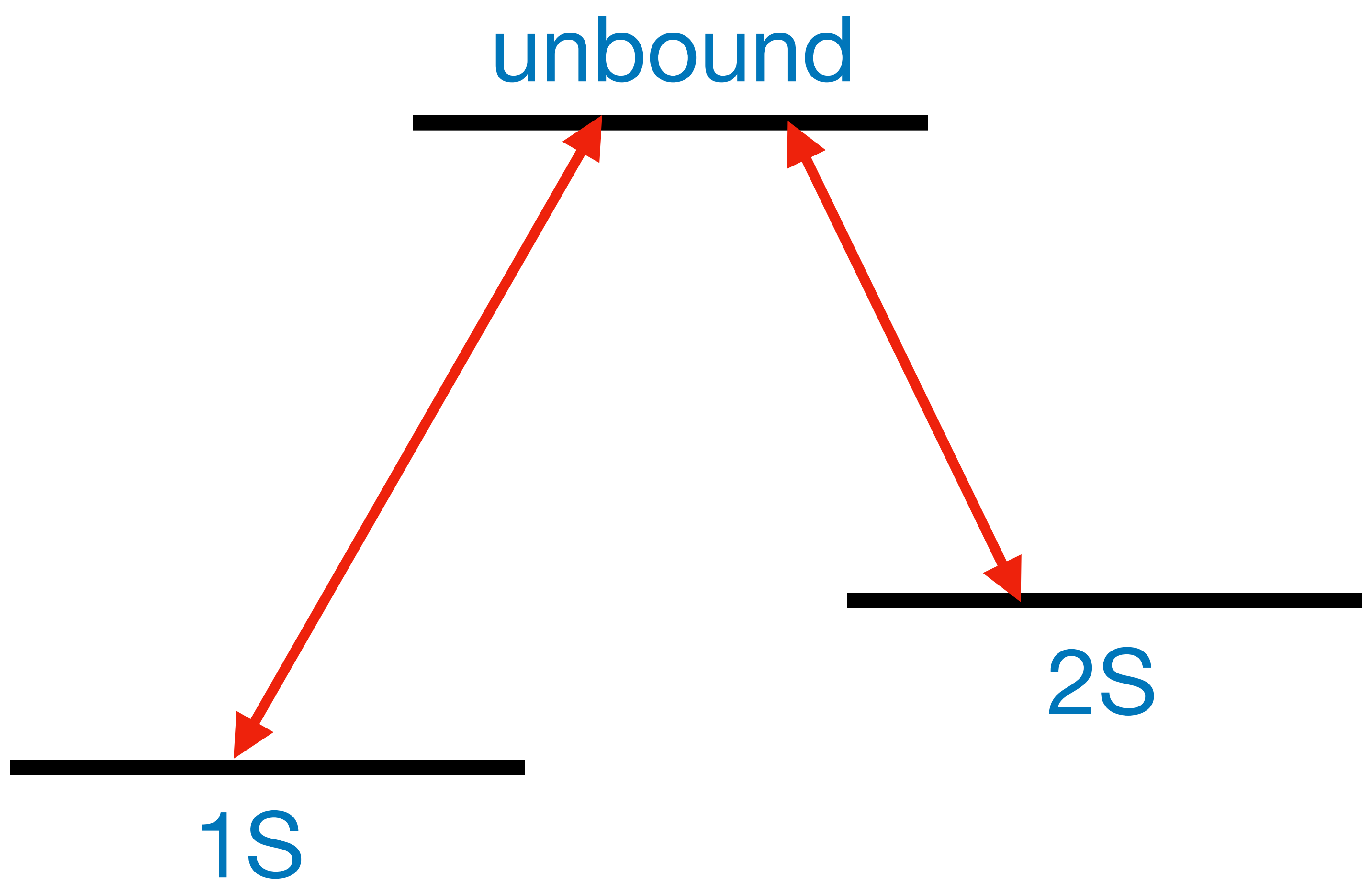
$$\frac{1}{\tau_I} \sim \frac{H_{\mathrm{int}}^2}{T} \sim T \frac{T^2}{(Mv)^2}$$

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\mathrm{pNRQCD}} = \mathscr{L}_{\mathrm{light\ quarks}} + \mathscr{L}_{\mathrm{gluon}} + \int d^3 r \mathrm{Tr}_{\mathrm{color}} \Big[S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O$$
$$+ V_A \left(O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \mathrm{h.c.} \right) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \cdots \Big]$$

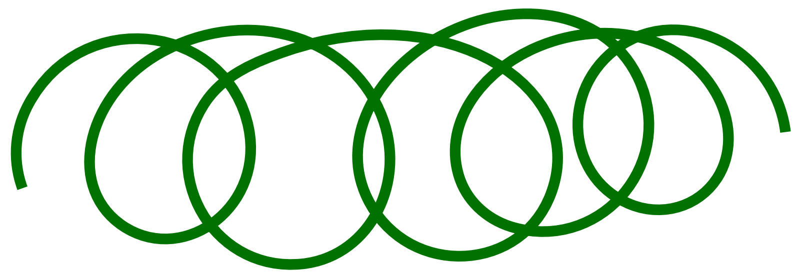
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\mathrm{int}}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

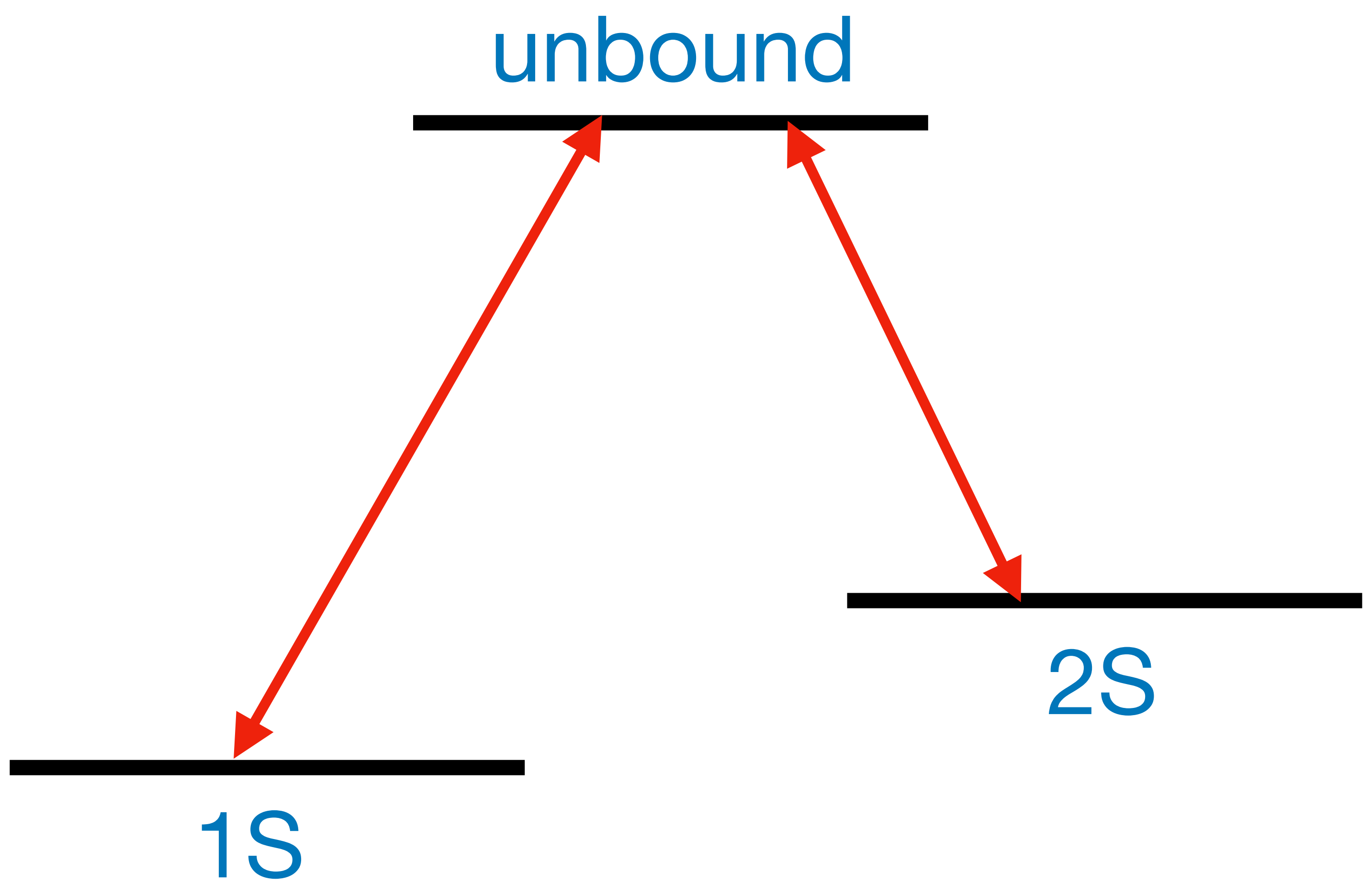
QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\mathrm{pNRQCD}} = \mathscr{L}_{\mathrm{light\ quarks}} + \mathscr{L}_{\mathrm{gluon}} + \int d^3r \mathrm{Tr}_{\mathrm{color}}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$
$$+ V_A\big(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + \mathrm{h.c.}\big) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

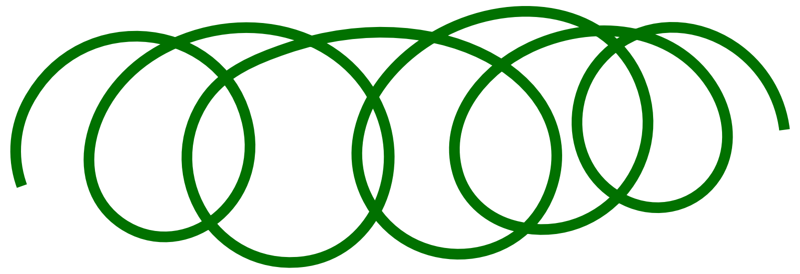
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r {\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

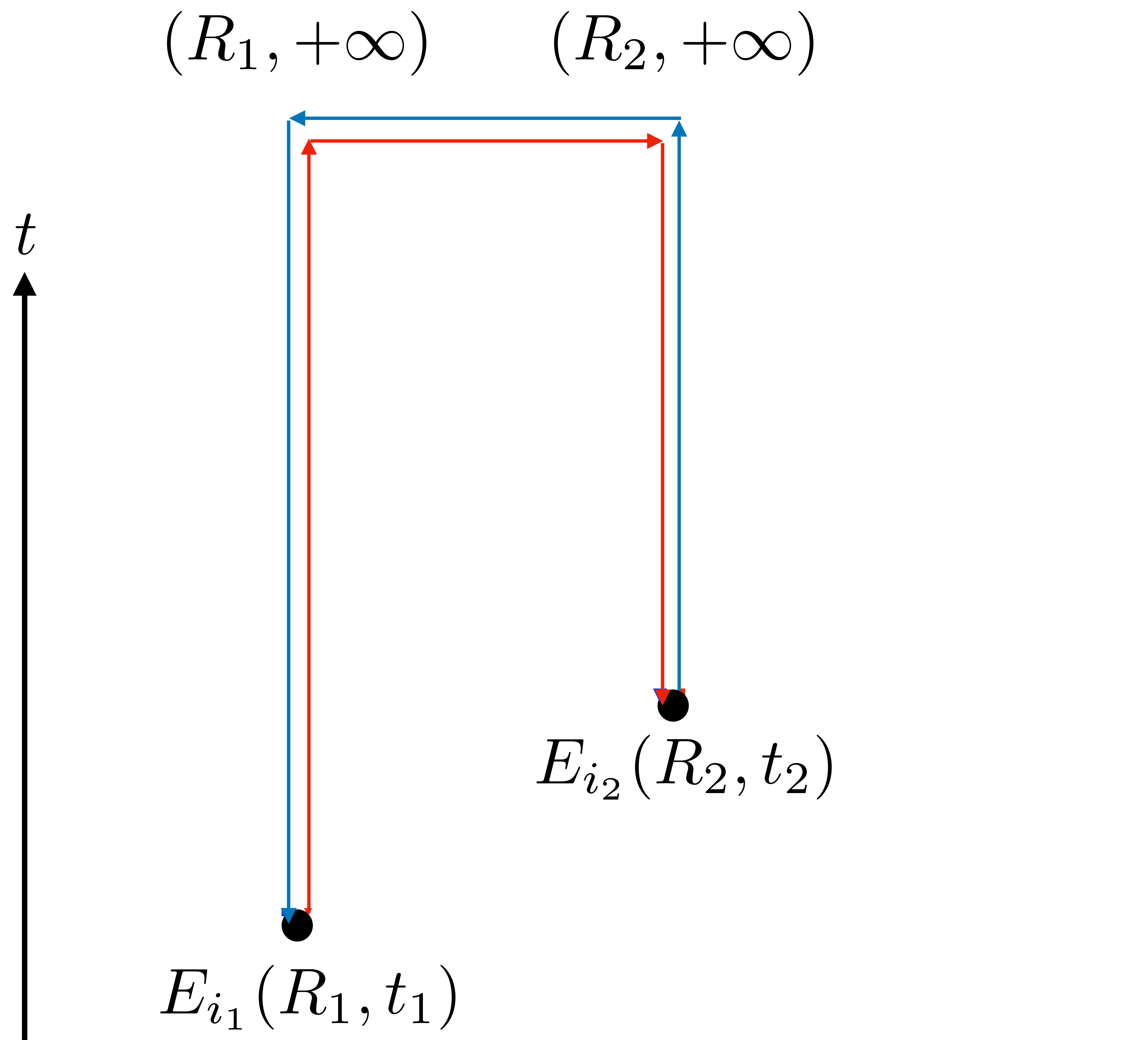
$$+V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\right) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

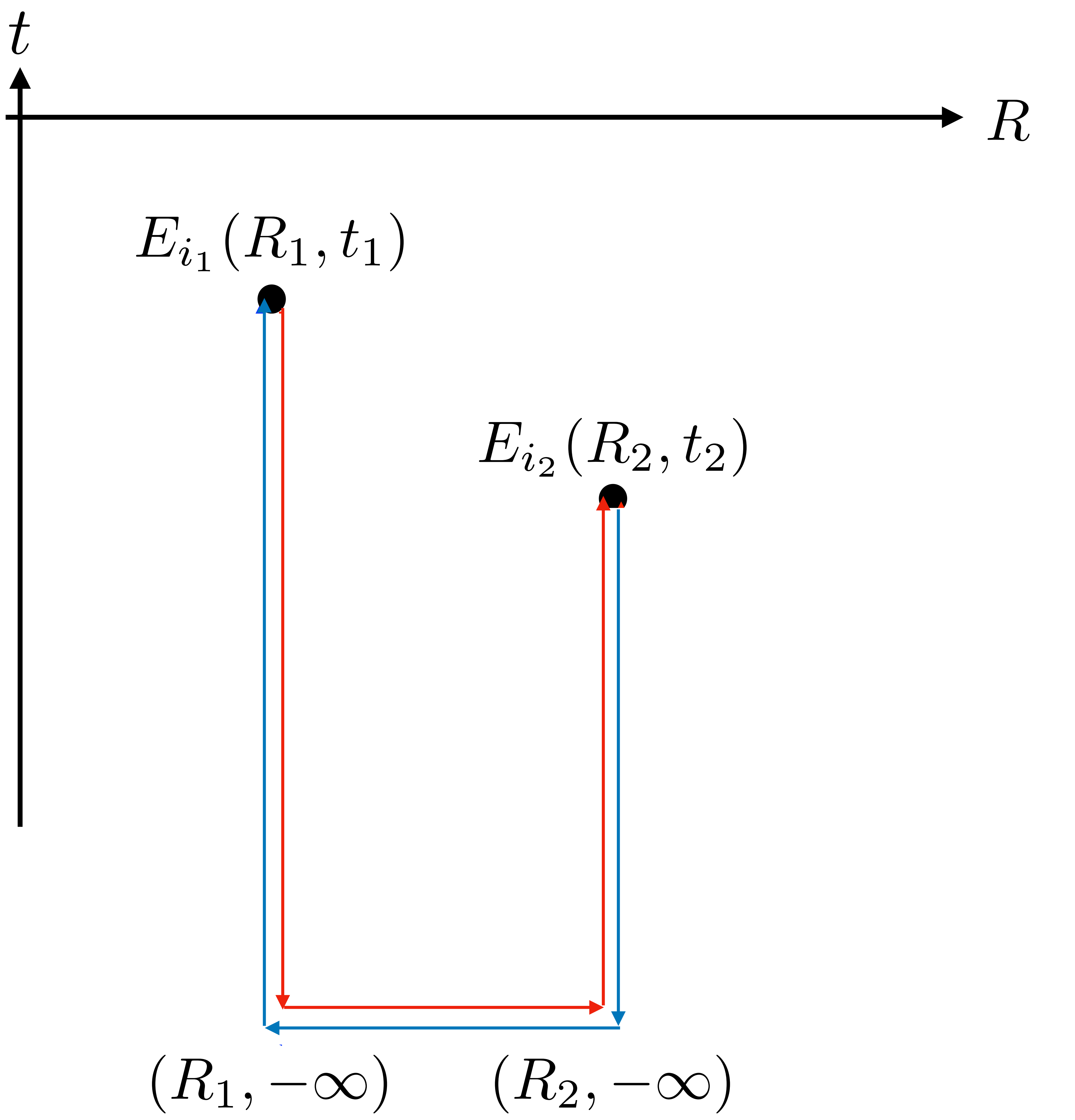
What do we need to calculate?

QGP chromoelectric correlators

for quarkonium transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





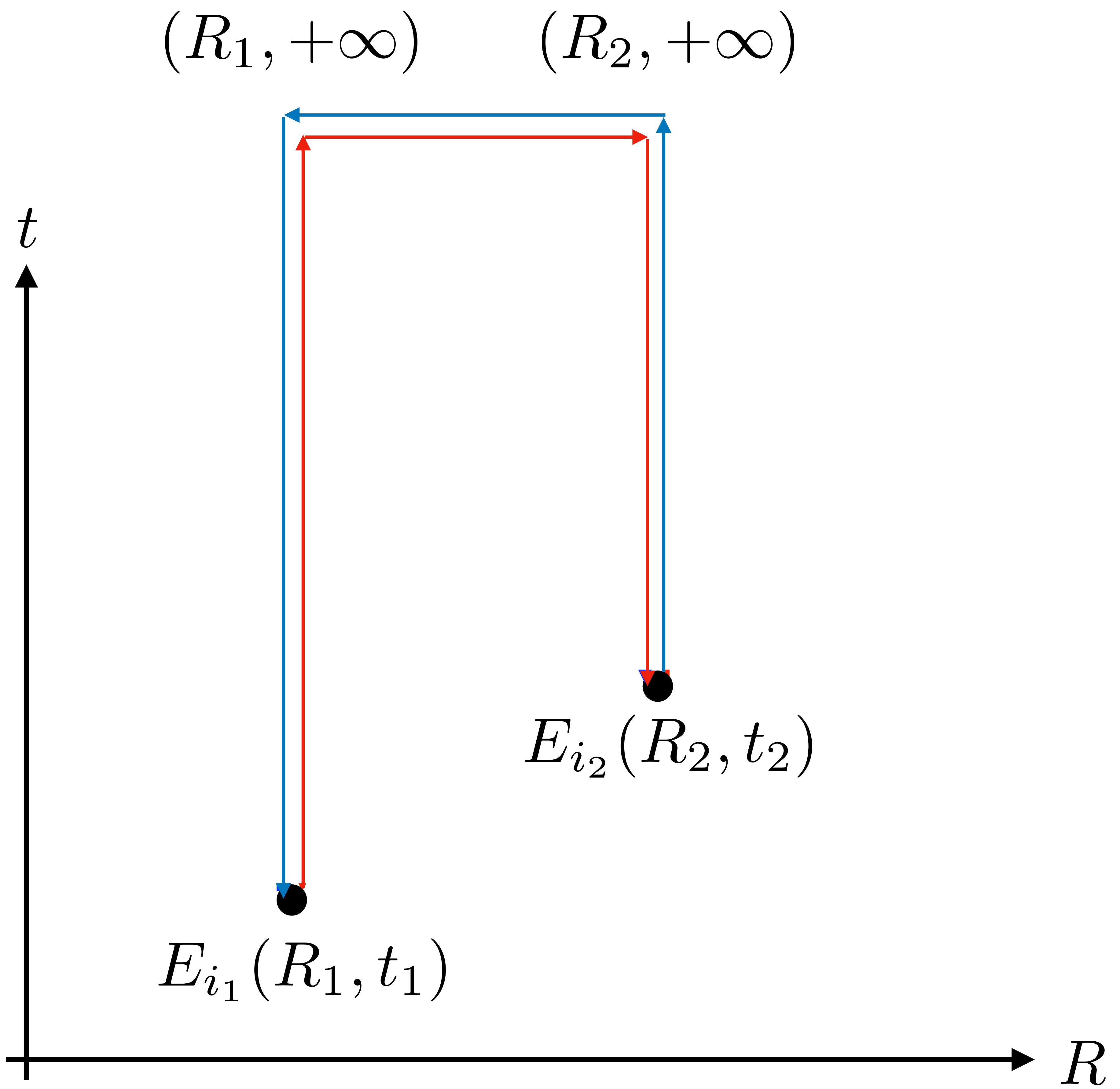
$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

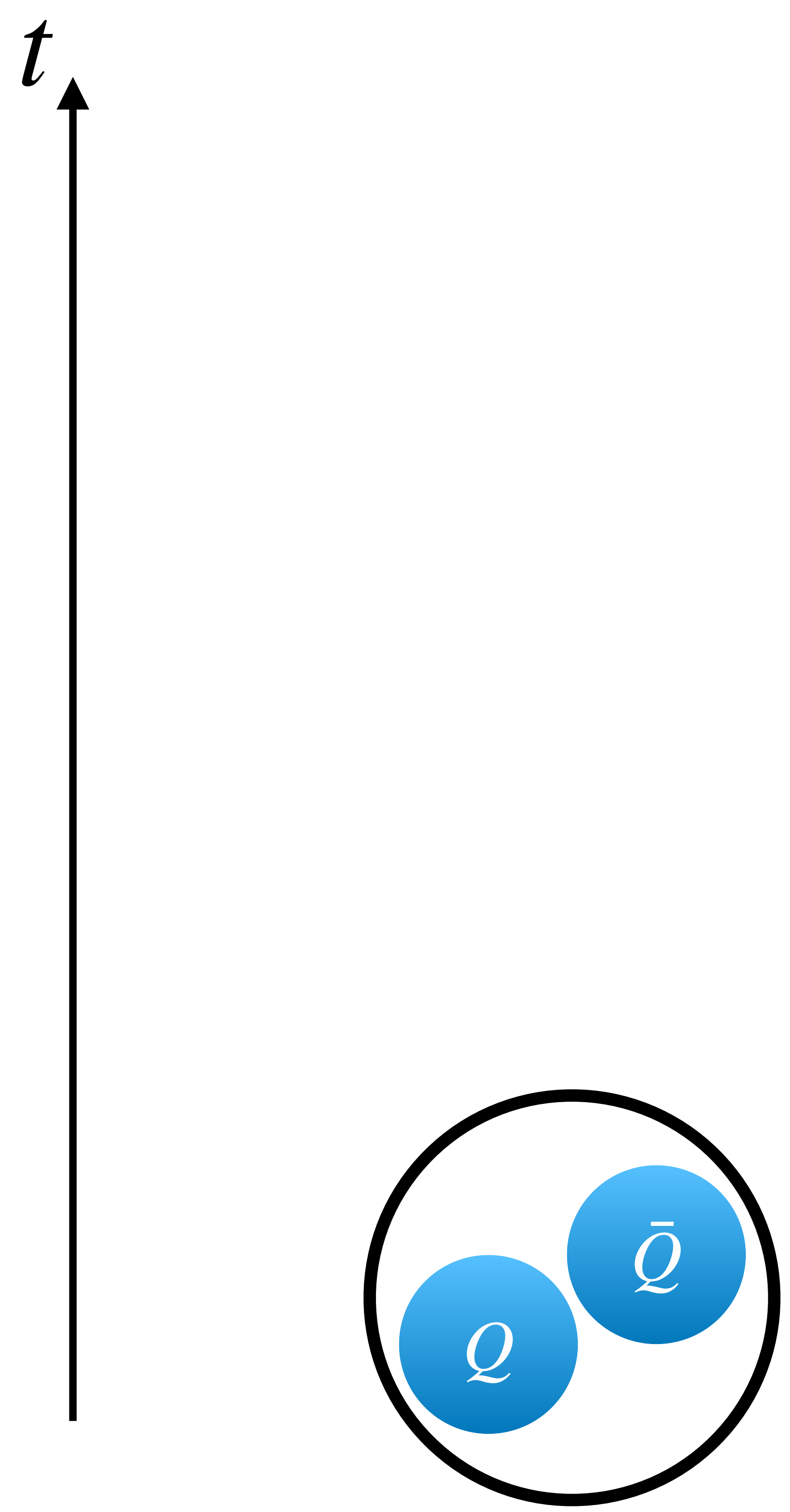
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonium transport





bound state:
color singlet

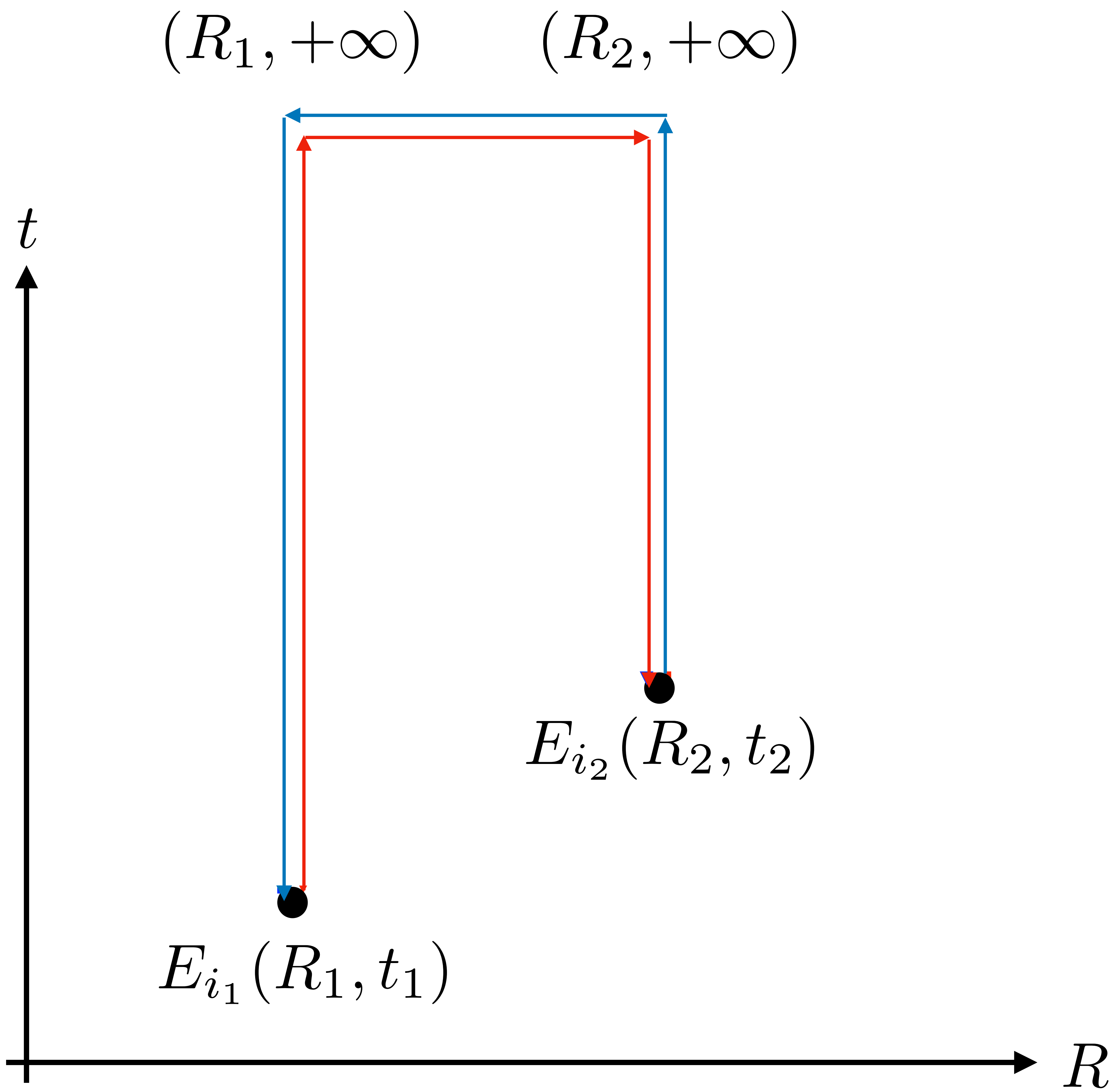
$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a_6 \right\rangle_T$$

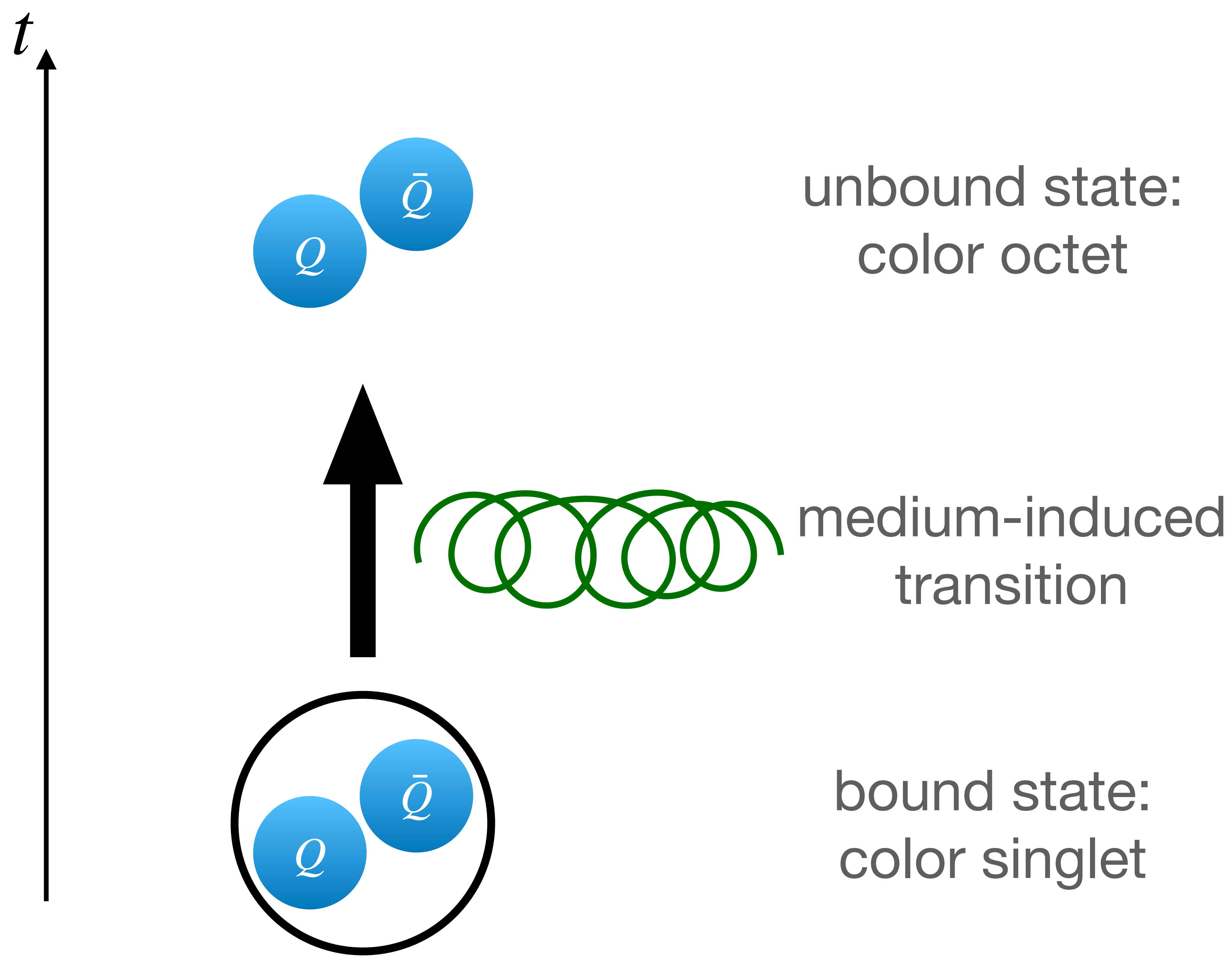
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonium transport

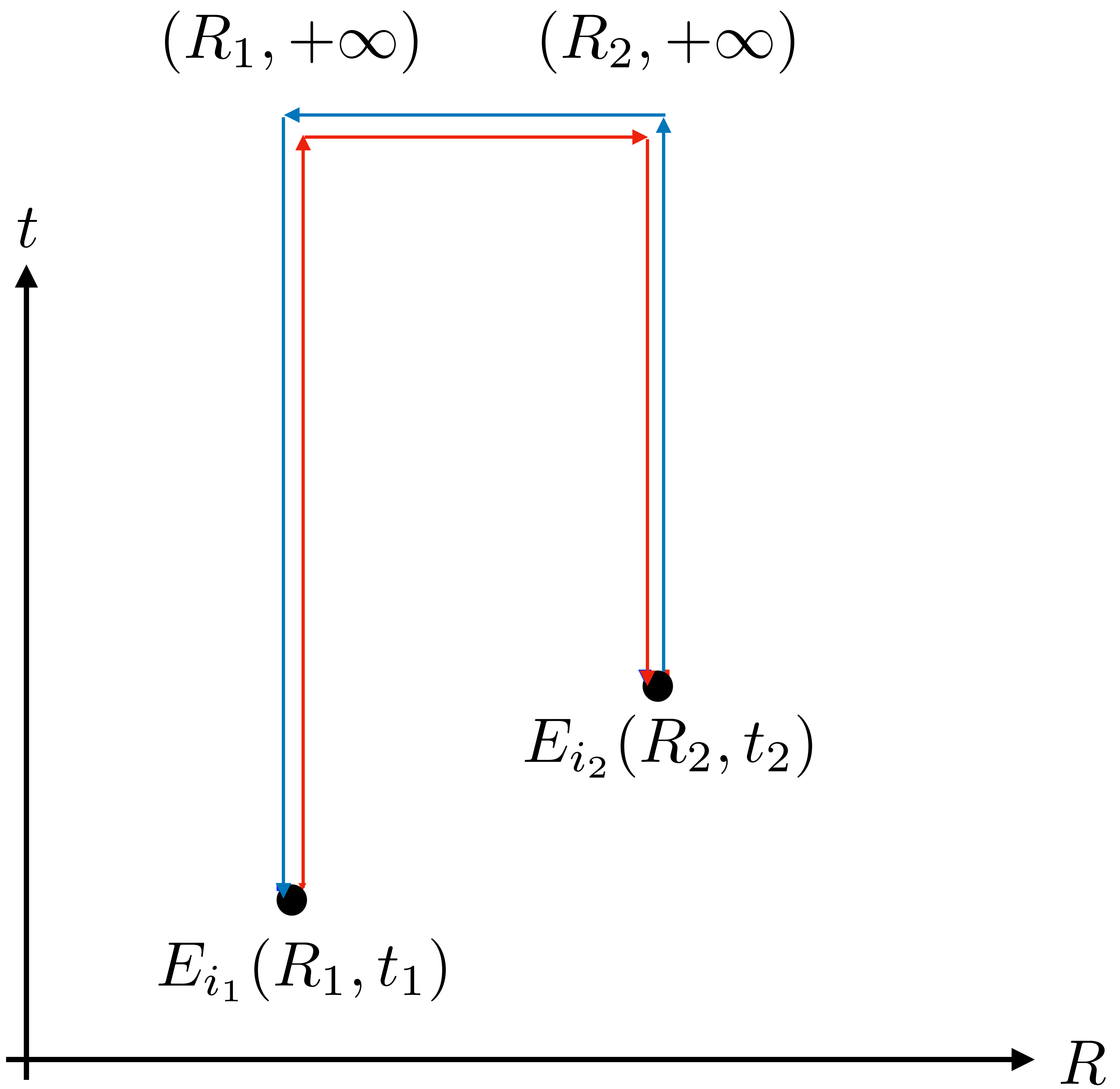


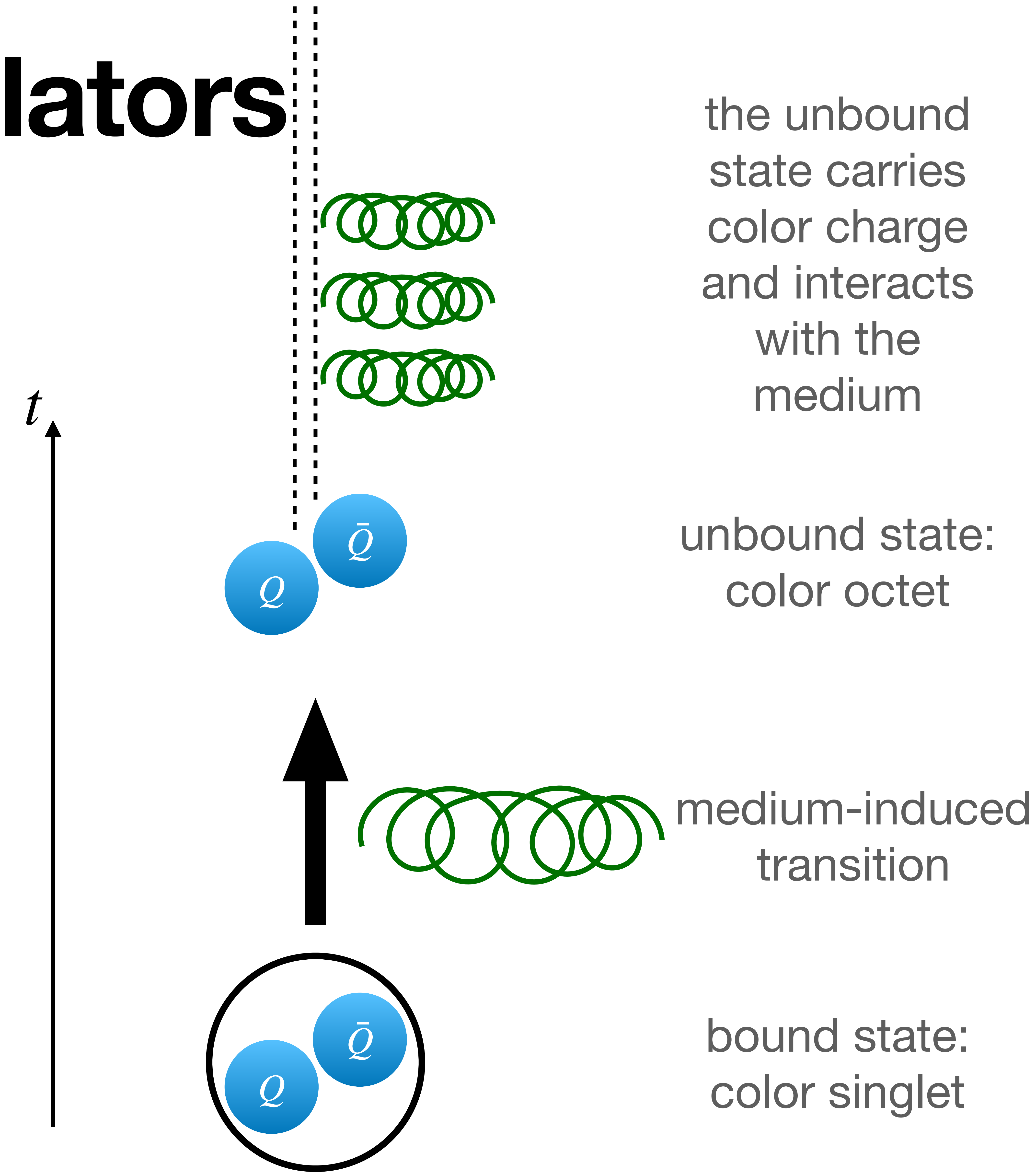


$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathcal{W}_2\right)^a \left(\mathcal{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

QGP chromoelectric correlators

for quarkonium transport





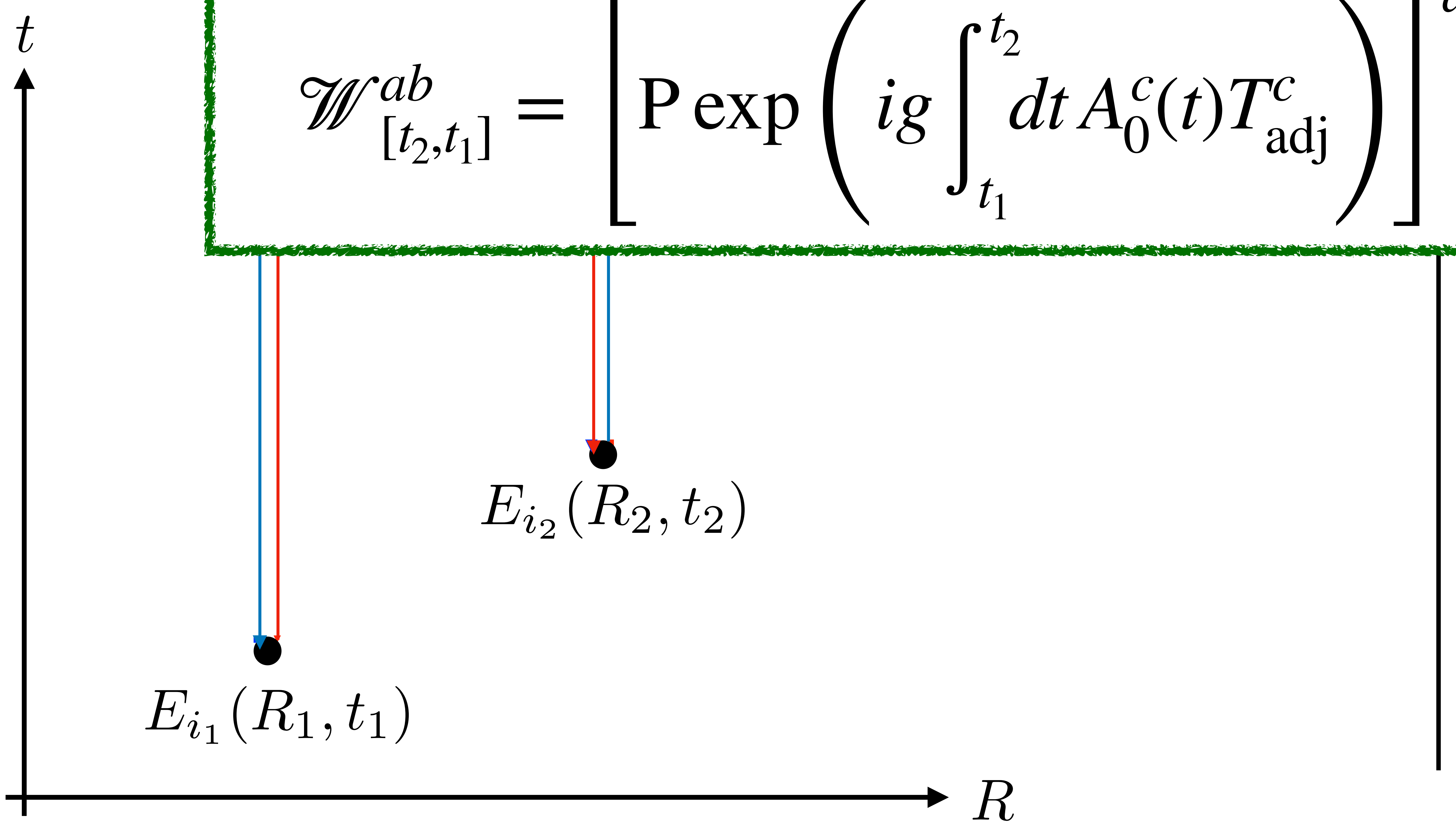
$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

QGP chromoelectric correlators

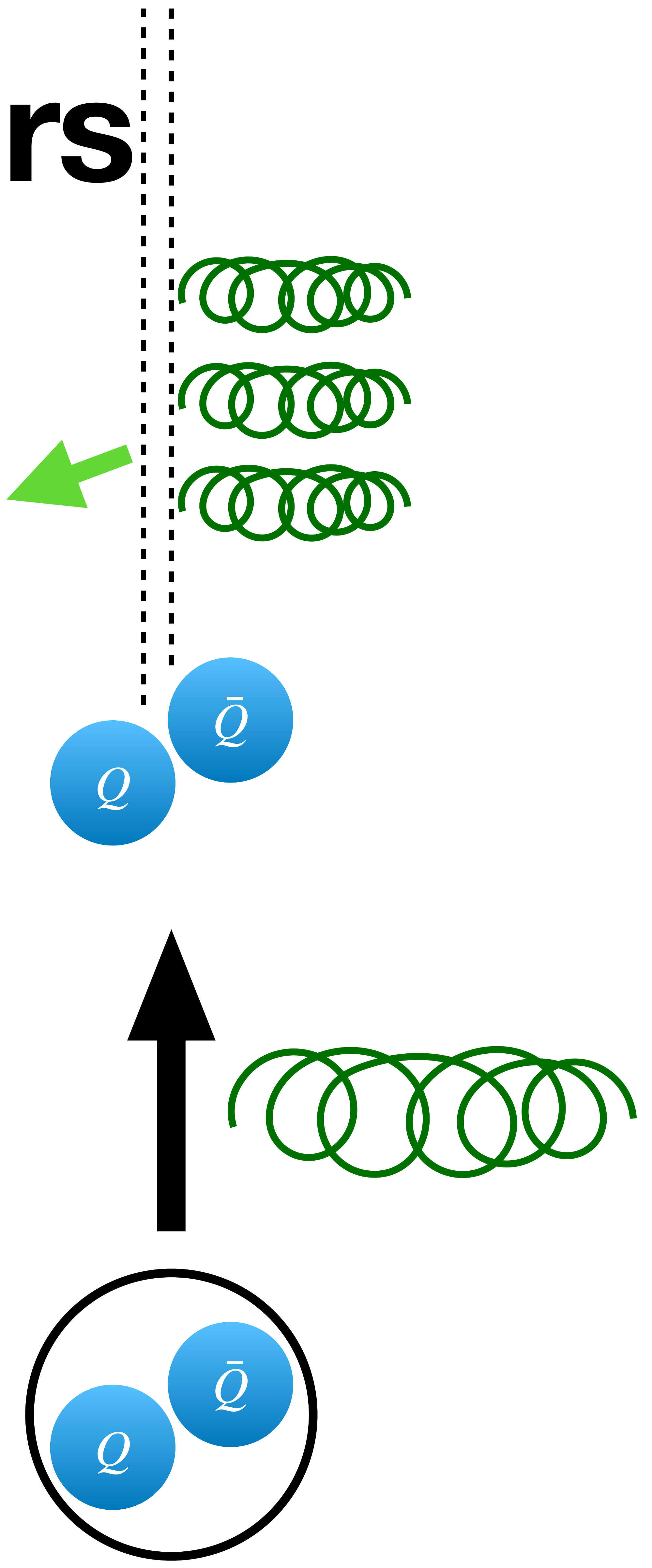
for quarkonium transport

Summing the one-gluon insertions along the heavy quark path generates a Wilson line:

$$\mathcal{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathcal{W}_2 \right)^a \left(\mathcal{W}_1 E_{i_1}(\mathbf{R}_1, t_1) \right)^a \right\rangle_T$$



the unbound state carries color charge and interacts with the medium

unbound state: color octet

medium-induced transition

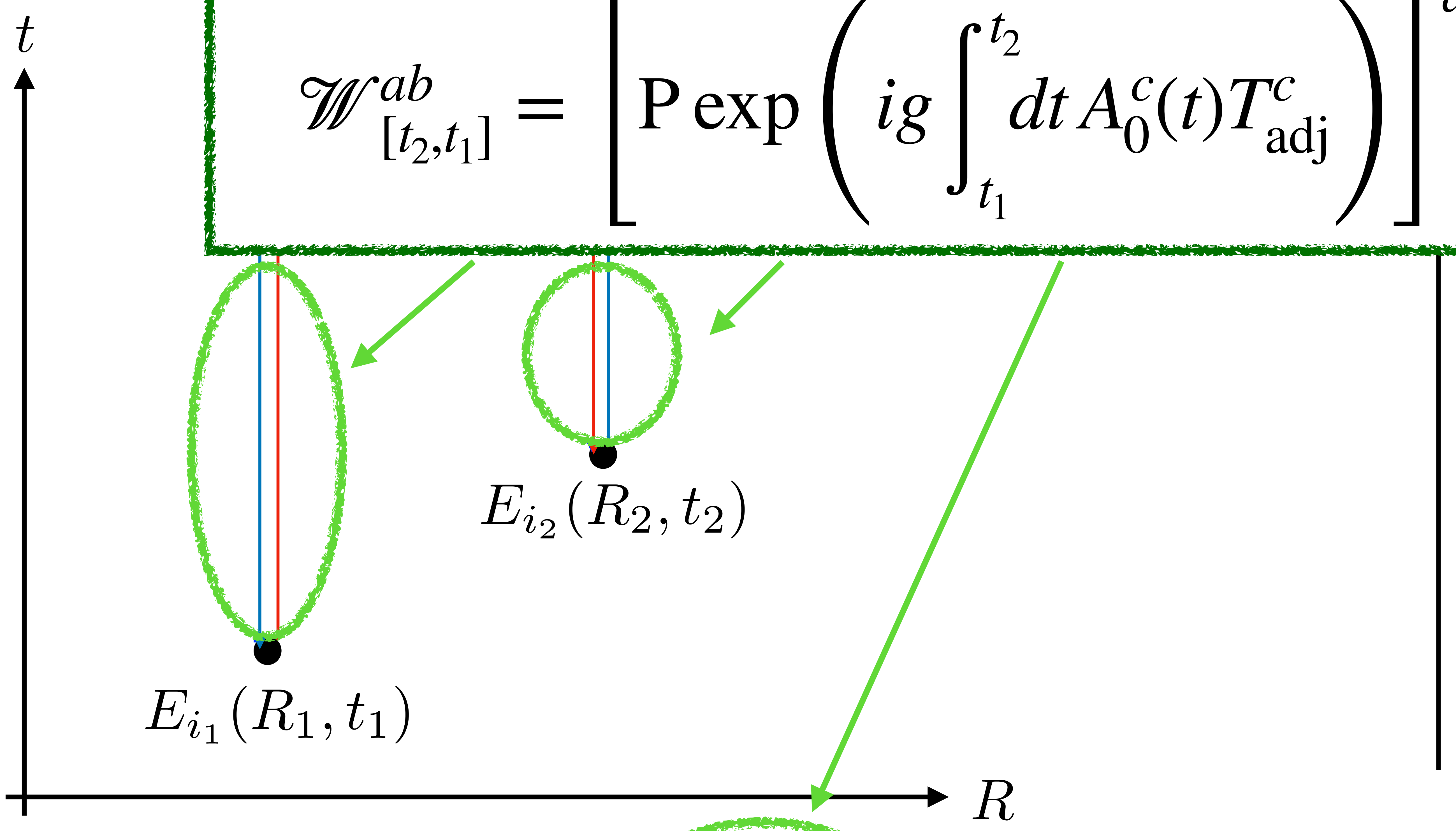
bound state: color singlet

QGP chromoelectric correlators

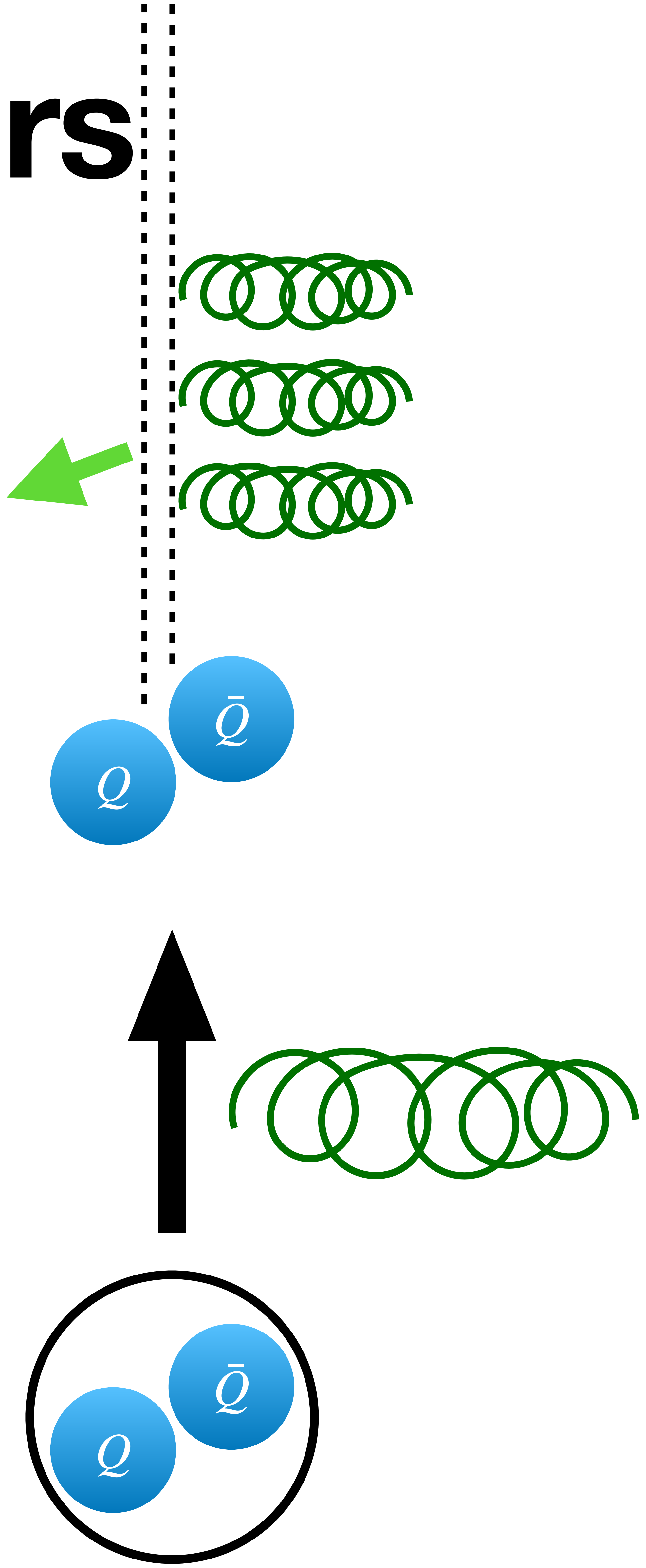
for quarkonium transport

Summing the one-gluon insertions along the heavy quark path generates a Wilson line:

$$\mathcal{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$

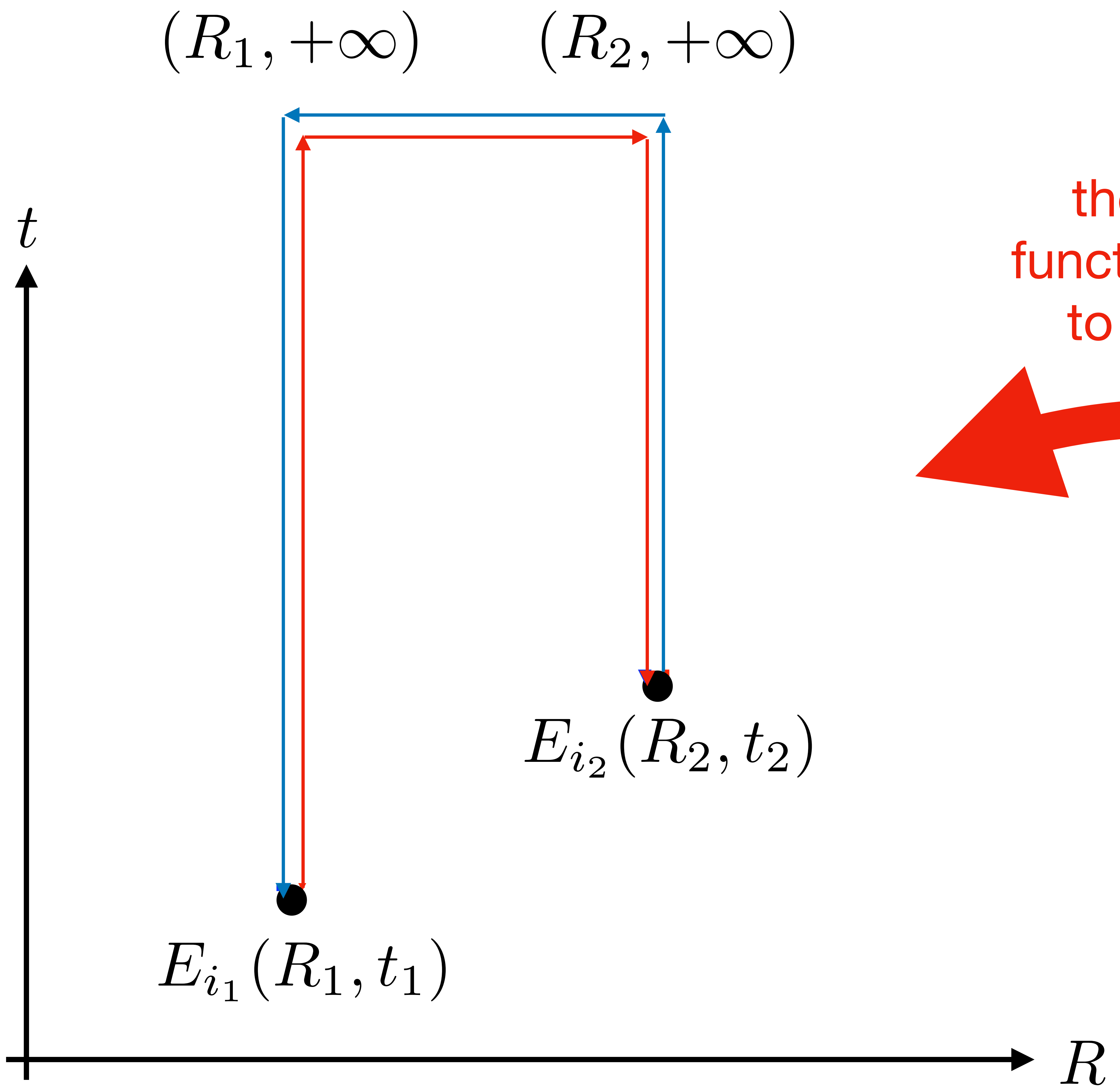


$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathcal{W}_2\right)^a \left(\mathcal{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

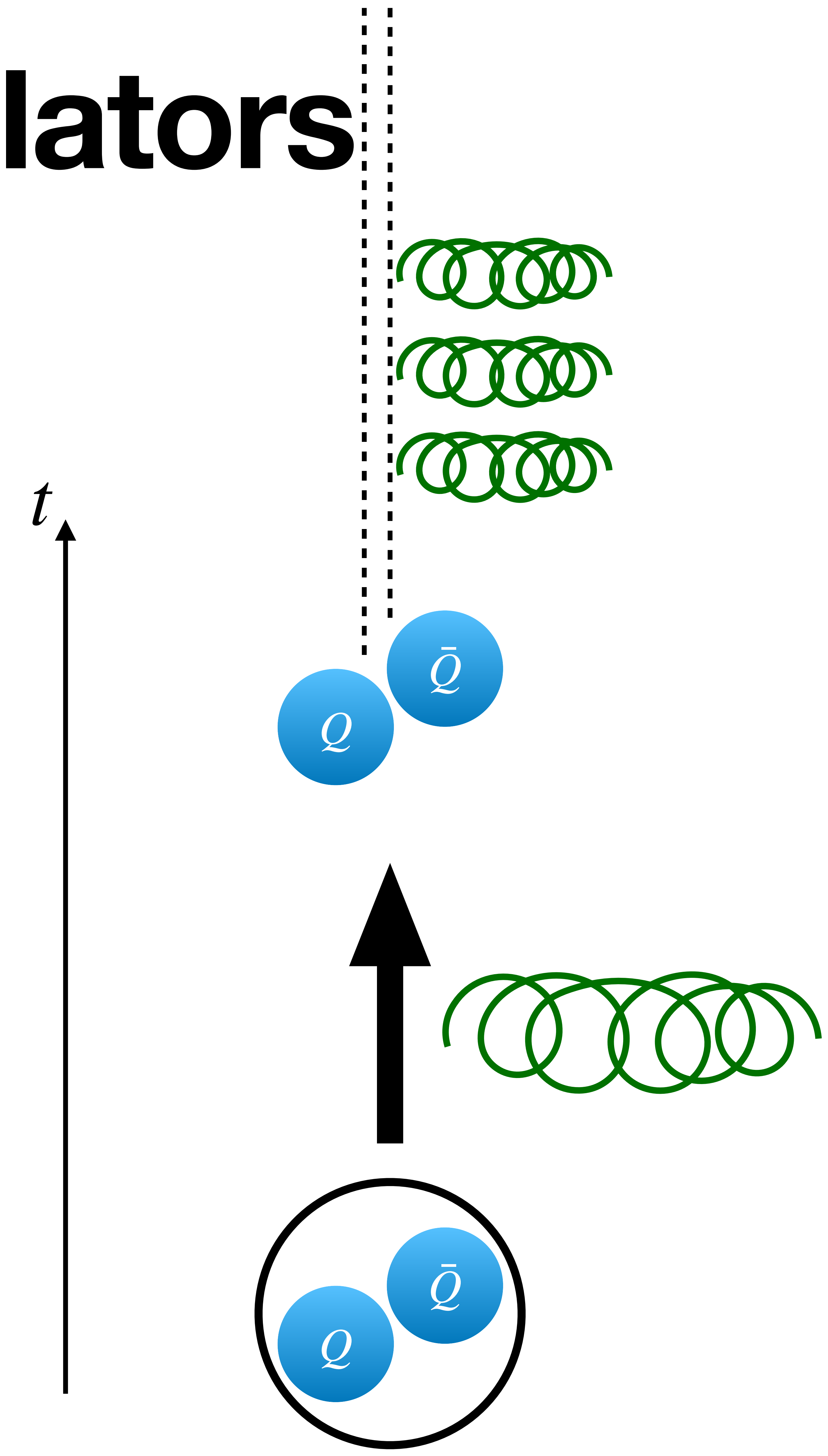


QGP chromoelectric correlators

for quarkonium transport



the correlation function associated to this process



the unbound state carries color charge and interacts with the medium

unbound state: color octet

medium-induced transition

bound state: color singlet

$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a_6 \right\rangle_T$$

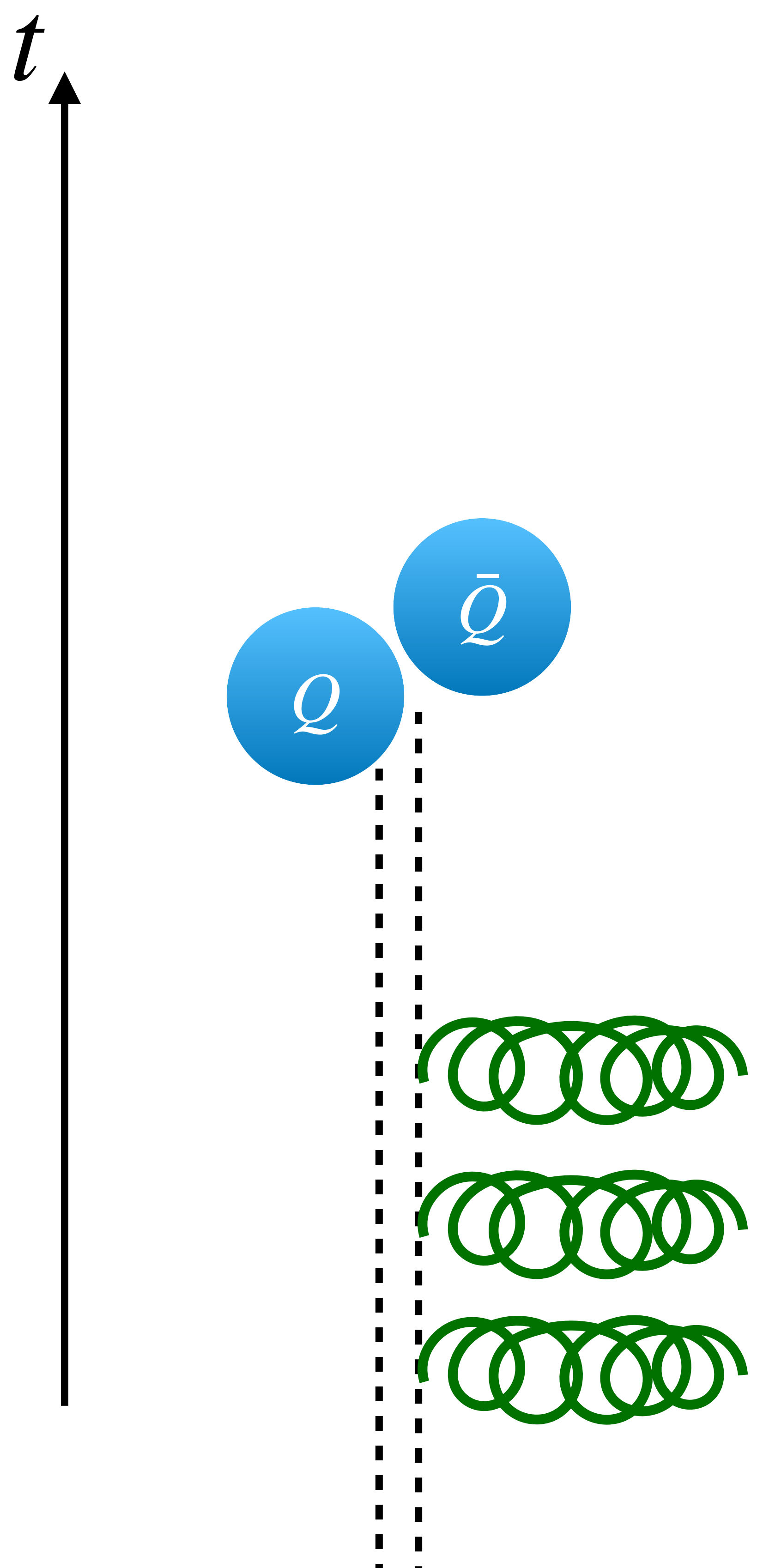
QGP chromoelectric correlators

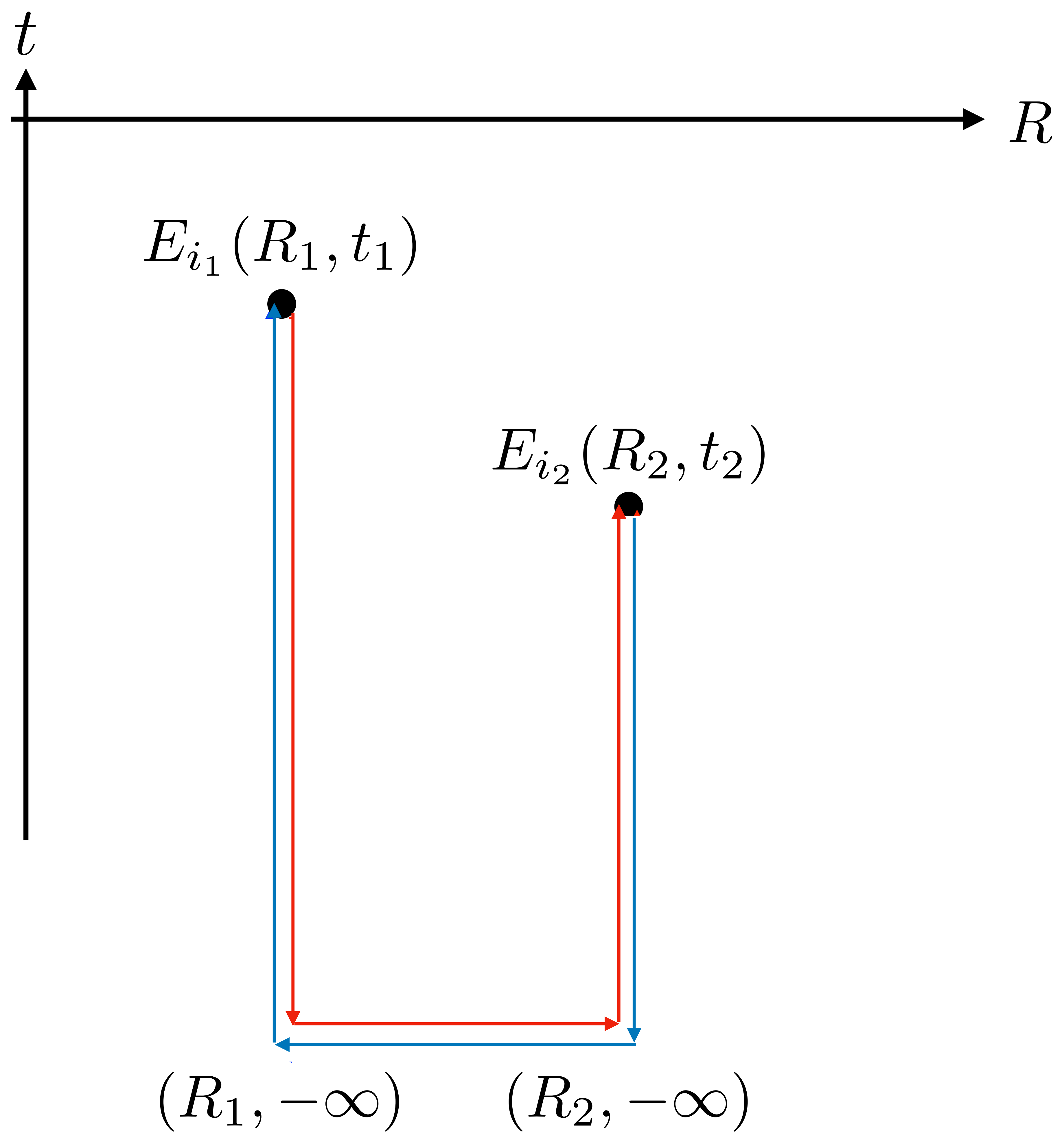
for quarkonium transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2) \right)^a \left(E_{i_1}(\mathbf{R}_1, t_1) \mathscr{W}_{1'} \right)^a \right\rangle_T$$

unbound state: color octet

the unbound state carries color charge and interacts with the medium





See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289 X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

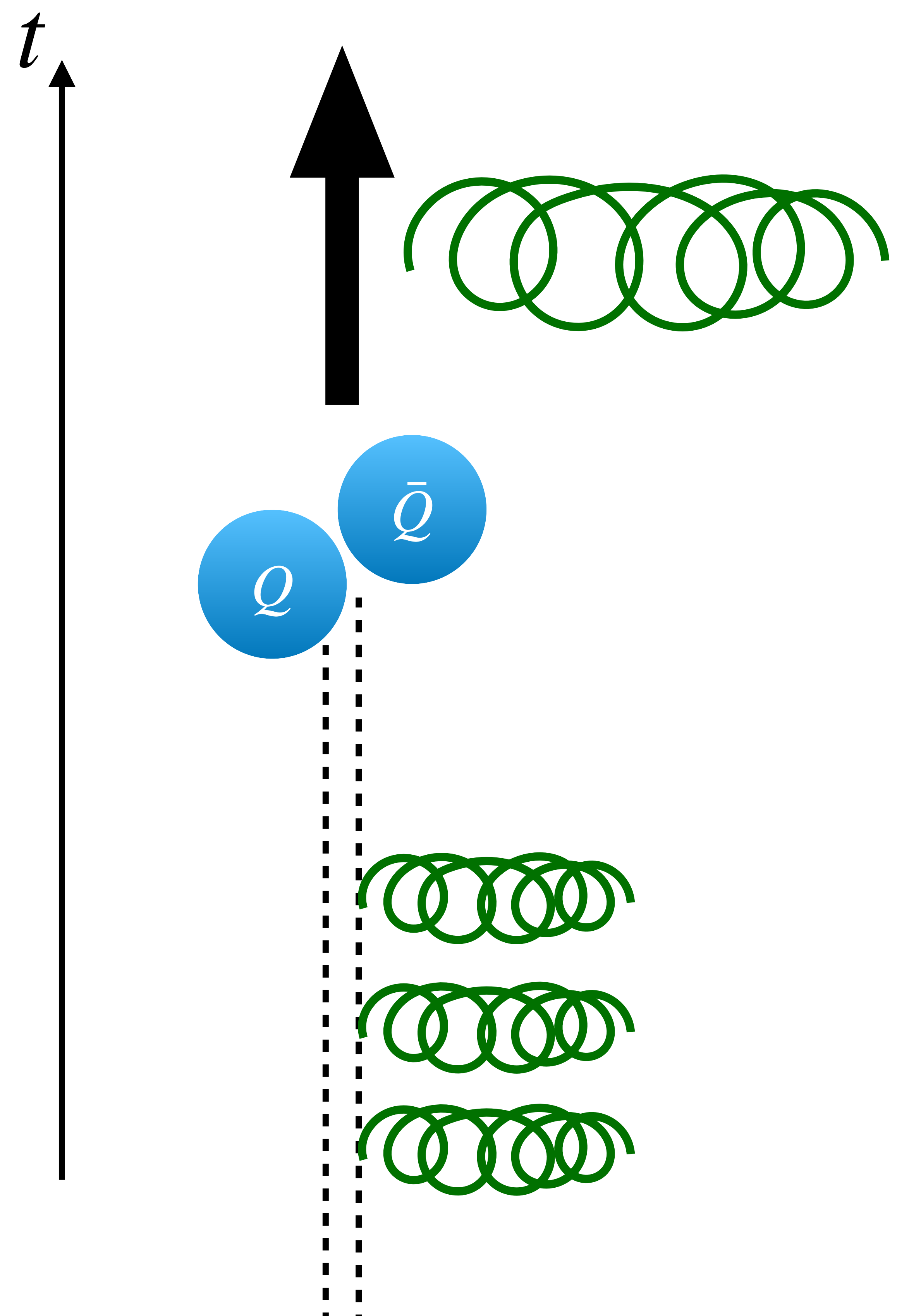
for quarkonium transport

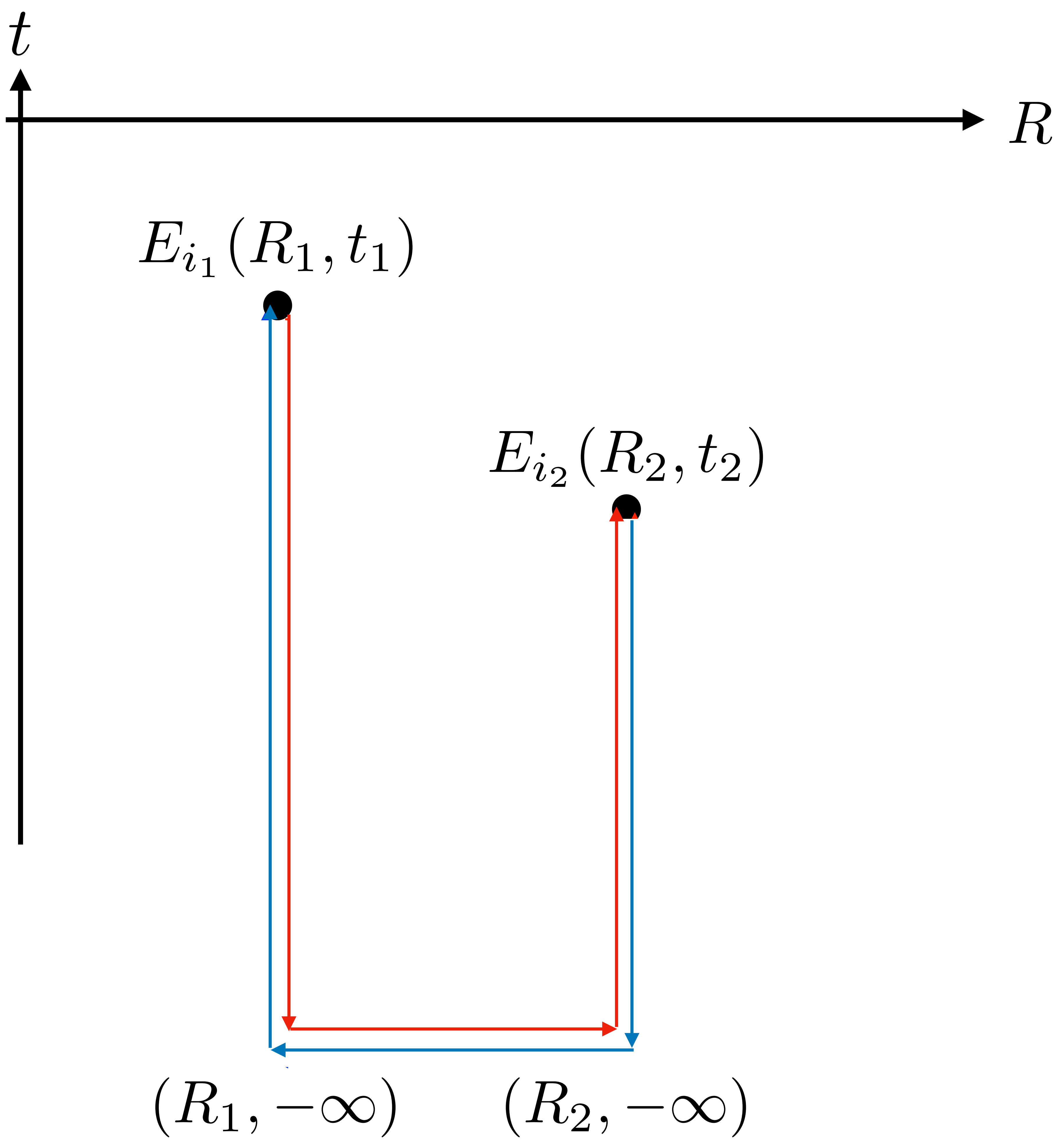
$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$

medium-induced transition

unbound state: color octet

the unbound state carries color charge and interacts with the medium





QGP chromoelectric correlators

for quarkonium transport

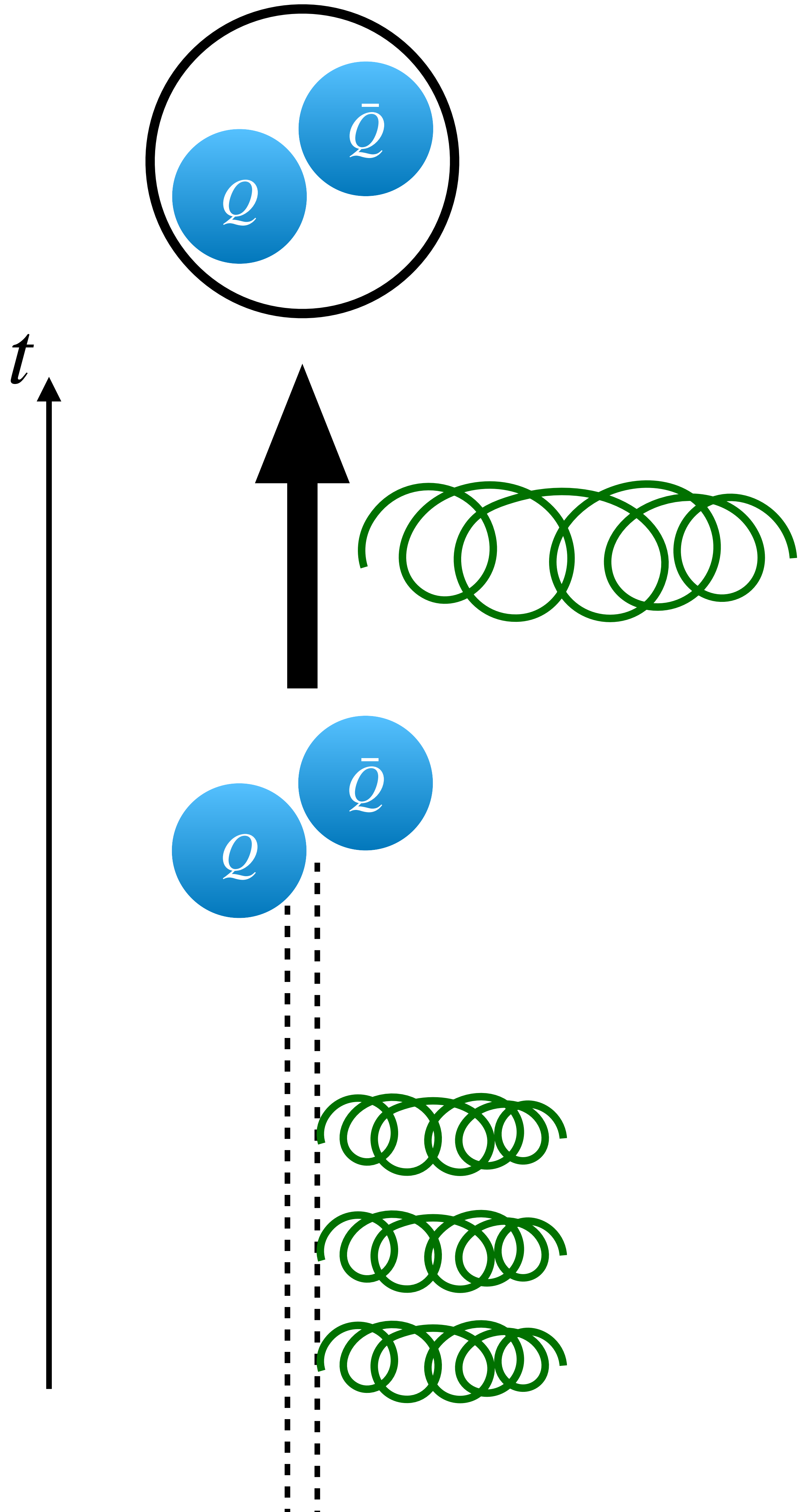
$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$

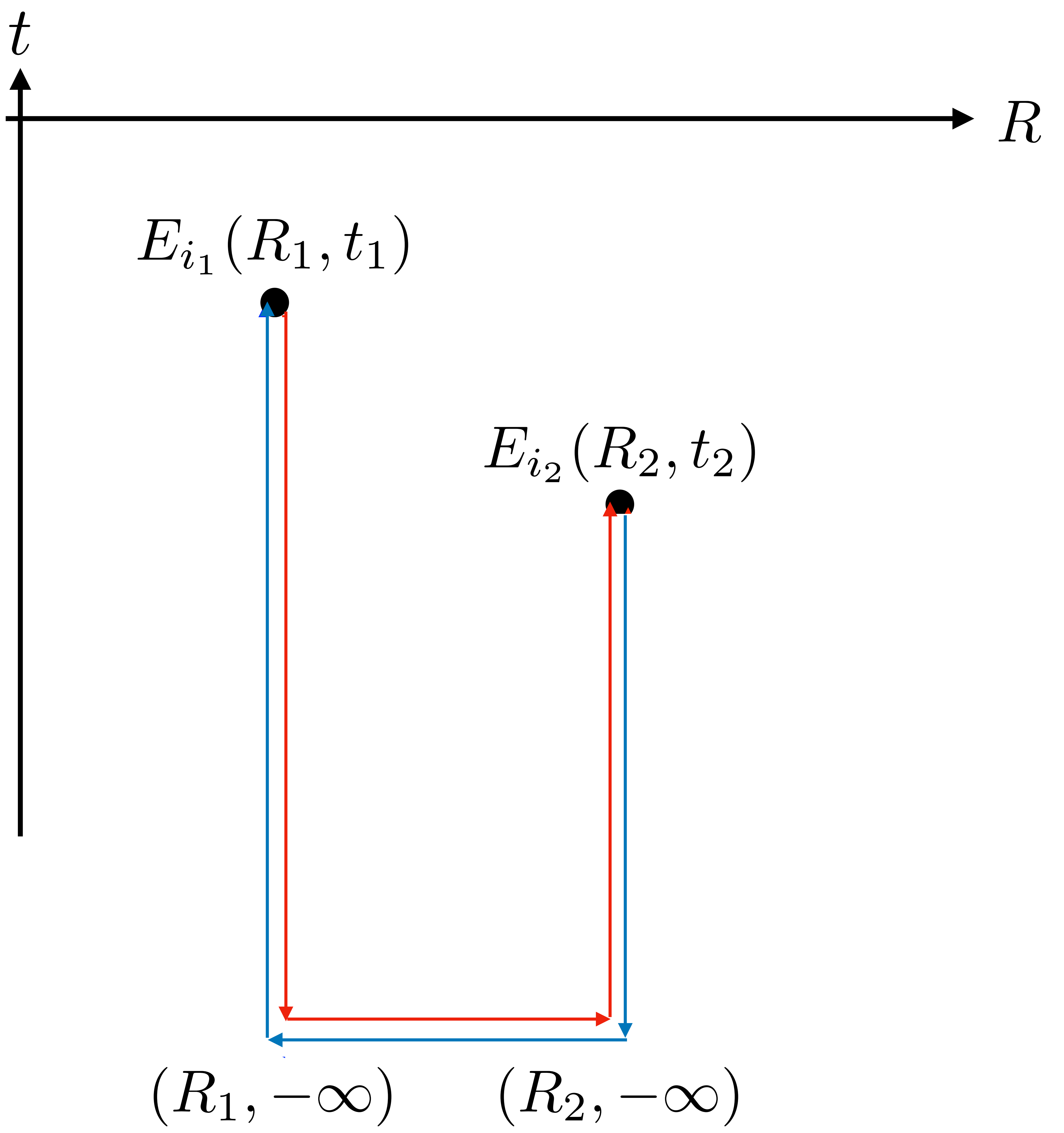
bound state:
color singlet

medium-induced
transition

unbound state:
color octet

the unbound
state carries
color charge
and interacts
with the
medium





QGP chromoelectric correlators

for quarkonium transport

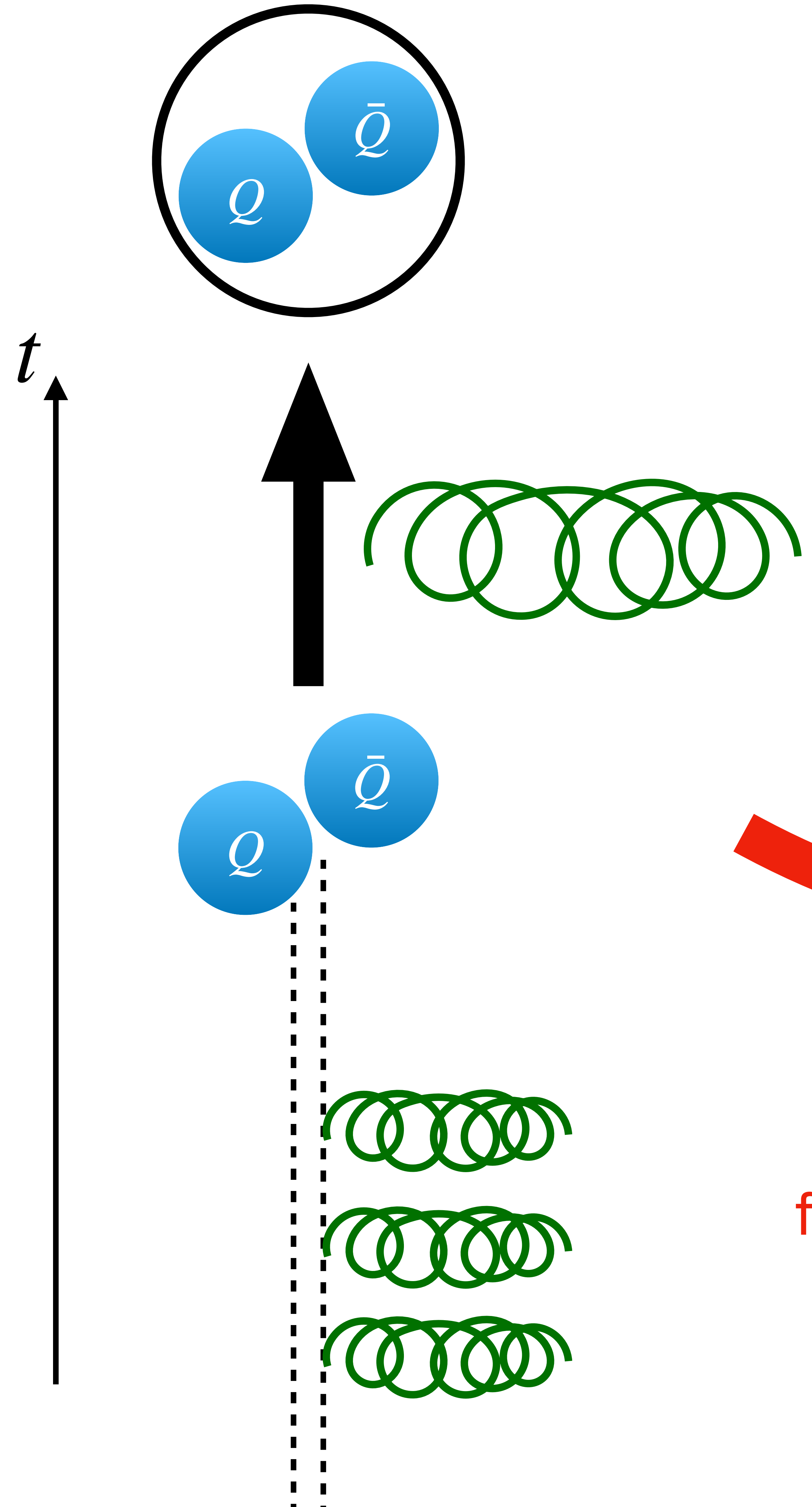
$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$

bound state:
color singlet

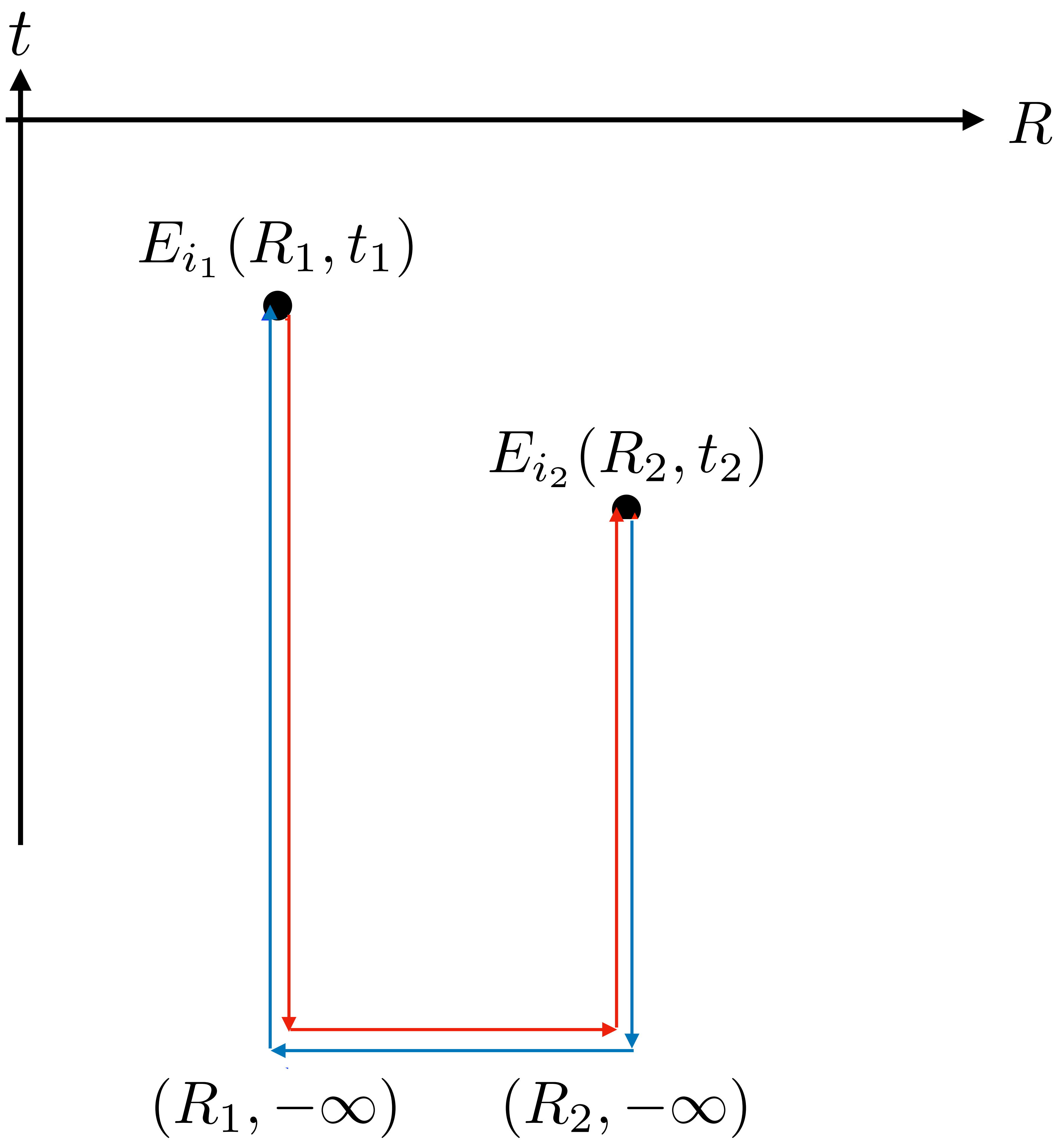
medium-induced
transition

unbound state:
color octet

the unbound
state carries
color charge
and interacts
with the
medium



the correlation
function associated
to this process



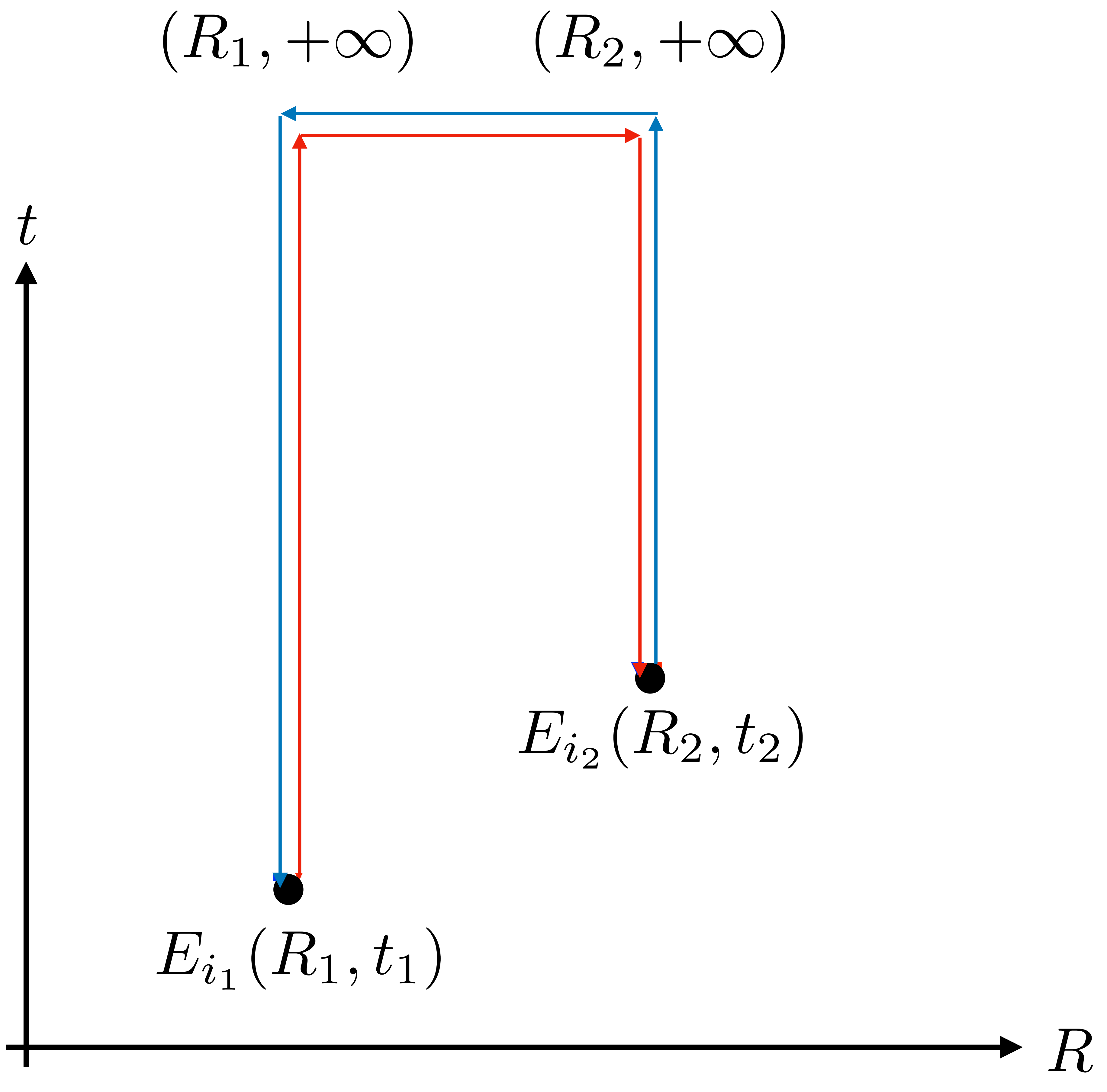
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

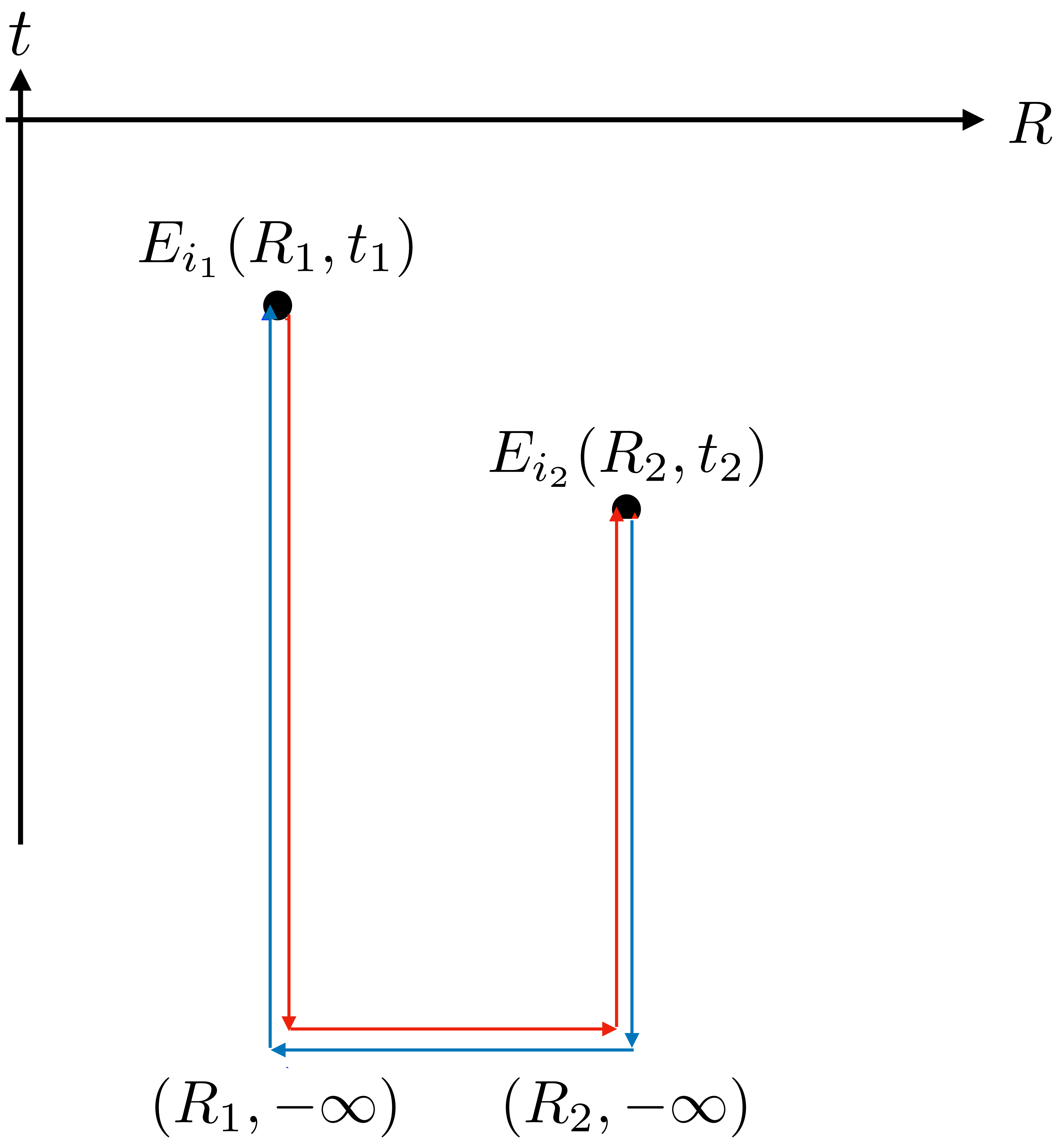
X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonium transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$





$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

Why are these correlators interesting?

These determine the dissociation and formation rates of quarkonia (in the quantum optical limit):

$$\Gamma^{\mathrm{diss}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3} \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \, |\langle \psi_{\mathscr{B}} | \mathbf{r} | \Psi_{\mathbf{p}_{\mathrm{rel}}} \rangle|^2 [g_E^{++}]^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}_{\mathrm{rel}}^2}{M}, \mathbf{q}\right),$$

$$\Gamma^{\mathrm{form}} \propto \int \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{cm}}}{(2\pi)^3} \frac{\mathrm{d}^3\mathbf{p}_{\mathrm{rel}}}{(2\pi)^3} \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \, |\langle \psi_{\mathscr{B}} | \mathbf{r} | \Psi_{\mathbf{p}_{\mathrm{rel}}} \rangle|^2 [g_E^{--}]^{>}_{ii}\left(q^0 = \frac{\mathbf{p}_{\mathrm{rel}}^2}{M} - E_{\mathscr{B}}, \mathbf{q}\right) \times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\mathrm{cm}}, \mathbf{r} = 0, \mathbf{p}_{\mathrm{rel}}, t)\,.$$

A comparison with heavy quark diffusion

Different physics with the same building blocks

[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

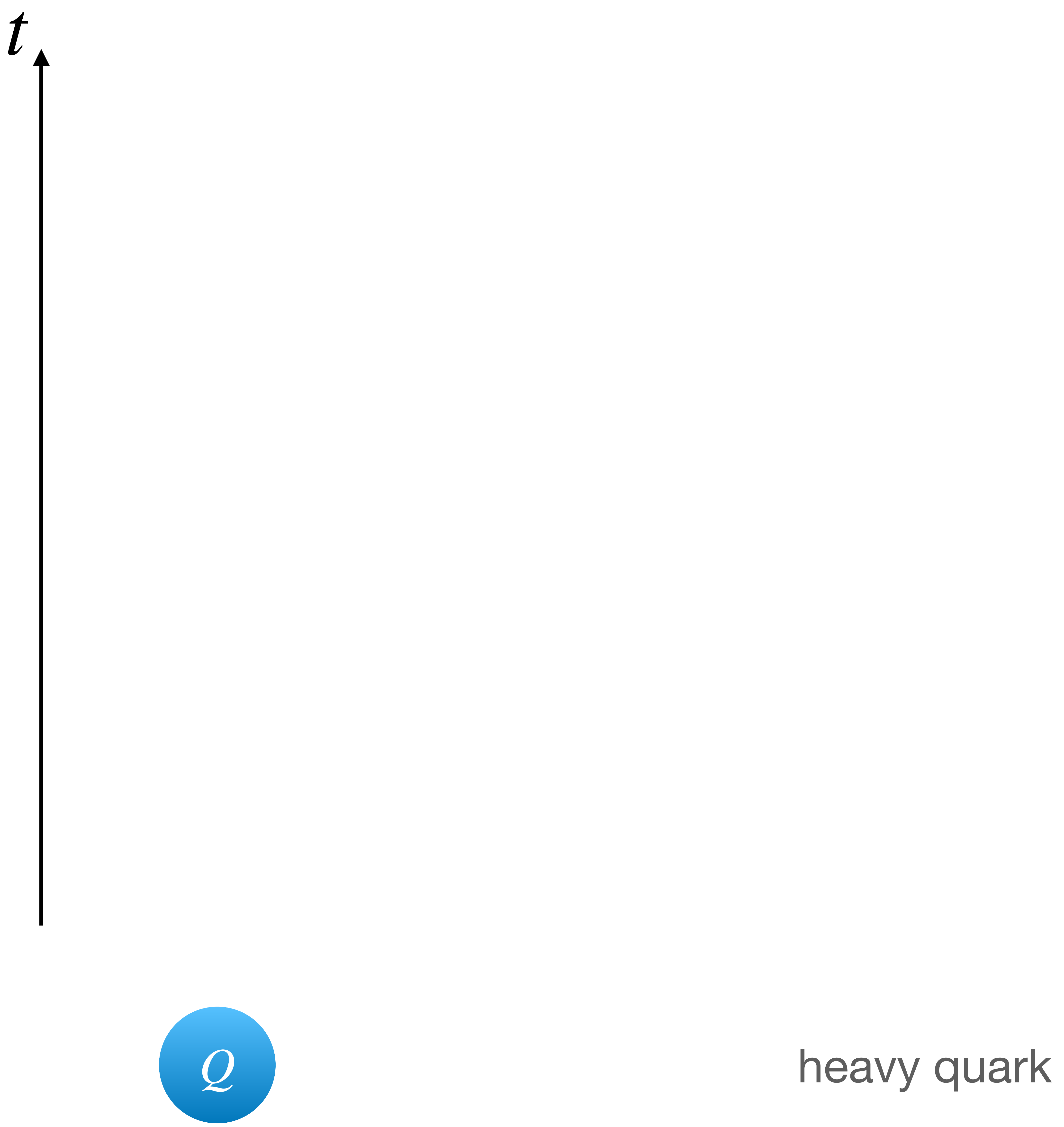
Heavy quark diffusion

an analogous picture

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



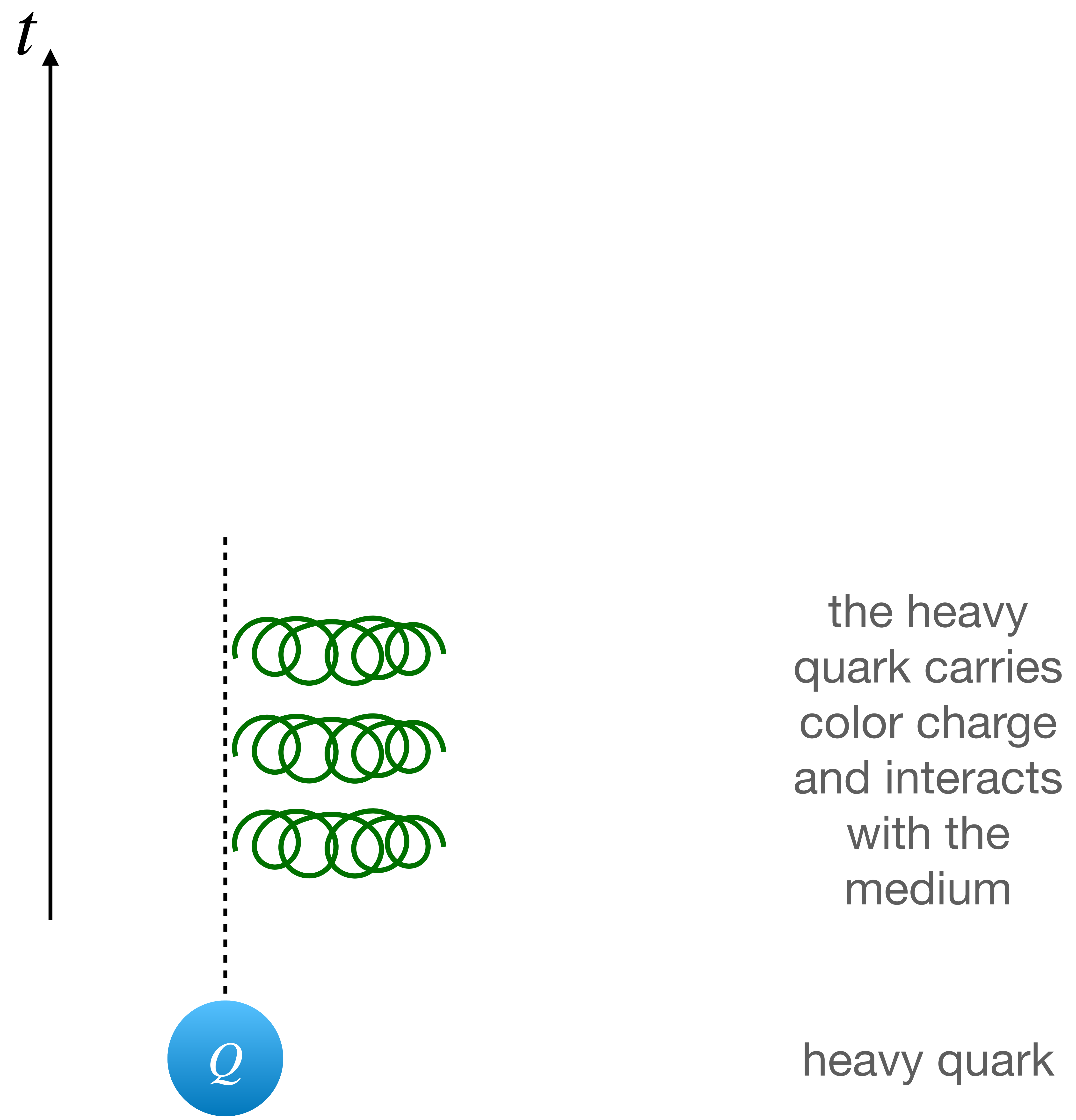
Heavy quark diffusion

an analogous picture

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



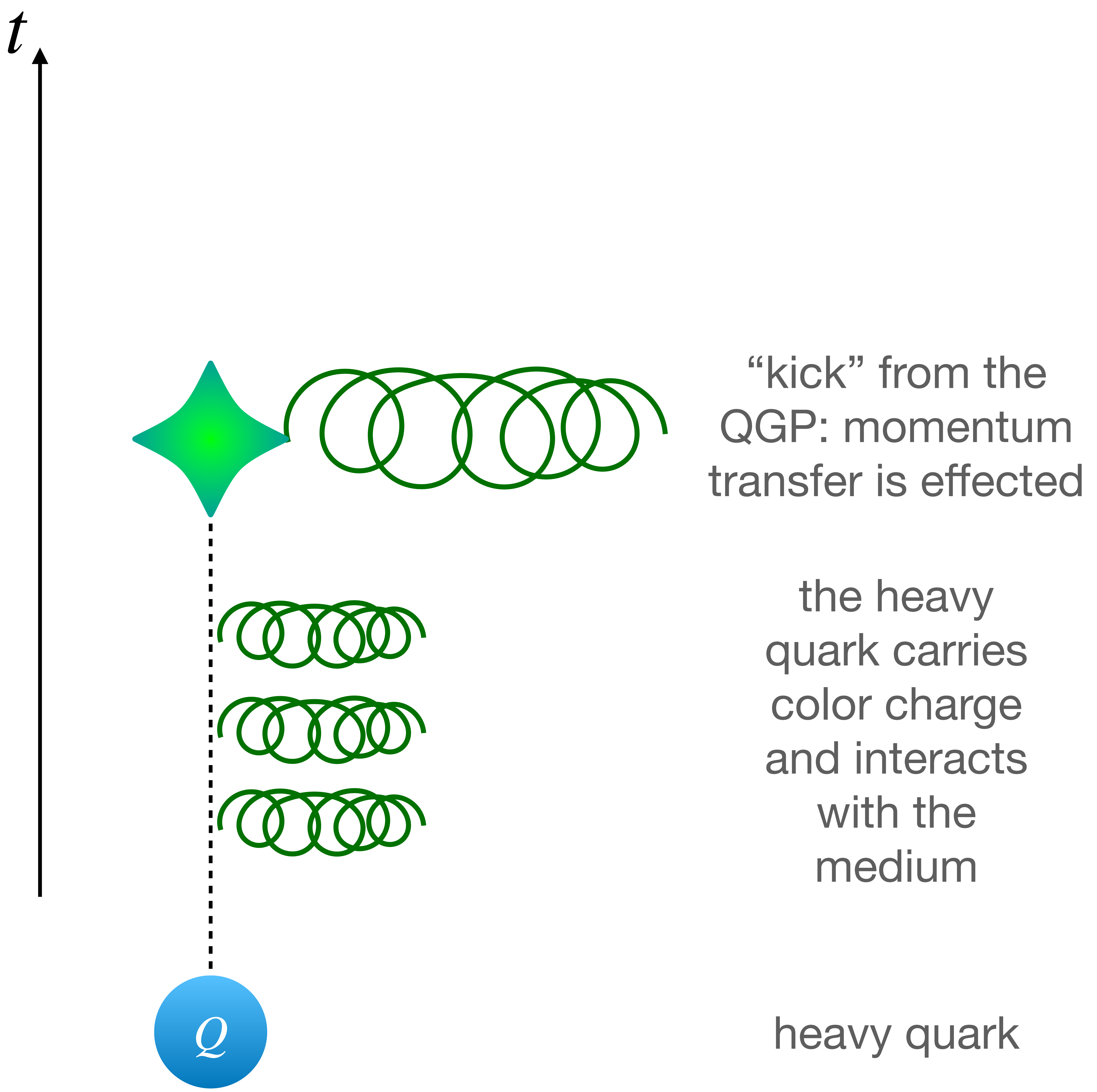
Heavy quark diffusion

an analogous picture

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

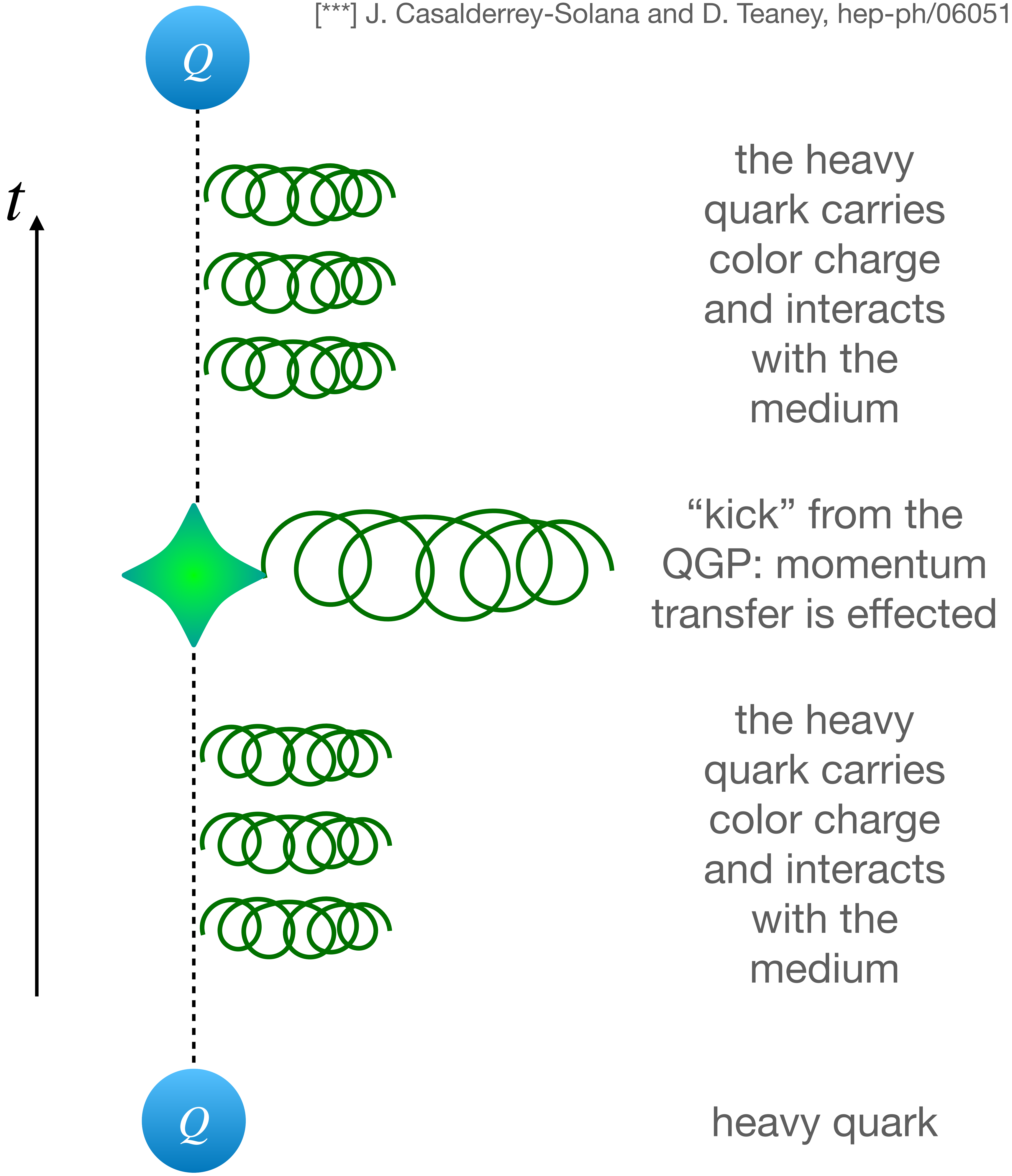
Heavy quark diffusion

an analogous picture

- The heavy quark diffusion coefficient is also defined from a correlation of chromoelectric fields:

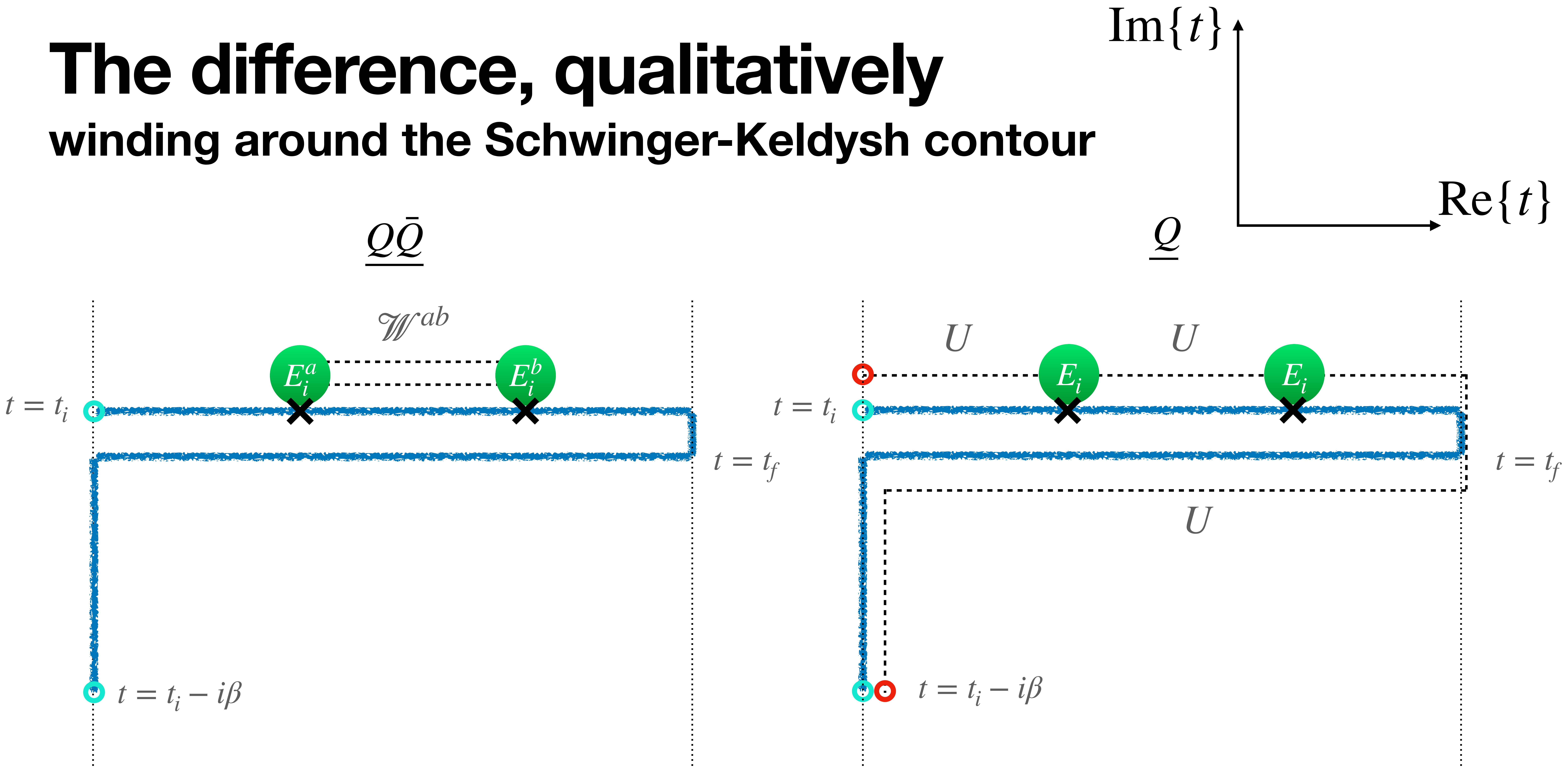
$$\langle \mathrm{Tr}\Big[(U_{[\infty,t]}E_i(t)U_{[t,-\infty]})^\dagger \times (U_{[\infty,0]}E_i(0)U_{[0,-\infty]})\Big]\rangle$$

- It reflects the typical momentum transfer $\langle p^2 \rangle$ received from "kicks" from the medium.



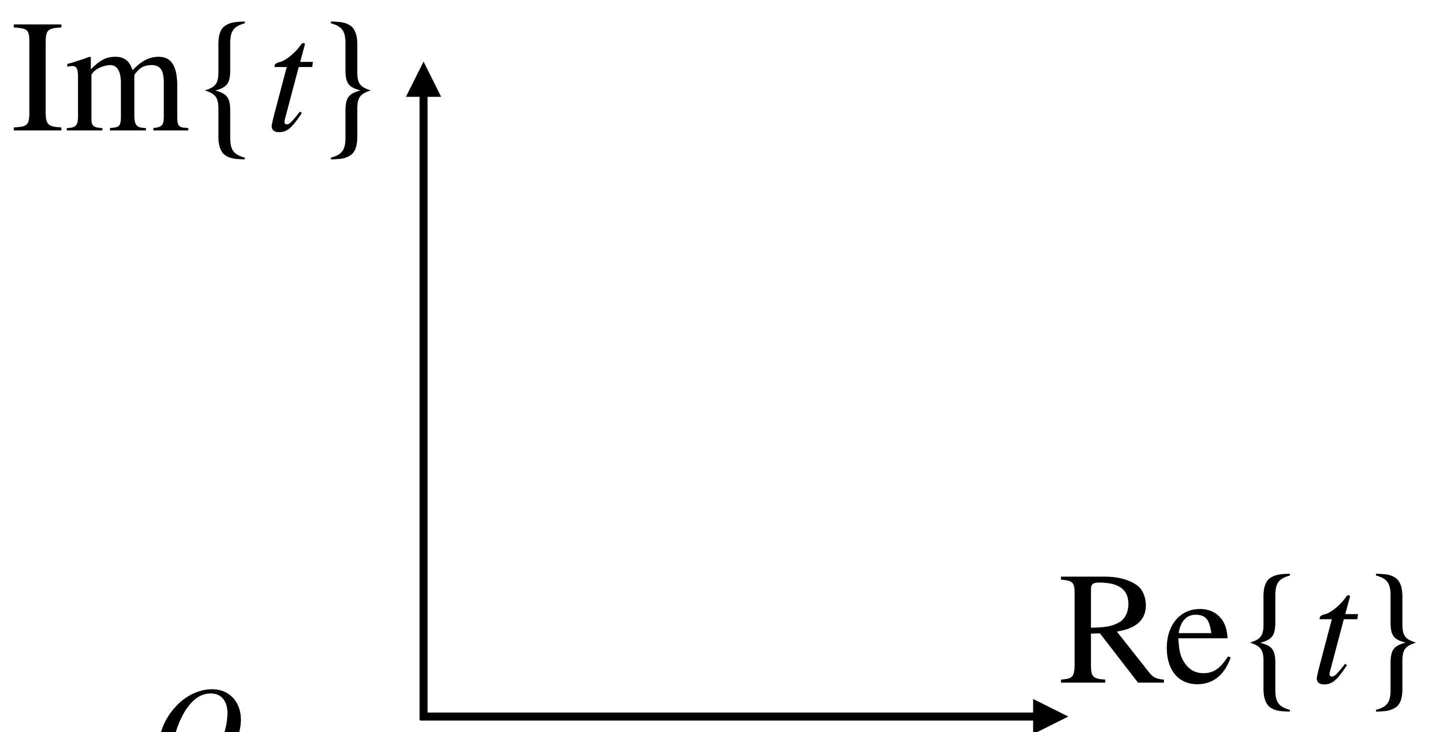
The difference, qualitatively

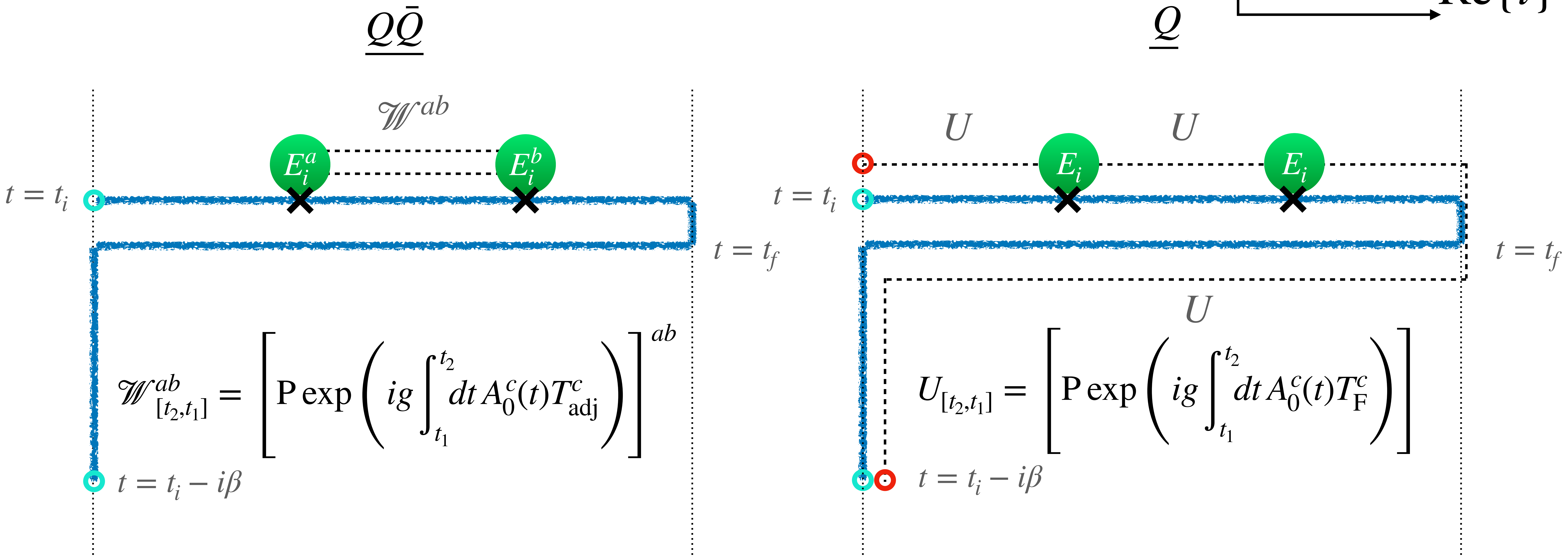
winding around the Schwinger-Keldysh contour



The difference, qualitatively

winding around the Schwinger-Keldysh contour





How to do calculations

the Schwinger-Keldysh contour

Path integral representations

- Imaginary time calculations:

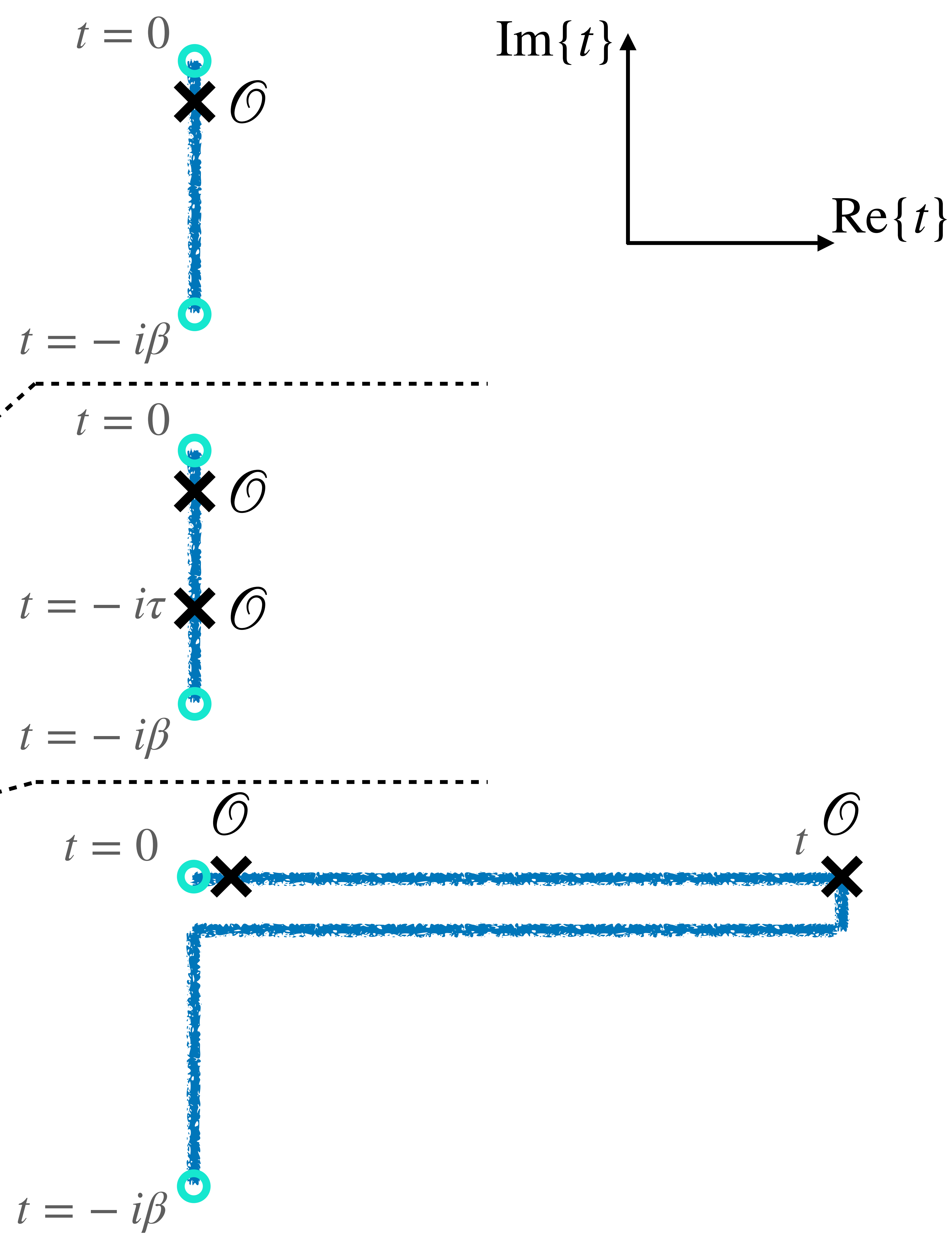
equilibrium #: $\langle \mathcal{O} \rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}e^{-\beta H}\right]$,

and also two-point functions:

$$\langle \mathcal{O}(\tau)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}(0)e^{-\tau H}\mathcal{O}(0)e^{-(\beta-\tau)H}\right]$$

- Real time calculations:

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[e^{iHt}\mathcal{O}(0)e^{-iHt}\mathcal{O}(0)e^{-\beta H}\right]$$



Path integral representations

How to do calculations

the Schwinger-Keldysh contour

- Imaginary time calculations:

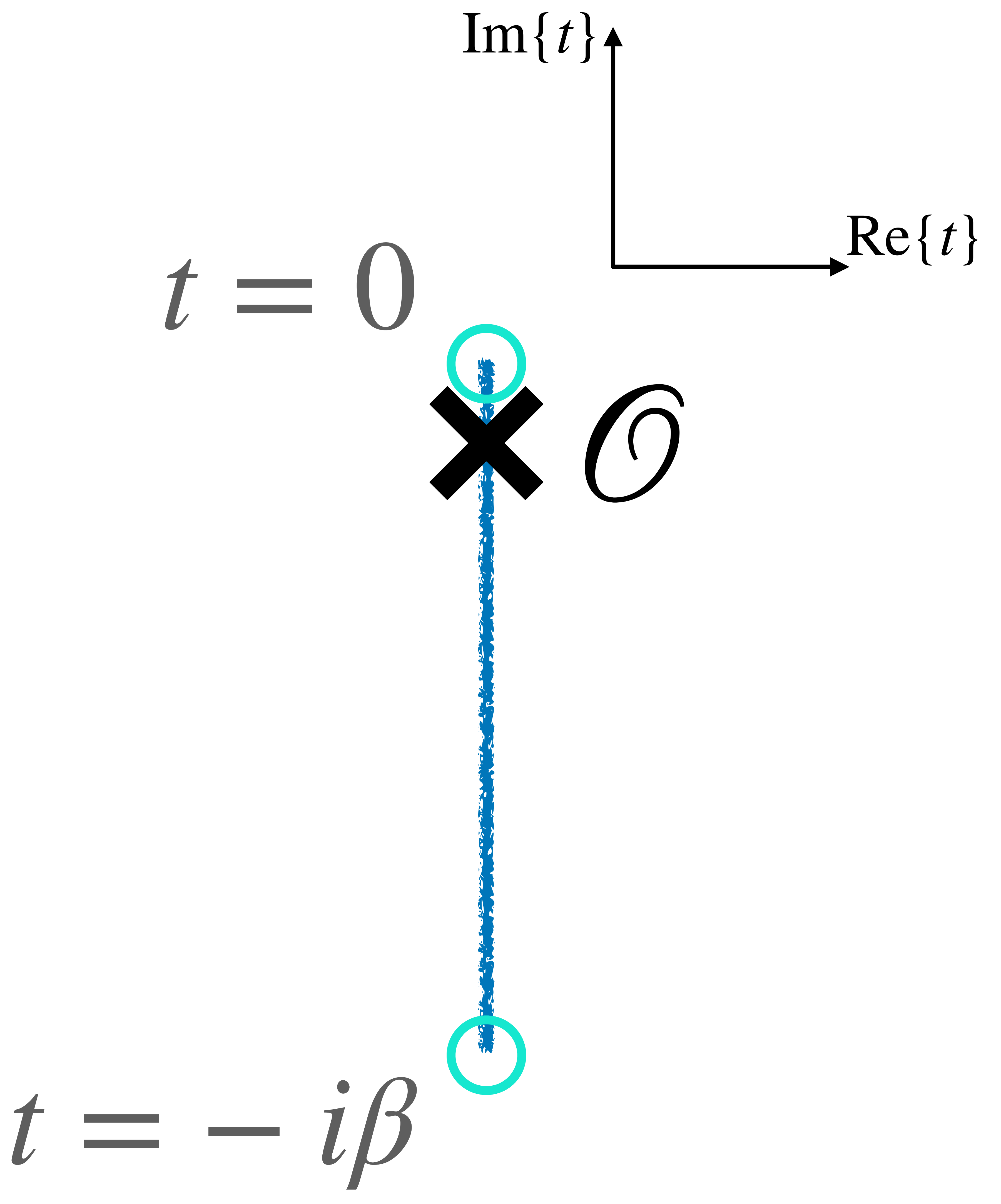
equilibrium #: $\langle \mathcal{O} \rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}e^{-\beta H}\right]$,

and also two-point functions:

$$\langle \mathcal{O}(\tau)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}(0)e^{-\tau H}\mathcal{O}(0)e^{-(\beta-\tau)H}\right]$$

- Real time calculations:

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[e^{iHt}\mathcal{O}(0)e^{-iHt}\mathcal{O}(0)e^{-\beta H}\right]$$



How to do calculations

the Schwinger-Keldysh contour

- Imaginary time calculations:

  equilibrium #: $\langle \mathcal{O} \rangle = Z^{-1} \mathrm{Tr}\left[\mathcal{O} e^{-\beta H}\right]$,

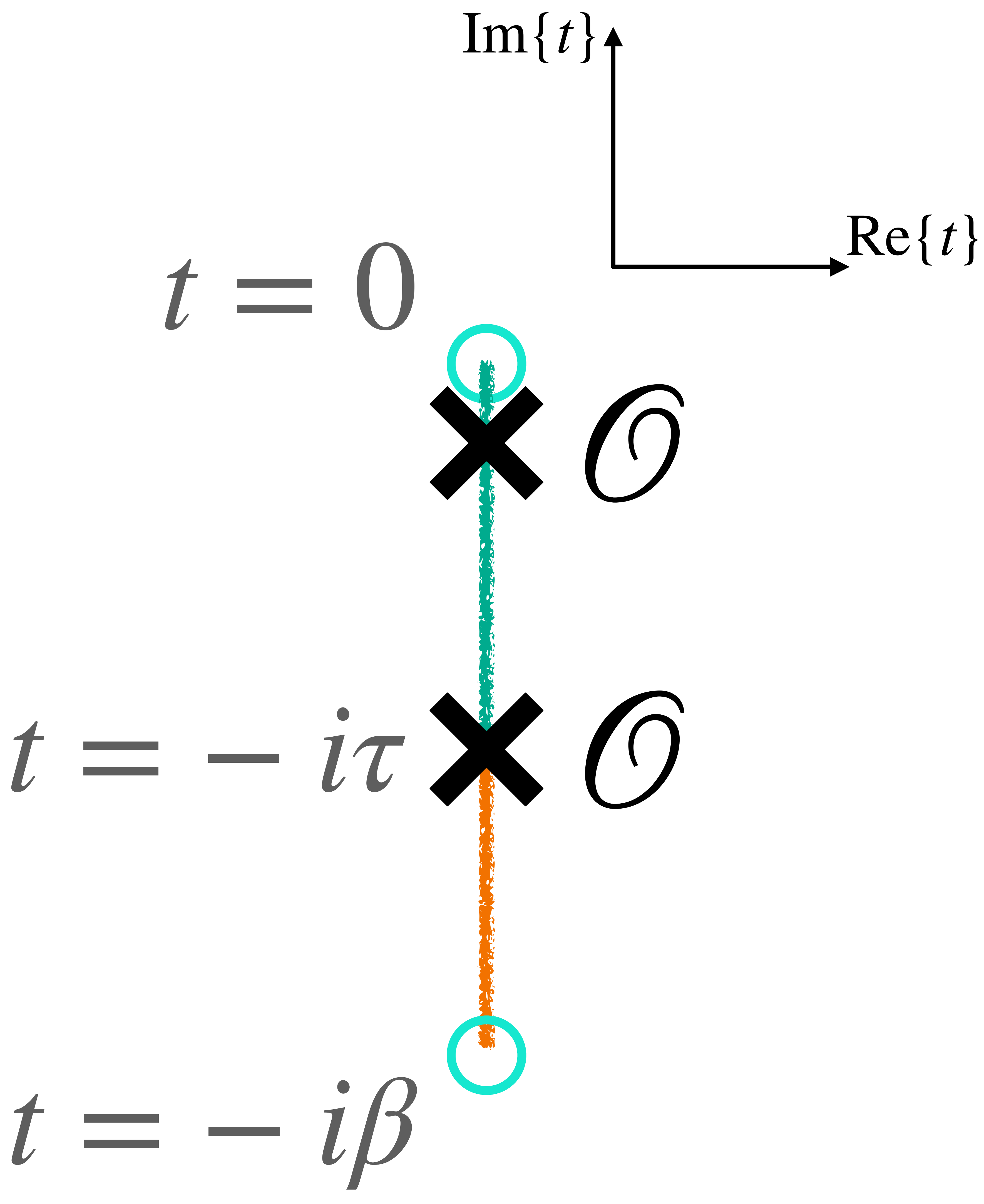
and also two-point functions:

$$\langle \mathcal{O}(\tau)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}(0)e^{-\tau H}\mathcal{O}(0)e^{-(\beta-\tau)H}\right]$$

- Real time calculations:

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[e^{iHt}\mathcal{O}(0)e^{-iHt}\mathcal{O}(0)e^{-\beta H}\right]$$

Path integral representations



How to do calculations

the Schwinger-Keldysh contour

- Imaginary time calculations:

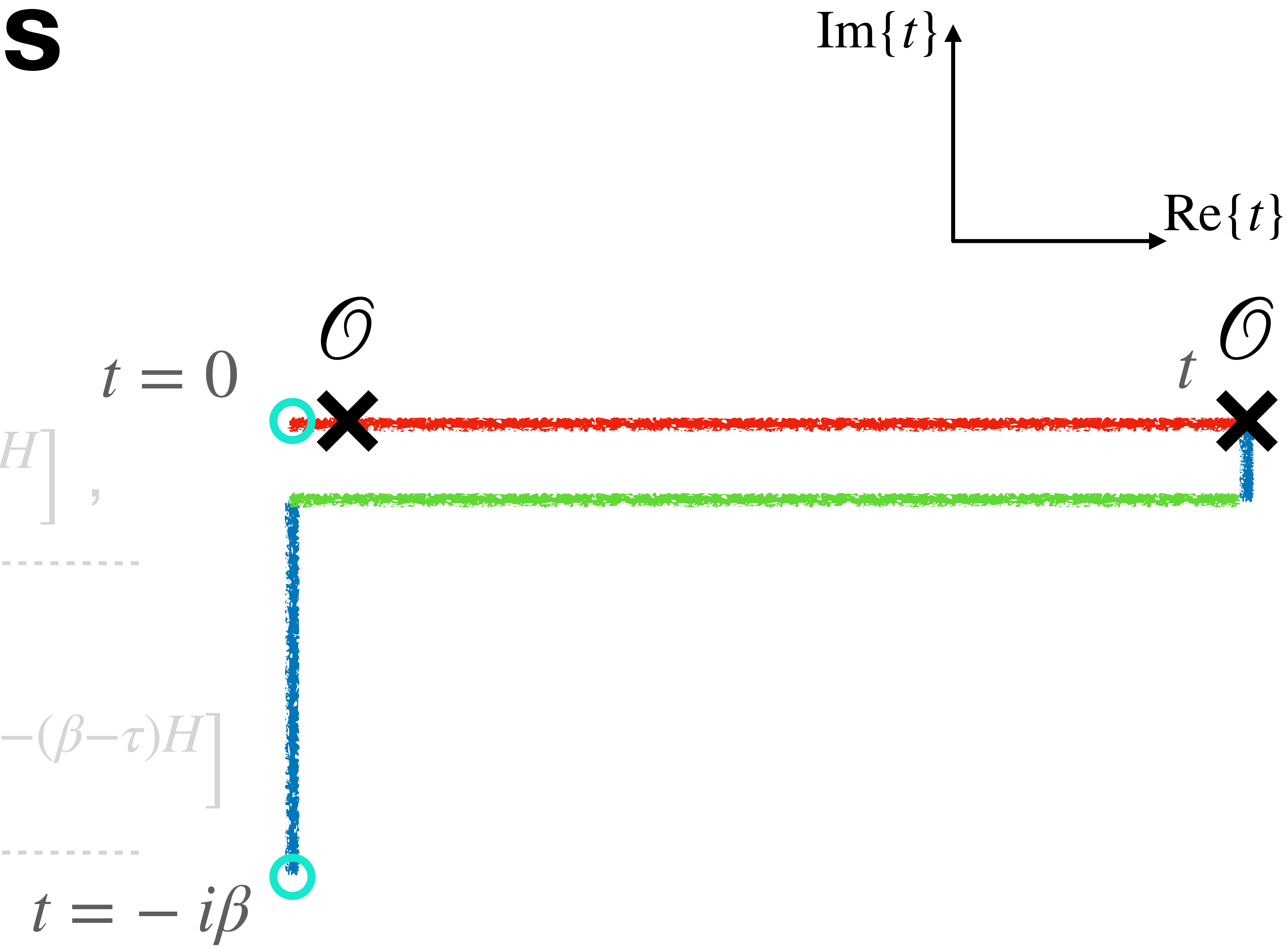
  equilibrium #: $\langle \mathcal{O} \rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}e^{-\beta H}\right]$,

and also two-point functions:

$$\langle \mathcal{O}(\tau)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[\mathcal{O}(0)e^{-\tau H}\mathcal{O}(0)e^{-(\beta-\tau)H}\right]$$

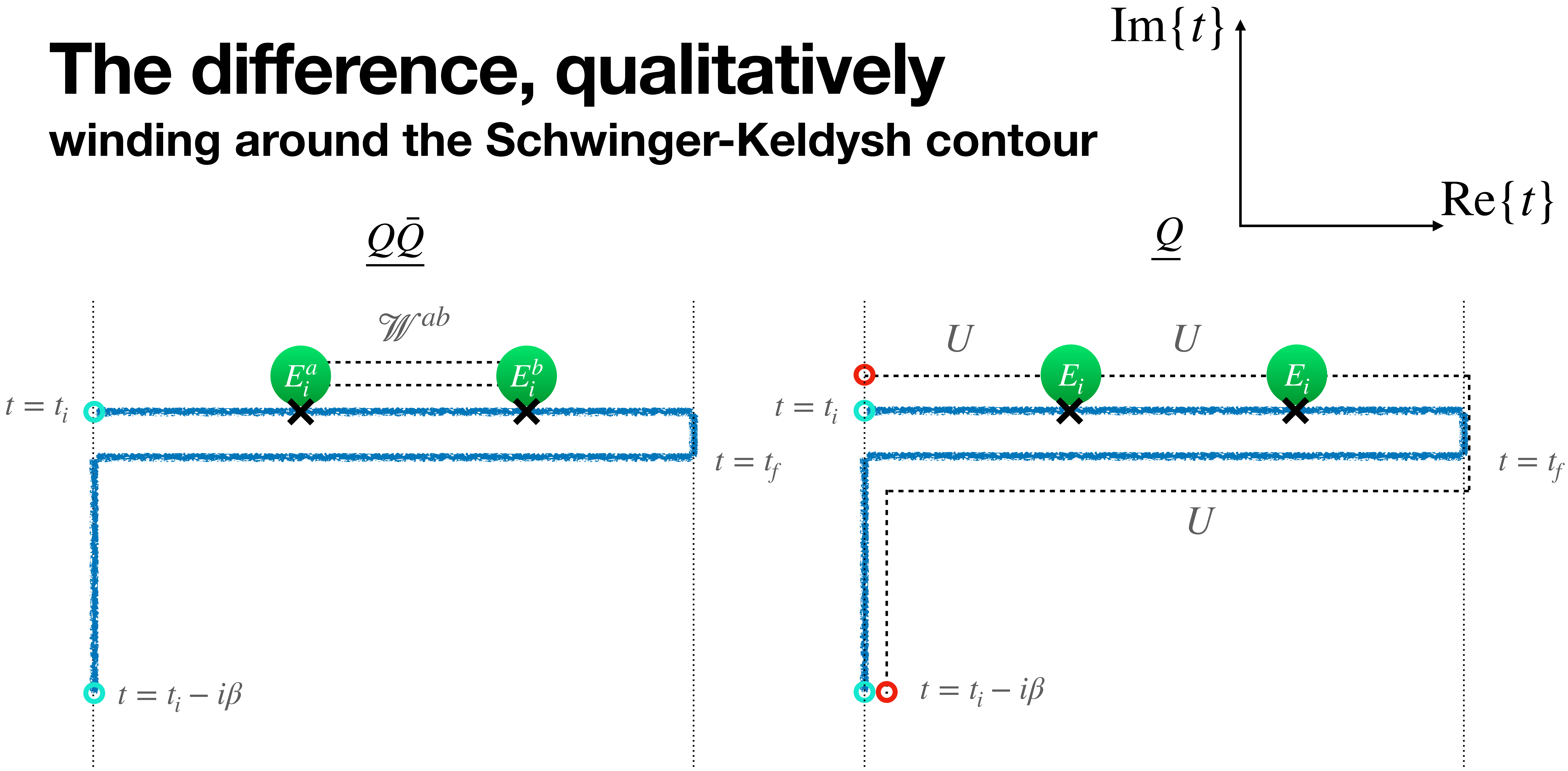
- Real time calculations:

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle = Z^{-1}\mathrm{Tr}\left[e^{iHt}\mathcal{O}(0)e^{-iHt}\mathcal{O}(0)e^{-\beta H}\right]$$



The difference, qualitatively

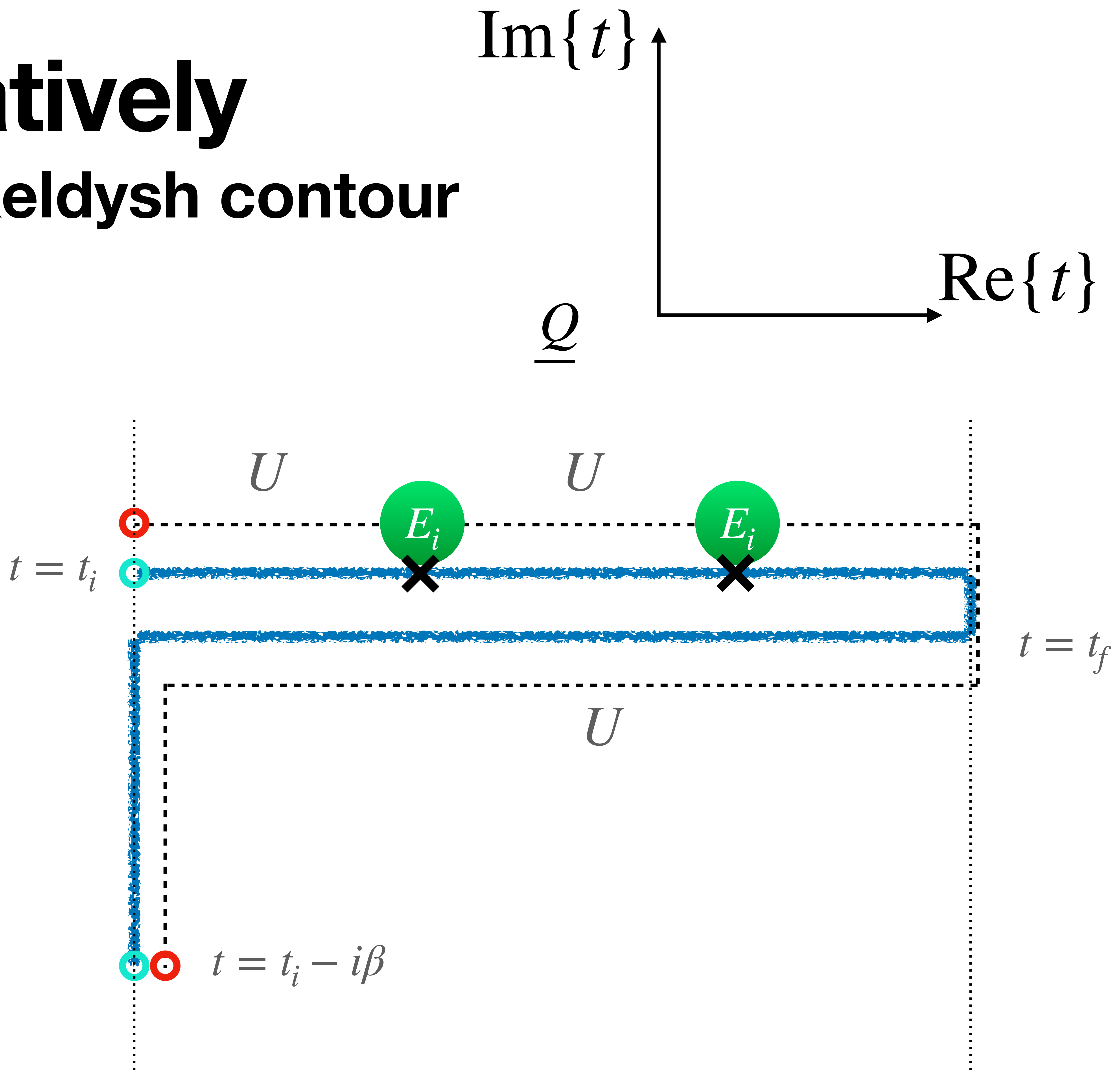
winding around the Schwinger-Keldysh contour



The difference, qualitatively

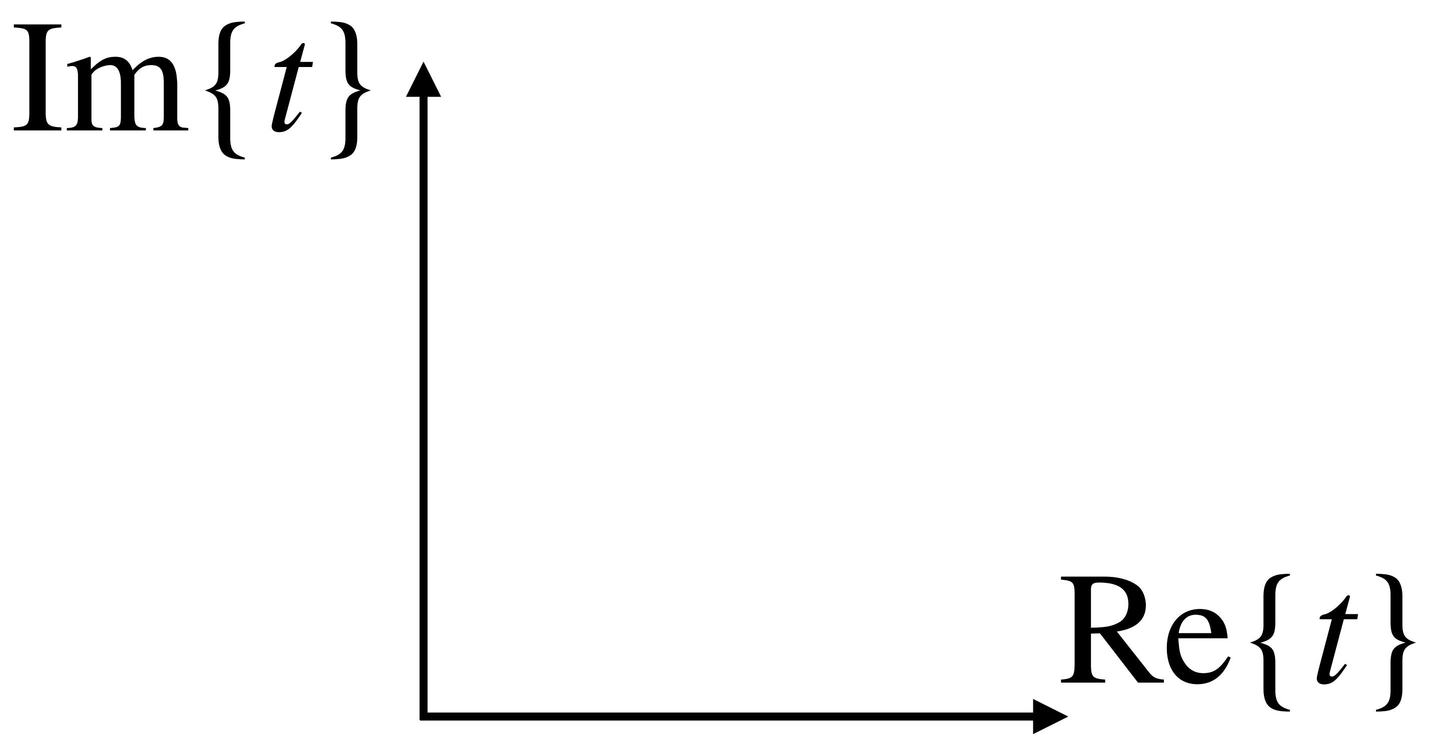
winding around the Schwinger-Keldysh contour

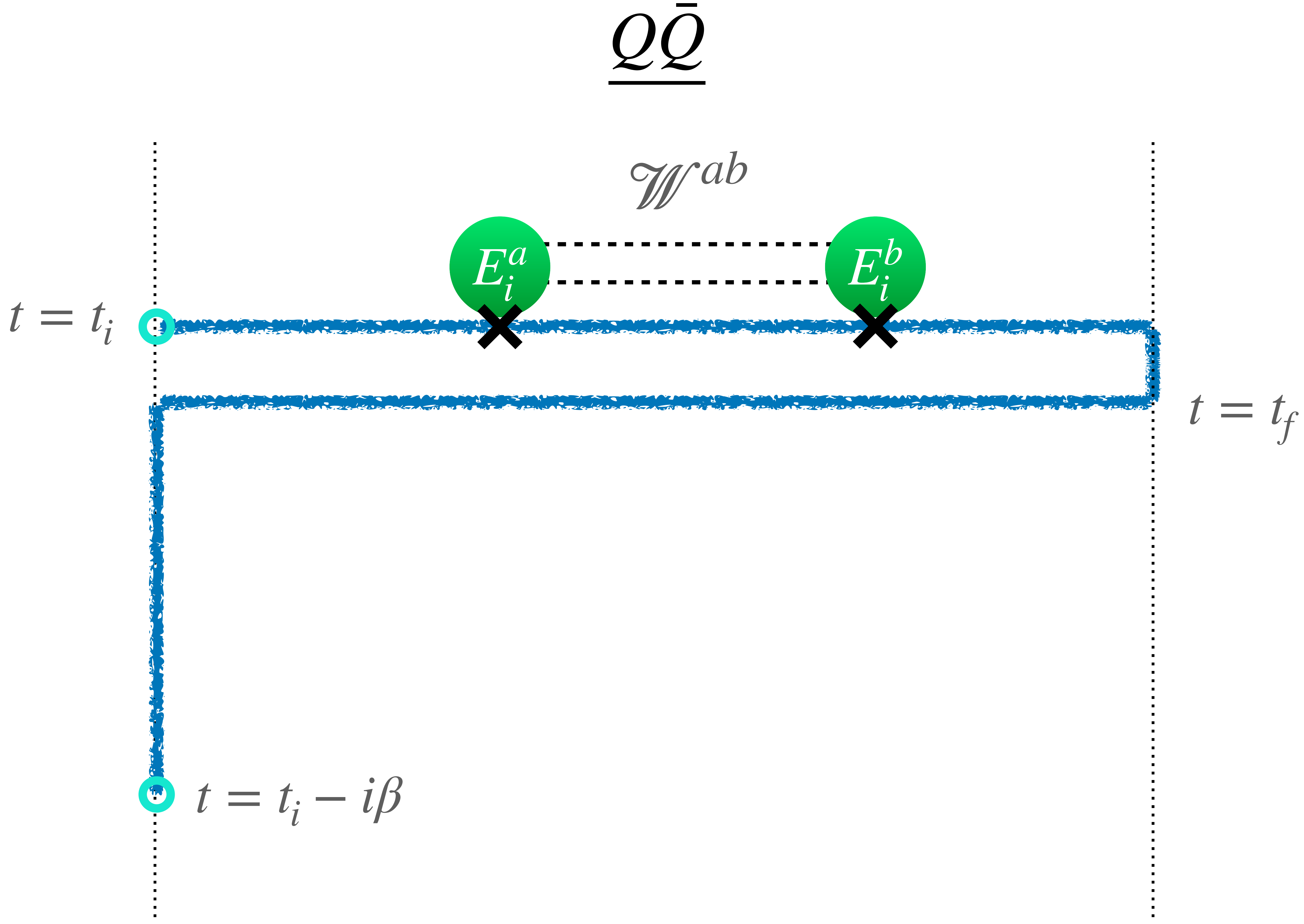
- The heavy quark is present at all times:
  - It is part of the construction of the thermal state of the QGP.
  - The Wilson line, which enforces the Gauss' law constraint due to the point charge, is also present on the Euclidean segment.



The difference, qualitatively

winding around the Schwinger-Keldysh contour

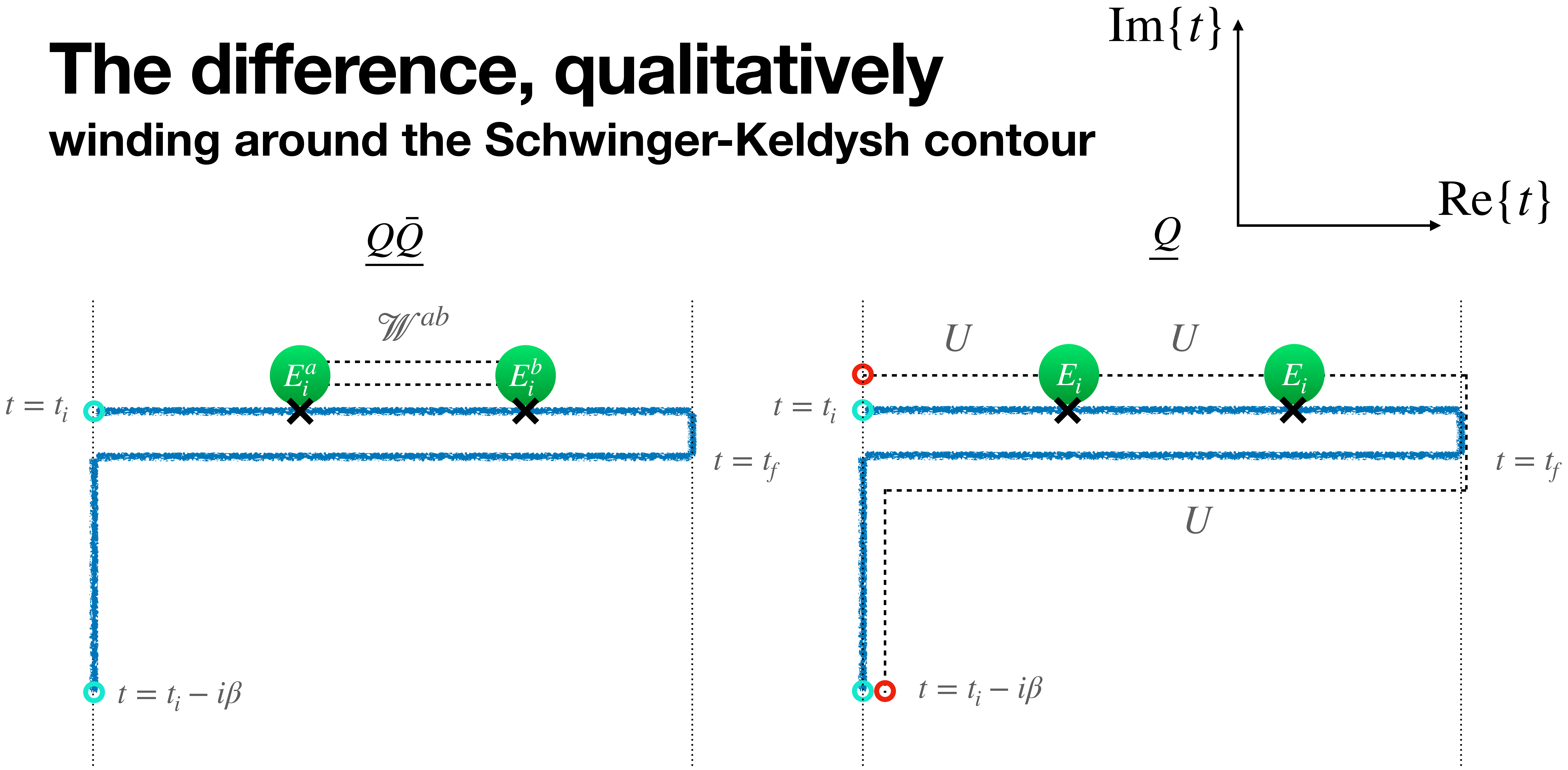




- In this correlator, the heavy quark pair is present at all times, but it is only color-charged for a finite time:
    - It is *not* part of the construction of the thermal state of the QGP.
    - The adjoint Wilson line, representing the propagation of unbound quarkonium (in the adjoint representation), is only present on the real-time segment.

The difference, qualitatively

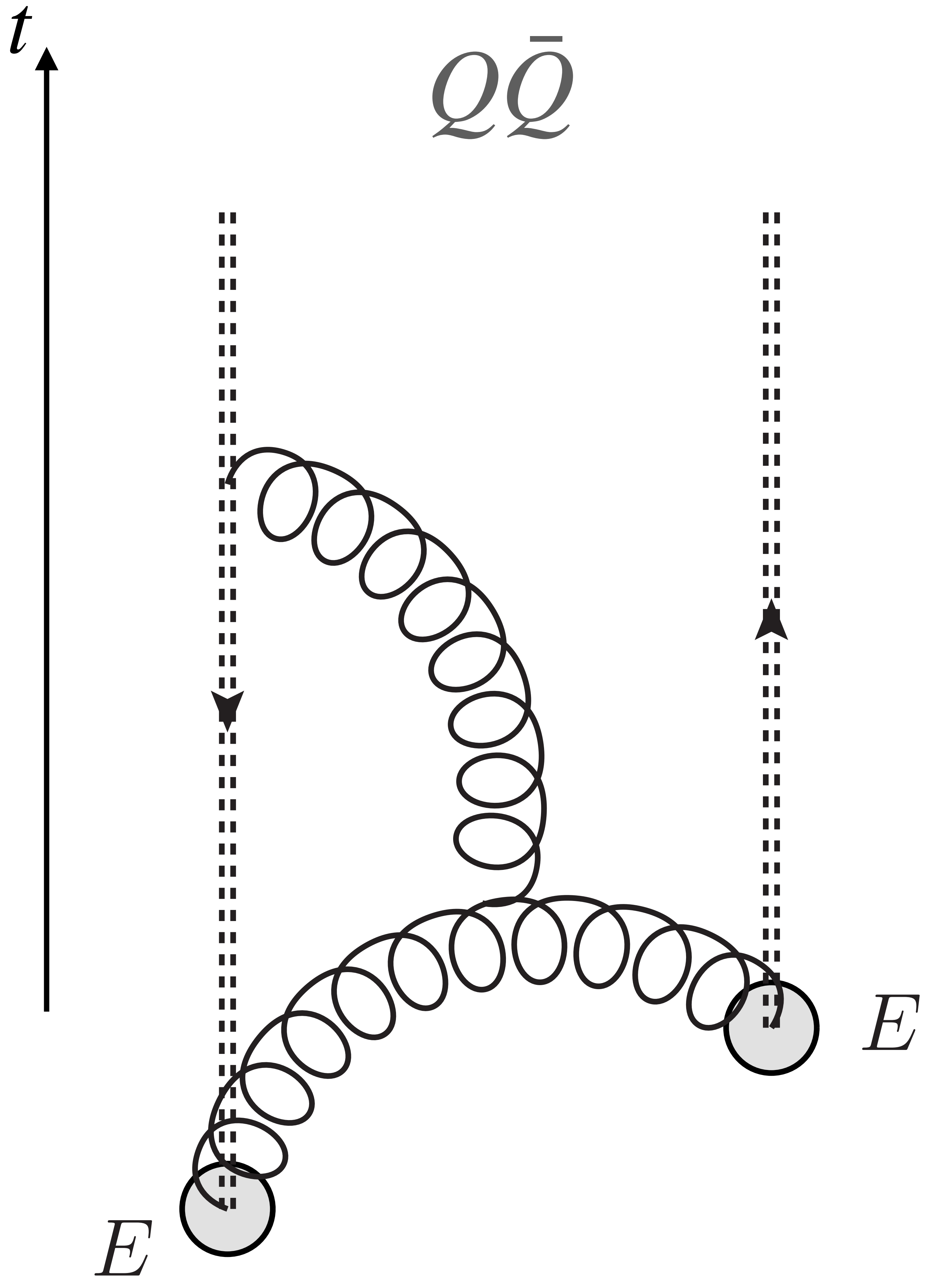
winding around the Schwinger-Keldysh contour



BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

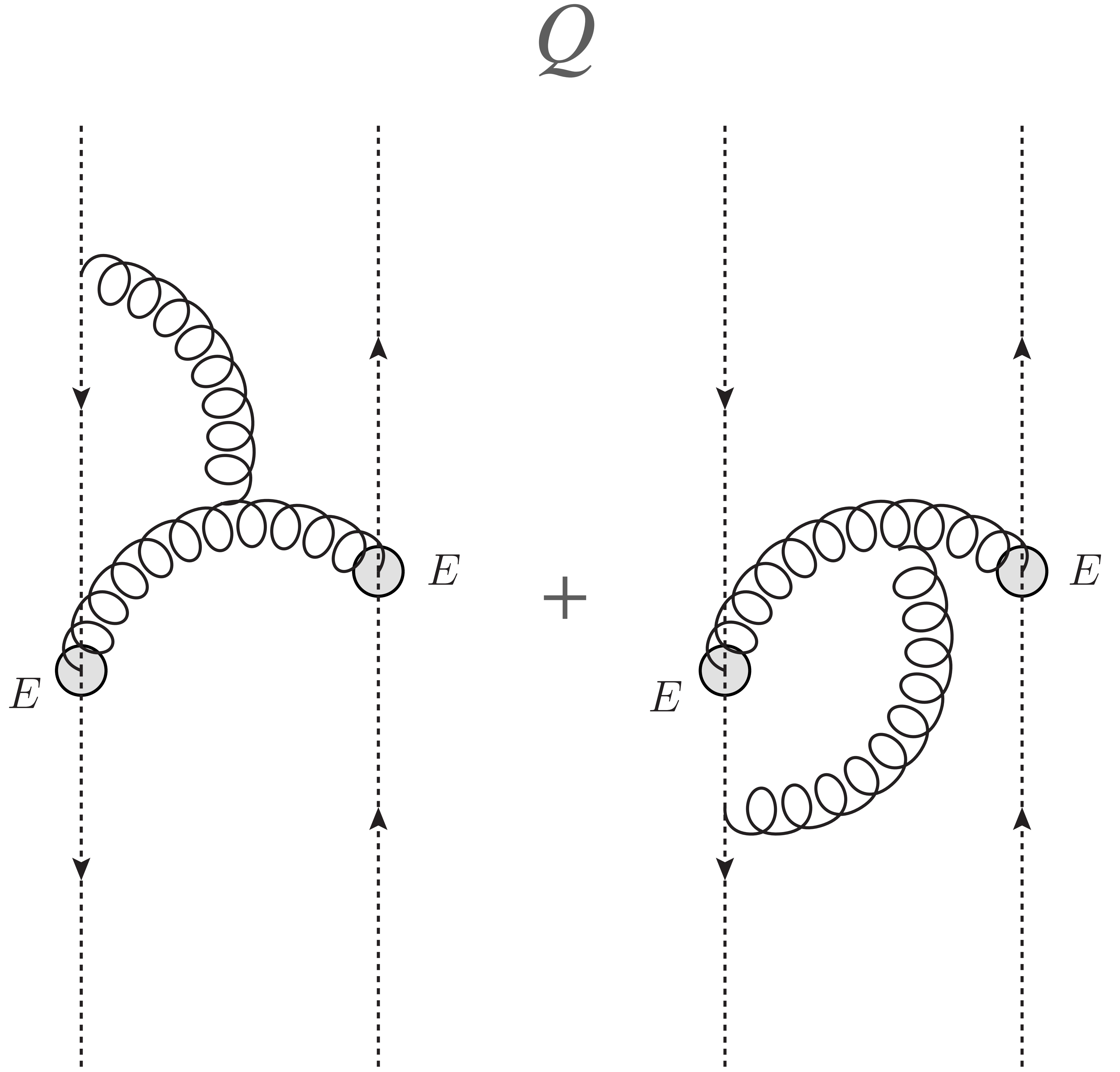
The difference in pQCD

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

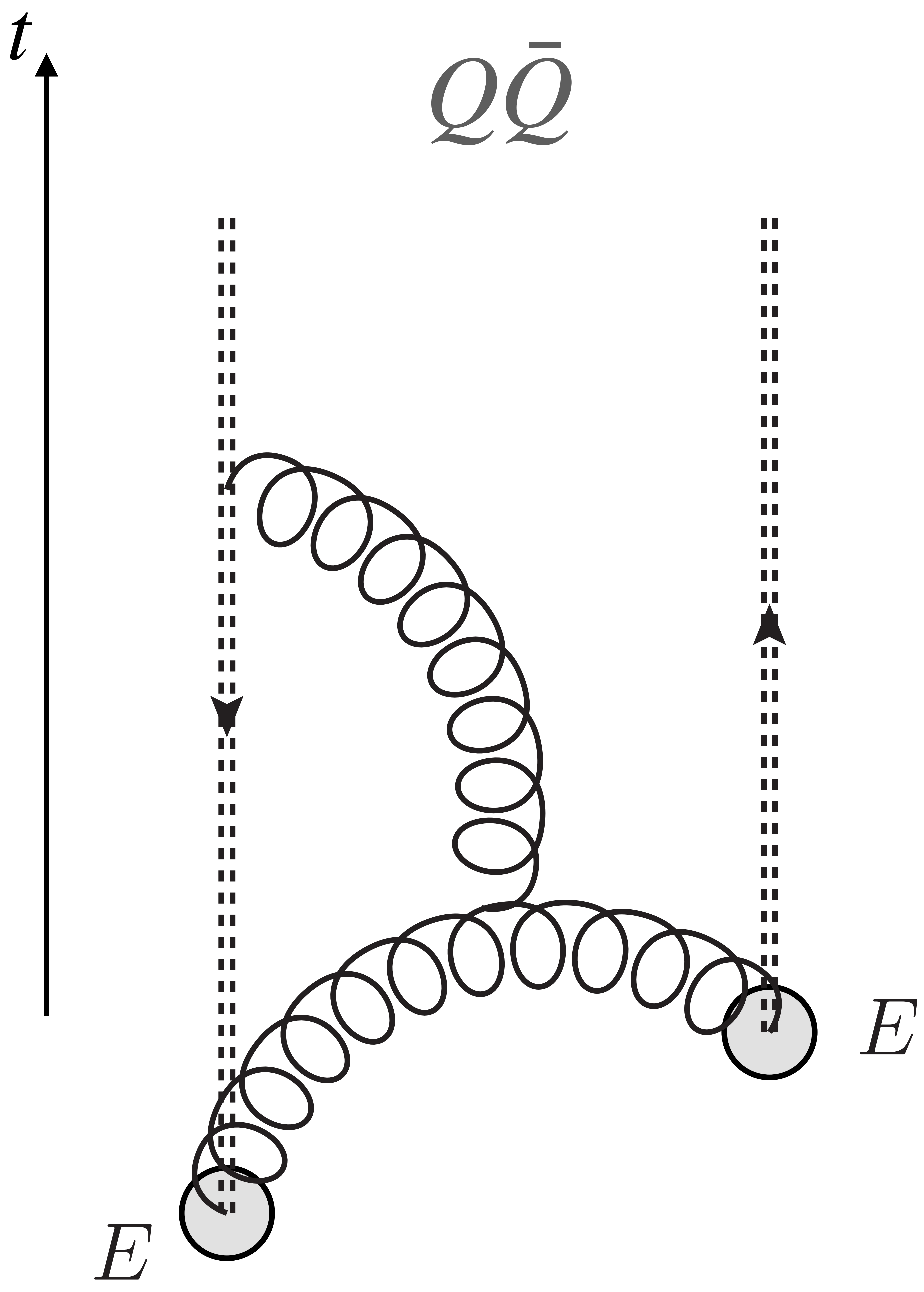
$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi}\omega^3$$



BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

The difference in pQCD

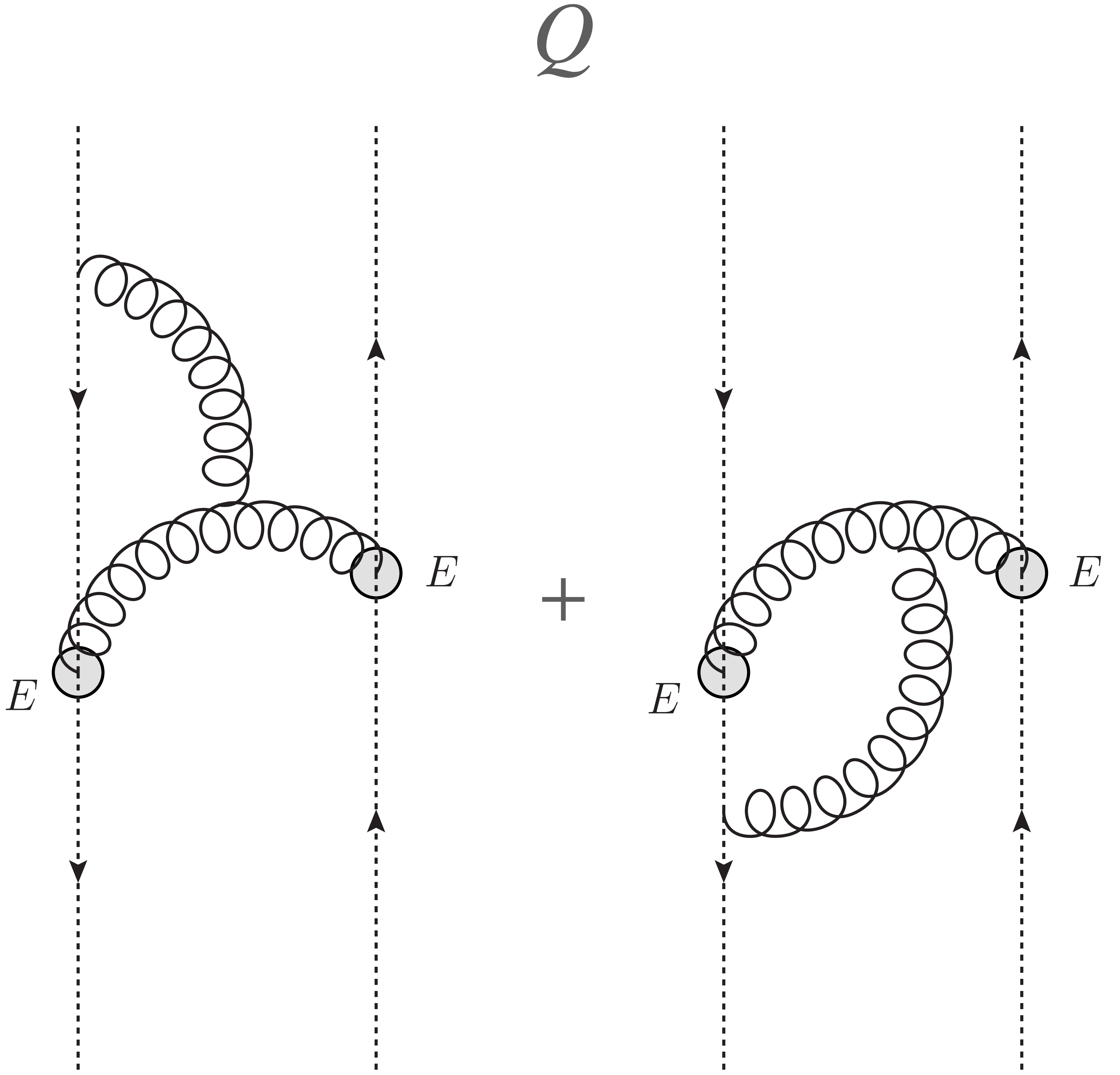
operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi}\omega^3$$

The difference is due to different operator orderings (different possible gluon insertions).

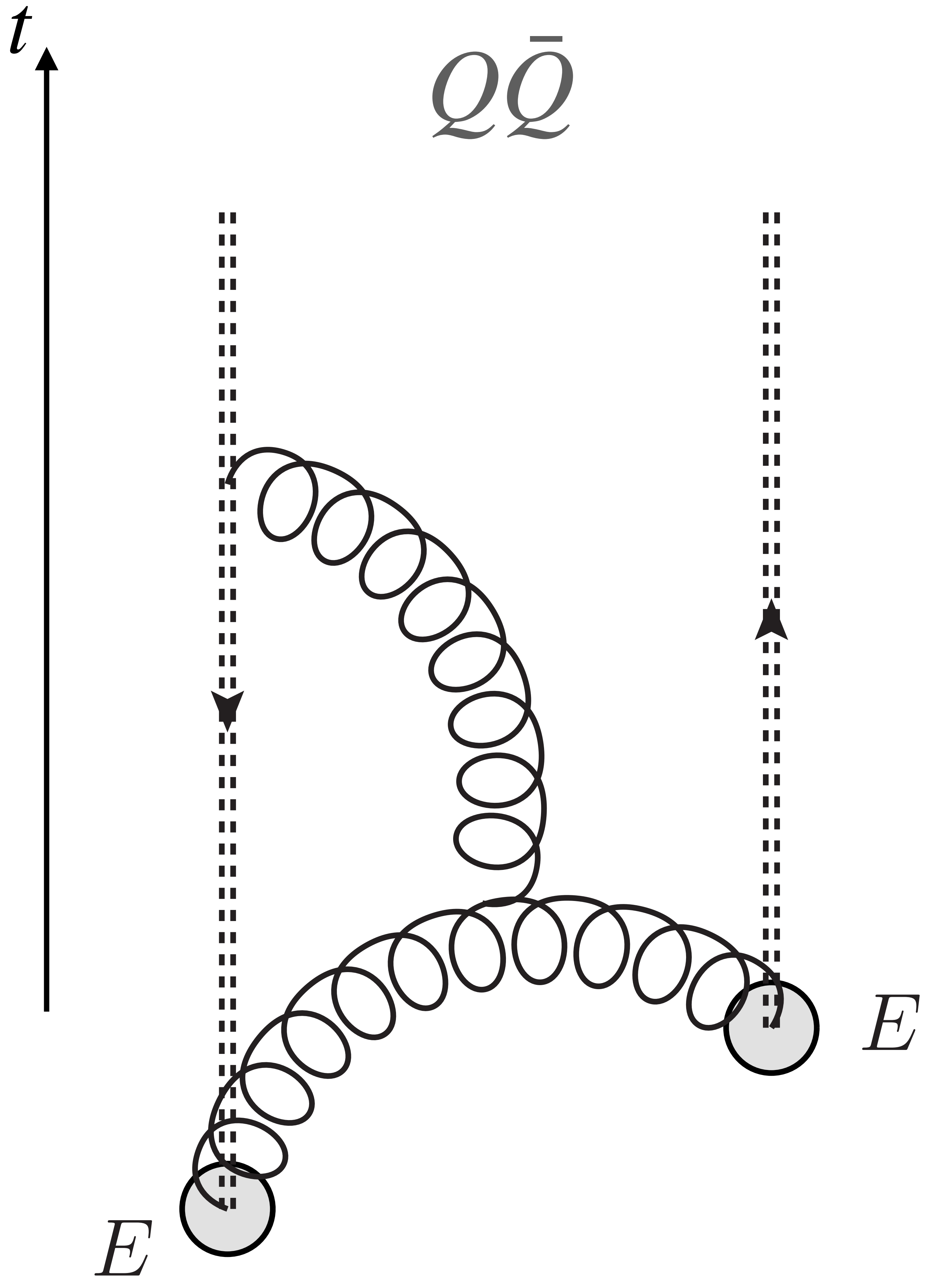


BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

The difference in pQCD

Gauge invariant!

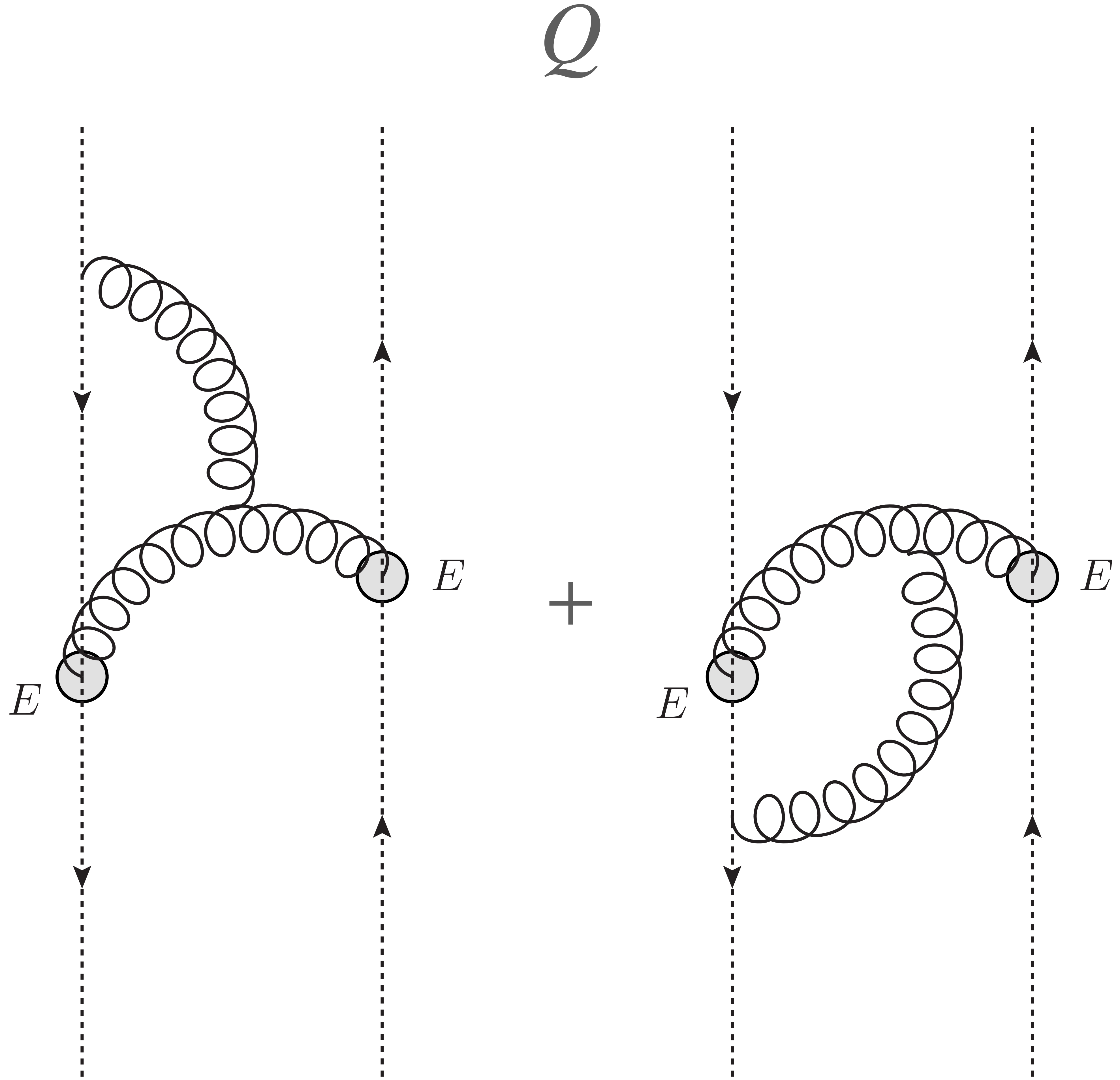
operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi}\omega^3$$

The difference is due to different operator orderings (different possible gluon insertions).

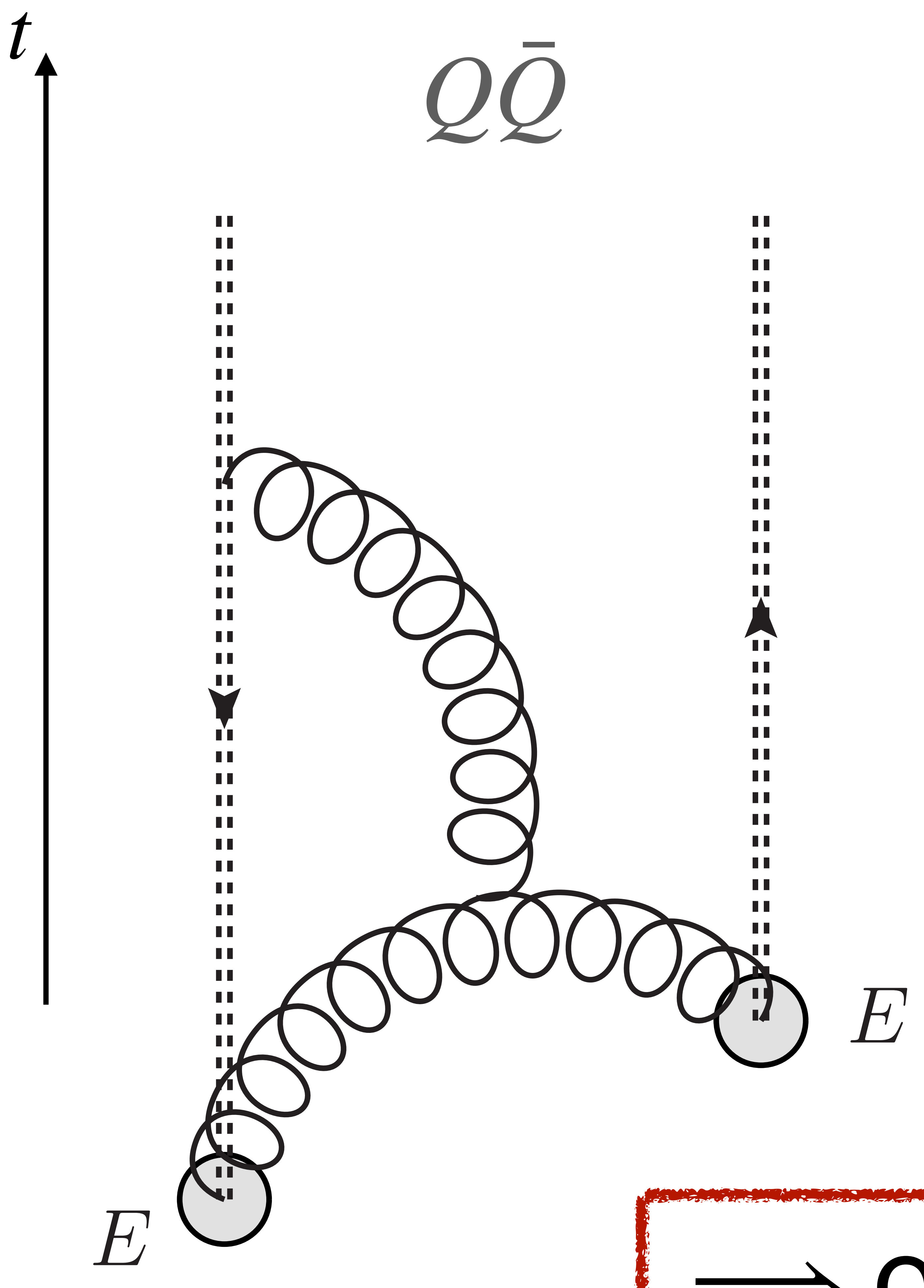


BS and X. Yao, hep-ph/2205.04477
Phys. Rev. Lett. 130, 052302

The difference in pQCD

Gauge invariant!

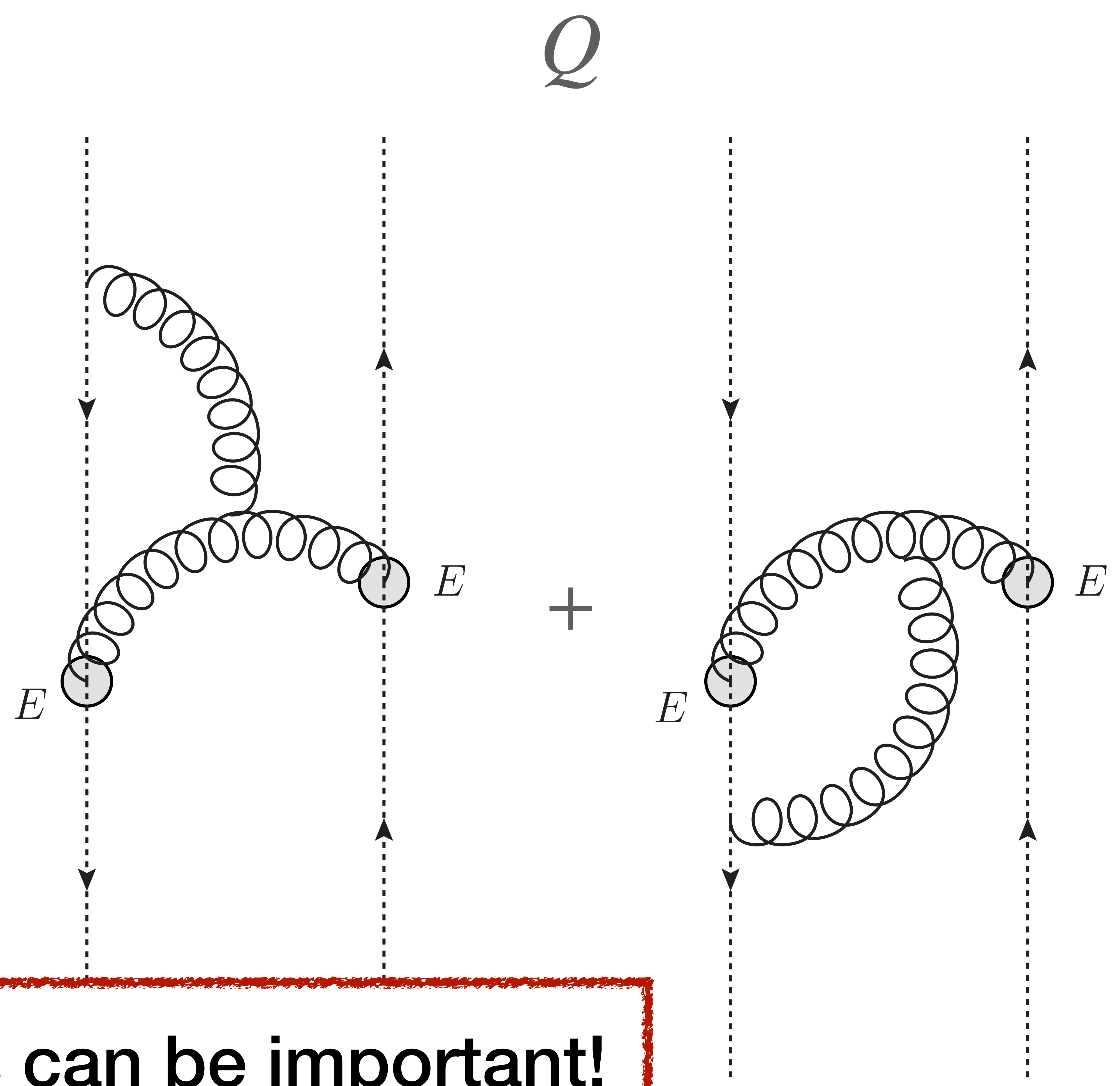
operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.

$$\Delta\rho(\omega) = \frac{g^4 N_c^2 C_F T_F}{4\pi}\omega^3$$

The difference is due to different operator orderings (different possible gluon insertions).



$\Longrightarrow$ Quantum color correlations can be important!

What about the difference at strong coupling?

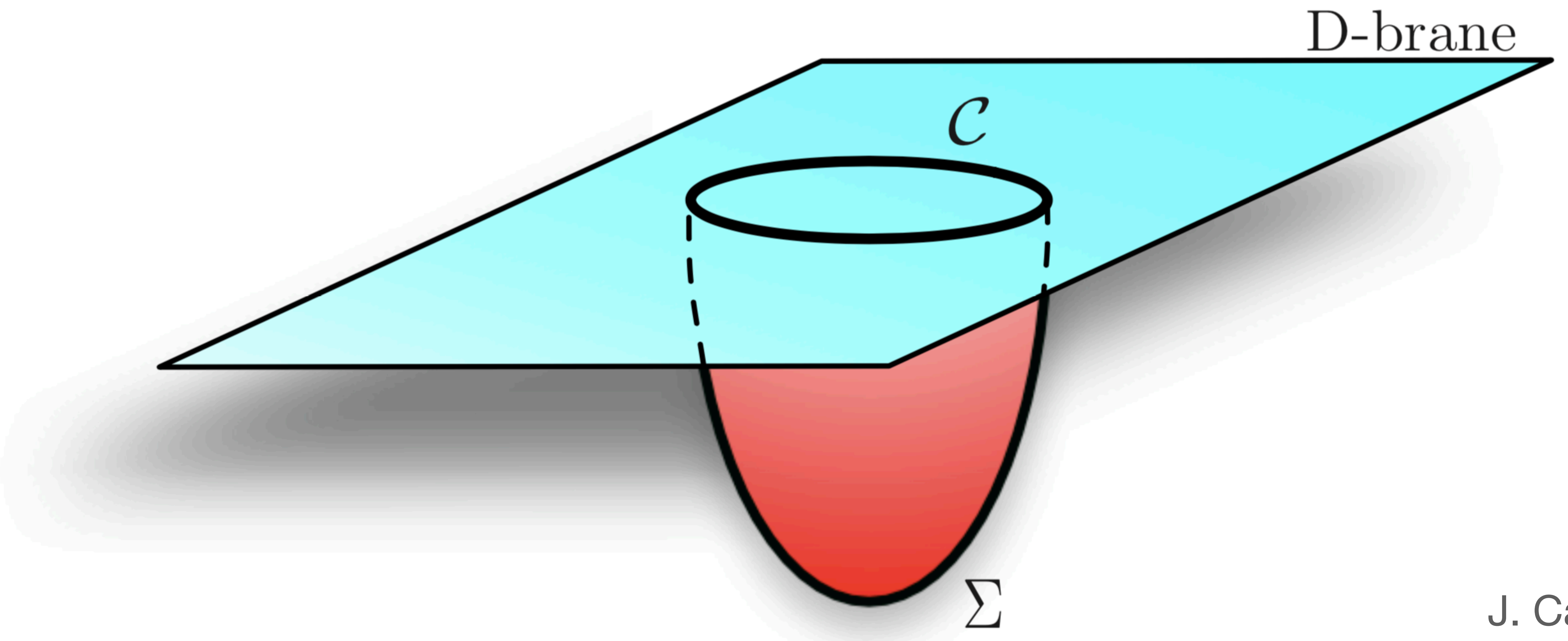
[**] J. Maldacena, hep-th/9711200

Wilson loops in AdS/CFT

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

How do Wilson loops help?

setup — pure gauge theory

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^{\mu}(s_2)}\frac{\delta}{\delta f^{\nu}(s_1)}W[\mathscr{C}_f]\bigg|_{f=0} = (ig)^2\mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^{\rho}(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^{\sigma}(s_1)U_{[s_1,0]}\Big]$$

How do Wilson loops help?

setup — pure gauge theory

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathscr{C}$:

$$\frac{\delta}{\delta f^\mu(s_2)}\frac{\delta}{\delta f^\nu(s_1)}W[\mathscr{C}_f]\Bigg|_{f=0} = (ig)^2 \mathrm{Tr}_{\mathrm{color}}\Big[U_{[1,s_2]}F_{\mu\rho}(\gamma(s_2))\dot{\gamma}^\rho(s_2)U_{[s_2,s_1]}F_{\nu\sigma}(\gamma(s_1))\dot{\gamma}^\sigma(s_1)U_{[s_1,0]}\Big]$$

- Same as the lattice calculation of the heavy quark diffusion coefficient:

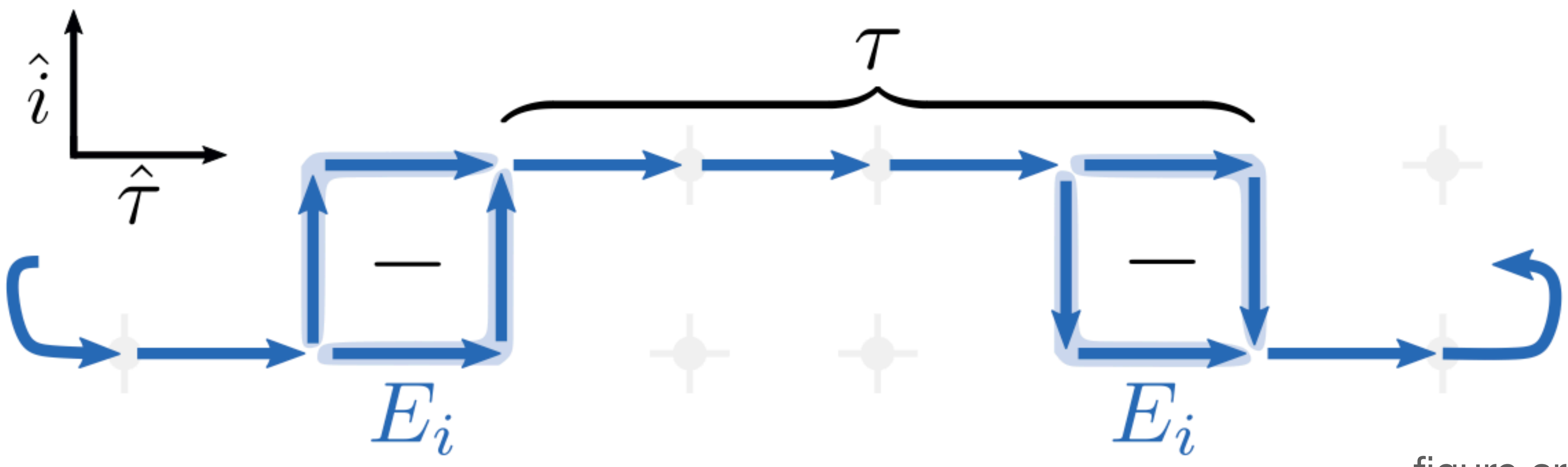


figure credit: L. Altenkort

Wilson loops in $\mathcal{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\text{BPS}}[\mathscr{C}; \hat{n}] = \frac{1}{N_c} \text{Tr}_{\text{color}} \left[\mathscr{P} \exp \left(ig \oint_{\mathscr{C}} ds \, T^a \left[A_\mu^a \, \dot{x}^\mu + \hat{n}(s) \cdot \overrightarrow{\phi}^a \sqrt{\dot{x}^2} \, \right] \right) \right],$$

which is *not* the standard Wilson loop.

Wilson loops in $\mathcal{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\text{BPS}}[\mathscr{C}; \hat{n}] = \frac{1}{N_c} \text{Tr}_{\text{color}} \left[\mathscr{P} \exp \left(ig \oint_{\mathscr{C}} ds \, T^a \left[A_\mu^a \, \dot{x}^\mu + \hat{n}(s) \cdot \vec{\phi}^{\,a} \sqrt{\dot{x}^2} \right] \right) \right],$$

which is *not* the standard Wilson loop.

- $\mathcal{N} = 4$ SYM has 6 scalar fields $\vec{\phi}^{\,a}$, which enter the above Wilson loop through a direction $\hat{n} \in \mathrm{S}_5$. Also, its dual gravitational description is $\mathrm{AdS}_5 \times \mathrm{S}_5$.

Wilson loops in $\mathcal{N} = 4$ SYM

a slightly different observable

- A holographic dual in terms of an extremal surface exists for

$$W_{\mathrm{BPS}}[\mathscr{C};\hat{n}] = \frac{1}{N_c}\mathrm{Tr}_{\mathrm{color}}\left[\mathscr{P}\exp\left(ig\oint_{\mathscr{C}} ds\, T^a\left[A^a_\mu\,\dot{x}^\mu + \hat{n}(s)\cdot\vec{\phi}^{\,a}\sqrt{\dot{x}^2}\,\right]\right)\right],$$

which is *not* the standard Wilson loop.

- $\mathcal{N} = 4$ SYM has 6 scalar fields $\vec{\phi}^{\,a}$, which enter the above Wilson loop through a direction $\hat{n} \in \mathrm{S}_5$. Also, its dual gravitational description is $\mathrm{AdS}_5 \times \mathrm{S}_5$.

- What to do with this extra parameter? For a single heavy quark, just set $\hat{n} = \hat{n}_0$.

Choosing $\hat{n}$

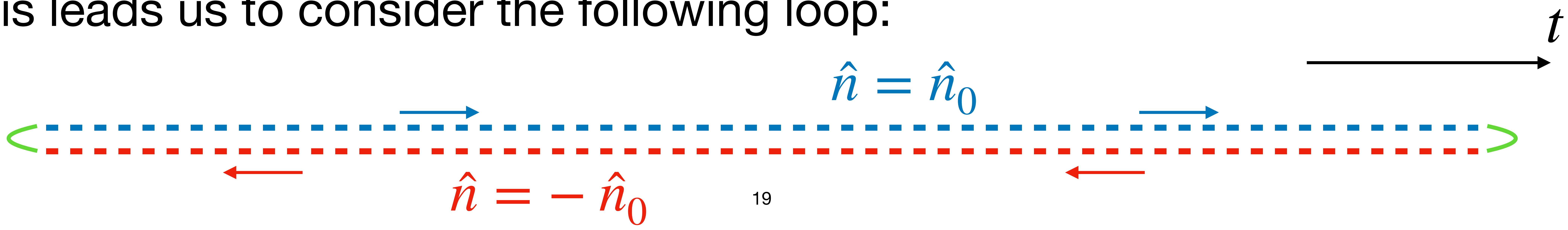
what is the best proxy for an adjoint Wilson line?

- A key property of the adjoint Wilson line is

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \frac{1}{T_F}\mathrm{Tr}\left[\mathscr{T}\{T^a U_{[t_2,t_1]} T^b U^\dagger_{[t_2,t_1]}\}\right],$$

which means that we can obtain the correlator we want by studying deformations of a Wilson loop of the form $W = \frac{1}{N_c}\mathrm{Tr}\left[UU^\dagger\right] = 1.$

- This leads us to consider the following loop:



Wilson loops in AdS/CFT

setup

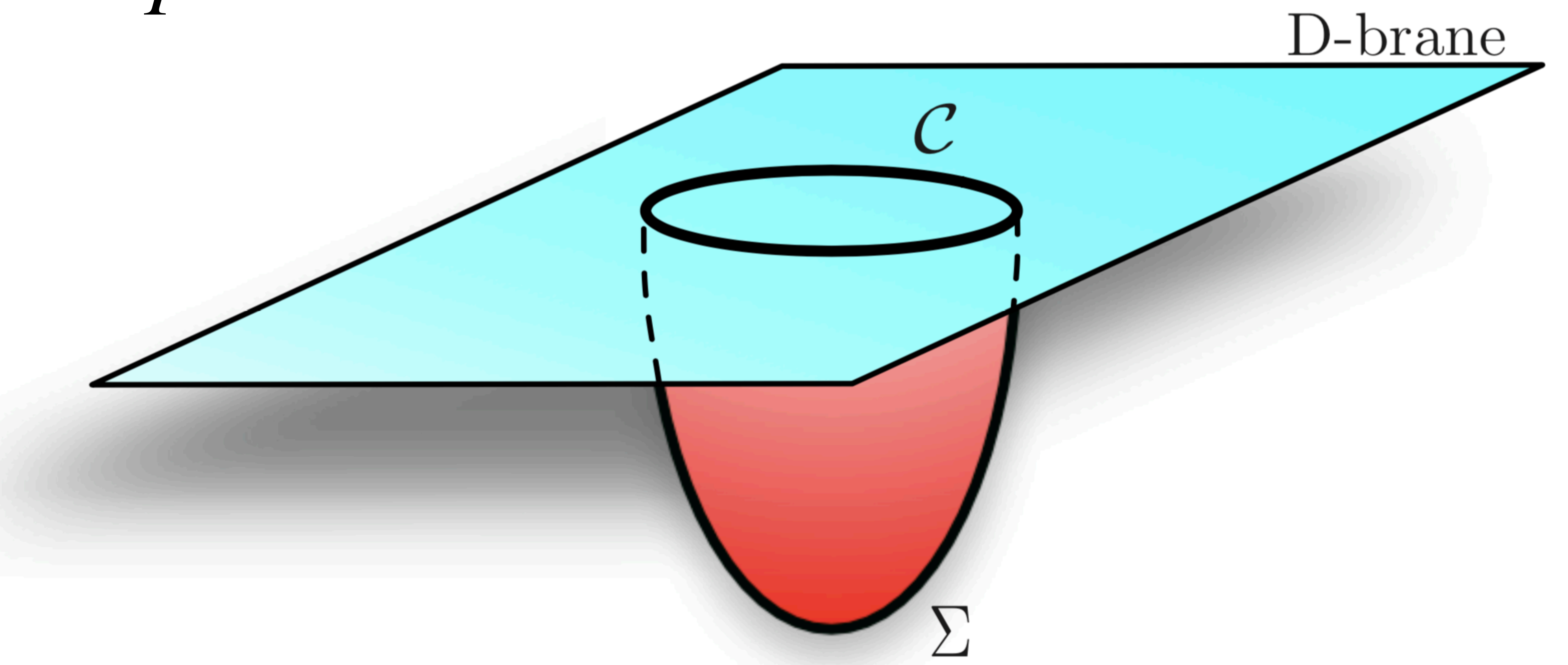
- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
  - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma] \rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$

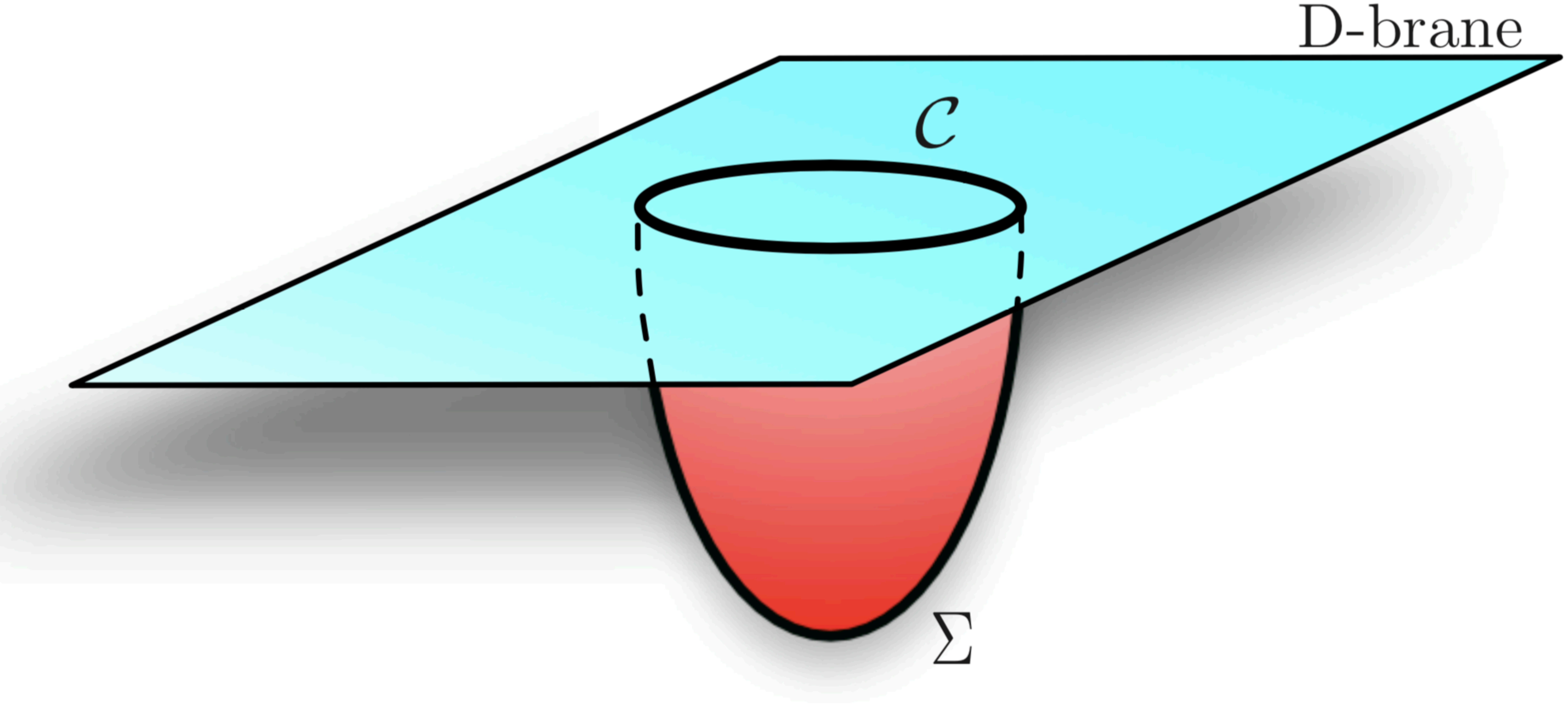
Metric of interest for finite T calculations:

$$ds^2 = \frac{R^2}{z^2}\left[-f(z)\,dt^2 + d\mathbf{x}^2 + \frac{1}{f(z)}dz^2 + z^2 d\Omega_5^2\right]$$

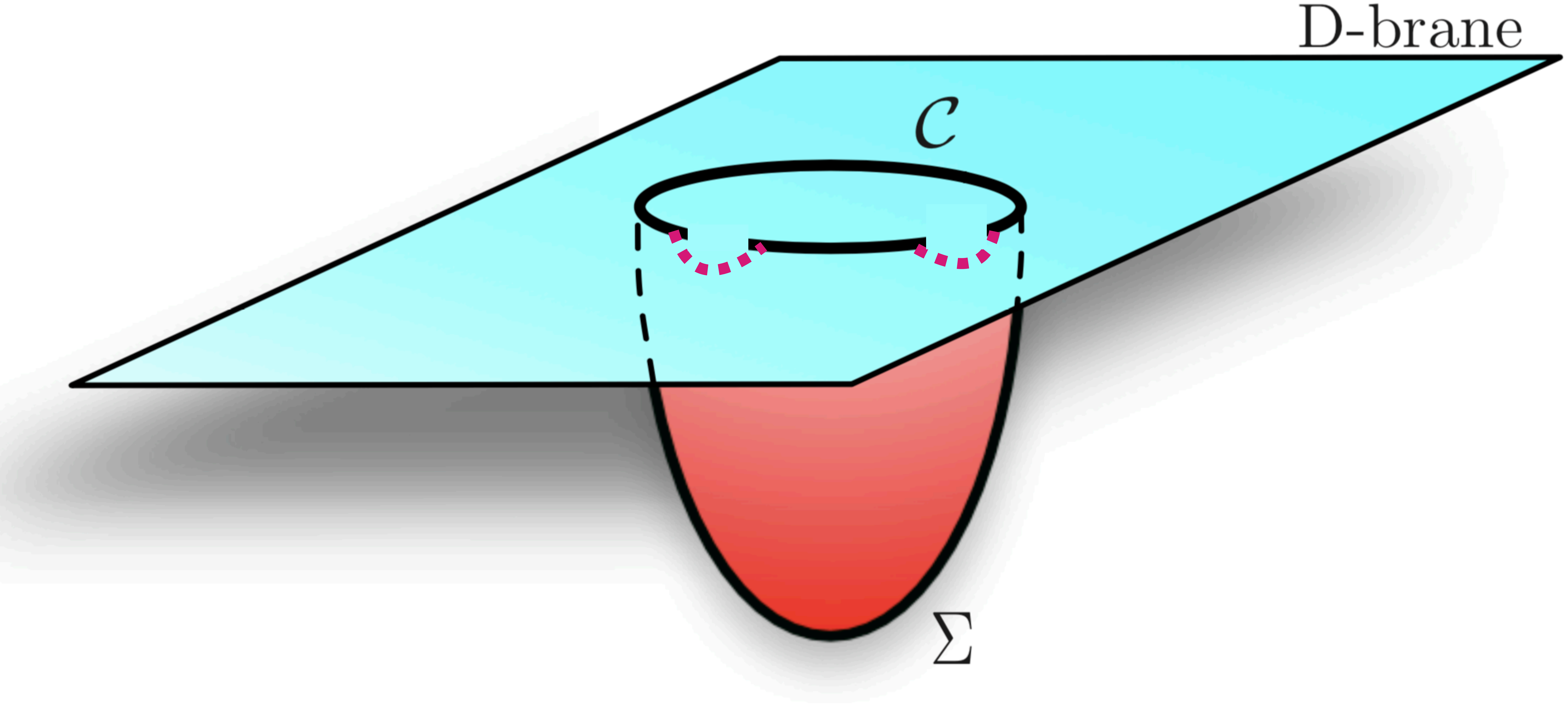
$$f(z) = 1 - (\pi T z)^4$$



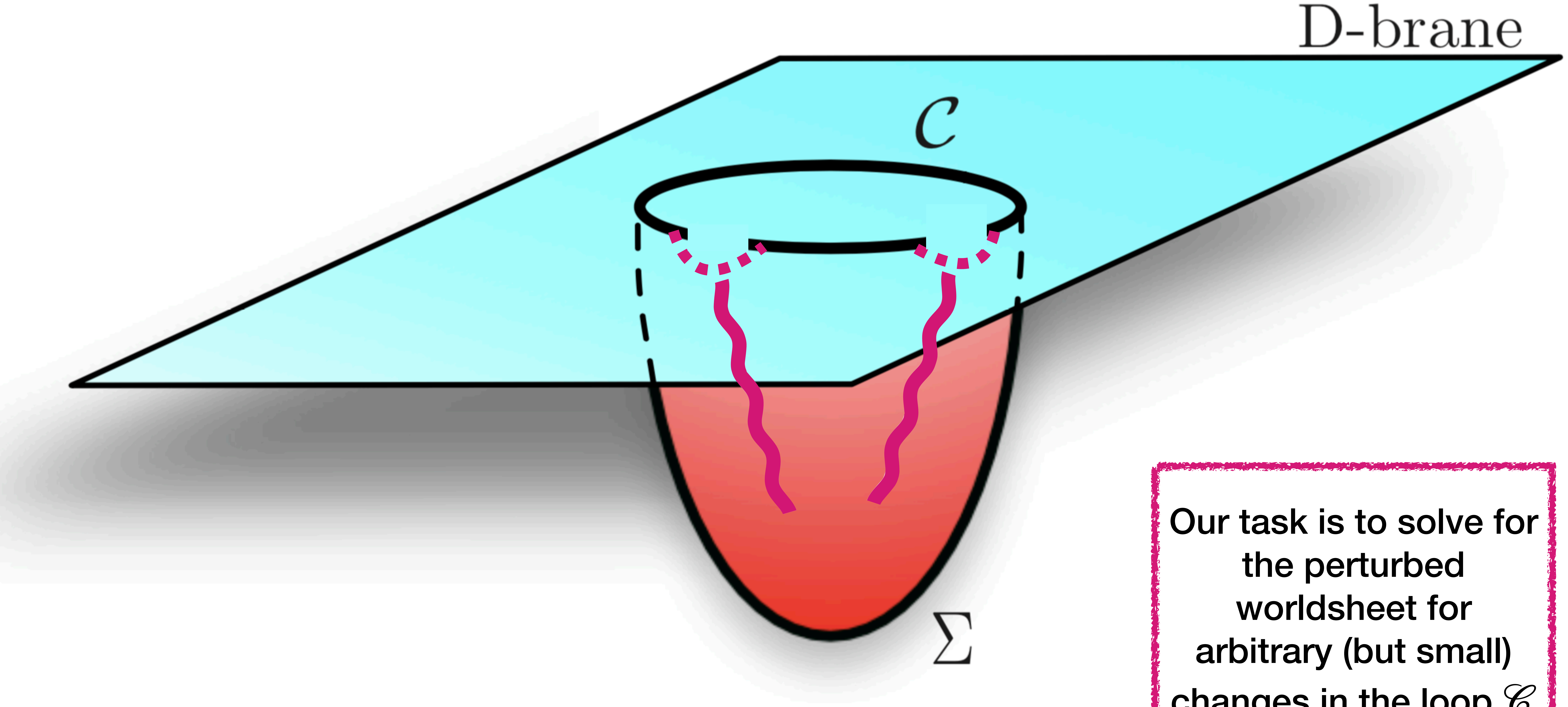
J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618



D-brane
$\mathcal{C}$
Σ


D-brane
$\mathcal{C}$
Σ


D-brane
$\mathcal{C}$
Σ



Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

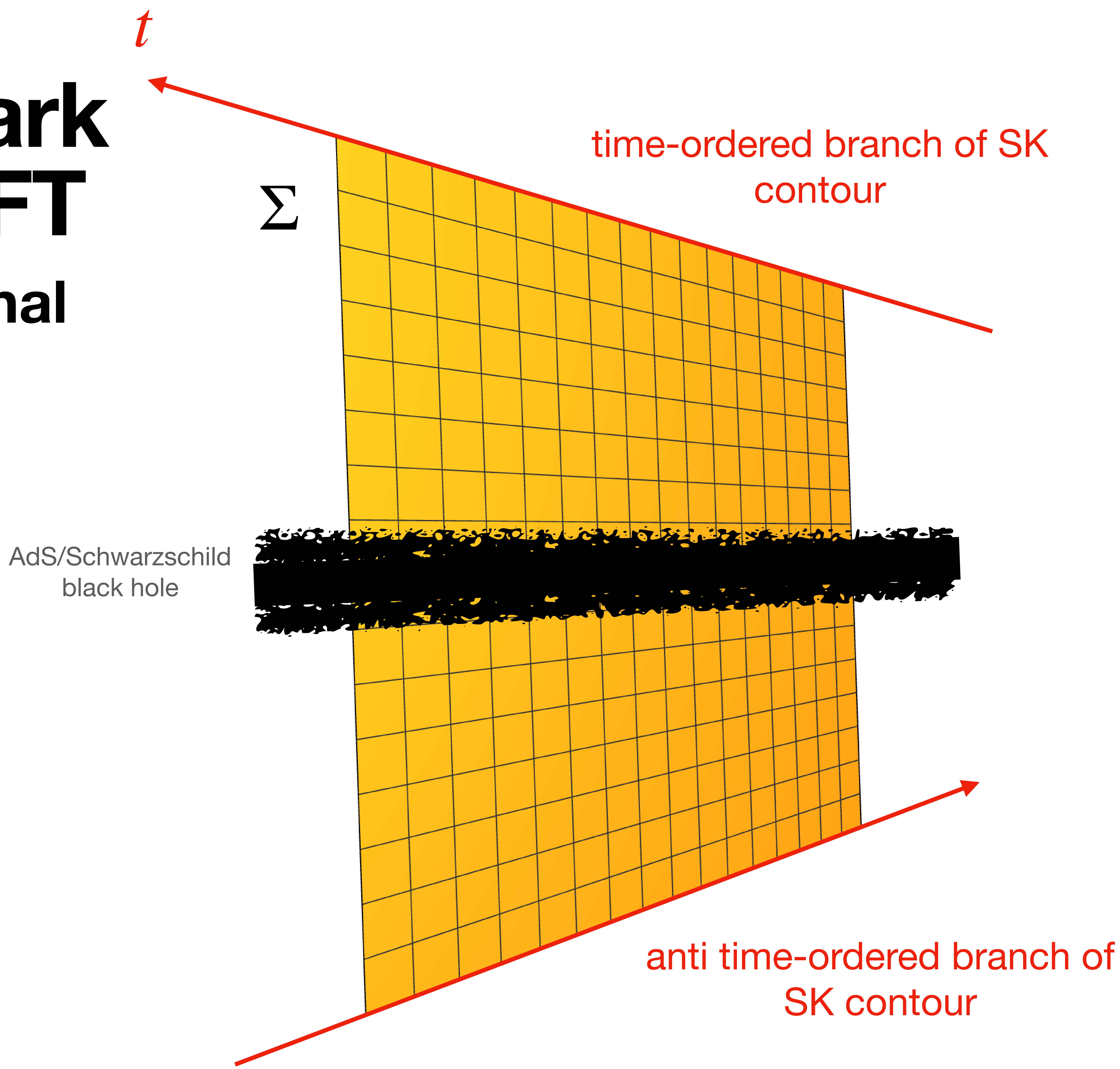
[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

Review: Heavy Quark Diffusion in AdS/CFT

using the same computational technique

Steps of the calculation:

1. Find the appropriate background solution



[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

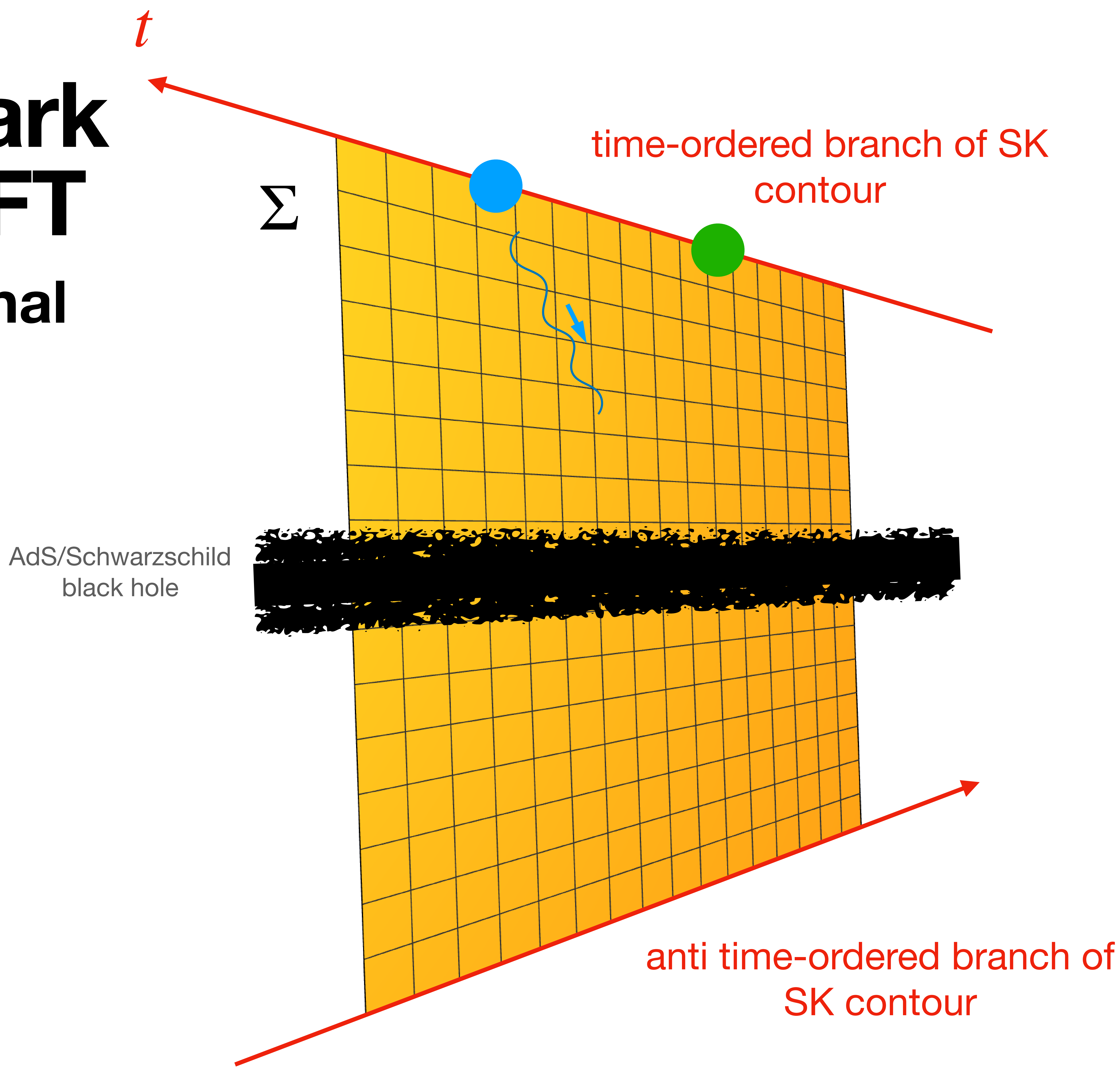
Review: Heavy Quark Diffusion in AdS/CFT

using the same computational technique

Steps of the calculation:

1. Find the appropriate background solution

2. Introduce perturbations



[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

Review: Heavy Quark Diffusion in AdS/CFT

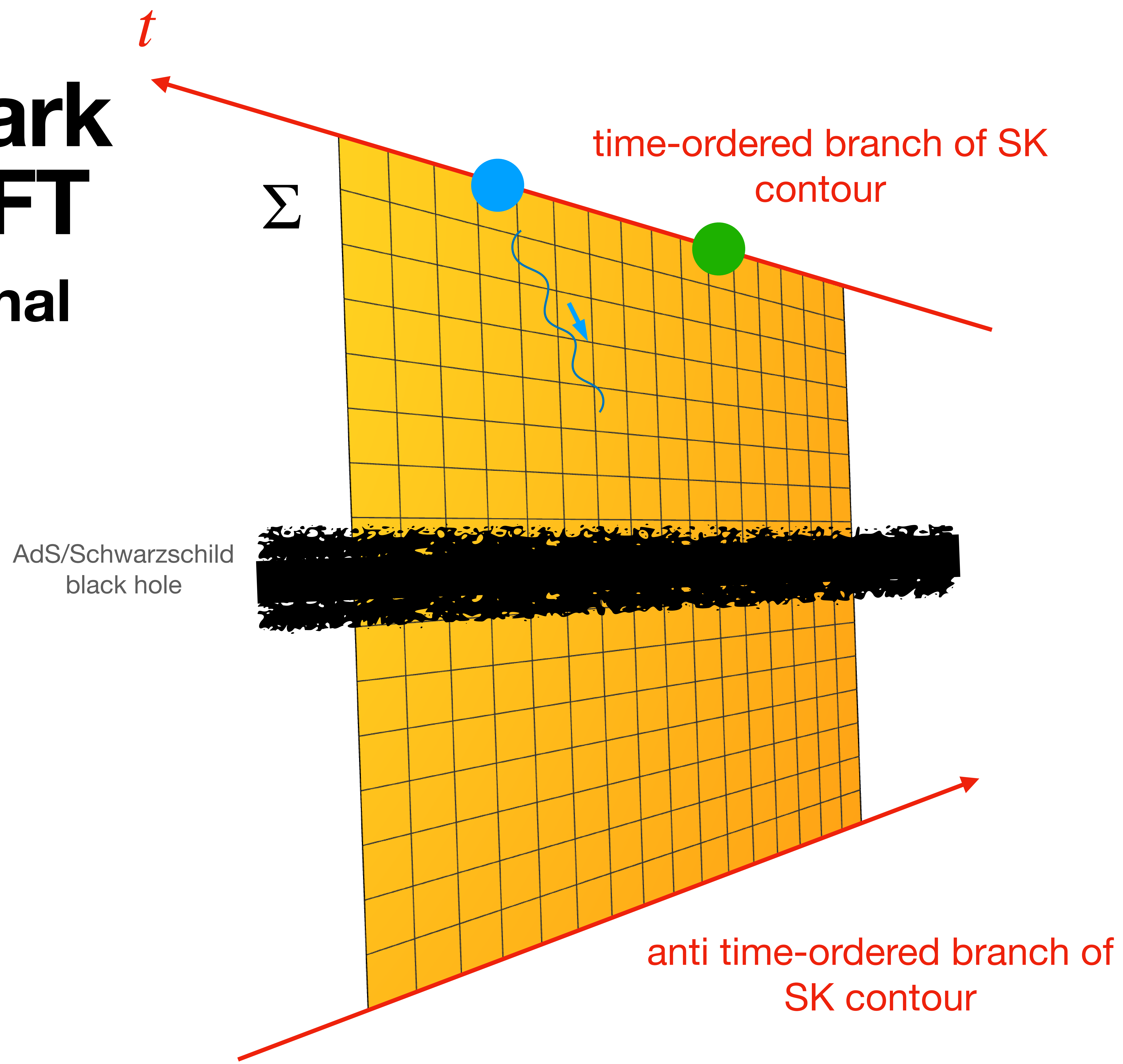
using the same computational technique

Steps of the calculation:

1. Find the appropriate background solution

2. Introduce perturbations

3. Evaluate the deformed Wilson loop and take derivatives



[***] J. Casalderrey-Solana and D. Teaney, hep-ph/0605199

Review: Heavy Quark Diffusion in AdS/CFT

using the same computational technique

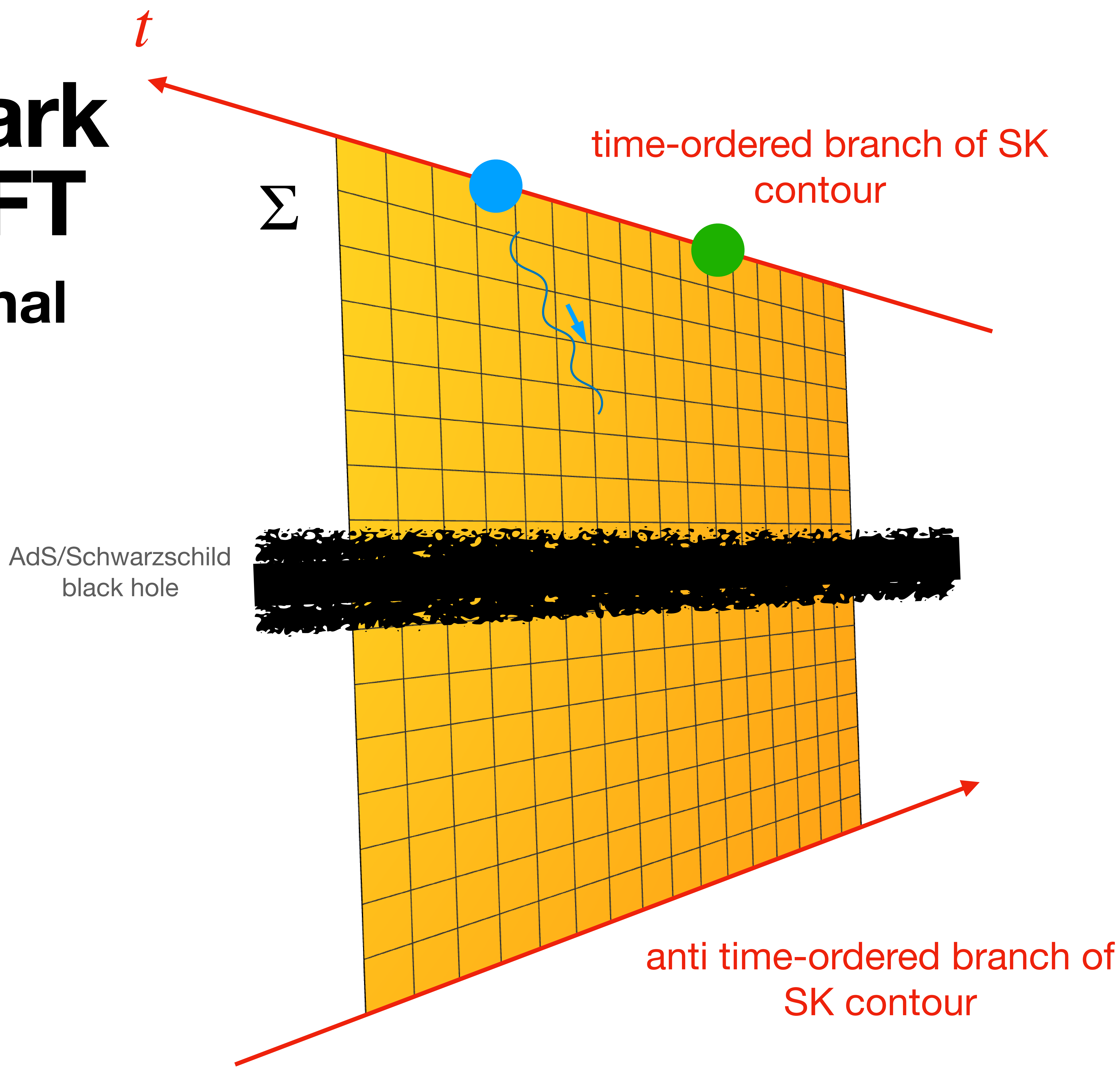
Steps of the calculation:

1. Find the appropriate background solution

2. Introduce perturbations

3. Evaluate the deformed Wilson loop and take derivatives

From here: $\kappa = \pi\sqrt{g^2 N_c}\, T^3$

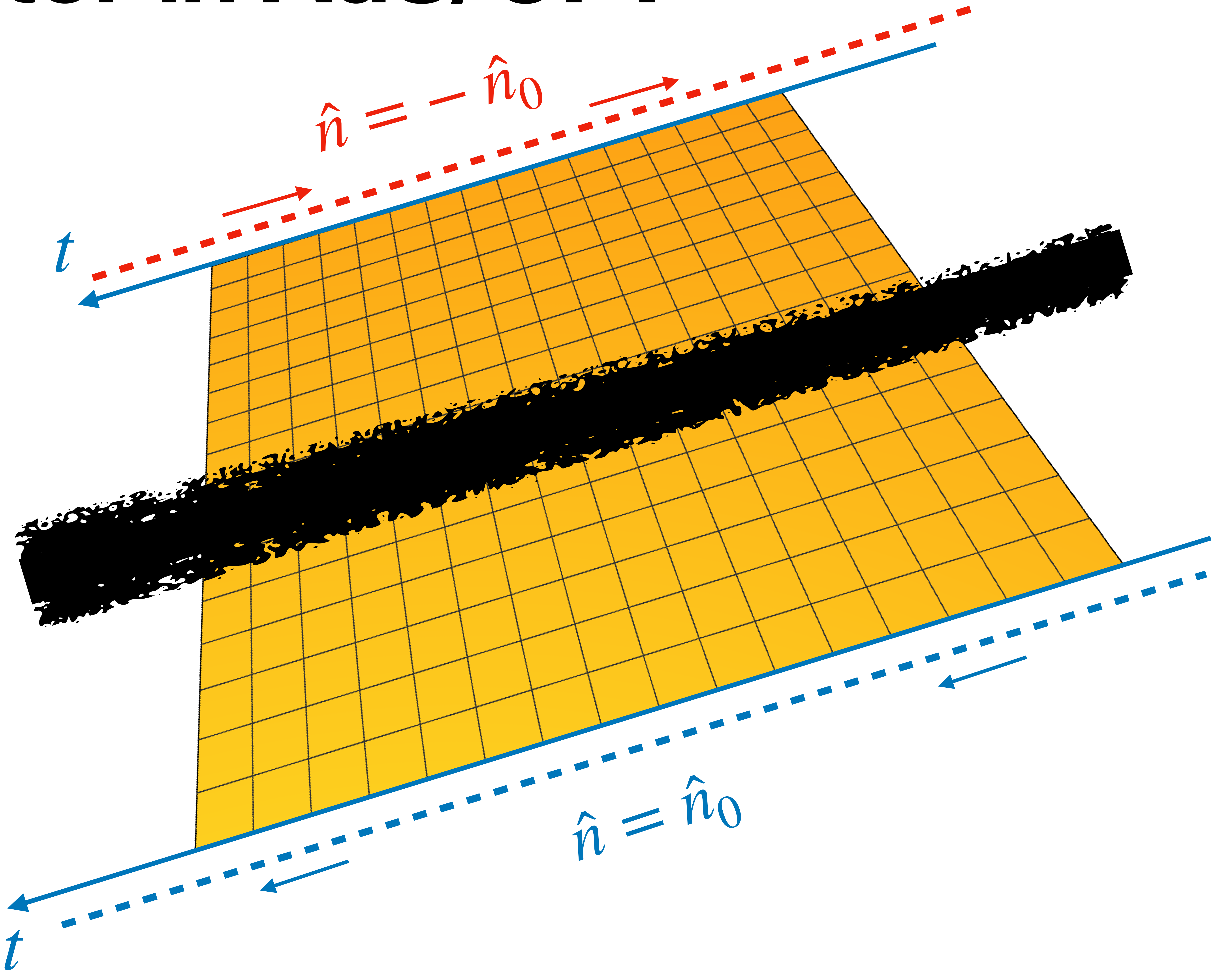


Quarkonium correlator in AdS/CFT

Quarkonium correlator in AdS/CFT

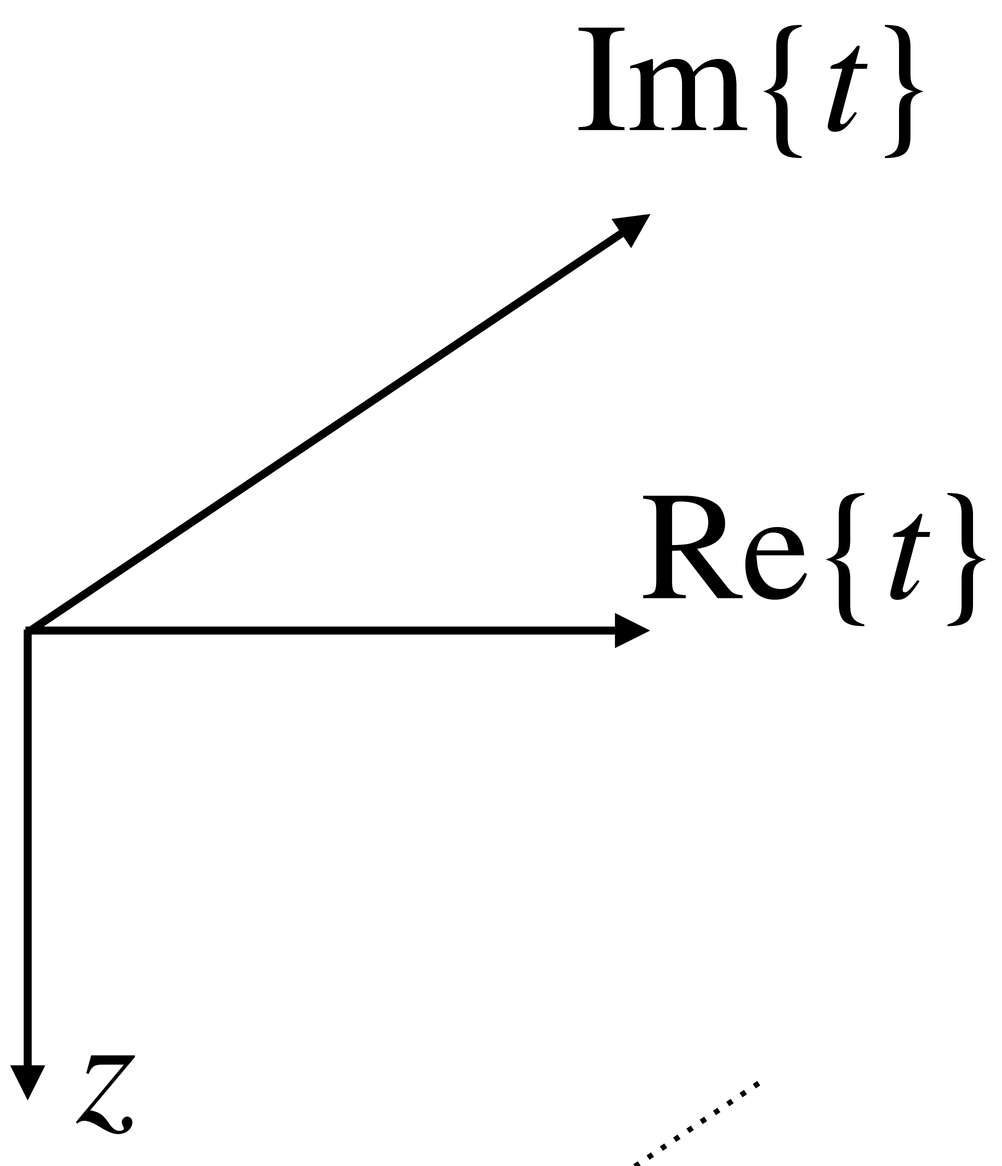
a very similar picture

- Same steps as before:
  1. Find background solution
  2. Introduce perturbations
  3. Evaluate the derivatives
- Differences:
  - Boundary conditions
  - Time-ordered correlator; not retarded

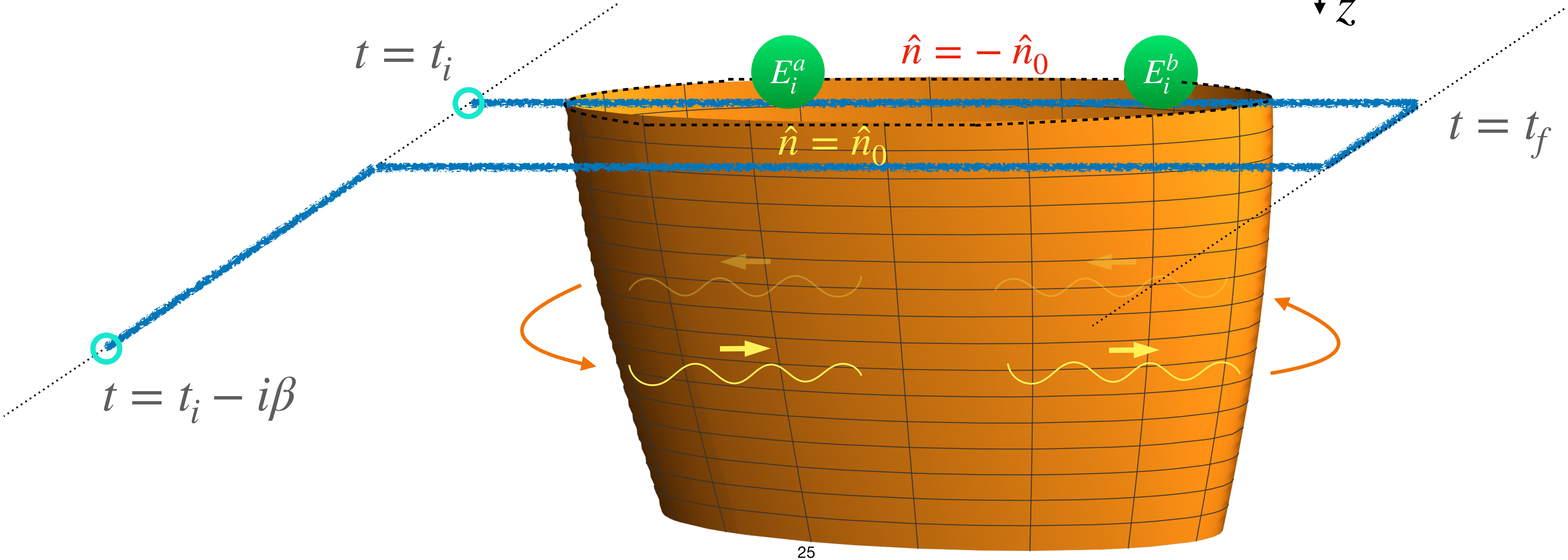


Boundary conditions

Quarkonium correlator



Fluctuations are matched at the turnaround points of the extremal surface. No direct sensitivity to the imaginary time segment.

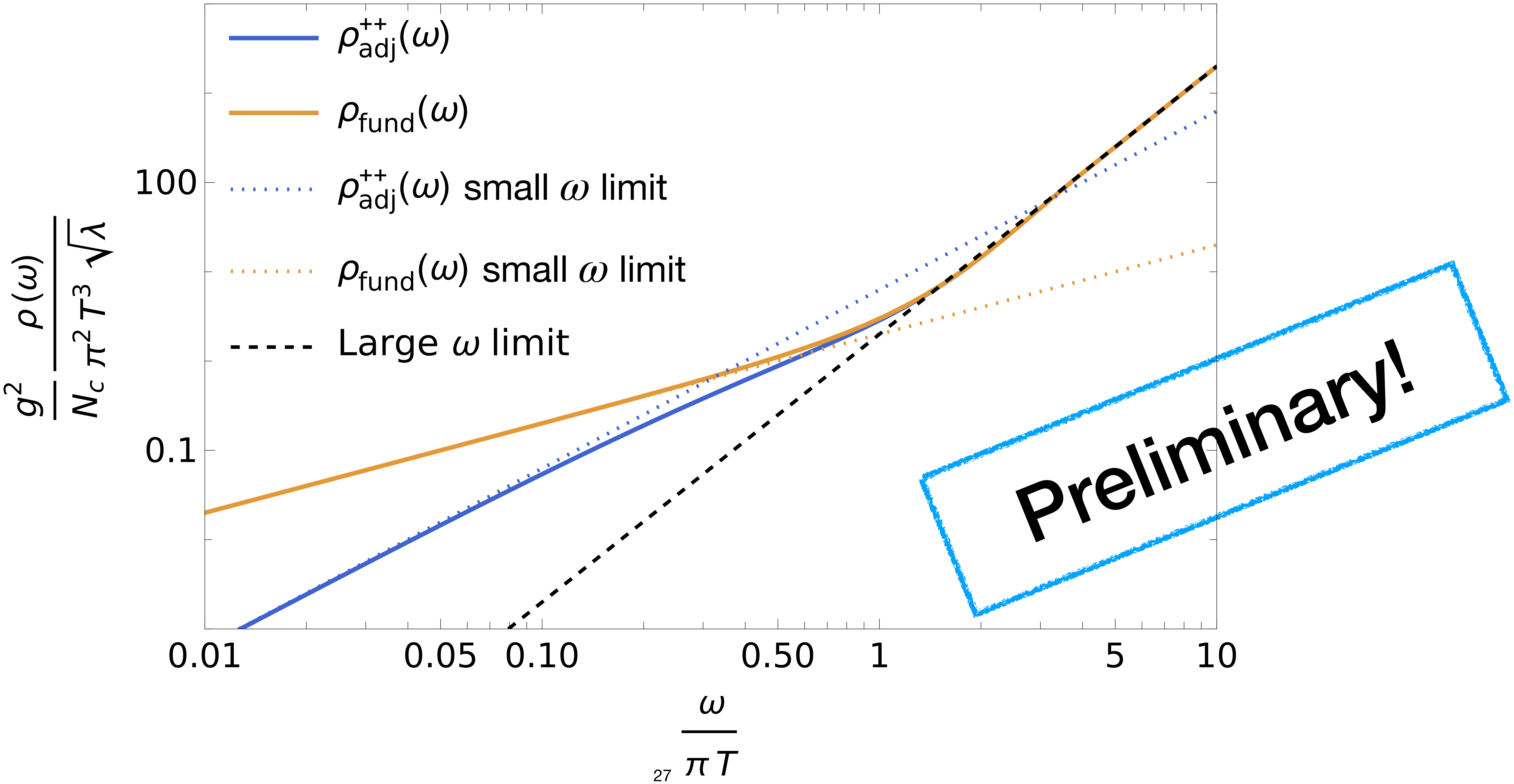


Boundary conditions

Quarkonium correlator

Fluctuations are matched through the imaginary time segment solving the equations of motion $\Longrightarrow$ factors of $e^{\beta\omega}$, KMS relations

Comparison of spectral functions



Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport

  A. at weak coupling in QCD

  B. at strong coupling in $\mathcal{N} = 4$ SYM

- Next steps:

  - Generalize the calculations to include a boosted medium

  - Use them as input for quarkonia transport codes

- Stay tuned!

Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport

  A. at weak coupling in QCD

  B. at strong coupling in $\mathcal{N} = 4$ SYM

- Next steps:
  - Generalize the calculations to include a boosted medium
  - Use them as input for quarkonia transport codes
- Stay tuned!

Thank you!

Extra slides

Open quantum systems

“tracing/integrating out” the QGP

- Given an initial density matrix $\rho_{\rm tot}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\rm tot}(t) = U(t)\rho_{\rm tot}(t=0)U^\dagger(t).$$

- We will only be interested in describing the evolution of quarkonium and its final state abundances

$$\implies \rho_S(t) = {\rm Tr}_{\rm QGP}\left[U(t)\rho_{\rm tot}(t=0)U^\dagger(t)\right].$$

- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\rm tot}(t=0) = \rho_S(t=0) \otimes e^{-H_{\rm QGP}/T}/\mathcal{Z}_{\rm QGP}$.

Open quantum systems

"tracing/integrating out" the QGP: semi-classic description

Unitary evolution of environment + subsystem

⇩ Trace out the environment degrees of freedom

OQS: ρ_S has non-unitary, time-irreversible evolution

⇩ Markovian approximation $\Longleftrightarrow$ weak coupling in H_I

OQS: Lindblad equation

⇩ Wigner transform: $f(\mathbf{x}, \mathbf{k}, t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k} + \frac{\mathbf{k}'}{2} \middle| \rho_S(t) \middle| \mathbf{k} - \frac{\mathbf{k}'}{2} \right\rangle$

Semi-classic subsystem: Boltzmann/Fokker-Planck equation

Lindblad equations for quarkonia at low T

quantum Brownian motion limit & quantum optical limit in pNRQCD

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial \rho}{\partial t} = -i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

$$\tau_I \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

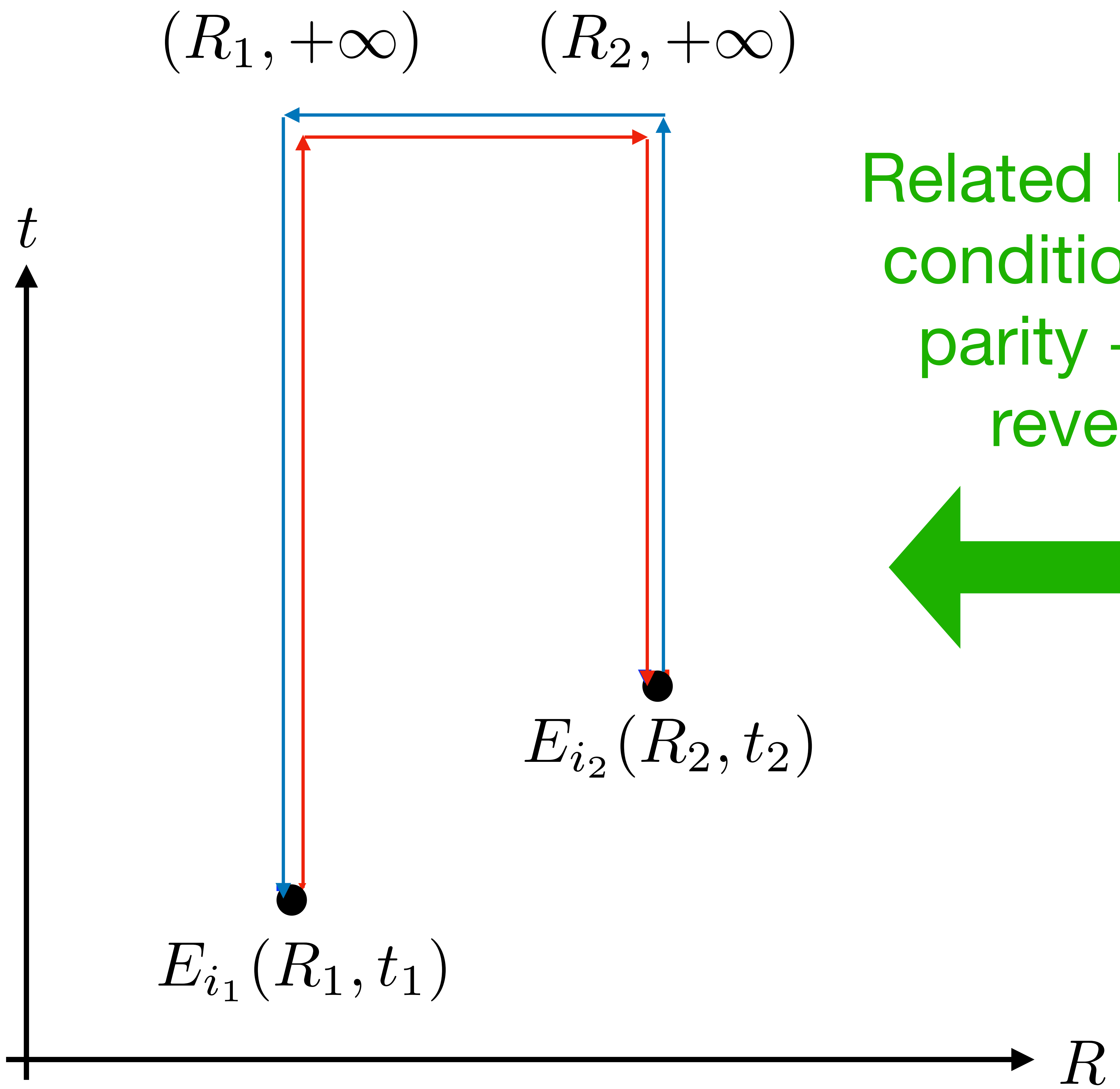
$$\tau_I \gg \tau_E$$
$$\tau_I \gg \tau_S$$

relevant for $Mv \gg Mv^2, T \gtrsim m_D$

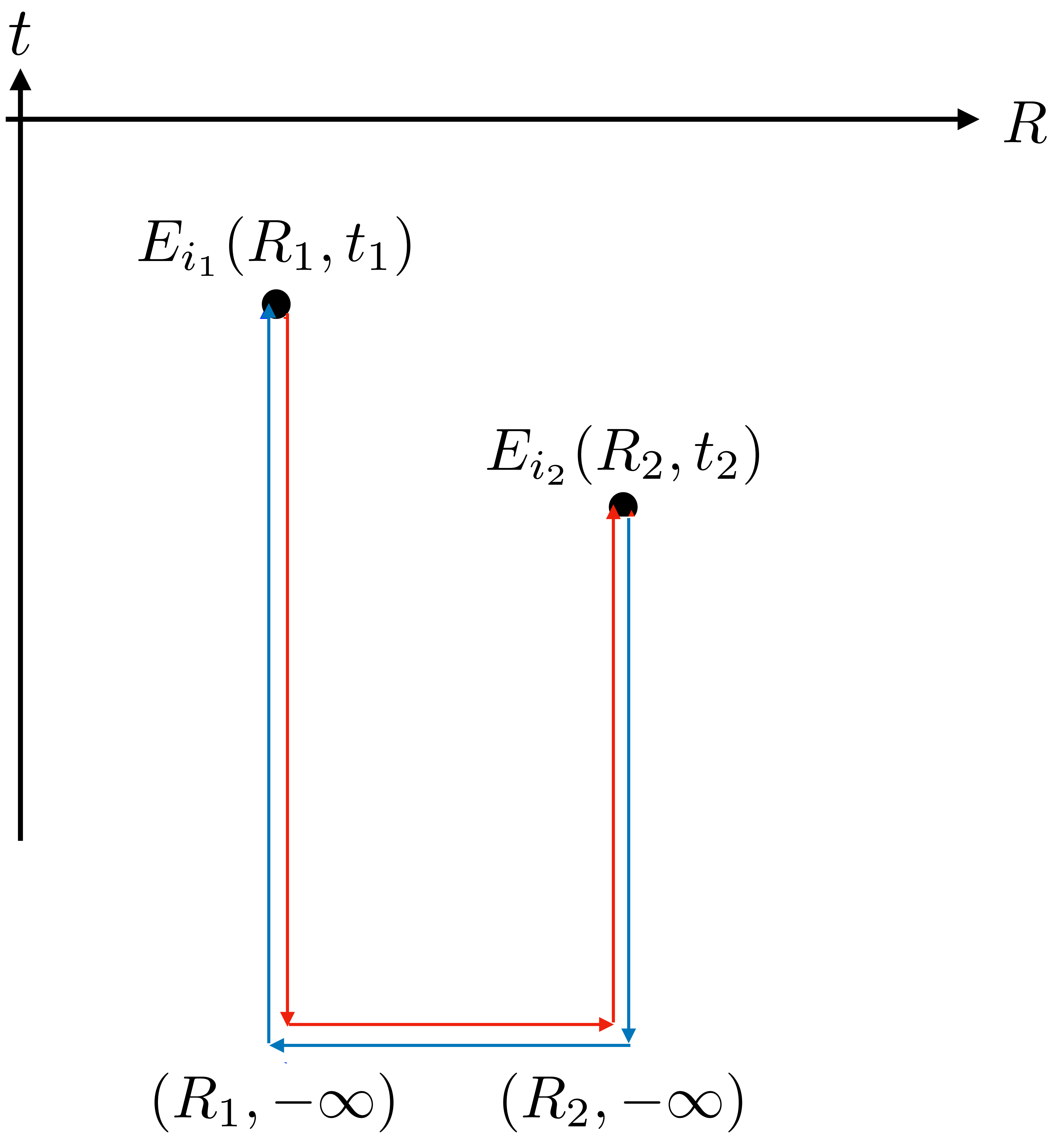
QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$



Related by KMS conditions and parity + time reversal



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

The correlators we discussed are also directly related to the correlators that define the transport coefficients in the quantum brownian motion limit:

$$\gamma \equiv \frac{g^2}{6N_c}\mathrm{Im}\int_{-\infty}^{\infty} ds\,\langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathcal{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,,$$

$$\kappa \equiv \frac{g^2}{6N_c}\mathrm{Re}\int_{-\infty}^{\infty} ds\,\langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathcal{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,.$$

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{++}]_{ji}^{<}(q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{--}]_{ji}^{<}(q)\,,$$

and one can show that they are related by

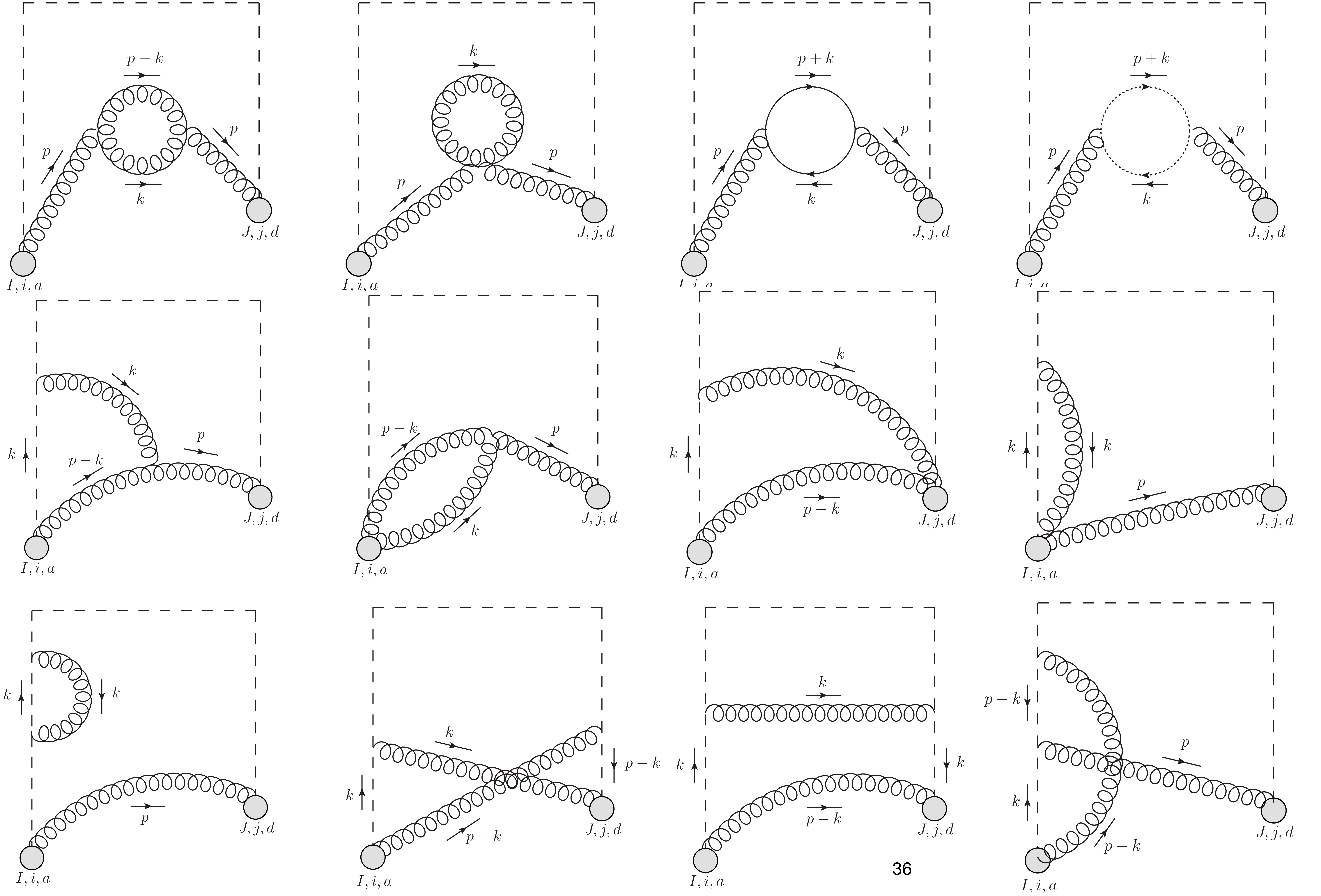
$$[g_E^{++}]_{ji}^{>}(q) = [g_E^{--}]_{ji}^{<}(-q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = [g_E^{++}]_{ji}^{<}(-q)\,.$$

The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]_{ji}^{>}(q) - [g_E^{++/--}]_{ji}^{<}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -\,[\rho_E^{--}]_{ji}(-q)\,.$$

T. Binder, K. Mukaida, BS and X. Yao, hep-ph/2107.03945

Weakly coupled calculation in QCD



The real-time calculation proceeds by evaluating these diagrams (+ some permutations of them) on the Schwinger-Keldysh contour

The spectral function at NLO

and a comparison with its heavy quark counterpart

It is simplest to write the integrated spectral function:

$$\varrho_E^{++}(p_0) = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\delta^{ad}\delta_{ij}\left[\rho_E^{++}\right]_{ji}^{da}(p_0,\mathbf{p})\,.$$

We found

$$g^2\varrho_E^{++}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36} + \frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

and the heavy quark counterpart is, with the same T-dependent function $F(p_0/T)$,

Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$g^2\rho_E^{\mathrm{HQ}}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36} - \frac{2\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\rm NG}[\Sigma_f]}$:

$$S_{\rm NG}[\Sigma_f] = S_{\rm NG}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\rm NG}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1) f(t_2) + O(f^3)\ .$$